

AVALANCHING AND SELF-ORGANISED CRITICALITY, A PARADIGM FOR GEOMAGNETIC ACTIVITY?

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Abstract. The characterization of global energy storage and release in the coupled solar wind-magnetosphere system remains one of the fundamental problems of space physics. Recently, it has been realised that a new paradigm in physics, that of Self Organised Criticality (SOC) may encapsulate the mixing and merging of flux on many scales in the magnetotail prompting bursty energy release and reconfiguration. SOC is consistent with qualitative measures such as power-law power spectra and bursty bulk flows and with more quantitative tests such as power law burst distributions in auroral indices and auroral optical activity. Here, we present a careful classification of the broad range of systems that fall under the general description of 'SOC'. We argue that some, but not all, of these are consistent with our current understanding of the magnetosphere. We discuss the observed low dimensionality of the dynamic magnetosphere in terms of both SOC model properties, and observables. Observations of burst statistics are highlighted; we show that these are currently suggestive but not sufficient to confirm SOC and in particular we find that auroral indices are not effective at distinguishing the internal dynamics of the magnetosphere from that of the intermittent solar wind driver. This may also elucidate the paradox of predictability and complexity of the coupled solar wind-magnetosphere system.

1. Introduction

An important avenue of substorm research has been the study of the state space of the coupled solar wind- magnetosphere-ionosphere system (see, e.g., the review of Klimas *et al.*, 1996) The auroral electrojet index (AE) has been used to estimate the energy released from the magnetosphere into the ionosphere. One approach has been the search for low dimensional chaos in AE, which motivated studies of its fractal dimension and Hurst exponent(s). These studies indicate that AE has two self-affine regions with a break in the scaling exponent at about 2 hours (Takalo *et al.*, 1993). Recent work in this area has been concerned with identification of chaos in systems for which the driver must also be considered; with provision of a robust suite of indicators of low dimensional chaos; and with the relationship of these methods to the successful nonlinear predictive filter techniques (see the review of Sharma, 1995).

However, as noted by Chang (1992, 1999), low dimensionality is a property of a system near criticality. Critical behaviour (such as scale invariant fluctuations on all length scales) had previously been known in systems at phase transitions (e.g.,



Huang, 1987) and, hence, for small regions of parameter space. The ‘sandpile’ cellular automaton constructed by Bak *et al.* (1987) (BTW) exhibited this behaviour as a natural consequence of its time evolution, without parameters having to be adjusted ‘by hand’, which was thus dubbed self-organised criticality (SOC). Such a non-equilibrium system dissipates energy by means of many avalanches of all sizes, but returns to its out of equilibrium critical stationary state rather than relaxing to a non critical one. Hence an alternative framework for explanation of the low dimensionality, the existence of localized current disruption and bursty bulk flows (e.g., Lui *et al.*, 1988; Angelopoulos *et al.*, 1996) and power law magnetic field spectra in the magnetotail (Hoshino *et al.*, 1994), has been given by postulating the magnetotail to be an open, dissipative dynamical system at a (forced or self organised) critical state (Chang, 1992, 1999; see also Klimas *et al.*, 2000). In particular, SOC implies that the probability distributions of energy release events within the system are power laws. Recent evidence for this has been found in auroral images (Lui *et al.*, 2000). Also Consolini (1997, 1999) has revealed a power law in a measure of burst size in AE. However, recent analysis by Freeman *et al.* (2000a) as we shall see, calls into question the usefulness of indices in this context.

Other indicators such as *Dst* and particle injection events in the near earth magnetosphere indicate the outflow of energy from the system due to large scale ‘systemwide’ reorganizations. These are typically not self similar, rather the intensity and time intervals between one substorm and the next have a probability distribution with a well defined mean (e.g., Borovsky *et al.*, 1993; Smith *et al.*, 1996). Consistent with this and the SOC conjecture a simple avalanche model (Chapman *et al.*, 1998, 1999) has been shown to exhibit global events with a characteristic mean and internal events showing two scale free regions.

We therefore have a mixture of requirements for any attempt to model the system, part observational and part theoretical. The concepts of self organisation and criticality will first be discussed in Section 2. We will then attempt to establish whether this idealized SOC state is in fact needed to account for the observed burstiness, self similarity and low dimensionality associated with magnetospheric dynamics. In Section 3 we contrast sandpile models with other models which display intermittency such as Coupled Map Lattices (CML) which may be understood in terms of dissipative chaos. Finally in Section 4, we discuss observables that may distinguish these approaches.

2. Self-Organization and Criticality

One difficulty in applying SOC is its broad definition in the literature. We identify three main classes relevant to magnetospheric physics. One is the original mechanism suggested by Bak *et al.* (1987) which we will call ‘SOC’. The second was introduced by Chang (1992) and referred to as (Forced) ‘F/SOC’. The third is

a phenomenological definition based on observation of some or all of a set of possible diagnostics of SOC such as bursty time series, '1/f' power spectra and avalanche distributions. We shall use the term 'SOC-like' for these. These distinctions are useful since they refer to different sets of phenomenology and underlying theoretical structure.

2.1. SELF-ORGANIZED CRITICALITY (SOC)

Noise with very long correlation times, the so-called '1/f' or 'flicker' noise is ubiquitous in nature. It is distinguished by a power-law, i.e., scale-free, power spectral density of the form $f^{-\beta}$, with β typically between 0.8 and 1.4. Its spatial counterpart is fractal structure. The apparent ubiquity of these phenomena led BTW to propose the SOC mechanism as their common origin. BTW showed that a discrete cellular automaton simulation ('sandpile model') of a spatially extended, slowly driven, many degree of freedom system in more than one dimension could exhibit both a scale-free spatial response to perturbation and bursty time evolution. They demonstrated the scale-free property in the spatial response by exhibiting power-law 'avalanche' distributions of size, duration and spatial extent of toppling events in the model. The mechanism that BTW proposed to explain this behaviour was an underlying fixed point in the dynamics ('criticality'), which was attractive ('self-organised'). Support for the presence of critical behaviour in the original BTW model was given mainly by their demonstration of finite-size scaling in the avalanche distributions for different lengths of the model system, as this was a property unique to critical systems.

