OBSERVATION OF UNIVERSALITY IN THE GENERALIZED SIMILARITY OF EVOLVING SOLAR WIND TURBULENCE AS SEEN BY *ULYSSES*

S. C. CHAPMAN[1], R. M. NICOL[1], E. LEONARDIS[1,3], K. KIYANI[1], AND V. CARBONE[2]

[1] Centre for Fusion, Space and Astrophysics, Physics Department, University of Warwick, Coventry, CV4 7AL, UK
[2] Dipartimento di Fisica, Università della Calabria, 87036 Rende, Italy

ABSTRACT

We perform statistical analysis of the fluctuating magnetic field observed in-situ by the *Ulysses* spacecraft, from the perspective of quantitative characterization of the evolving magnetohydrodynamic (MHD) turbulence. We focus on two successive polar passes around solar minimum which provide extended intervals of quiet, fast solar wind at a range of radial distances and latitudes: the south polar pass of 1994 and the north polar pass of 1995. Fully developed inertial range turbulence has a characteristic statistical similarity property of quantities that characterize the flow, such as the magnetic field components $B_k(t)$, so that the pth moment of fluctuations has power-law dependence on scale τ such that $< |B_k(t + \tau) - B_k(t)|^p > \sim \tau^{\zeta(p)}$. We instead find a generalized similarity $< |B_k(t + \tau) - B_k(t)|^p > \sim g(\tau/\tau_0)^{\zeta(p)}$ consistent with extended self-similarity; and in particular all of these *Ulysses* observations, from both polar passes, share the same single function $g(\tau/\tau_0)$. If these observations are indeed characteristic of MHD turbulence evolving in-situ, then this quantifies for the first time a key aspect of the *universal* nature of evolving MHD turbulence in a system of finite size, with implications both for theoretical development, and for our understanding of the evolving solar wind.

Key words: MHD – plasmas – solar wind – turbulence

1. INTRODUCTION

In-situ satellite observations of bulk plasma parameters in the solar wind provide an unparalleled opportunity to study a large-scale, high magnetic Reynolds number (Matthaeus et al. 2005; see also Goldstein & Roberts 1999) magnetized plasma flow over a range of timescales from seconds to years. Solar wind monitors such as *WIND* and *ACE* provide in-situ observations at ∼1 AU in the ecliptic, and are complemented by observations over a range of heliographic distances and latitudes, most notably from *Ulysses* which has provided several polar passes.

The statistical properties of fluctuations in the magnetic field and plasma flow suggest that the solar wind flow is turbulent (see e.g., the reviews of Tu & Marsch 1995; Goldstein 2001; Bruno & Carbone 2005). There is typically a clear range over which the power spectral density of the magnetic field and velocity components have power-law dependence; fluctuations have non-Gaussian probability densities, and the flow is intermittent (Sorriso-Valvo et al. 1999; Burlaga 2001; Hnat et al. 2002). This power-law dependence of the power spectral density which is typically obtained from the time series of single-point in-situ observations of the plasma field and flow is observed over a broad range of timescales. It extends from the characteristic (kinetic) timescales of departures from an ideal magnetohydrodynamic (MHD) flow to an outer scale where there is a crossover at longer timescales to an approximately $1/f$ power spectral density understood to be a signature of direct coronal origin (see, e.g., Matthaeus et al. 2007 and references therein). This outer scale evolves to successively lower frequencies as the solar wind flow develops (see, e.g., Bavassano et al. 1982) and there is also a latitudinal dependence (see, e.g., Horbury et al. 1996a, 1996b; Marsch & Tu 1996). Since the observed fluctuations on these timescales are presumed to be predominately Alfvénic, this

scaling range then corresponds to an inertial range of (anisotropic) MHD turbulence (Matthaeus et al. 1990; see also Milano et al. 2004; Chapman & Hnat 2007).

