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Whistler mode wave coupling effects on electron dynamics in the near Earth magnetosphere

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Abstract. Nonlinear electron whistler mode wave particle interactions in the near earth magnetosphere suggest several candidate mechanisms for phase space diffusion of electrons into the loss cone to provide a source of auroral precipitating electrons. A unidirectional whistler mode wave propagating parallel to the background magnetic field and interacting with an electron can be shown to produce resonance (trapping of electrons) but the process is not stochastic. Chaos can be introduced by resonance between electrons that mirror many times in the earth's field, and a whistler wavefield. Here, we show that an electron interacting with two oppositely directed, parallel propagating whistler mode waves exhibits stochastic behaviour in addition to resonance. The background field is uniform, so that stochasticity arises solely due to the presence of the wavefield without requiring bounce motion of electrons between mirror points. Stochasticity first appears for wave amplitudes not inconsistent with observations at disturbed times in the near earth magnetosphere and therefore may contribute to pitch angle diffusion.

1. Introduction

A number of mechanisms have been proposed for pitch angle scattering of electrons by whistler mode waves in the near earth magnetosphere. As well as processes associated with waves that are strongly oblique (e.g. Inan, 1992), a candidate mechanism is cyclotron resonance processes with near parallel propagating waves (Kennel and Petschek 1966; Lyons and Williams, 1984; Villalon and Burke, 1991). Stochasticity has been introduced by considering the bounce motion of electrons interacting with a single unidirectional whistler mode wave (Faith, 1997). Here we highlight the possibility that reflected, bidirectional, waves interact with single electron motion to introduce chaos in the electron trajectories in the whistler wavefield. Ray tracing studies (Thorne and Horne, 1994; Thorne and Horne, 1996) suggest that reflected bidirectional and for much of their ray paths, parallel propagating whistler waves are present in the near earth magnetosphere. The electron interaction with an omnidirectional parallel propagating wave is not chaotic, and we shall see, for bidirectional waves this is still the case sufficiently close to resonance. Away from resonance we find that in the presence of oppositely directed parallel propagating waves, much of phase space becomes dominated by stochastic orbits which act to enhance pitch angle diffusion.

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2. Theoretical approach

We consider the case of an electron in the presence of a wave field gyrating around a uniform background magnetic field $B_o\hat{x}$ with vector potential $\underline{A}_o = B_o y\hat{z}$. The wave field $\underline{B}_w(x,t)$ is the superposition of two oppositely directed, parallel propagating whistler mode waves $\underline{B}_w^+$, $\underline{B}_w^-$

$$\underline{B}_w^+ = B_w[\cos(kx-\omega t)\hat{y} - \sin(kx-\omega t)\hat{z}] \quad (1)$$

$$\underline{B}_w^- = B_w[\cos(-kx-\omega t+\vartheta)\hat{y} - \sin(-kx-\omega t+\vartheta)\hat{z}] \quad (2)$$

Each one of the above right hand polarized waves is in resonance with electrons moving antiparallel to the wavenumbers $\underline{k}$, and satisfying the resonance condition

$$\omega - \underline{k}\cdot\underline{v} = n\Omega \quad (3)$$

where $\underline{v}$ is the particle velocity ω is the wave frequency and $\Omega = eB_o/m$ is the gyrofrequency. For whistler mode waves $\omega < n\Omega$. The vector potential for the above wave field can be expressed as:

$$\underline{A}_w = \underline{A}_w^+ + \underline{A}_w^- = 2\frac{B_w}{k}\sin\left(kx - \frac{\vartheta}{2}\right) \cdot \left[\sin\left(\omega t + \frac{\vartheta}{2}\right)\hat{y} - \cos\left(\omega t + \frac{\vartheta}{2}\right)\hat{z}\right] \quad (4)$$

where ϑ is the initial difference in phase between the two waves and B_w each wave amplitude. The particle velocity components parallel and perpendicular to the background field are v_x and $v_\perp$ respectively. The phase angles between $\underline{v}_\perp$ and the wave vectors $\underline{B}_w^+$,$\underline{B}_w^-$ are ψ

and $\psi - (2kx - \vartheta)$, (see Figure 1). $\Delta\psi = \vartheta - 2kx$ is the total difference in phase between the two opposite propagating waves. The equations of motion from the Lorentz force in a coordinate system of $(v_x, v_\perp, \psi)$ are

$$\frac{dv_x}{dt} = -v_\perp \Omega_w[\sin(\psi) + \sin(\psi - (2kx - \vartheta))] \quad (5)$$

$$\frac{dv_\perp}{dt} = -(v_x - \frac{\omega}{k})\Omega_w \sin(\psi) -(v_x + \frac{\omega}{k})\Omega_w \sin(\psi - (2kx - \vartheta)) \quad (6)$$