Demonstration (particularly by renormalisation group (RG) methods) that sandpile models can indeed exhibit attractive fixed points has thus been an important thread in SOC research for the past decade (see Jensen, 1998, for a brief review). Such fixed points have been observed in the conservative BTW model and others, hence SOC has been shown to exist as a mechanism. A parallel thread has been the experimental search for SOC in real sandpiles, ricepiles, and other rather less controllable systems such as seismic faults, solar reconnection events and astrophysical accretion disks (see the reviews of Jensen, 1998; Turcotte, 1999; Dendy and Helander, 1997). The only signature of criticality per se so far tested for in data is finite-size scaling, which requires the system size to be changed, so only the Oslo ricepile (Frette *et al.*, 1996) among natural systems can be said to demonstrate SOC behaviour in this sense. The relevance of criticality in particular for magnetospheric dynamics is that such a critical system effectively becomes low dimensional, as noted by Chang (1992). In addition it may have implications for the presence or absence of time correlation between successive energy release events, as conjectured by Watkins *et al.* (2000), Consolini (1999) and Freeman *et al.* (2000b) following Boffetta *et al.* (1999).

2.2. F/SOC

Although BTW hypothesised a mechanism (SOC) to explain the avalanche phenomenology they observed, it remains possible that many natural or model systems may share all or part of the avalanche phenomenology without necessarily being self-organised. One difference from SOC comes with some model systems including the ‘Forest Fire’ models which are controlled by repulsive rather than attractive fixed points, and so have to be tuned to exhibit scaling, rather than being attracted there from arbitrary initial conditions in the control parameters. Chang has called this ‘forced criticality’, and it differs from SOC as it is critical without self-organisation. It has relevance to the problem of magnetospheric dynamics, as argued by Chang (1992, 1999) and also Consolini and de Michelis (2000) since the magnetospheric system may be driven to a critical or near-critical state as a result of the continuous loading process that it undergoes. Indeed, the simple model used by Chapman *et al.* (1998) has been shown to possess a repulsive fixed point by RG (Tam *et al.*, 2000).

2.3. SOC-LIKE

The description ‘SOC-like’ may be usefully applied for systems which have not yet been analytically treated and for which the classic tests of criticality such as finite size scaling either cannot be or have not yet been performed. A good example of the latter are the thresholded diffusion equations which have received extensive study (see Section 5.4 of Jensen, 1998) as continuous space and time differential models for avalanche phenomenology. An important example is that of Lu (1995) who demonstrated that the differenced diffusion equation for an arbitrary one dimensional field ϕ

$$\frac{\partial \phi(x, t)}{\partial t} = \frac{\partial}{\partial x} \left[D(x, t) \frac{\partial \phi}{\partial x} \right] + S(x, t) \quad (1)$$

with a noise-like driver $S(x, t)$ and a particular choice of nonlinear diffusion coefficient $D(x, t)$ generated avalanching with power law statistics. One can then (Vassiliadis *et al.*, 1998) cast the MHD equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \wedge (\eta(x, t)\mathbf{J}) + \nabla \wedge (\mathbf{v} \wedge \mathbf{B}) \quad (2)$$

in terms of diffusion dominated MHD, by replacing the advection term by a source term without explicit velocity dependence giving

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \wedge (\eta(x, t)\mathbf{J}) + S(\mathbf{x}, t), \quad (3)$$

then using $\mu_0 \mathbf{J} = \nabla \wedge \mathbf{B}$ and solving for \mathbf{B} using a discrete space-time but continuous field scheme. This method then retains the phenomenology of resistive MHD provided that diffusion initiated mixing and merging dominates convection

– it has yet to be demonstrated that this is an appropriate approximation for the magnetosphere. The above model has been further developed by Klimas and co-workers (see Takalo *et al.*, 1999; Klimas *et al.*, 2000, and references therein). However, Equation (3) in one dimension corresponds essentially to Burgers’ equation with a noise drive, depending upon the form of η . Care hence needs to be taken with solutions of this system to distinguish Burgers turbulence-type solutions from those that could only be SOC-like (Krommes, 2000). Solutions of the diffusion equation are naturally long-range in space and time without invoking criticality. However the avalanche distributions that are also observed may indeed indicate critical or near-critical behaviour and we may thus call them SOC-like pending analytic study. Such questions have recently become more relevant since several authors (in particular Boffetta *et al.*, 1999) have noticed the similarity of avalanche distributions in SOC systems to the distributions of amplitudes and lifetimes in shell models of turbulence (see Section 3.1 below).

Finally, it should be noted that the low dimensionality of the dynamic magnetosphere supports phenomenology that may be ‘SOC like’ rather than SOC. Singular spectrum analysis of previously identified substorm events in auroral indices (see, for example, Sitnov *et al.*, 2000) reveals a state space that is low dimensional and may be described reasonably well by a cusp catastrophe, as first suggested by (Lewis, 1991). In this context it is critical to understand to what extent measures of the system dynamics such as auroral indices also measure the solar wind drive directly and hence to quantify their appropriateness for such studies; we shall see in Section 4 this remains an open question.

