Introduction to Set Operations

CIS008-2 Logic and Foundations of Mathematics

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Outline

1. Recap
2. Introduction to sets
3. Class Exercises

Recap

- Reviewed the definition of:
  - Natural numbers $\mathbb{N}$
  - Integers $\mathbb{Z}$
  - Rational numbers $\mathbb{Q}$
  - Real numbers $\mathbb{R}$.
- Introduced Base systems.
- Showed how to change between different Bases without loss of generallity.
- Guided examples of pseudocoded change of Base.

Notation

Notation	Example	Reads
$\{,\ldots,\}$	$\{x_1, x_2, \ldots x_n\}$	Set with elements $x_1, x_2 \ldots x_n$
$\in$	$x \in A$	x belongs to A
$\notin$	$x \notin A$	x does not belong to A
$\ni$	$A \ni x$	The set A contains the x as an element
$\not\ni$	$A \not\ni x$	The set A does not contain x as an element

Also, we define a set builder with an example notation as $\{x \in A \mid p(x)\}$, which reads the set of those elements x of A for which the proposition $p(x)$ is true.

Subsets and Proper Subsets

If X and Y are sets. If every element of X is and element of Y, we say X is a **subset** of Y and write $X \subseteq Y$.

Any set of X is a subset of itself, also the empty set $\varnothing$ is a subset of every set. If X is a subset of Y and X does not equal Y, we say that X is a **proper subset** of Y and write $X \subset Y$.

The set of all subsets (proper or not) of a set X, denoted $\mathcal{P}(X)$, is called the **power set** of X.

Universe

Sometimes when dealing with sets, all of which are subsets of a set U, the set U is called a **universal set** (or simply a **universe**).

Union

The **union** of X and Y consists of all elements belonging to either X or Y (or both).

$$X \cup Y = \{x \mid x \in X \text{ or } x \in Y\}$$

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Intersection

The **intersection** of X and Y consists of all elements belonging to both X and Y.

$$X \cap Y = \{x \mid x \in X \text{ and } x \in Y\}$$

X and Y are **disjoint** if

$$X \cap Y = \varnothing \qquad (1)$$

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Complement & Difference

The **complement** of X, being a subset of a universal set U, consists of all elements **not** belonging to X.

$$U - X = \{x \mid x \notin X\}$$

$U - X$ is called the compliment of X and is written $\overline{X}$.

The **difference** (or **relative complement**) $X - Y$ consists of all elements belonging to X and not Y.

$$X - Y = \{x \mid x \in X \text{ and } x \notin Y\}$$

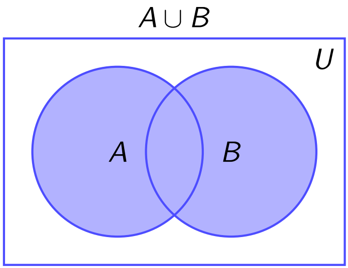
$X - Y$ is called the relative compliment of Y.

Venn diagrams

Venn diagrams provide a pictorial views of sets

- A rectangle depicts a universal set.
- Subsets of a universal set are drawn as circles within the rectangle
- The inside of a circle represents the members of that set.

Union



Example

If $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$ then $A \cup B = \{1, 2, 3, 4, 6\}$

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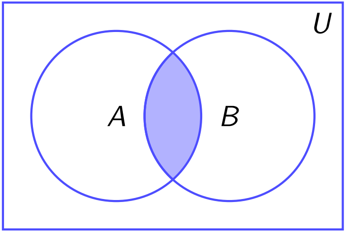
Intersection

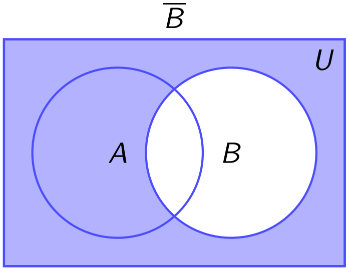
$A \cap B$



Example

If $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$ then $A \cap B = \{2\}$

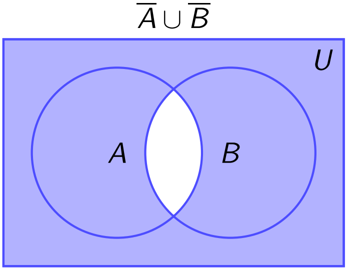
Complement



Example

If $U = \mathbb{Z}$ and $B = \mathbb{N}$ then $\overline{B} = \{0, -1, -2, -3, \dots\}$.