In an infinite, homogeneous medium, turbulence is characterized in the context of the statistical properties, as a function of spatial scale, of fluctuations that capture the dynamics of the flow (Frisch 1995). Spatial scale is implied from single spacecraft in-situ observations from the Taylor hypothesis (Taylor 1938; see also Matthaeus et al. 2005), so that variation along a lengthscale along the bulk flow direction is obtained from comparing consecutive observations over a time interval τ. In common with much of the literature on the turbulent solar wind, we will focus on fluctuations in components of the magnetic field $B_k(t + \tau) - B_k(t)$ on a given timescale τ. Fully developed inertial range turbulence in an infinite medium would then imply that the (generalized) structure functions have statistical similarity, that is, power-law dependence on scale τ:

$$S_p(\tau) = \langle |B_k(t + \tau) - B_k(t)|^p \rangle \sim \tau^{\zeta(p)}. \quad (1)$$

The existence of statistical similarity is a *universal* feature of the turbulence, that is, it is not sensitive to details of the flow. In practical realizations of turbulence, such as finite domain numerical simulations and laboratory experiments (Merrifield et al. 2007; Fujisaka & Grossmann 2001), or the solar wind (Grossmann et al. 1997; Pagel & Balogh 2001; Sorriso-Valvo et al. 2006; Nicol et al. 2008), one often finds that the domain is too small, or the turbulence is not sufficiently evolved, for power-law scaling to be established. A generalization of (1), extended self-similarity (ESS; Benzi et al. 1993) has been found to hold for the structure functions:

$$S_p(\tau) = [S_q(\tau)]^{\zeta(p)/\zeta(q)}. \quad (2)$$

Intriguingly, this result, although confirmed experimentally over a wide variety of turbulent flows, has yet to find a theoretical basis. ESS implies a generalized similarity:

$$S_p(\tau) = \langle |B_k(t + \tau) - B_k(t)|^p \rangle = S_p(\tau_0) [g(\tau/\tau_0)]^{\zeta(p)}, \quad (3)$$
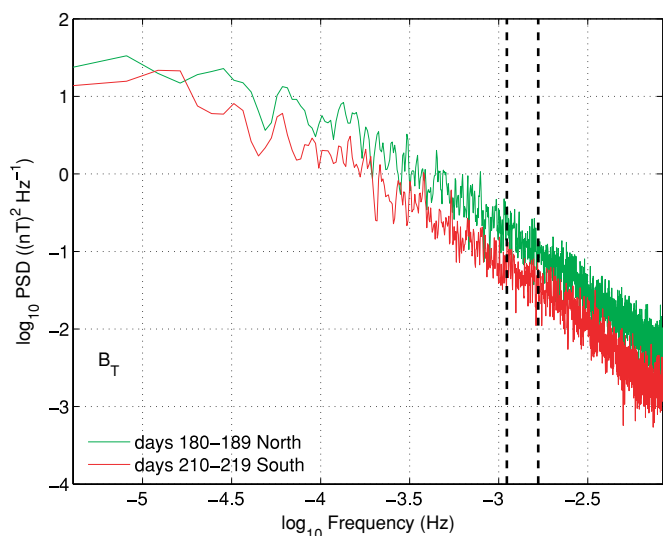

Figure 1. Power spectral density of two representative 10-day intervals from the north and south polar passes, T component of magnetic field.

where the crossover function g is also anticipated to be a universal property of turbulence (Fujisaka & Grossmann 2001). Here, we will explicitly test Equation (3) using successive north and south polar passes at solar minimum.