3. Models

3.1. NUMERICAL MODELS FOR AVALANCHING AND INTERMITTENCY

A generic approach is to obtain a reduced description of a given system that can be implemented on a lattice. Briefly, lattice based numerical models may be classified (Bohr *et al.*, 1998) as (i) reduced forms of the governing PDE as in the case of turbulent shell models that are obtained by approximate truncation of the Navier Stokes equations, (ii) Coupled Map Lattices that spatially couple systems with prescribed nonlinear time evolution to give simple models of spatiotemporal chaos and (iii) avalanche models that combine simple redistribution rules (spatial coupling) with thresholded time evolution.

Many variants of turbulence shell models exist and we refer the reader to Bohr *et al.* (1998) for a comprehensive discussion. Briefly, these consist of difference equations in k space that couple k modes locally thus allowing energy to cascade in k which can generate power law power spectra with Kolmogorov scaling. It is relevant to note here that shell models such as that of Gledzer, Ohkitani and Yamada (the GOY model, see Bohr *et al.*, 1998) generate intermittency and power

law power spectra and yet must be distinguished from other numerical systems with bursty evolution. Power-law distributions in lifetime and burst size have been observed in a shell model by Boffetta *et al.* (1999). The observation by Einaudi and Velli (1999) of an avalanche distribution arising from MHD plasma simulations of anisotropic turbulence suggests that the avalanche distribution may be largely insensitive to the underlying physics of the turbulent system.

Coupled Map Lattices (CML), on the other hand, model spatiotemporal chaos in configuration space. Although many variants have been explored (see Kaneko, 1993, for a review) the essential philosophy is to decompose the processes underlying the phenomena of interest into independent components (e.g., convection, diffusion) each of which may be nonlinear, and then to reduce each of these to simple parallel dynamics on a lattice. A typical CML may then comprise a local nonlinear map (logistic map, say) evolving each variable in time at each lattice point on the grid, plus coupling between the points (discretized, and therefore truncated, diffusion, say) which is linear. Spatiotemporal chaos in this sense then implies deterministic chaos that is high dimensional; because the number of dimensions increases as we increase the number of coupled nonlinear nodes on the lattice. The deterministic nature of the local evolution means that Lyapunov exponents are well defined for the coupled system in contrast to most 'sandpile' algorithms. In addition certain universality classes, including that of diffusive coupling, can be constructed to be tractable via RG. The generation of intermittency via this 'classic' CML should therefore be distinguished from other grid based real variable sandpile models which are also sometimes dubbed 'CML' but which, due either to the introduction of randomness, or thresholds for diffusion, or both, do not have well defined Lyapunov exponents and cannot be treated as high dimensional chaotic systems in this sense. This definition of the 'classic' CML can also be seen to be related to, but distinct from, low dimensional chaos which can also generate intermittency; both show intermittency for particular values of control parameters in their underlying nonlinear maps. The low-dimensional system may in principle be distinguished via phase space reconstruction, as has been attempted for the magnetosphere (Sharma, 1995).

Several avalanche models have been studied in the magnetospheric context, and we briefly recap here. In addition to the Lu-type models discussed in Section 2.3. Consolini (1997, 1999), studied the 1D Hwa-Kardar 'running' algorithm which gives a broken power-law power spectrum similar to that of AE. A 2D modification of the BTW model was used by Uritsky and Pudovkin (1998), in which the thresholding rule was 'driven' using the AE index. Chapman *et al.* (1998, 1999) have studied a 1D model with a continuous variable, in distinction to the cellular automata this may be called a 'discrete space time' model. This model had nonlocal redistribution rules and thus manifested nontrivial distributions even in one dimension (see also Uritsky and Semenov, 1998). More recently Consolini and de Michelis (2000) have studied a 2D modified Forest Fire model driven by a 1D coupled map lattice.

3.2. MODELS FOR SELF-ORGANISED LOW DIMENSIONALITY

One resolution of the observational evidence for low dimensionality in the dynamic magnetosphere, and bursty evolution that is also robust (and hence less easy to explain in terms of deterministic chaos) is to note that systems at criticality can also be low dimensional. This key point was raised by Chang (1992), and the implication is the following. Avalanche (sandpile) models, forest fire models and so forth have robust emergent phenomenology that yields bursty time evolution with power law burst statistics as required but these systems are by construction high dimensional, in the same sense as CML. If in addition these systems exhibit fixed points, then close to the fixed points, that is, close to criticality, the behavior is low dimensional. For avalanche models to describe the low dimensional magnetosphere they must have fixed points. This places a rather strong restriction on the models which may, for example, exclude those proposed to more closely resemble diffusion dominated MHD.

In this context it is intriguing to note that, just as classical CML, avalanche models can be modified to exhibit low dimensional behaviour. We give an example here by modifying the sandpile model used by (Chapman *et al.*, 1998). Briefly, this model is similar to others in that sand is redistributed when a critical gradient is exceeded locally. Unlike other models however, redistribution occurs across all sites within an ongoing avalanche by construction; the sand is redistributed conservatively such that within the avalanche the gradients are all set to zero. We can consider a generalization of this model by introducing a ‘fluidisation parameter’ that is, by flattening back the sand behind the leading edge of an ongoing avalanche for a fixed distance L_f . This has been extensively investigated by Chapman (2000) and corresponds to moving the system away from the repulsive fixed point. Here we present a summary of the results which are germane to the discussion. The behaviour of the system with L_f essentially has two regimes, when L_f is of order the system size, the behaviour is just that of the original $L_f = N$ (system length) sandpile model considered in Chapman *et al.* (1998, 1999), that is, evolution is bursty and burst statistics are power law. However, if we reduce L_f to much less than a quarter of the system size, although energy can still only be released by avalanches, the evolution becomes quasiregular, with a distinct loading-unloading cycle and statistics that are power law only over a restricted range. The essential point here is that an originally high-dimensional sandpile model can, for appropriate parameters, exhibit low dimensional dynamics. We show in Figure 1 the time series $E(t)$ for these two cases. For $L_f = 2000$, $N = 4096$, the system evolution is ‘intermittent’ whereas for $L_f = 4$, $N = 4096$ it is quasiregular, and for this latter case we show (in Figure 2) the limit cycle for the system obtained by embedding. The implication is that low dimensionality can either be a signature of a system close to criticality or of certain classes of avalanching systems that can be tuned to give either intermittent, or quasiregular, time evolution.