$$\frac{d\psi}{dt} = kx - \omega t + \Omega t - (v_x - \frac{\omega}{k})\Omega_w \cos(\psi) -(v_x + \frac{\omega}{k})\Omega_w \cos(\psi - (2kx - \vartheta)) \quad (7)$$

where $\Omega_w = \frac{eB_w}{m}$. For $\Omega_w \ll \Omega$, $\psi = kx + (\Omega - \omega)t$. One obtains from (5)

$$\frac{d^2\psi}{dt^2} + kv_\perp\Omega_w[\sin(\psi) + \sin(\psi - (2kx - \vartheta))] = 0 \quad (8)$$

Equation (8) has the basic form of the pendulum equation where the phase angles oscillate around an equilibrium position, with frequency $\varpi^2 = k\Omega_w v_\perp$. (8) can be expressed in a more simplified way as

$$\frac{d^2\psi}{dt^2} + kv_\perp\Omega_w[\sin(\psi) + \sin(\psi - \gamma)] = 0 \quad (9)$$

where $\gamma(t) = 2[(\Omega - \omega)t - \frac{\vartheta}{2}]$. (9) can be expressed as

$$\frac{d^2\Theta}{dt^2} - 2kv_\perp\Omega_w \sin(\Theta)\sin(\frac{\gamma}{2}) = 0 \quad (10)$$

where $\Theta = \frac{\gamma}{2} + \frac{\pi}{2} - \psi$. This is of the form of a simple pendulum with a time modulated length or gravitational accelaration. The above pendulum equation can be derived from Hamiltonian

$$H = \frac{1}{2}(\frac{d\Theta}{dt})^2 + 2kv_\perp\Omega_w \cos(\Theta)\sin(\frac{\gamma}{2}) \quad (11)$$

which simply has the form of the standard Hamiltonian. We can assume that around the resonance fixed points, and for sufficiently small wave amplitudes from (5)

$$x = \xi + \beta t \quad (12)$$

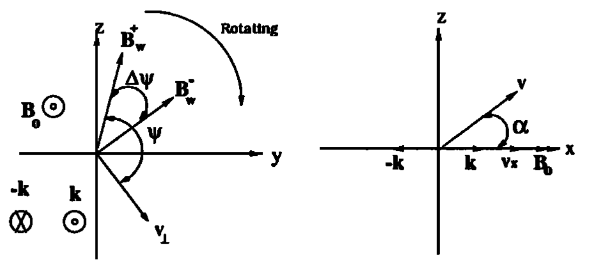

Figure 1. Coordinate system for a single electron and two oppositely directed whistler mode waves with different wave phases. ψ is the phase angle (rotating frame) between the electron perpendicular velocity and the magnetic field vector of one of the waves, and α the pitch angle between the electron parallel velocity and the background magnetic field vector.

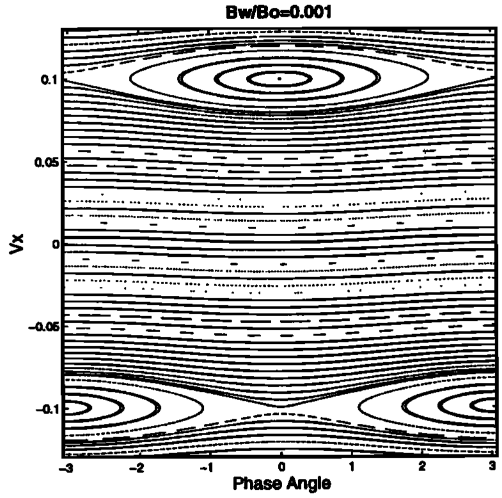

Figure 2. For a $\omega = \Omega/2$, $B_w = 0.001B_o$, the trajectories are regular for a particle of 100 keV.

where β is equal to the parallel velocity near resonance. By substituting this value in (9) one finds that this equation is satisfied if (3) is obeyed and we shall see the resonances appear in the numerical solutions of the full equations of motion (Figure 2 onwards). Equation (9) has resonances at $\psi = 0$, $n\pi$ and at $(\psi - \gamma) = 0$, $n\pi$. At these points (9) reduces to the pendulum equation of motion of a particle in a single wave (Dysthe, 1970; Gendrin, 1974).The motion is integrable in this one degree of freedom system with coordinates $\frac{d\psi}{dt}$, ψ or $\frac{d(\psi-\gamma)}{dt} = \frac{d\psi}{dt} - 2(\Omega - \omega)$, $\psi - \gamma$. Close to these resonant points coupling will be weak and the motion is near integrable. As we move away from resonance the system now has two degrees of freedom ψ, $\gamma(t)$. This gives the possibility for chaos and enhanced diffusion in phase space, and we will examine this numerically in the next section.

