CIS009-2, MECHATRONICS ROBOT COORDINATION SYSTEMS II

David Goodwin

Department of Computer Science and Technology University of Bedfordshire

 07^{th} February 2013



Outline

Mechatronics

David Goodwir

Mappin

- Translat
- Rotatio

Operate

- Translatio
- Rotatio

Transforma

Inverting

Inverting

transformation equations Mapping
 Translation
 Rotational

2 Operators

Translation Rotation

3 Transformation

multiplication Inverting

transformation equations



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Mappings involving translated frames

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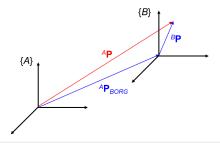
Transformation

multiplication

transform

A translated frame shifts without rotation

$${}^{A}P = {}^{B}P + {}^{A}P_{BORG}$$





Mappings involving translated frames

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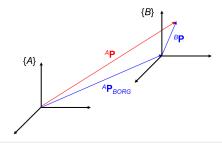
multiplication

transform

A translated frame shifts without rotation

 \bullet Translation takes place when $^{A}X\parallel^{B}X$, $^{A}Y\parallel^{B}Y$ and $^{A}Z\parallel^{B}Z$

$${}^{A}P = {}^{B}P + {}^{A}P_{BORG}$$





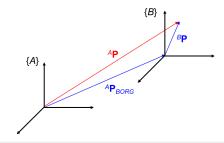
Mappings involving translated frames

Mechatronics

A translated frame shifts without rotation

- Translation takes place when ${}^AX \parallel {}^BX$, ${}^AY \parallel {}^BY$ and ${}^{A}Z \parallel {}^{B}Z$
 - Mapping in this case means representing BP in $\{B\}$ in $\{A\}$ in the form of ${}^{A}P$

$${}^{A}P = {}^{B}P + {}^{A}P_{BORG}$$





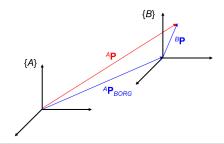
Mappings involving translated frames

Mechatronics

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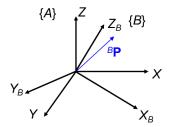


Mappings involving rotated frames

Mechatronics

Rotate a vector about an axis means the projection to that axis remains the same

$${}^AP = {}^B_BR + {}^BP$$





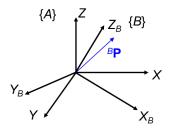
Mappings involving rotated frames

Mechatronics

Rotate a vector about an axis means the projection to that axis remains the same

Mapping BP in $\{B\}$ to AP in $\{A\}$ is

$${}^AP = {}^B_BR + {}^BP$$





Mappings involving rotated frames

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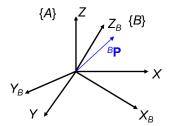
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transformatio equations Rotate a vector about an axis means the projection to that axis remains the same

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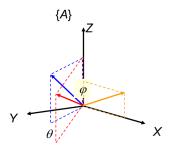


MAPPINGS Example

Mechatronics

• A vector AP is rotated about Z-axis by θ and is subsequently rotated about X-axis by ϕ . Give rotation matrix that accomplishes these rotations in the given order

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





MAPPINGS Example

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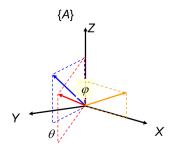
Rotation

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Mappings involving general frames

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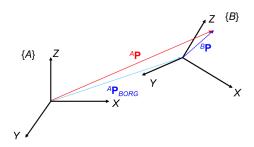
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These mappings involve both translation and rotation

$${}^{A}P = {}^{A}_{B}R^{B}P + {}^{A}P_{BORG}$$

$$\begin{bmatrix} ^AP \\ 1 \end{bmatrix} = \begin{bmatrix} & ^ABR & ^AP_{BORG} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ^BP \\ 1 \end{bmatrix}$$