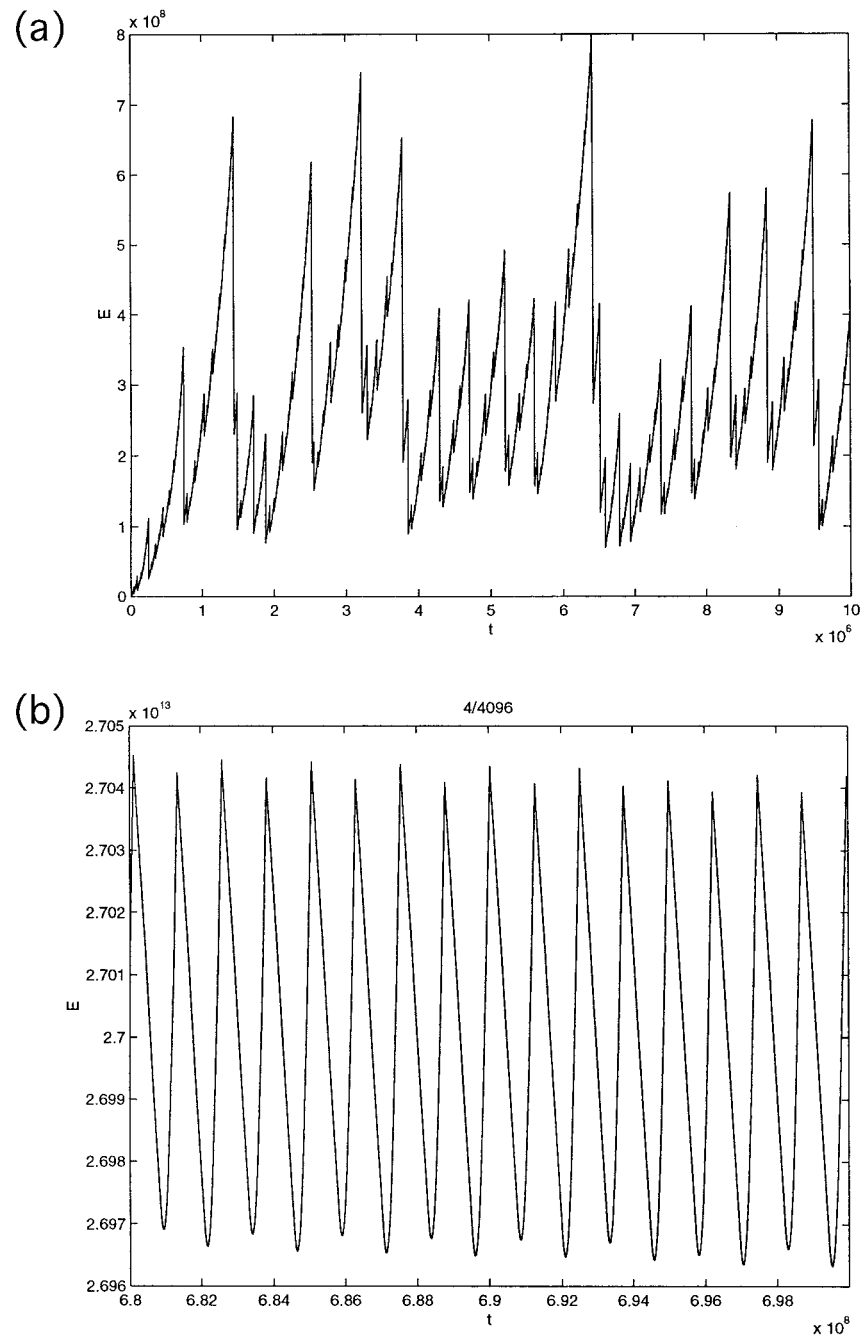


Figure 1. Timeseries of energy released during avalanches with flattening back length (a) $L_f = 2000$ and (b) $L_f = 4$.

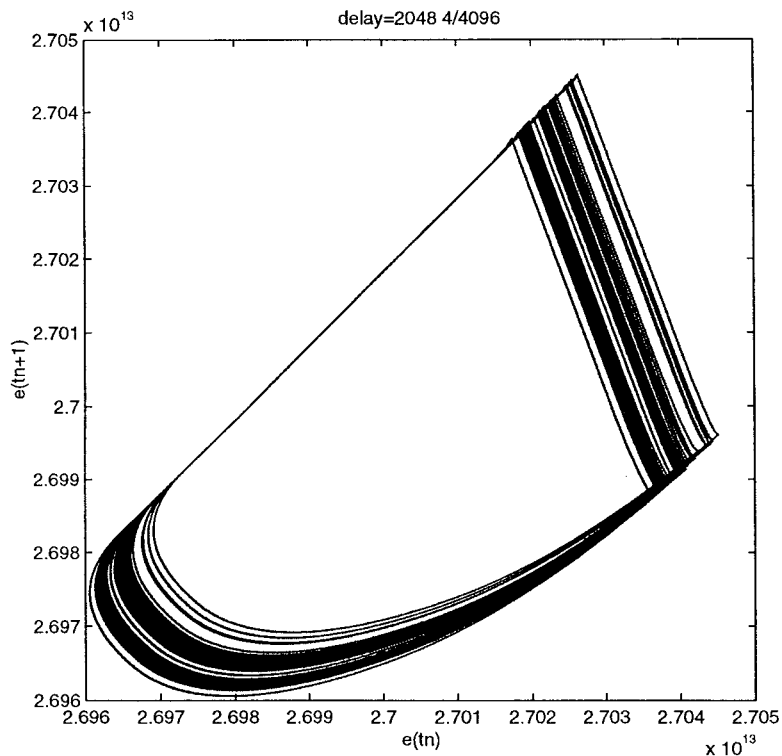


Figure 2. Phase space reconstruction for the time series shown in part in Figure 1(b).

