Matrix Operation Algorithms

CIS008-2 Logic and Foundations of Mathematics

David Goodwin

david.goodwin@perisic.com

University of Bedfordshire

11:00, Tuesday 6th February 2012

Outline

1. Solution to Linear Equations - examples with numbers
   - Method of inverses
   - Gaussian Elimination
   - Gauss-Jordan Elimination
2. Forming an algorithm for Addition
3. Forming an algorithm for Multiplication
4. Forming an algorithm to find Cofactor

Outline

1. Solution to Linear Equations - examples with numbers
   - Method of inverses
   - Gaussian Elimination
   - Gauss-Jordan Elimination
2. Forming an algorithm for Addition
3. Forming an algorithm for Multiplication
4. Forming an algorithm to find Cofactor

The Problem

We could have the following Matrix equation to be solved:

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 5 & 6 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 5 \end{bmatrix}$$

which could be represented as

$$\mathbf{Ax} = \mathbf{b}$$

Considering the interpretation, opposite, this matrix equation could also be written as

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 4 & 3 \\ 3 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 7 \end{bmatrix}$$

Notice that we have changed the position of the rows in the augmented matrix, and left the **x** column matrix alone.

The interpretation of the opposite matrix equation can be a set of linear equations:

$$\begin{aligned} 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \\ x + 3y - 2z &= 5 \end{aligned}$$

It is worth noting that each equation has no perticular hierarchical ranking, The set of equations could equally be arranged in any order. i.e.

$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

Finding the cofactors

One solution lay in using the inverse the the matrix, **A**, to find **x**

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$
$$\mathbf{A}^{-1}\mathbf{A}\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$
$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

So this general method will need to find an inverse of a matrix and then perform a matrix multiplication. The matrix of cofactors, described as a signed version of a matrix of minors, is

$$\mathbf{C} = \begin{bmatrix} 9 & -3 & -2 \\ -28 & 12 & 4 \\ 17 & -7 & -2 \end{bmatrix}$$

The inverse of a matrix envolves finding a matrix of cofactors, **C**.

$$c_{11} = (-1)^{1+1}((4 \cdot 6) - (5 \cdot 3))$$
$$c_{12} = (-1)^{1+2}((2 \cdot 6) - (3 \cdot 3))$$
$$c_{13} = (-1)^{1+3}((2 \cdot 5) - (3 \cdot 4))$$
$$c_{21} = (-1)^{2+1}((3 \cdot 6) - (5 \cdot -2))$$
$$c_{22} = (-1)^{2+2}((1 \cdot 6) - (3 \cdot -2))$$
$$c_{23} = (-1)^{2+3}((1 \cdot 5) - (3 \cdot 3))$$
$$c_{31} = (-1)^{3+1}((3 \cdot 3) - (4 \cdot -2))$$
$$c_{32} = (-1)^{3+2}((1 \cdot 3) - (2 \cdot -2))$$
$$c_{33} = (-1)^{3+3}((1 \cdot 4) - (2 \cdot 3))$$

University of Bedfordshire

Finding the adjoint and the determinant

The matrix of cofactors is

$$\mathbf{C} = \begin{bmatrix} 9 & -3 & -2 \\ -28 & 12 & 4 \\ 17 & -7 & -2 \end{bmatrix}$$

but we require the adjoint of $\mathbf{A} = adj(A) = C^T$, where the transpose is simply the reflection of the matrix about it's diagonal:

$$adj(\mathbf{A}) = \mathbf{C}^T = \begin{bmatrix} 9 & -28 & 17 \\ -3 & 12 & -7 \\ -2 & 4 & -2 \end{bmatrix}$$