3. Numerical Results

The regular and stochastic behaviour of the electron motion is investigated via effective Poincare surface of section plots of $v_x = [\frac{d\psi}{dt} - (\Omega - \omega)]/k$, versus ψ (see for example Tabor, 1989; Chen, 1992; Ram et al, 1993 and references therein). The plots are made by solving the equations of motion forward in time using a variable order, variable stepsize ordinary differential equations integrator (e.g. Chapman and Watkins, 1993). The surface of section is $v_z = 0$, where v_z is the z component of the perpendicular velocity. Each trajectory was integrated for 100000 gyroperiods, in order to allow particles to fully explore all regions of accessible phase space. For stochastic trajectories significant diffusion in phase space is found to occur on tens of electron gyroperiods. All particle trajectories have the same energy and phase angle ψ at $t = 0$ whilst the initial pitch angle was varied from 0 to π giving 180 sets of initial velocities so as to cover the interval $[-v_R \; v_R]$ where v_R is the resonant v_x (see Figure 2). Several runs with different wave amplitudes, particle energies and wave frequencies have been performed to investigate the level of stochasticity in the particle dynamics. Unnormalized, the selected parameters correspond to in the near earth magnetosphere; the range of wave frequencies covers the region below the typical gyrofrequency of 2 kHz at L=6 (Parrot et al, 1994), the plasma frequency at L=6 is around 8 kHz. Electron energies investigated are in the range 4-100 keV. We find that the particle dynamics is sensitive in the variation of the wave amplitude as well as the particle energy.

The change in the dynamics of electrons of a given energy as we increase the wave amplitude is illustrated in figures 2-4, which show Poincare SOS for a 100keV electron. For $B_w/B_0 = 0.001$ the behaviour is essentially regular, that is, it is predicted by the simple pendulum equation of motion. As we increase B_w/B_0 to 0.005 (Figure 3) regions of stochasticity appear, these allow particles to explore all phase angles and a range of pitch angles (that is, a range of $v_x = v\cos(\alpha)$). Onset of weak stochasticity can be seen just around the separatrices of the resonances. At $B_w/B_o = 0.008$ (Figure 4) we see stochasticity over significant regions of phase space. The particle motion is still regular close to resonance, and particles cannot diffuse between the regions surrounding the resonances associated with the forward and backward propagating waves. If we increase B_w/B_0 further then phase space diffusion becomes global and stochastic particles can move across $v_x = 0$.

Change of the relative phase between the waves does not affect this behaviour qualitatively, simply moving the location of the resonances as we would expect from equation (9)(see Figure 5).

This behaviour is qualitatively the same for different electron energies in that there will be a threshold value for B_w/B_0 when stochasticity appears; however the value of this threshold is energy dependent. These

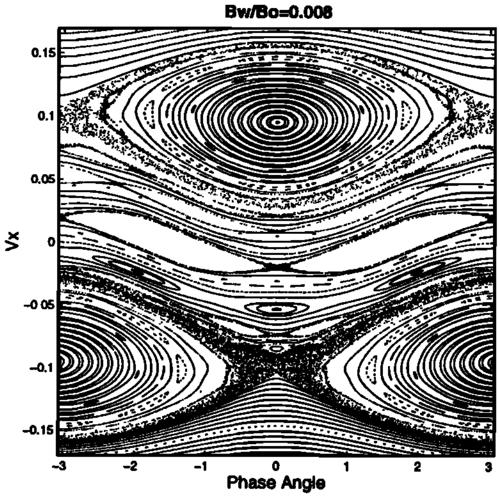

Figure 4. Enhanced diffusion for $B_w = 0.008B_o$ for a 100 keV electron and a wave frequency of $\Omega/2$.

wave amplitudes correspond to 100s of pT for $L = 6$ (ie taking 2kHz gyrofrequency) and are within the range of strong emissions seen at disturbed times (Parrot et al, 1994) and individual wavepackets near the magnetopause (Nagano et al, 1996). The change in the detailed dynamics of the particles with frequency is complex and will be adressed in a future paper. Here we note that the wave amplitude at which stochasticity first appears in phase space is similar to within an order of magnitude. We also anticipate that the more realistic situation where several wave modes are considered (and that the wavenumbers of the oppositely directed waves are not the same) will increase the complexity of the detailed dynamics; this is beyond the scope of this initial study and will be adressed in future.

Finally, Figure 6 shows that the particle energy oscillates with time so that the net energy change is zero. Hence, this Wave Particle Interaction mechanism will lead to pure pitch angle diffusion rather than particle energization.