4. What Can We Measure?

For models and laboratory experiments the identification of SOC or SOC-like phenomenology is fairly straightforward. One would expect evidence in burst size and life time distributions of power laws, evidence of fractal structures and long range correlations, and finite size scaling as one varies the system size. Unfortunately, dealing with observations rather than carefully constructed experiments presents some nontrivial problems and we highlight these next. Of principal concern in the magnetosphere is the variability of the driver and the extent to which any given observable yields the output of the system, the system's internal dynamics, or a mix of these with the driver superimposed. The problem of deciding when an observation is characterising the driver, or the system's response to it, is problematic since the driver in this case, that is, the solar wind, is known to be turbulent and exhibit intermittency. One approach in terms of modelling (Consolini and de Michelis, 2000) is to couple the avalanche model to an intermittent drive; this may mimic the overall coupled driver-system behaviour but does not easily allow one to unravel the complex dynamics of the drive from that of the system.

Auroral indices have received considerable attention as a means for testing the predictions of models showing complexity. These indices have the advantage

that they provide long time series; essential to test the hypothesis of scale free (power law) behaviour. The broken power law form of the power spectrum of AE (Tsurutani *et al.*, 1990) and later, its burst lifetime and size distribution (Consolini, 1997, 1999) have been taken to be strong indicators of complexity and SOC in the magnetosphere's dynamic evolution. Comparisons between the indices and the solar wind drive, with an emphasis on predicting magnetospheric activity have however implied that the dynamics is to some extent predictable (Sharma, 1995). Care must be taken to clearly define what actually constitutes successful prediction of some observed activity; this also relies on some clear definition of substorm onset and differentiation between substorms and other sporadic events (BBFs, pseudobreakups). In terms of SOC phenomenology the question is more clearly addressed if we compare burst distributions from the magnetosphere and the solar wind.

Consolini (1997, 1999) and Takalo *et al.* (1999, see also references therein) have defined a measure of energy bursts, i.e., integrated dissipation rate with respect to an arbitrary constant threshold level and applied it to the auroral electrojet index *AE*. They found power laws in both size and burst duration, but Consolini (1999), has since shown the presence of a small 'bump' with characteristic values of burst magnitude e and duration T . Freeman *et al.* (2000a) showed that the burst lifetime distributions $P(T)$ for *AU* and *AL* between January 1978 and June 1988 share this form. In addition observations from the WIND satellite's particle and magnetic field instruments between January 1995 and December 1998 were used to show that the power law component of $P(T)$ but not the 'bump' is also present in two measures (Akasofu's ε and vB_s) which estimate the energy delivered by the solar wind to the Earth's magnetopause region. This close correspondence is illustrated in Figure 3 from Freeman *et al.* (2000a) comparing curves for *AU* and ε . Freeman *et al.* (2000a) presented arguments that the 'bump' corresponds to the effect of the DP1 substorm 'unloading' current system.

Evidence has since been given (Freeman *et al.*, 2000b) suggesting that the scaling region common to *AU/AL* and ε is in fact present in the energy flow within the solar wind itself, as measured by $P(e)$ and $P(T)$ for the Poynting vector. In addition (Freeman *et al.*, 2000b) a power law inter-burst interval distribution has been exhibited for the Poynting vector which in turn has implications for the predictability of fluctuations in the DP2 ionospheric current system, if later work confirms that the bursts in *AU/AL* are causally related to those in ε and vB_s . We note that Consolini (1999) has shown AE to also have a power-law rather than exponential inter-burst interval distribution.

Another measure of magnetospheric output is auroral optical activity. POLAR UVI allows almost the entire auroral oval to be imaged. These images show substorms plus smaller events which may be related to BBFs and other bursty configurations within the magnetotail. One can construct a burst distribution for these and such a study was performed by Lui *et al.* (2000). The key results are shown in Figure 4. Here 'blobs' of brightness in the auroral oval have been identified and,

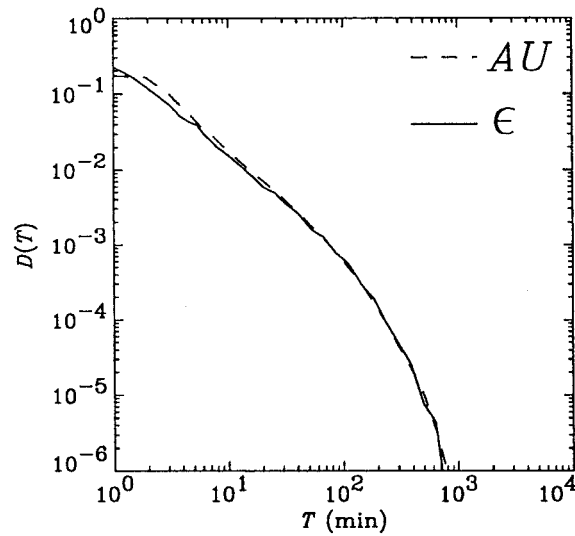


Figure 3. A direct comparison of the burst lifetime PDFs of AU (for January 1978–June 1988) and ϵ (calculated from WIND SWE and MFI data for 1984–1987). Thresholds have been chosen to give similar exponential cut off to times. From Freeman *et al.* (2000a).

after suitable background subtraction, both the intensity summed over a blob, and its area, can be obtained. When the probability distributions of integrated intensity and size were plotted, Lui *et al.* (2000) found a power law slope plus a ‘bump’ at large values corresponding to substorm breakups. Looking at quiet times where no substorms occur they found the power law with unchanged index only. This appears to be strong evidence that the non-substorm (the internal) events are power law whereas the substorm (systemwide) events are not, consistent with simple avalanche models such as Chapman *et al.* (1998).

