Polynomial Evaluation and Factorisation

CIS002-2 Computational Alegrba and Number Theory

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Outline

1. Polynomial functions
2. Division
3. Nesting Polynomial
4. Class exersises
5. Remainder Theorem
6. factor theorem

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2. Division
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- A polynomial of the second degree is a quadratic expression
- A polynomial of the third degree is a cubic expression
- A polynomial of the fourth degree is a quartic expression
- etc

Outline

Division of one expression by another

- Let us consider $(12x^3 - 2x^2 - 3x + 28)$ divided by $(3x + 4)$
- We set this out as for long division with numbers.
- To make $12x^3$, $3x$ must be multiplied by $4x^2$, so we insert this as the first term in the quotient
- we multiply the divisor $(3x + 4)$ by $4x^2$ and subtract this from the first two terms.
- Perform the addition (here we add $-2x^2$ and $-16x^2$) and bring down the next term (here $-3x$)
- To make $-18x^2$, $3x$ must be multiplied by $-6x$, so we insert this as the next term in the quotient
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$$3x + 4\overline{)\; 12x^3 \quad - 2x^2 \quad - 3x + 28}$$

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$$\begin{array}{r|l} & \quad 4x^2 \\ \hline 3x+4 & 12x^3 - 2x^2 - 3x + 28 \end{array}$$

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$$
\begin{array}{r|llll}
 & 4x^2 & & & \\
\hline
3x+4) & 12x^3 & -2x^2 & -3x & +28 \\
 & -12x^3 & -16x^2 & & \\
\hline
 & & -18x^2 & -3x & \\
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\hline
3x+4 & 12x^3 & -2x^2 & -3x & +28 \\
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\hline
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\hline
\end{array}
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\hline
3x+4) & 12x^3 \; -2x^2 & -3x & +28 \\
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\hline
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 & 18x^2 & +24x & \\
\hline
 & & 21x & +28
\end{array}
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$$
\begin{array}{r|l}
 & 4x^2 \quad -6x \quad +7 \\
\hline
3x+4) & 12x^3 \quad -2x^2 \quad -3x + 28 \\
 & -12x^3 - 16x^2 \\
\hline
 & -18x^2 \quad -3x \\
 & 18x^2 + 24x \\
\hline
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\end{array}
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\hline
 & & -18x^2 & -3x & \\
 & & 18x^2 & +24x & \\
\hline
 & & & 21x & +28 \\
 & & & -21x & -28 \\
\hline
 & & & & 0
\end{array}
$$

$$
\begin{aligned}
f(x) &= (12x^3 - 2x^2 - 3x + 28) \\
&= (3x + 4)(4x^2 - 6x + 7)
\end{aligned}
$$

Class Exercise

If an expression has a power missing, insert the power with zero coefficient.
Determine the following:

$(4x^3 + 13x + 33)$ divided by $(2x + 3)$

$(6x^3 - 7x^2 + 1)$ divided by $(3x + 1)$

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$$
\begin{array}{rrrrr}
 & 2x^2 & -3x & +11 & \\
\cline{2-5}
2x+3 \big) & 4x^3 & & +13x & +33 \\
 & -4x^3 & -6x^2 & & \\
\cline{2-3}
 & & -6x^2 & +13x & \\
 & & 6x^2 & +9x & \\
\cline{3-4}
 & & & 22x & +33 \\
 & & & -22x & -33 \\
\cline{4-5}
 & & & & 0
\end{array}
$$

$$
\begin{aligned}
f(x) &= (4x^3 + 13x + 33) \\
&= (2x + 3)(2x^2 - 3x + 11)
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 & 6x^2 & +9x & & \\ \cline{2-3}
 & & 22x & +33 & \\
 & & -22x & -33 & \\ \cline{3-4}
 & & & 0 &
\end{array}
$$

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f(x) &= (4x^3 + 13x + 33) \\
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$(6x^3 - 7x^2 + 1)$ divided by $(3x + 1)$

$$
\begin{array}{rrrrr}
 & 2x^2 & -3x & +1 \\ \cline{2-5}
3x+1) & 6x^3 - 7x^2 & & +1 \\
 & -6x^3 - 2x^2 & & \\ \cline{2-2}
 & -9x^2 & & \\
 & 9x^2 & +3x & \\ \cline{2-3}
 & & 3x & +1 \\
 & & -3x & -1 \\ \cline{3-4}
 & & & 0
\end{array}
$$

$$
\begin{aligned}
f(x) &= (6x^3 - 7x^2 + 1) \\
&= (3x + 1)(2x^2 - 3x + 1)
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Outline

Evaluation of a polynomial by 'nesting'

- Consider the polynomial $f(x) = 5x^3 + 2x^2 - 3x + 6$.