The other ingredient in finding the inverse is finding the determinant of matrix $\mathbf{A}$. Here we can use the signed mionors of the top row of matrix $\mathbf{A}$ which we have already worked out, and we multiply them by the elements in the top row, then to be summed:

$$c_{11} = (-1)^{1+1}((4 \cdot 6) - (5 \cdot 3))$$
$$c_{12} = (-1)^{1+2}((2 \cdot 6) - (3 \cdot 3))$$
$$c_{13} = (-1)^{1+3}((2 \cdot 5) - (3 \cdot 4))$$

$$det(\mathbf{A}) = (9 \cdot 1) + (-3 \cdot 3) + (-2 \cdot -2) = 4$$

University of Bedfordshire

Finding the inverse then using it

No we have the adjoint and the determinant, we simply divide the adjoint by the determinant to find the inverse matrix:

$$\mathbf{A}^{-1} = \frac{adj(\mathbf{A})}{det(\mathbf{A})}$$

$$= \frac{1}{4}\begin{bmatrix} 9 & -28 & 17 \\ -3 & 12 & -7 \\ -2 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2.25 & -7 & 4.25 \\ -0.75 & 3 & -1.75 \\ -0.5 & 1 & -0.5 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

$$= \frac{1}{4}\begin{bmatrix} 9 & -28 & 17 \\ -3 & 12 & -7 \\ -2 & 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -15 \\ 8 \\ 2 \end{bmatrix}$$

Therefore $x = -15$, $y = 8$, and $z = 2$

University of Bedfordshire

The Problem

We could have the following Matrix equation to be solved:

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 5 & 6 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 5 \end{bmatrix}$$

which could be represented as

$$\mathbf{Ax} = \mathbf{b}$$

Considering the interpretation, opposite, this matrix equation can be augmented and written as

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 2 & 4 & 3 & 8 \\ 3 & 5 & 6 & 7 \end{array}\right]$$

Notice that we have changed the position of the rows in the augmented matrix, and left the **x** column matrix alone.

The interpretation of the opposite matrix equation can be a set of linear equations:

$$\begin{aligned} 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \\ x + 3y - 2z &= 5 \end{aligned}$$

It is worth noting that each equation has no perticular hierarchical ranking, The set of equations could equally be arranged in any order. i.e.

$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

Proceed to reduced row Echelon form

We first want to eliminate the first element of the second row. This is done by subtracting some multiple of the first row from the whole of second row to give 0 as the first element. Logically it can be seen that the multiple will be $\frac{\text{1st element 2nd row}}{\text{1st element 1st row}}$. In this case it is $\frac{2}{1}$, giving a row $\begin{bmatrix} 2 & 6 & -4 & | & 10 \end{bmatrix}$. This row is subtracted from the second row, element wise, to give a new second row, $\begin{bmatrix} 0 & -2 & 7 & | & -2 \end{bmatrix}$. We can then form the new augement matrix:

$$\begin{bmatrix} 1 & 3 & -2 & | & 5 \\ 0 & -2 & 7 & | & -2 \\ 3 & 5 & 6 & | & 7 \end{bmatrix}$$

$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

We can look at the elimination of x in the second equation by multiplying the first equation by 2

$$2x + 6y - 4z = 10$$

Then we would subtract this new equation from the second equation to give:

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

Proceed to reduced row Echelon form

Now want to eliminate the first element of the third row. This is done by subtracting some multiple of the first row from the whole of third row to give 0 as the first element. Logically it can be seen that the multiple will be $\frac{\text{1st element 3rd row}}{\text{1st element 1st row}}$. In this case it is $\frac{3}{1}$, giving a row $\begin{bmatrix} 3 & 9 & -6 & | & 15 \end{bmatrix}$. This row is subtracted from the third row, element wise, to give a new third row, $\begin{bmatrix} 0 & -4 & 12 & | & -8 \end{bmatrix}$. We can then form the new augement matrix:

$$\begin{bmatrix} 1 & 3 & -2 & | & 5 \\ 0 & -2 & 7 & | & -2 \\ 0 & -4 & 12 & | & -8 \end{bmatrix}$$

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

We can look at the elimination of x in the third equation by multiplying the first equation by 3

$$3x + 9y - 6z = 15$$

Then we would subtract this new equation from the second equation to give:

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ -4y + 12z &= -8 \end{aligned}$$