These two observational results may not be in conflict if we consider that both the drive and the internal magnetospheric dynamics are SOC-like, because it has also been shown (Chapman *et al.*, 1999) that avalanche models may tend to absorb information about detailed fluctuations in the drive. However, the striking coincidence between the burst distributions of the indices and of the solar wind found by Freeman *et al.* (2000a), for all except the largest events is suggestive that auroral indices are, for the small scale (non-systemwide) events, strongly influenced directly by the drive itself, and are therefore (i) easy to predict and (ii) not a good method for uniquely testing the SOC hypothesis for magnetospheric dynamics.

5. Conclusions

Here we have elucidated the different classes of SOC description, and related descriptions of turbulent and other high dimensional systems such as CML. This has

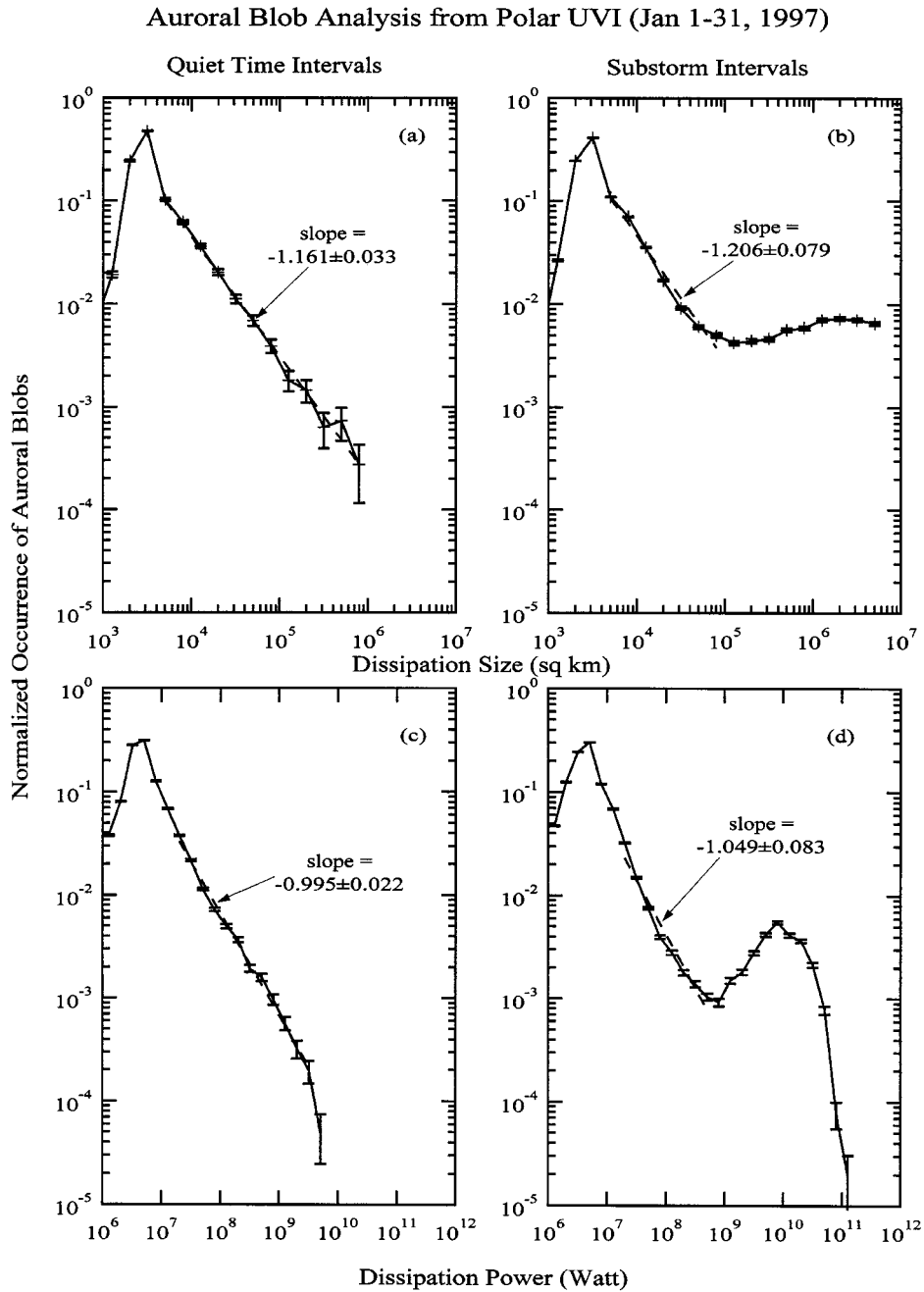


Figure 4. Probability distributions of the size, and energy dissipated, in auroral 'blobs' during quiet and active times, from Lui *et al.* (2000).

suggested that most, if not all, of the current evidence for SOC, F/SOC or SOC-like behaviour in the magnetosphere, both quantitative (avalanche distributions, power law spectra, low dimensionality and self-affinity), and more qualitative (burstiness and inherent multiscale nature) could be obtained from either shell models for turbulence, *bona fide* coupled map lattices or avalanche models. A particular ambiguity is that the low dimensionality, which Chang (1992) pointed out was a feature of forced criticality, can also arise from low dimensional chaos, or from a 'tuned' high dimensional chaotic system under certain conditions. These all however, embody different aspects of the underlying physics and therefore future work needs to be directed specifically at distinguishing them; in particular multispacecraft missions such as CLUSTER II will allow unambiguous turbulence measures, as well as avalanche statistics to be determined in situ.