2. OBSERVATIONS

We focus here exclusively on *Ulysses* observations that provide contiguous intervals comprised of fast, quiet, solar wind from polar passes at solar minimum, which are free from large-scale features of coronal origin. As well as the $\sim 1/f$ range found at lower frequencies, there is also recent evidence of a scaling signature *within* the inertial range that correlates with the level of magnetic complexity in the corona (Hnat et al. 2007; Kiyani et al. 2007); and that the observed turbulent solar wind in the ecliptic is ordered w.r.t. magnetic structures of coronal origin (Borovsky et al. 2008). During each of the south polar passes in 1994, and the subsequent north polar passes in 1995, the *Ulysses* spacecraft spent approximately three months above a polar coronal hole in an interval of quiet, fast, solar wind. We analyze days 210–269 of 1994, the south polar pass, during which *Ulysses* moved from a heliospheric distance of 2.6141–2.2041 AU and from an heliographic latitude of $-75°\!.17$ to $-79°\!.60$. We will compare these with days 180–239 of 1995, the north polar pass, previously analyzed by Nicol et al. (2008), during which *Ulysses* moved from 1.7926 AU to 2.2043 AU heliospheric distance and through $73°\!.76$ to $77°\!.03$ heliographic latitude (both passes peak at $80°\!.22$). We parse each of these polar pass time series into six, 10-day contiguous intervals, thus providing 12 "snapshots" of the statistics of fluctuations at a range of heliographic latitudes and distances from the corona. We use one minute averaged measurements of the radial (R), tangential (T), and normal (N) magnetic field components (where R is the Sun–*Ulysses* axis, T is the cross product of R with the solar rotation axis, and N is the cross product of R and T). Each of these intervals then comprises approximately $13,000$ data points, sufficient to explore the scaling properties of the turbulence and its crossover to $\sim 1/f$ scaling.

In Figure 1, we show the power spectral densities of the T component of the magnetic field for a representative 10-day interval from the north and south polar passes. The approximate location of the crossover from the inertial range (with $\sim -5/3$
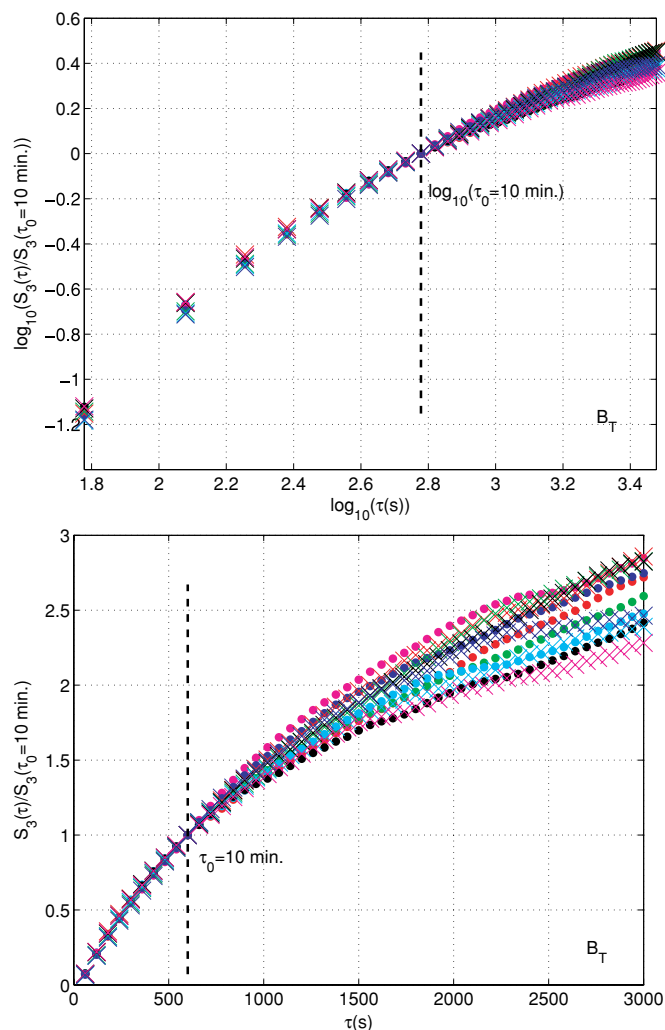

Figure 2. Third-order structure function S_3 of the T component of the magnetic field plotted vs. differencing timescale τ, on (a) logarithmic and (b) linear axes, for all 12 10-day intervals during *Ulysses* north ($\circ$) and south ($\times$) polar passes. Each trace is normalized to $S_3(\tau_0 = 10)$ minutes.