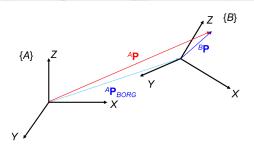
Mappings involving general frames

Mechatronics

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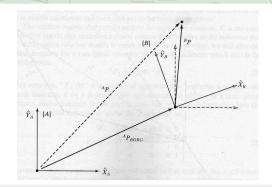
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• Given $\{A\}$, $\{B\}$, BP and ${}^AP_{BORG}$, calculate AP , where $\{B\}$ is rotated relative to $\{A\}$ about Z_A -axis by 30 degrees, translated 10 units in X_A -axis and translated 5 units in Y_A -axis, and ${}^BP = \begin{bmatrix} 3.0 & 7.0 & 0.0 \end{bmatrix}$





MAPPINGS Example

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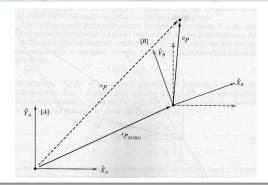
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• Calculate rotation matrix

$${}_{B}^{A}\mathbf{R} = \begin{bmatrix} \cos 30 & -\sin 30 & 0\\ \sin 30 & \cos 30 & 0\\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{R}^{B}\mathbf{P} + {}^{A}\mathbf{P}_{BORG}$$

$$= \begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{B}\mathbf{P} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 & 0 & | & 10 \\ \sin 30 & \cos 30 & 0 & | & 5 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 7.0 \\ 0 \\ 1 \end{bmatrix}$$

MAPPINGS exercises

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Calculate rotation matrix

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MAPPINGS exercises

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Calculate rotation matrix

$${}_{B}^{A}\mathbf{R} = \begin{bmatrix} \cos 30 & -\sin 30 & 0\\ \sin 30 & \cos 30 & 0\\ 0 & 0 & 1 \end{bmatrix}$$

Calculate ^AP

$${}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{R}^{B}\mathbf{P} + {}^{A}\mathbf{P}_{BORG}$$

$$= \begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{B}\mathbf{P} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 & 0 & | & 10 \\ \sin 30 & \cos 30 & 0 & | & 5 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 7.0 \\ 0 \\ 1 \end{bmatrix}$$

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Calculate rotation matrix

$${}_{B}^{A}\mathbf{R} = \begin{bmatrix} \cos 30 & -\sin 30 & 0\\ \sin 30 & \cos 30 & 0\\ 0 & 0 & 1 \end{bmatrix}$$

• Calculate AP

$${}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{R}^{B}\mathbf{P} + {}^{A}\mathbf{P}_{BORG}$$

$$= \begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{B}\mathbf{P} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 & 0 & | & 10 \\ \sin 30 & \cos 30 & 0 & | & 5 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 7.0 \\ 0 \\ 1 \end{bmatrix}$$



Mechatronics

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T matrix

$$\begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} = {}^{A}_{B}\mathbf{T}$$

Trans(a, b, c) =
$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
Translation



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T matrix

$$\begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} = {}^{A}_{B}\mathbf{T}$$

Translation matrix of displacements a, b, and c along

Trans
$$(a, b, c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No rotation

Translation



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$$\begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} = {}^{A}_{B}\mathbf{T}$$

No rotation

Translation

Two special cases:

Trans $(a, b, c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$

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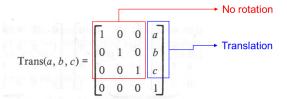


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T matrix

$$\begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} = {}^{A}_{B}\mathbf{T}$$

- Two special cases:
 - Translation matrix of displacements a, b, and c along X, Y and Z axes





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T matrix

$$\begin{bmatrix} {}^{A}_{B}\mathbf{R} & {}^{A}\mathbf{P}_{BORG} \\ \mathbf{0} & 1 \end{bmatrix} = {}^{A}_{B}\mathbf{T}$$

- Two special cases:
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Trans
$$(a, b, c)$$
 =
$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
Translation



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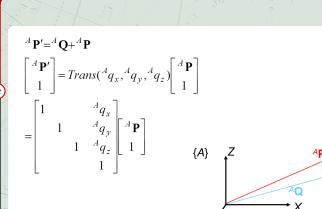
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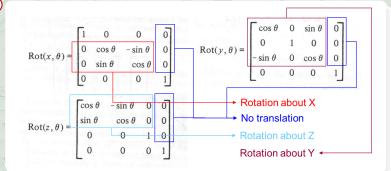
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• Rotation about X-axis