A difficulty is that to test for power law dependence, long timeseries or large sets of observations are needed. In principle, geomagnetic indices are suitable in this respect, but recent work suggests that the power law part of the avalanche distributions shown by magnetospheric indices may in fact be a directly driven manifestation of solar wind control of the DP2 current. This also prompts a re-examination of other uses of indices to unravel the relationship between the solar wind driver and the response of the dynamic magnetosphere.

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References

- Angelopoulos, V., Coroniti, F. V., Kennel, C. F., Kivelson, M. G., Walker, R. J., Russell, C. T., McPherron, K. L., Sanchez, E., Meng, C. I., Baumjohann, W., Reeves, G. D., Belian, R. D., Sato, N., Friis-Christensen, E., Sutcliffe, P. R., Yumoto, K. and Harris, T.: 1996, *J. Geophys. Res.* **101**, 4967.
- Bak, P., Tang, C. and Wiesenfeld, K.: 1987, 'Self-Organized Criticality: An Explanation of $1/f$ noise', *Phys. Rev. Lett.* **50**, 381.
- Bohr, T., Jensen, M., Paladin, G. and Vulpiani, A.: 1998, *Dynamical Systems Approach to Turbulence*, Cambridge University Press, p. 350.
- Bofetta, G., Carbone, V., Giuliani, P., Veltri, P. and Vulpiani, A.: 1999, 'Power Laws in Solar Flares: Self-Organized Criticality or Turbulence?', *Phys. Rev. Lett.* **83**, 4662.
- Borovsky, J. E., Nemzek, R. J. and Belian, R. D.: 1993, 'The Occurrence Rate of Magnetospheric-Substorm Onsets: Random and Periodic Substorms', *J. Geophys. Res.* **98**, 3807.
- Chang, T. S.: 1992, 'Low-Dimensional Behaviour and Symmetry Breaking of Stochastic Systems Near Criticality – Can These Effects Be Observed in Space and in the Laboratory?', *IEEE Trans. Plasma Sci.* **20**, 691.

- Chang, T. S.: 1999, 'Self-Organized Criticality, Multi-Fractal Spectra, Sporadic Localized Reconnections and Intermittent Turbulence in the Magnetotail', *Phys. Plasmas* **6**, 4137.
- Chapman, S. C.: 2000, 'Inverse cascade Avalanche Model with Limit Cycle Exhibiting Period Doubling, Intermittency and Self Similarity', *Phys. Rev. E*, **62**, 1905.
- Chapman, S. C., Watkins, N. W., Dendy, R. O., Helander, P. and Rowlands, G.: 1998, 'A Simple Avalanche Model as an Analogue for Magnetospheric Activity', *Geophys. Res. Lett.* **25**, 2397.
- Chapman, S. C., Dendy, R. O. and Rowlands, G.: 1999, 'A Sandpile Model with Dual Scaling Regimes for Laboratory, Space and Astrophysical Plasmas', *Phys. Plasmas* **6**, 4169.
- Christensen, K., Olami, Z. and Bak, P.: 1992, 'Deterministic $1/f$ Noise in Nonconservative Models of Models of Self-Organized Criticality', *Phys. Rev. Lett.* **68**, 2417.
- Consolini, G.: 1997, 'Sandpile Cellular Automata and Magnetospheric Dynamics', in S. Aiello, N. Iucci, G. Sironi, A. Treves and U. Villante (eds), *Proc. vol. 58, 'Cosmic Physics in the Year 2000'*, SIF, Bologna, Italy.
- Consolini, G.: 1999, 'Avalanches, Scaling and Criticality in Magnetospheric Dynamics', *Phys. Rev. Lett.*, submitted.
- Consolini, G. and de Michelis, P.: 2000, 'A Revised Forest-Fire Automaton for the Nonlinear Dynamics of the Earth's Magnetotail', *J. Atmospheric Solar Terrest. Phys.*, in press.
- Dendy, R. O. and Helander, P.: 1997, 'Sandpiles, Silos and Tokamak Phenomenology: a Brief Review', *Plasma Phys. Controlled Fusion* **39**, 1947.
- Einaudi, G. and Velli, M.: 1999, 'The Distribution of Flares, Statistics of Magneto-hydrodynamic Turbulence and Coronal Heating', *Phys. Plasmas* **6**, 4146.
- Freeman, M. P., Watkins, N. W. and Riley, D. J.: 2000a, 'Evidence for a Solar Wind Origin of the Power Law Burst Lifetime Distribution of the AE Indices', *Geophys. Res. Lett.* **27**, 1087.
- Freeman, M. P., Watkins, N. W. and Riley, D. J.: 2000b, 'An SOC-Like Avalanche Distribution Observed in an MHD Turbulent Cascade in the Solar Wind', *Phys. Rev. E*, submitted.
- Frette, V., Christensen, K., Malthe-Sorensen, A., Feder, J., Jossang, T. and Meakin, P.: 1996, 'Avalanche Dynamics in a Pile of Rice', *Nature* **379**, 49.
- Hoshino, M., Nishida, A., Yamamoto, T. and Kokubun, S.: 1994, 'Turbulent Magnetic Field in the Distant Magnetotail: Bottom-Up Process of Plasmoid Formation', *Geophys. Res. Lett.* **21**, 2935.
- Huang, K.: 1987., *Statistical Mechanics*, second ed., Wiley, New York.
- Jensen, H. J.: 1998 *Self-Organized Criticality: Emergent Complex: Behaviour in Physical and Biological Systems*, Cambridge University Press, Cambridge, p. 153.
- Kaneko, K.: 1993, *Theory and Applications of Coupled Map Lattices*, Wiley, New York.
- Klimas, A. J., Vassiliadis, D., Baker, D. N. and Roberts, D. A.: 1996, 'The Organised Nonlinear Dynamics of the Magnetosphere', *J. Geophys. Res.* **101**, 13089.
- Klimas, A. J., Valdivia, J. A., Vassiliadis, D., Baker, D. N., Hesse, M. and Takalo, J.: 2000, 'The Role of Self-Organized Criticality in the Substorm Phenomenon and Its Relation to Localized Reconnection in the Magnetospheric Plasma Sheet', *J. Geophys. Res.*, submitted.
- Krommes, J. A.: 2000, 'Renormalized Dissipation in the Nonconservatively Forced Burgers Equation', *Phys. Plasmas* **7**, 1064.
- Lewis, Z. V.: 1991, 'On the Apparent Randomness of Substorm Onsets', *Geophys. Res. Lett.* **18**, 1627.
- Lu, E. T.: 1995, 'Avalanches in Continuum Dissipative Systems', *Phys. Rev. Lett.* **74**, 2511.
- Lui, A. T. Y., Lopez, R. E., Krimigis, S. M., McEntire, R. W., Zanetti, L. J. and Potemra, T. A.: 1988, 'A Case Study of Magnetotail Current Sheet Disruption and Diversion', *Geophys. Res. Lett.* **15**, 721.
- Lui, A. T. Y., Chapman, S. C., Liou, K., Newell, P. T., Meng, C. I., Brittnacher, M. and Parks, G. K.: 2000, 'Is the Dynamic Magnetosphere an Avalanching System?', *Geophys. Res. Lett.* **27**, 911.
- Sharma, A. S.: 1995, 'Assessing the Magnetosphere's Nonlinear Behaviour: Its Dimension is Low, Its Predictability High', *Rev. Geophys. Suppl. Series* **33** (1), 645.