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- i.e. $(5x + 2)x - 3$

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- i.e. $(5x + 2)x - 3$
- Repeat the process: enclose the whole of this in brackets, multiply by x and add on the next coefficient
- i.e. $f(x) = ((5x + 2)x - 3)x + 6$ in nested form.

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- Repeat the process: enclose the whole of this in brackets, multiply by x and add on the next coefficient
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- i.e. $5x + 2$
- Enclose these in brackets, multiply by x and add on the next cofficient.
- i.e. $(5x + 2)x - 3$
- Repeat the process: enclose the whole of this in brackets, multiply by x and add on the next coefficient
- i.e. $f(x) = ((5x + 2)x - 3)x + 6$ in nested form.
- This nesting process allows exaluation of the expression with less operations.
- *Note: If any power is missing from the polynomial, it must be included with a zero coefficiant before nesting is carried out*

Outline

Class Exersises

For each of the following cases, express the polynomial in nested form and evaluate the function for the given value of x.

1. $f(x) = 4x^3 + 3x^2 + 2x - 4$ evaluated for $[x = 2]$.
2. $f(x) = 2x^4 + x^3 - 3x^2 + 5x - 6$ evaluated for $[x = 3]$.
3. $f(x) = x^4 - 3x^3 + 2x - 3$ evaluated for $[x = 5]$.
4. $f(x) = 2x^4 - 5x^3 - 3x^2 + 4$ evaluated for $[x = 4]$.

Class Exersises

For each of the following cases, express the polynomial in nested form and evaluate the function for the given value of x.

1. $f(x) = 4x^3 + 3x^2 + 2x - 4$ evaluated for $[x = 2]$. $((4x + 3)x + 2)x - 4$, $f(2) = 44$
2. $f(x) = 2x^4 + x^3 - 3x^2 + 5x - 6$ evaluated for $[x = 3]$.
3. $f(x) = x^4 - 3x^3 + 2x - 3$ evaluated for $[x = 5]$.
4. $f(x) = 2x^4 - 5x^3 - 3x^2 + 4$ evaluated for $[x = 4]$.

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2. $f(x) = 2x^4 + x^3 - 3x^2 + 5x - 6$ evaluated for $[x = 3]$. $(((2x + 1)x - 3)x + 5)x - 6$, $f(3) = 171$
3. $f(x) = x^4 - 3x^3 + 2x - 3$ evaluated for $[x = 5]$.
4. $f(x) = 2x^4 - 5x^3 - 3x^2 + 4$ evaluated for $[x = 4]$.

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3. $f(x) = x^4 - 3x^3 + 2x - 3$ evaluated for $[x = 5]$. $(((x - 3)x + 0)x + 2)x - 3,\ f(5) = 257$
4. $f(x) = 2x^4 - 5x^3 - 3x^2 + 4$ evaluated for $[x = 4]$.

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4. $f(x) = 2x^4 - 5x^3 - 3x^2 + 4$ evaluated for $[x = 4]$. $(((2x - 5)x - 3)x + 0)x + 4$, $f(4) = 148$

Outline

Remaineder Theorem

- The **remainder theorem** states that if a polynomial $f(x)$ is divided by $(x - a)$, the quotient will be a polynomial $g(x)$ of degree one less than that of $f(x)$, together with a remainder R still to be divided by $(x - a)$.

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- therefore $f(x) = (x-a)g(x) + R$
- When $x = a$, $f(a) = 0 \cdot g(a) + R$ i.e. $R = f(a)$.
- If $f(x)$ were to be divided by $(x-a)$, the remainder would be $f(a)$.

Example

- If $(x^3 + 3x^2 - 13x - 10)$ were divided by $(x - 3)$ the remainder would be ...

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- We can verify this by the long division:

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- If $(x^3 + 3x^2 - 13x - 10)$ were divided by $(x - 3)$ the remainder would be ...
- $R = f(3) = 5$, where it may be useful to use the nested form, $f(x) = ((x + 3)x - 13)x - 10$.
- We can verify this by the long division:
- $$\begin{array}{r} x^2 \quad + 6x \quad + 5 \\ x-3\overline{)\; x^3 + 3x^2 - 13x - 10} \\ \underline{-\, x^3 + 3x^2 \qquad\qquad\quad} \\ 6x^2 - 13x \qquad \\ \underline{-\, 6x^2 + 18x \qquad} \\ 5x - 10 \\ \underline{-\, 5x + 15} \\ 5 \end{array}$$

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- For example, if $f(x) = x^3 + 2x^2 - 14x + 12$ and we substitute $x = 2$, then $f(2) = 0$.
- Therefore $(x - 2)$ is a factor of $f(x)$.