exponent) and the $\sim 1/f$ range is marked with a vertical dashed line on the plot, at $\tau = 15$ minutes, a second dashed line at 10 minutes denotes the reference timescale that we use below. We can see that in this respect the power spectra are quite similar; this is also seen in the other components. We will now probe this more carefully with structure function analysis. For each of the 10-day contiguous intervals, and for each of the $k = R, T$, and N components of the magnetic field, we compute the generalized (or modulus) structure functions $S_p(\tau) = \langle |B_k(t+\tau) - B_k(t)|^p \rangle$ (in common with the most turbulence studies we compute the modulus for reasons of accuracy; see, e.g., Chapman et al. 2005 and Nicol et al. 2008). For processes that have non-Gaussian statistics (are "heavy tailed"), outlying points, which have poor statistical representation in our finite length samples, can bias the estimates of the structure functions. We have used the method developed by Kiyani et al. (2006) to verify that our results are robust against this source of uncertainty (for details see Nicol et al. 2008).

In Figure 2, we focus on the third-order ($p = 3$) structure function for both north and south polar passes. We overplot, on both linear (lower panel) and logarithmic (upper panel) axes, $S_3(\tau)/S_3(\tau_0 = 10)$ minutes versus τ for the T component for
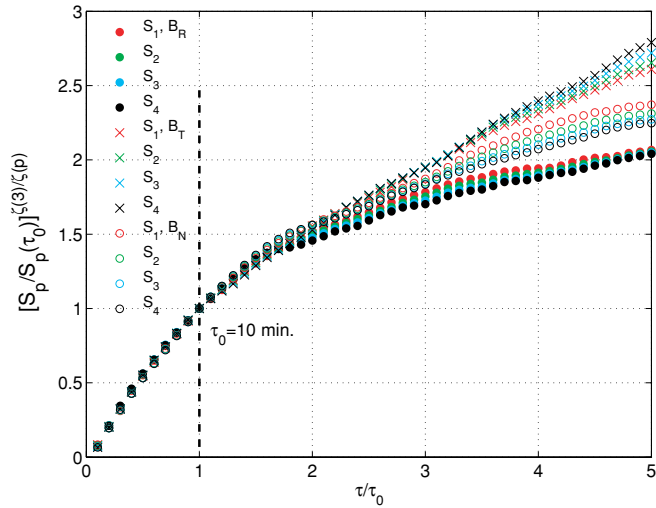
Figure 3. Check of universality of the function $g(\tau/\tau_0)$ which is obtained for each of the first four order $p = 1, \ldots, 4$ structure functions for all components of the magnetic field, for a representative 10-day interval during *Ulysses* north polar pass (see text for details).

each of the twelve 10-day intervals of data (six from each polar pass). The reference timescale $\tau_0 = 10$ minutes, which we will use to fix the normalizing constants throughout our analysis is selected to be within the timescale at which there is a crossover to $\sim 1/f$ scaling (and is otherwise arbitrary).

For timescales up to $\sim$15–20 minutes we can immediately see that the $S_3(\tau)$ do not show power-law scaling with τ for any of the intervals, but do show a remarkable collapse onto a single function. This was found previously by Nicol et al. (2008) for consecutive intervals within a single (north) polar pass; we now find the same functional dependence here for observations separated by roughly one year. The crossover from this range of turbulence to the $\sim 1/f$ range can be clearly seen at times beyond $\sim$15–20 minutes where the curves diverge from each other. As found previously (Horbury et al. 1996a; Nicol et al. 2008), the $\sim 1/f$ range shows latitudinal and radial variations in its scaling exponent when quantified through structure functions.