- Sitnov, M. I., Sharma, A. S., Papadopoulos, K., Vassiliadis, D., Valdivia, J. A., Klimas, A. J. and Baker, D. N.: 2000, 'Phase Transition-Like Behavior of the Magnetosphere During Substorms', *J. Geophys. Res.* **105**(12), 955.
- Smith, A. J., Freeman, M. P. and Reeves, C. D.: 1996, 'Postmidnight VLF Chorus Events, a Substorm Signature Observed at the Ground Near $L=4$ ', *J. Geophys. Res.* **101**, 24641.
- Takalo, J., Timonen, J. and Koskinen, H.: 1993, 'Correlation Dimension and Affinity of *ae* Data and Bicolored Noise', *Geophys. Res. Lett.* **20**, 1527.
- Takalo, J., Timonen, J., Klimas, A., Valdivia, J. and Vassiliadis, D.: 1999, 'Nonlinear Energy Dissipation in a Cellular Automaton Magnetotail Field Model', *Geophys. Res. Lett.* **26**, 1813.
- Tam, W. Y., Chang, T. S., Chapman, S. C. and Watkins, N. W.: 2000, 'Analytical Determination of Power Law Index for the Chapman *et al.* Sandpile (FSOC) Analog for Magnetospheric Activity-Renormalization Group Analysis', *Geophys. Res. Lett.* **27**, 1367.
- Tsurutani, B., Sugiura, M., Iyemori, T., Goldstein, B. E., Gonzalez, W. D., Akasofu, S.-I. and Smith, E.J.: 1990, 'The Nonlinear Response of AE to the IMF B_y driver: A spectral break at 5 hours', *Geophys. Res. Lett.* **17**, 279.
- Turcotte, D. L.: 1999, 'Self-Organized Criticality', *Rep. Prog. Phys.* **62**, 1.
- Uritsky, V. M. and Semenov, V. S.: 1998, 'A Sandpile Model for Global Statistics of Reconnection Events in the Magnetotail', *Proc. International Workshop on 'The Solar Wind-Magnetosphere System 3'*, 23–25 September, Graz Austria.
- Uritsky, V. M. and Pudovkin, M.: 1998, 'Low Frequency $1/f$ -like Fluctuations of the AE-Index as a Possible Manifestation of Self-Organised Criticality in the Magnetosphere', *Ann. Geophys.* **10**, 1580.
- Vassiliadis, D., Anastasiadis, A., Georgioulis, M. and Vlahos, L.: 1998, 'Derivation of Fine Cellular Automata Models from a Subset of the Magnetohydrodynamic Equations', *Astrophys. J.* **509**, L53.
- Watkins, N. W., Freeman, M. P., Chapman, S. C. and Dendy, R. O.: 2000, 'Testing the SOC Hypothesis for the Magnetosphere', *J. Astron. Solar Terrest. Phys.*, in press.

Note in Proof

It is important to distinguish the meaning of an 'attractive fixed point' in the *time evolution* (phase space or parameter space) of a system from that in the *flow of the real space renormalisation group* transformation applied to a critical or near critical system. Insofar as it evolves to criticality from arbitrary initial conditions in parameter space, an *attractive* fixed point in the former sense has been used to explain the 'self-organization' of SOC systems (e.g. Jensen, 1998). A *repulsive* fixed point under a real-space RG transformation, however, is generally found for critical systems and is a diagnostic for phase transitions (e.g. page 79 and chapter 11 of Sornette, 2000).

Sornette, D.: 2000, *Critical Phenomena in Natural Sciences. Chaos, Fractals, Self-organization and Disorder: Concepts and Tools*, Springer-Verlag, Berlin.