Factor Theorem

- If $f(x)$ is a polynomial and substituting $(x = a)$ gives a zero remainder i.e. $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.
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-
$$
\begin{array}{r|l}
 & x^2 \quad + 4x \quad - 6 \\
\hline
x - 2) & x^3 + 2x^2 - 14x + 12 \\
 & -x^3 + 2x^2 \\
\hline
 & 4x^2 - 14x \\
 & -4x^2 + 8x \\
\hline
 & -6x + 12 \\
 & 6x - 12 \\
\hline
 & 0
\end{array}
$$

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- $$\begin{array}{r} x^2 \quad + 4x \quad - 6 \\ x - 2 \overline{) \; x^3 + 2x^2 - 14x + 12} \\ -\, x^3 + 2x^2 \qquad\qquad\quad \\ \hline 4x^2 - 14x \qquad \\ -\, 4x^2 \quad + 8x \qquad \\ \hline -\, 6x + 12 \\ 6x - 12 \\ \hline 0 \end{array}$$
- The quadratic factor obtained may be factorised further into two linear factors.
- We apply a $b^2 - 4ac$ test, where $f(x) = ax^2 + bx + c$, and if $b^2 - 4ac$ is a perfect square, then further factorisation is possible.

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Class Exersise

Factorise $f(x) = x^3 - 5x^2 - 2x + 24$ as far as possible, $(f(3) = 0)$

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Factorise $f(x) = x^3 - 5x^2 - 2x + 24$ as far as possible, $(f(3) = 0)$

$$
\begin{array}{r|l}
 & x^2 - 2x - 8 \\
\hline
x - 3 & x^3 - 5x^2 - 2x + 24 \\
 & -x^3 + 3x^2 \\
\hline
 & -2x^2 - 2x \\
 & 2x^2 - 6x \\
\hline
 & -8x + 24 \\
 & 8x - 24 \\
\hline
 & 0
\end{array}
$$

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x - 3) & x^3 - 5x^2 - 2x + 24 \\
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 & -8x + 24 \\
 & 8x - 24 \\
 & 0
\end{array}
$$

$b^2 - 4ac = 36 = 6^2$ so linear factors exist

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Class Exersise

Factorise $f(x) = x^3 - 5x^2 - 2x + 24$ as far as possible, $(f(3) = 0)$

$$
\begin{array}{r|rrrr}
 & x^2 & -\,2x & -\,8 & \\
\hline
x-3 & x^3 & -\,5x^2 & -\,2x & +\,24 \\
 & -\,x^3 & +\,3x^2 & & \\
\hline
 & & -\,2x^2 & -\,2x & \\
 & & 2x^2 & -\,6x & \\
\hline
 & & & -\,8x & +\,24 \\
 & & & 8x & -\,24 \\
\hline
 & & & & 0
\end{array}
$$

$x = -2$ gives $(x^2 - 2x - 8) = 0$

$b^2 - 4ac = 36 = 6^2$ so linear factors exist

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Factorise $f(x) = x^3 - 5x^2 - 2x + 24$ as far as possible, $(f(3) = 0)$

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\begin{array}{r|l}
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 & 2x^2 - 6x \\
\hline
 & -8x + 24 \\
 & 8x - 24 \\
\hline
 & 0
\end{array}
$$

$b^2 - 4ac = 36 = 6^2$ so linear factors exist

$x = -2$ gives $(x^2 - 2x - 8) = 0$

$$
\begin{array}{r|l}
 & x - 4 \\
\hline
x + 2) & x^2 - 2x - 8 \\
 & -x^2 - 2x \\
\hline
 & -4x - 8 \\
 & 4x + 8 \\
\hline
 & 0
\end{array}
$$

Class Exersise

Factorise $f(x) = x^3 - 5x^2 - 2x + 24$ as far as possible, $(f(3) = 0)$

$$
\begin{array}{r|l}
 & x^2 - 2x - 8 \\
\hline
x - 3) & x^3 - 5x^2 - 2x + 24 \\
 & -x^3 + 3x^2 \\
\hline
 & -2x^2 - 2x \\
 & 2x^2 - 6x \\
\hline
 & -8x + 24 \\
 & 8x - 24 \\
\hline
 & 0
\end{array}
$$