University of Bedfordshire

Proceed to reduced row Echelon form

Finally we want to eliminate the second element of the third row. This is done by subtracting some multiple of the second row from the whole of third row to give 0 as the second element. Logically it can be seen that the multiple will be $\frac{\text{2nd element 3rd row}}{\text{2nd element 2nd row}}$. In this case it is $\frac{-4}{-2}$, giving a row $\begin{bmatrix} 0 & -4 & 14 & | & -4 \end{bmatrix}$. This row is subtracted from the third row, element wise, to give a new third row, $\begin{bmatrix} 0 & 0 & -2 & | & -4 \end{bmatrix}$. We can then form the new augement matrix:

$$\begin{bmatrix} 1 & 3 & -2 & | & 5 \\ 0 & -2 & 7 & | & -2 \\ 0 & 0 & -2 & | & -4 \end{bmatrix}$$

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ -4y + 12z &= -8 \end{aligned}$$

We can look at the elimination of y in the third equation by multiplying the second equation by 2

$$-4y + 14z = -4$$

Then we would subtract this new equation from the second equation to give:

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ -2z &= -4 \end{aligned}$$

Proceed to reduced row Echelon form

$$\begin{bmatrix} 1 & 3 & -2 & | & 5 \\ 0 & -2 & 7 & | & -2 \\ 0 & 0 & -2 & | & -4 \end{bmatrix}$$

We can now find our answers by reading off the last row to find z, then back substitution to find y from the second row, then back substitution again to find x from the first row.

$$\begin{aligned} -2z &= -4 \\ z &= 2 \\ -2y + 7z &= -2 \\ -2y &= -2 - (7 \times 2) \\ y &= 8 \\ x + 3y - 2z &= 5 \\ x &= 5 - (3 \times 8) - (-2 \times 2) \\ x &= -15 \end{aligned}$$

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ -2z &= -4 \end{aligned}$$

We can now find our answers by reading off the last equation to find z, then back substitution to find y from the second equation, then back substitution again to find x from the first equation.

$$\begin{aligned} -2z &= -4 \\ z &= 2 \\ -2y + 7z &= -2 \\ -2y &= -2 - (7 \times 2) \\ y &= 8 \\ x + 3y - 2z &= 5 \\ x &= 5 - (3 \times 8) - (-2 \times 2) \\ x &= -15 \end{aligned}$$

The Problem

We could have the following Matrix equation to be solved:

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 5 & 6 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 5 \end{bmatrix}$$

which could be represented as

$$\mathbf{Ax} = \mathbf{b}$$

Considering the interpretation, opposite, this matrix equation can be augmented and written as

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 2 & 4 & 3 & 8 \\ 3 & 5 & 6 & 7 \end{array}\right]$$

Notice that we have changed the position of the rows in the augmented matrix, and left the **x** column matrix alone.

The interpretation of the opposite matrix equation can be a set of linear equations:

$$\begin{aligned} 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \\ x + 3y - 2z &= 5 \end{aligned}$$

It is worth noting that each equation has no perticular hierarchical ranking, The set of equations could equally be arranged in any order. i.e.

$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

We first want to eliminate the first element of the second row. This is done by subtracting some multiple of the first row from the whole of second row to give 0 as the first element. Logically it can be seen that the multiple will be $\frac{\text{1st element 2nd row}}{\text{1st element 1st row}}$. In this case it is $\frac{2}{1}$, giving a row $\begin{bmatrix} 2 & 6 & -4 & | & 10 \end{bmatrix}$. This row is subtracted from the second row, element wise, to give a new second row, $\begin{bmatrix} 0 & -2 & 7 & | & -2 \end{bmatrix}$. We can then form the new augement matrix:

$$\begin{bmatrix} 1 & 3 & -2 & | & 5 \\ 0 & -2 & 7 & | & -2 \\ 3 & 5 & 6 & | & 7 \end{bmatrix}$$

$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x + 4y + 3z &= 8 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

We can look at the elimination of x in the second equation by multiplying the first equation by 2

$$2x + 6y - 4z = 10$$

Then we would subtract this new equation from the second equation to give:

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

University of Bedfordshire

Now want to eliminate the first element of the third row. This is done by subtracting some multiple of the first row from the whole of third row to give 0 as the first element. Logically it can be seen that the multiple will be $\frac{\text{1st element 3rd row}}{\text{1st element 1st row}}$. In this case it is $\frac{3}{1}$, giving a row $\begin{bmatrix} 3 & 9 & -6 & | & 15 \end{bmatrix}$. This row is subtracted from the third row, element wise, to give a new third row, $\begin{bmatrix} 0 & -4 & 12 & | & -8 \end{bmatrix}$. We can then form the new augement matrix:

$$\begin{bmatrix} 1 & 3 & -2 & | & 5 \\ 0 & -2 & 7 & | & -2 \\ 0 & -4 & 12 & | & -8 \end{bmatrix}$$

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

We can look at the elimination of x in the third equation by multiplying the first equation by 3

$$3x + 9y - 6z = 15$$

Then we would subtract this new equation from the second equation to give:

$$\begin{aligned} x + 3y - 2z &= 5 \\ -2y + 7z &= -2 \\ -4y + 12z &= -8 \end{aligned}$$

University of Bedfordshire

We now want to find $1y$ in the second row, so in this case we divide the whole of the second row by -2 to give

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -7/2 & 1 \\ 0 & -4 & 12 & -8 \end{array}\right]$$

We now want to find $0y$ in the third row, so in this case we subtract the -4 times the whole of the second row from the whole of the third row give

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -7/2 & 1 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

then divide the whole of the third row by -2 to give

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -7/2 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

We now want to find $0z$ in the second row, so in this case we subtract $-7/2$ times the third row from the second row

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

We now want to find $0z$ in the first row, so in this case we subtract the -2 times the whole of the third row from the whole of the first row give

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 9 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

We now want to find $0y$ in the first row, so in this case we subtract the 3 times the whole of the second row from the whole of the first row give

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

We can simply read off the answers since the main matrix is an identity matrix, giving $x = -15$, $y = 8$, and $z = 2$.

Outline

1. Solution to Linear Equations - examples with numbers
   - Method of inverses
   - Gaussian Elimination
   - Gauss-Jordan Elimination
2. Forming an algorithm for Addition
3. Forming an algorithm for Multiplication
4. Forming an algorithm to find Cofactor

Forming an algorithm for Addition

- If we begin an algorithm construction, we identify the inputs and then outputs, and identify how we create each element of the answer.
- For matrix addition I know there are going to be two inputs, i.e. two matrices to add.
- Similarly I know there will be one output, since two matrices added together will give one matrix.
- The answer will be a matrix of the same size as either of the inputs, so I know I can make each element of the output by adding corresponding.
- If we remember the axioms of matrix addition, two matrices must be the same size to be able to be added. It would be useful to contain an if else statement to account for this.

1. Input(A,B), Output(C)
2. To form the answer we must index every element of the answer, i.e. two for loops, looping the elements in rows and then columns
3. A message appears when matrices aren't the same size, and additon is not possible.

Forming an algorithm for Addition

```
function C=Matrix_add(A,B)
% and this will form the first line of the function
```

```
for row=1:row_max
  for col=1:col_max
    %code to find resulting matrix C, element by element
  end
end
```

```
if size(A)~=size(B)
  message='matrices are different sizes'
else
  % do the code to find the answer
end
```

Forming an algorithm for Addition

The next stage would be to find all the variables and constants we have used in the basic construction blocks of the algorithms so far, then explicitly define them, calculate them, or pre-allocate their structure. Make sure not to overwrite the inputs, and make sure to define the ouput somewhere, here I will pre-allocate it's structure with a view to performing a calculation within a for loop. Also, there is no use defining all the variables used, if the algorithm is not able to perform it's function, so I would make sure to define them after the else part of the if statement, when we know we can perform the addition.

```
row_max=size(A,1); %A or B could have been used since
col_max=aize(A,2); %we know they are the same size
C=zeros(row_max,col_max); %pre-define the answer
```

Finally, we put the blocks of code together in the correct places and correct order to form our algorithm. Also, we must explicitly calculate the answer within the nested for loops. Since the answer is complete when we have looped over all rows and all columns, there is no need to perform further operations on it. Make sure to suppress outputs with ; when we dont want the operation printed to screen.