 $\cos \theta$ $\cos \theta$ $-\sin\theta$ 0 $Rot(y, \theta) =$ $Rot(x, \theta) =$ $\sin \theta$ $\cos \theta$ 0 0 Rotation about X $\cos \theta$ $-\sin\theta$ 0 No translation $\sin \theta$ $\cos \theta$ $Rot(z, \theta) =$ Rotation about Z Rotation about Y •



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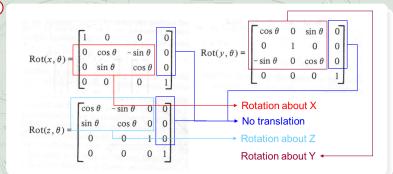
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Rotation

- Rotation about X-axis
- Rotation about Y-axis

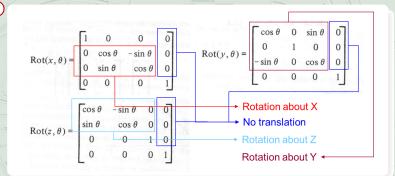




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Rotation

- Rotation about X-axis
- Rotation about Y-axis
- Rotation about Z-axis

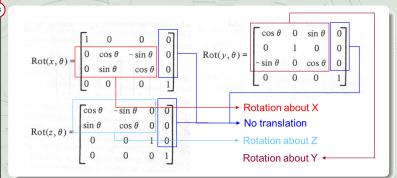




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Rotation

- Rotation about X-axis
- Rotation about Y-axis
- Rotation about Z-axis





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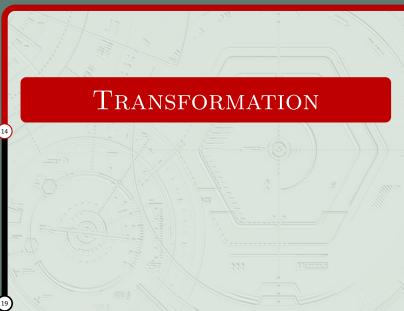
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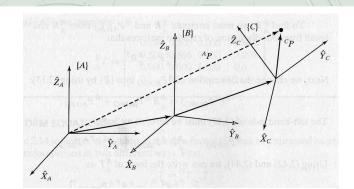
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• Multiplication transformation





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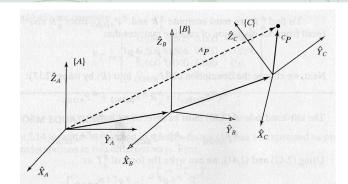
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Multiplication transformation

• Known a point in $\{C\}$ and B_CT matrix from $\{C\}$ to $\{B\}$ and A_BT matrix from $\{B\}$ to $\{A\}$





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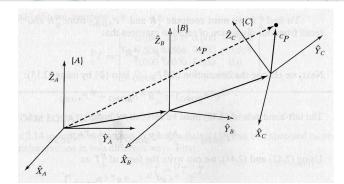
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Multiplication transformation

- Known a point in $\{C\}$ and B_CT matrix from $\{C\}$ to $\{B\}$ and A_BT matrix from $\{B\}$ to $\{A\}$
- ullet Find its position and orientation in $\{A\}$





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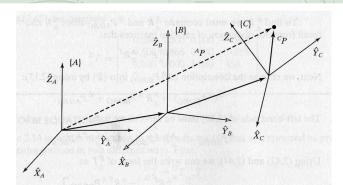
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Multiplication transformation

- Known a point in $\{C\}$ and $_C^BT$ matrix from $\{C\}$ to $\{B\}$ and $_B^AT$ matrix from $\{B\}$ to $\{A\}$
- ullet Find its position and orientation in $\{A\}$