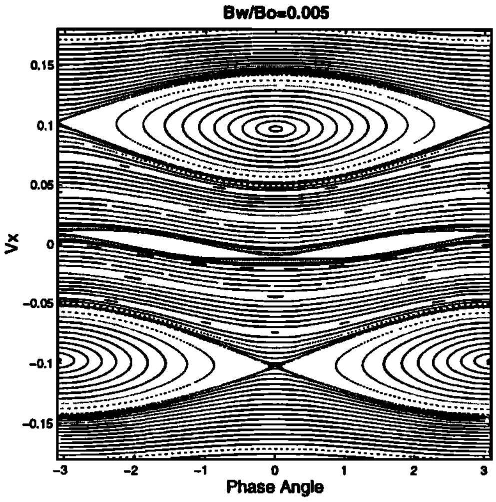

Figure 3. Diffusion around the separatrices first appears for $B_w \simeq 0.005B_o$, shown for electron energy of 100 keV and wave frequency of $\Omega/2$.

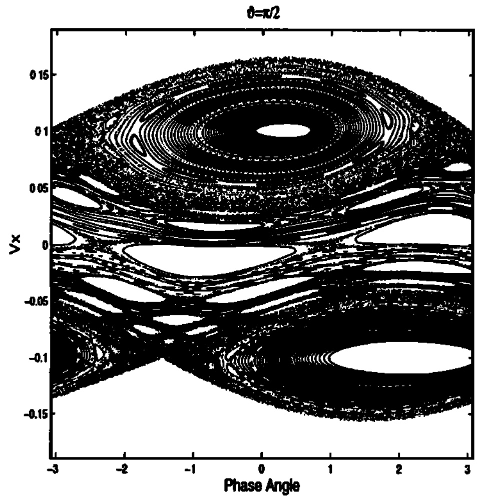

Figure 5. Effect of varying the phase between the two waves, for a 100 keV electron, wave frequency $\omega = \Omega/2$, $B_w = 0.008B_o$ and $\vartheta = \frac{\pi}{2}$.

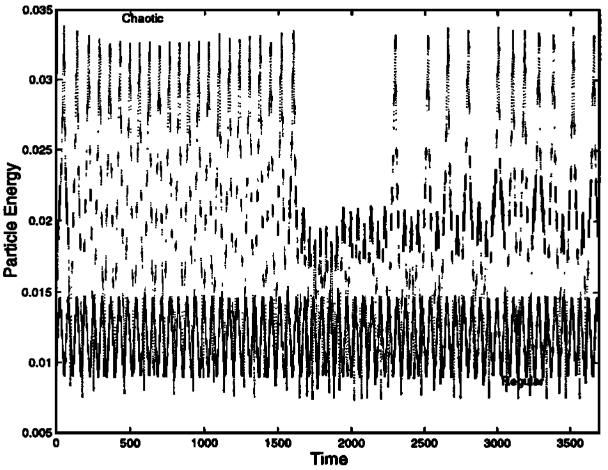

Figure 6. Dependence of the particle energy on time for $B_w = 0.008B_o$ and $\omega = \Omega/2$, showing that the net particle energy remains constant.

4. Conclusions

It has been shown that coupling between two oppositely propagating whistler mode waves of the same amplitude and wavenumber can lead to stochastic electron dynamics. Specifically, it is shown that chaotic phenomena occur due to the presence of the second wave. For sufficiently small wave amplitudes, however, the deviations from the single whistler mode wave electron interactions is not significant and the motion of the particle in phase space can be considered as identical to that of a simple pendulum. As the amplitude increases the effects of the other wave on the particle by means of the Lorentz force become important, the integrals of motion are not conserved and regular dynamics can be maintained only in the vicinity of resonances; the KAM surfaces break up successively. It is also found that the value of the wave phase does not change the electron dynamics in a qualitative manner. Chaotic dynamics of electrons may allow significant pitch angle scattering into the loss cone (phase space diffusion) in the magnetosphere under certain circumstances. Results suggest that for example for 100 keV electrons, stochasticity is evident for wave amplitudes observed at $L = 6$ during disturbed times.

The diffusion coefficient for the process discussed here will be calculated in a future paper. However, we anticipate that pitch angle diffusion will scale as the local gyrofrequency. This is to be compared with stochastic diffusion due to bounce resonance (e.g. Faith et al, 1997) of electrons with whistler mode waves which will scale with the bounce frequency, this may be reflected in the much larger wave amplitudes required in Faith et al 1997 for diffusion. Also, there is some evidence that this process will still occur when the fields and electron motion are evolved selfconsistently (Devine and Chapman, 1996).

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