Algorithm for Addition

```
function C=Matrix_add(A,B)

if size(A)~=size(B)
  message='matrices are different sizes'
else
   row_max=size(A,1);
   col_max=aize(A,2);
   C=zeros(row_max,col_max);
   for row=1:row_max
     for col=1:col_max
       C(row,col)=A(row,col)+B(row,col);
     end
   end
end
```

University of Bedfordshire

Outline

1. Solution to Linear Equations - examples with numbers
   - Method of inverses
   - Gaussian Elimination
   - Gauss-Jordan Elimination
2. Forming an algorithm for Addition
3. Forming an algorithm for Multiplication
4. Forming an algorithm to find Cofactor

University of Bedfordshire

Forming an algorithm for Multiplication

- Algorithm for multiplication would have similar building blocks as addition, so we start with that code.
- If the inputs are $A(i,k)$ and $B(j,i)$, since the rows in A must be the same as the columns in B, we know the answer will be of size $C(j,k)$.
- If we remember the axioms of matrix multiplcation, The order of multiplication is important. We will leave this to the user of the code to notice.

1. Input(A,B), Output(C)
2. To form the answer we must index every element of the inputs to form the answer i.e. three for loops
3. A message appears when matrices aren't cannot be multiplied.

Forming an algorithm for Multiplication

```
function C=Matrix_mult(A,B)
% and this will form the first line of the function
```

```
for i=1:i_max %rows of A and columns of B
  for j=1:j_max %rows of C and B
    for k=1:k_max %columns of C and A
      %code to find resulting matrix C, element by element
    end
  end
end
```

```
if size(A,1)~=size(B,2)
  message='matrices cannot be multiplied'
else
  % do the code to find the answer
end
```

University of Bedfordshire

FORMING AN ALGORITHM FOR MULTIPLICATION

The next stage would be to find all the variables and constants we have used in the basic construction blocks of the algorithms so far, then explicitly define them, calculate them, or pre-allocate their structure.

```
i_max=size(A,1); %A or B could have been used
j_max=aize(B,1); %since rows in A = cols in B
k_max=aize(A,2);
C=zeros(j_max,k_max); %pre—define the answer
```

Finally, we put the blocks of code together in the correct places and correct order to form our algorithm. Also, we must explicitly calculate the answer within the nested for loops. I must self-reference C to find the answer. To do this I must have pre-defined C, and I do it because there is more than one operation for each element of C, specifically i operations per element of C.

University of Bedfordshire

Algorithm for Addition

```
function C=Matrix_mult(A,B)

if size(A,1)~=size(B,2)
  message='matrices cannot be multiplied'
else
  i_max=size(A,1); %or size(B,2) would work
  j_max=aize(B,1);
  k_max=aize(A,2);
  C=zeros(j_max,k_max);
  for i=1:i_max %rows of A and columns of B
    for j=1:j_max %rows of C and B
      for k=1:k_max %columns of C and A
        C(j,k)=C(j,k)+(A(i,k)*B(j,i));
      end
    end
  end
end
```

University of Bedfordshire

Outline

1. Solution to Linear Equations - examples with numbers
   - Method of inverses
   - Gaussian Elimination
   - Gauss-Jordan Elimination
2. Forming an algorithm for Addition
3. Forming an algorithm for Multiplication
4. Forming an algorithm to find Cofactor

University of Bedfordshire

Forming an algorithm to find Cofactor

- Finding a cofactor of an element of a matrix is a different structure to additon or multiplication of matrices
- We will have one input, square matrix A, and one output, a number being the cofactor c.
- If we remember the axioms of matrix cofactors, the matrix must be square, so we should test this at the start of the algorithm. If we didn't, it would produce an error from the compiler, out of matrix bounds, or something similar, it may even crash the software in certain circumstances (i.e. if we made an executable the program would break at the error. If it were in a queue system like distributed computing, it may cause memory leakage and keep trying to execute.)