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• A_CT matrix from $\{C\}$ to $\{A\}$ is the multiplication of B_CT matrix from $\{C\}$ to $\{B\}$ and A_BT matrix from $\{B\}$ to $\{A\}$

$${}^{B}\mathbf{P} = {}^{B}_{C}\mathbf{T}^{C}\mathbf{P}$$
 and ${}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{T}^{B}\mathbf{P}$

$${}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{T}^{B}_{C}\mathbf{T}^{C}\mathbf{P}$$

$${}_{C}^{A}\mathbf{T} = {}_{B}^{A}\mathbf{T}{}_{C}^{B}\mathbf{T}$$

$${}_{C}^{A}\mathbf{T} = \begin{bmatrix} & {}_{B}^{A}\mathbf{R}_{C}^{B}\mathbf{R} & {}_{B}^{A}\mathbf{R}^{B}\mathbf{P}_{CORG} + {}^{A}\mathbf{P}_{BORG} \\ 0 & 0 & 1 \end{bmatrix}$$



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multiplication

 ${}_C^AT$ matrix from $\{C\}$ to $\{A\}$ is the multiplication of ${}_C^BT$ matrix from $\{C\}$ to $\{B\}$ and A_BT matrix from $\{B\}$ to $\{A\}$



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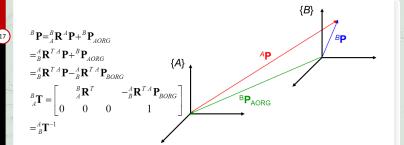
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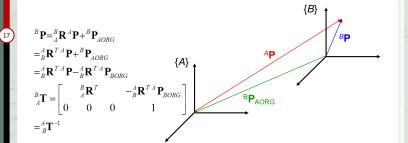
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- Inverting a transformation
 - Known T matrix from $\{B\}$ to $\{A\}$





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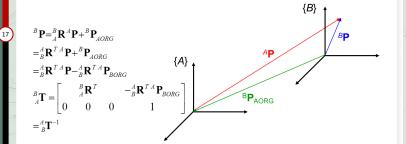
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Inverting a transformation

- \bullet Known T matrix from $\{B\}$ to $\{A\}$
- \bullet Find T matrix from $\{A\}$ to $\{B\}$





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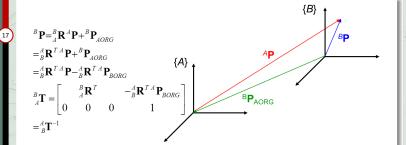
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Inverting a transformation

- \bullet Known T matrix from $\{B\}$ to $\{A\}$
- Find T matrix from $\{A\}$ to $\{B\}$





TRANSFORMATION Example

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Translation:

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Translation

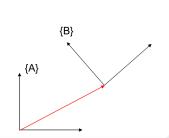
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• Frame $\{B\}$ is rotated relative to Frame $\{A\}$ about Z-axis by 30 degrees and translated 4 units in X-axis and 3 units in Y-axis. Find T matrix from $\{A\}$ to $\{B\}$.

$${}^{A}_{B}\mathbf{T} = \begin{bmatrix} 0.866 & -0.500 & 0 & 4 \\ 0.500 & 0.866 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$${}^{B}_{A}\mathbf{T} = {}^{A}_{B}\mathbf{T}^{-1} = \begin{bmatrix} 0.866 & 0.500 & 0 & -4.964 \\ -0.500 & 0.866 & 0 & -0.598 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





TRANSFORMATION Example

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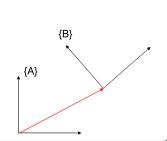
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Transform equations

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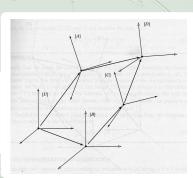
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Transform equation

$$:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$$

$$: \quad {}^{U}_{D}\mathbf{T} = {}^{U}_{B}\mathbf{T}^{B}_{C}\mathbf{T}^{C}_{D}\mathbf{T}$$

$$\therefore {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T} = {}^{U}_{B}\mathbf{T}_{C}^{B}\mathbf{T}_{D}^{C}\mathbf{T}$$



$${}^{U}_{B}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}_{D}^{C}\mathbf{T}^{-1}{}^{B}_{C}\mathbf{T}^{-1}$$