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x + 2) & x^2 - 2x - 8 \\
 & -x^2 - 2x \\
\hline
 & -4x - 8 \\
 & 4x + 8 \\
\hline
 & 0
\end{array}
$$

Therefore

$$f(x) = x^3 - 5x^2 - 2x + 24 = (x-3)(x^2 - 2x - 8) = (x-3)(x+2)(x-4)$$

Class Exersise

Factorise $f(x) = x^3 - 6x^2 - 7x + 60$ as far as possible, $(f(4) = 0)$

Class Exersise

Factorise $f(x) = x^3 - 6x^2 - 7x + 60$ as far as possible, $(f(4) = 0)$

$$
\begin{array}{r|rrrr}
 & & x^2 & -2x & -15 \\
\hline
x-4) & x^3 & -6x^2 & -7x & +60 \\
 & -x^3 & +4x^2 & & \\
\hline
 & & -2x^2 & -7x & \\
 & & 2x^2 & -8x & \\
\hline
 & & & -15x & +60 \\
 & & & 15x & -60 \\
\hline
 & & & & 0
\end{array}
$$

Class Exersise

Factorise $f(x) = x^3 - 6x^2 - 7x + 60$ as far as possible, $(f(4) = 0)$

$$
\begin{array}{r|l}
 & x^2 \quad - 2x - 15 \\
\hline
x - 4 & x^3 - 6x^2 \quad - 7x + 60 \\
 & -x^3 + 4x^2 \\
\hline
 & -2x^2 \quad - 7x \\
 & 2x^2 \quad - 8x \\
\hline
 & -15x + 60 \\
 & 15x - 60 \\
\hline
 & 0
\end{array}
$$

$b^2 - 4ac = 8^2$ so linear factors exist

Class Exersise

Factorise $f(x) = x^3 - 6x^2 - 7x + 60$ as far as possible, $(f(4) = 0)$

$$
\begin{array}{r|l}
 & x^2 - 2x - 15 \\
\hline
x - 4 & x^3 - 6x^2 - 7x + 60 \\
 & -x^3 + 4x^2 \\
\hline
 & -2x^2 - 7x \\
 & 2x^2 - 8x \\
\hline
 & -15x + 60 \\
 & 15x - 60 \\
\hline
 & 0
\end{array}
$$

$x = -3$ gives $(x^2 - 2x - 15) = 0$

$b^2 - 4ac = 8^2$ so linear factors exist

Class Exersise

Factorise $f(x) = x^3 - 6x^2 - 7x + 60$ as far as possible, $(f(4) = 0)$

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 & & 15x & -60 \\
\hline
 & & & 0
\end{array}
$$

$b^2 - 4ac = 8^2$ so linear factors exist

$x = -3$ gives $(x^2 - 2x - 15) = 0$

$$
\begin{array}{rrrr}
 & x & -5 & \\
\hline
x+3) & x^2 & -2x & -15 \\
 & -x^2 & -3x & \\
\hline
 & & -5x & -15 \\
 & & 5x & +15 \\
\hline
 & & & 0
\end{array}
$$

Class Exersise

Factorise $f(x) = x^3 - 6x^2 - 7x + 60$ as far as possible, $(f(4) = 0)$

$$
\begin{array}{r|rrr}
 & x^2 & -2x & -15 \\
\hline
x-4) & x^3 - 6x^2 & -7x & +60 \\
 & -x^3 + 4x^2 & & \\
\hline
 & -2x^2 & -7x & \\
 & 2x^2 & -8x & \\
\hline
 & & -15x & +60 \\
 & & 15x & -60 \\
\hline
 & & & 0
\end{array}
$$

$b^2 - 4ac = 8^2$ so linear factors exist

$x = -3$ gives $(x^2 - 2x - 15) = 0$

$$
\begin{array}{r|rr}
 & x & -5 \\
\hline
x+3) & x^2 - 2x & -15 \\
 & -x^2 - 3x & \\
\hline
 & -5x & -15 \\
 & 5x & +15 \\
\hline
 & & 0
\end{array}
$$

Therefore

$$f(x) = x^3 - 6x^2 - 7x + 60 = (x-4)(x^2 - 2x - 15) = (x-4)(x+3)(x-5)$$