Figure 2 is then highly suggestive that all of the intervals under study, from both north and south polar passes, share the same function $g(\tau/\tau_0)$ (3) which does not vary as conditions in the flow change with heliographic distance or latitude. An expression for the functional form of $g(\tau/\tau_0)$ has already been obtained in Nicol et al. (2008); see also Sorriso-Valvo et al. (2006). Here, we now test whether the $g(\tau/\tau_0)$ is a *universal* function, capturing the dynamics of the crossover from the inertial to the outer scale of the turbulence. We can test Expression (3) to reasonable precision at least for low orders, $p = 1, \ldots, 4$; for higher orders, the structure functions become less reliable (Dudok de Wit 2004). To obtain $g(\tau/\tau_0)$ we in principle simply need to invert Equation (3); in practice however we do not have independent measurements of the exponents, $\zeta(p)$, but rather their ratios from Equation (2) obtained by plotting structure functions of one order against another. In Figure 3, we plot

$$\left[\frac{S_p(\tau)}{S_p(\tau_0)} \right]^{\frac{\zeta(3)}{\zeta(p)}} = g(\tau/\tau_0)^{\zeta(3)} \qquad (4)$$

versus τ/τ_0, for orders $p = 1, \ldots, 4$, where $\tau_0 = 10$ minutes as above, and the ratios $\zeta(p)/\zeta(3)$ are obtained from plots of S_p

versus S_3 (see, e.g., Nicol et al. 2008). We show results for the R, T, and N components for a representative 10-day interval from the north polar pass. We indeed see that the curves coincide to good precision for all field components.

Finally, we estimate the variation in the Reynolds number explored by these intervals as follows. A measure of the Reynolds number R_E varies with the outer scale L and the dissipative, or dispersive lengthscale η as $R_E \sim (L/\eta)^{4/3}$ (see, e.g., Frisch 1995). From Figure 2, we can see that L, the timescale at which there is a crossover from turbulent to $\sim 1/f$ behavior, does not vary significantly from one interval to the next. We do not have a direct measurement of η; however, we can anticipate that η scales with either the ion Larmor radius, or the ion inertial length, or both, since these parameterize departure from MHD at small scales. These scales in turn depend on the inverse of the magnetic field magnitude, and the density, respectively, which on average vary with the heliospheric distance r as $\sim 1/r^2$ (Goldstein et al. 1996). The polar passes analyzed here then have $r = [1.8–2.6]$ AU giving a variation in R_E by a factor of ~ 2.5.

3. CONCLUSIONS

We have used magnetic field observations of evolving MHD turbulence in the solar wind from successive *Ulysses* 1994 and 1995 polar passes at solar minimum to test a proposed universal property of turbulence, that of generalized similarity. Fully evolved inertial range turbulence in an infinite medium has a universal property of similarity, that is, power-law dependence of the statistical properties of fluctuations, such as structure functions, with lengthscale (here, expressed as a timescale τ). In a system of finite size, or where the turbulence is not fully evolved, this similarity is lost, instead one generally observes an ESS, that is, power-law dependence on τ of the ratios of the structure functions, and this is typically seen in solar wind turbulence. ESS has yet to find a theoretical basis; however, it could arise if the property of similarity, that is, power-law dependence on τ, is generalized to power-law dependence on a function $g(\tau/\tau_0)$. A first step toward a formal understanding of ESS phenomenology is then to establish whether this generalized similarity is in fact universal. We have shown that this is indeed the case in the quiet fast polar solar wind flow at ~ 2 AU; we find the same function $g(\tau/\tau_0)$ orders the statistical properties of fluctuations in all components of the magnetic field, over intervals from north and south polar passes separated by one year with background magnetic field magnitude and density varying with distance by a factor of ~ 2.

The timescales over which we have studied $g(\tau/\tau_0)$ are at approximately an order of magnitude longer than that of the dissipation scale of the turbulence, but include the outer scale where there is a crossover to $\sim 1/f$ fluctuations of the coronal origin. The function $g(\tau/\tau_0)$, which we discuss here, therefore gives information on the essential dynamics of the largest scales of the MHD turbulence, for example, their anisotropy (Kurien & Sreenivasan 2000; Sorriso-Valvo et al. 2006) and interaction with the macroscopic heliospheric fields and flows. It would thus be intriguing to perform comparative studies with other astrophysical MHD turbulent flows in systems of finite size, for example in the boundary layers of planetary magnetospheres, as well as with other experimental realizations of turbulence.

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