1. Input(A), Output(c)
2. To form the answer we must find the minor and loop until the minor is a 2×2
3. A message appears when matrix isn't square.

Forming an algorithm for to find Cofactor

```
function c=Matrix_mult(A)
% and this will form the first line of the function
```

```
if size(A,1)~=size(A,2)
  message='matrix isnt square so cofactors arent defined'
else
  % do the code to find the answer
end
```

University of Bedfordshire

Forming an algorithm for to find Cofactor

Finding the minor would be simple, finding the minor reduced to 2×2 requires more thinking. The minor of a certain element $A(i,j)$ would be found by finding four separate matrices and concatenating them. The easiest way is to find them separately.

```
if i~=1
  if j~=1
    M_11=A(1:i-1,1:j-1);
  else
    M_11=A(1:i-1,1);
  end
elseif j~=1
  M_11=A(1,1:j-1);
else
  M_11=[];
end
```

which is the upper left matrix elements, where we have defined the the four possible outcomes and defined as the null matrix if we are finding the minor of the first element in A.

Forming an algorithm for to find Cofactor

The lower right element is the next eaisest to find, and is found in a similar way:

```
if i~=i_max
  if j~=j_max
    M_22=A(i+1:i_max,j+1:j_max);
  else
    M_22=A(i+1:i_max,1);
  end
elseif j~=j_max
  M_22=A(1,j+1:j_max);
else
  M_22=[];
end
```

which is the upper left matrix elements, where we have defined the the four possible outcomes and defined as the null matrix if we are finding the minor of the last element in A.

University of Bedfordshire

Forming an algorithm for to find Cofactor

The upper right element is found in a similar way:

```
if i~=1
  if j~=j_max
    M_12=A(1:i-1,j+1:j_max);
  else
    M_12=A(1:i-1,1);
  end
elseif j~=j_max
  M_12=A(1,j+1:j_max);
else
  M_12=[];
end
```

Forming an algorithm for to find Cofactor

The lower left element is found in a similar way:

```
if i~=i_max
  if j~=1
    M_21=A(i+1:i_max,1:j-1);
  else
    M_21=A(i+1:i_max,1);
  end
elseif j~=1
  M_21=A(1,1:j-1);
else
  M_21=[];
end
```

Forming an algorithm for to find Cofactor

Now we have found the four matrices that make up the minor matrix, we concatinate them to form a single matrix, M:

```
M=[M_11 M_12 ; M_21 M_22];
```

It would be useful to contain all this code in a stand-alone function, for if we want to use it multiple times in another larger code:

```
function M=Minor_mat(A,i,j)
i_max=size(A,1);
j_max=size(A,2);
if i~=1
  if j~=1
    M_11=A(1:i-1,1:j-1);
  else
    M_11=A(1:i-1,1);
  end
elseif j~=1
  M_11=A(1,1:j-1);
else
  M_11=[];
end
if i~=i_max
  if j~=j_max
    M_22=A(i+1:i_max,j+1:j_max);
  else
    M_22=A(i+1:i_max,1);
  end
elseif j~=j_max
  M_22=A(1,j+1:j_max);
else
  M_22=[];
end
```

```
if i~=1
  if j~=j_max
    M_12=A(1:i-1,j+1:j_max);
  else
    M_12=A(1:i-1,1);
  end
elseif j~=j_max
  M_12=A(1,j+1:j_max);
else
  M_12=[];
end
if i~=i_max
  if j~=1
    M_21=A(i+1:i_max,1:j-1);
  else
    M_21=A(i+1:i_max,1);
  end
elseif j~=1
  M_21=A(1,1:j-1);
else
  M_21=[];
end
M=[M_11 M_12 ; M_21 M_22];
```