Transform equations

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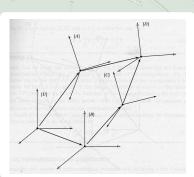
Operat

- Translatio
- Transformati
- multiplicat
- transformatio

 $:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$

$$: \quad {}^{U}_{D}\mathbf{T} = {}^{U}_{B}\mathbf{T}^{B}_{C}\mathbf{T}^{C}_{D}\mathbf{T}$$

$$\therefore {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T} = {}^{U}_{B}\mathbf{T}_{C}^{B}\mathbf{T}_{D}^{C}\mathbf{T}$$



$$_{B}^{U}\mathbf{T} = _{A}^{U}\mathbf{T}_{D}^{A}\mathbf{T}_{D}^{C}\mathbf{T}_{D}^{-1}_{C}^{B}\mathbf{T}^{-1}$$



Transform equations

Mechatronics

David Goodwii

Mappin

- Translat
- Operato

Орстат

- Rotation
- Transformat

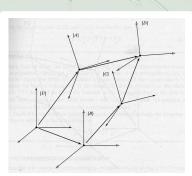
multiplicati

transformati equations • Transform equation

$$:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$$

$$: \quad {}^{U}_{D}\mathbf{T} = {}^{U}_{B}\mathbf{T}^{B}_{C}\mathbf{T}^{C}_{D}\mathbf{T}$$

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Transform equations

Mechatronics

David Goodwii

Mappin

- Translat
- .

Орстас

- Potation
- Transformat

multiplication

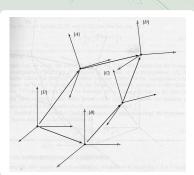
Inverting

transformatio equations • Transform equation

$$:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$$

$$: \quad {}^{U}_{D}\mathbf{T} = {}^{U}_{B}\mathbf{T}^{B}_{C}\mathbf{T}^{C}_{D}\mathbf{T}$$

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$$_{B}^{U}\mathbf{T} = _{A}^{U}\mathbf{T}_{D}^{A}\mathbf{T}_{D}^{C}\mathbf{T}_{D}^{-1}_{C}^{B}\mathbf{T}^{-1}$$



Transform equations

Mechatronics

David Goodwin

Mappin

- Translat Rotation
- .

Operat

- Translatio
- Kotation

multiplication

Invert

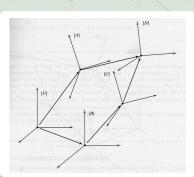
transformatio

• Transform equation

$$:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$$

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Transform equations

Mechatronics

David Goodwir

Mappin

- Translat Rotation
- 0----
- Transla
- Rotatio

Transformat

Inverti

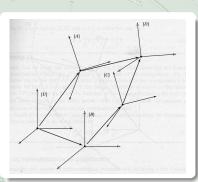
transformatio

• Transform equation

$$:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$$

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$${}^{U}_{B}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}_{D}^{C}\mathbf{T}^{-1}{}^{B}_{C}\mathbf{T}^{-1}$$



Transform equations

Mechatronics

David Goodwii

Mappin

Translat Rotation

Operat

Translation

Transformati

multiplication

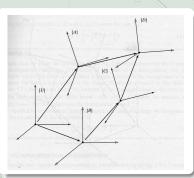
transformation equations

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$$:: {}^{U}_{D}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}$$

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 Any unknown T matrix can then be calculated from the ones given

$${}^{U}_{B}\mathbf{T} = {}^{U}_{A}\mathbf{T}_{D}^{A}\mathbf{T}_{D}^{C}\mathbf{T}^{-1}{}^{B}_{C}\mathbf{T}^{-1}$$



Transform equations

Mechatronics

David Goodwii

Mappin

- Translat Rotation
- .

Operat

- Translatio
- Transformat

multiplicati

Inverting

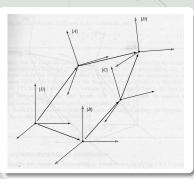
transformation equations

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