Union of Complements



Example

If $U = \mathbb{R}$, $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$ then
$\overline{A} \cup \overline{B} = \{x \mid x \in \mathbb{R} \text{ and } x \neq 2\}$

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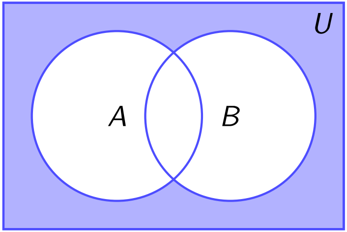
Intersection of Complements

$\overline{A} \cap \overline{B}$

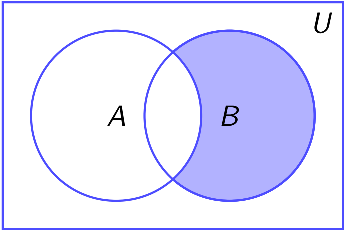


Example

There are 4 students in a class U, 2 pass the first coursework assignment A and 2 pass the second coursework assignment B. If only 1 student passes both coursework assignments, the amount of students that did not pass either coursework is $\overline{A} \cap \overline{B} = 1$.

Intersection of Complement and Subset

$$(\overline{A}) \cap B$$



Example

If $A = \{a, b, c\}$ and $B = \{c, d, e\}$ then $(\overline{A}) \cap B = \{d, e\}$

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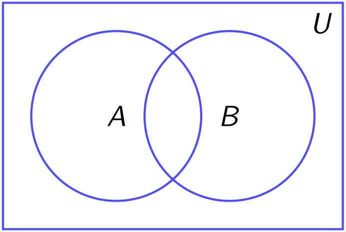
Intersection of Subset and its Complement

$B \cap \overline{B}$



Example

If $B =$ *"my only red apple"* and $U =$ *"all of my apples"* then $B \cap \overline{B} =$ *"none of my apples"*

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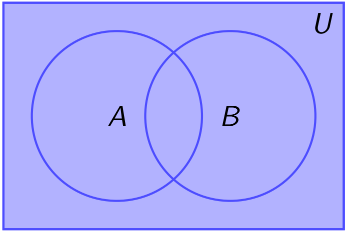
Union of Subset and its Complement

$B \cup \overline{B}$



Example

If A and B is each 1 bit, $A = 10$, $B = 01$, $A \cap B = 11$, and $\overline{A} \cap \overline{B} = 00$, then $B \cup \overline{B} = \{00, 10, 01, 11\}$

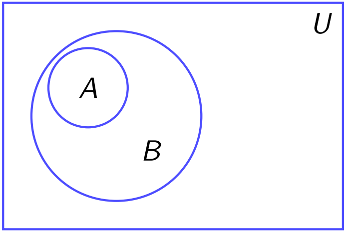
Questions

1. Draw a Venn diagram of two disjoint sets, A and B in a universe U.
2. Draw a Venn diagram of A being a proper subset of B, both within a universal set, U.
3. Draw a Venn diagram of $(\overline{A}) \cup B$.
4. Draw a Venn diagram of the complement of union of subsets, where A and B are subsets of the universal set U.
5. Write question (4) in a symbolic way.
6. Construct Venn diagrams for the $A \cap B = \varnothing$.
7. Construct Venn diagrams for the $A \cup B = A$.
8. Construct Venn diagrams for the $A - B = A$.

Questions (II)

Consider the Venn diagram opposite, then match each of the following

9. $A \cup B$
10. $A \cap B$
11. $A \cap \overline{B}$
12. $A - B$
13. $\overline{A} \cup B$
14. $\overline{A} \cap \overline{B}$
15. $\overline{A} \cup \overline{B}$
16. $\overline{U}$



with one of

- $\varnothing$
- A
- B
- U
- $\overline{B}$
- $\overline{A}$

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Questions (III)

Of 160 individuals with a skin disorder, 100 had been exposed to chemical **A** (individuals A), 50 to chemical **B** (individuals B), and 30 to chemicals **A** and **B**.

Use symbols and Venn diagrams to describe the number of individuals exposed to:

17. chemicals **A** and **B**
18. chemical **A** but not chemical **B**
19. chemical **B** but not chemical **A**
20. chemical **A** or chemical **B**
21. neither chemical **A** nor chemical **B**