CIS009-2, Mechatronics
Static Kinematics

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Outline

STATIC KINEMATICS

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Definition Science of motion that treats a subject without considering forces that cause the motion positions (x) velocity (x') accelerate (x") high order of derivatives of position (x(n)) Static kinematics of a manipulator Study of position of a manipulator (x)

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Structure of a manipulator

STRUCTURE OF A MANIPULATOR

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A manipulator consists of a set of links and joints that connect the links Links – rigid bodies Joints – different types

STRUCTURE OF A MANIPULATOR

Degree of freedom (DOF) DOF are the set of independent displacements that specify completely the displaced or deformed position of the body. In general, a rigid body in d-dimensions has d(d+1)/2 degrees of freedom (d translations + d(d-1)/2 rotations).

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STRUCTURE OF A MANIPULATOR

DOF of a robot arm A robot with 4 axes of motion but 3 DOF 3-DOF allows a robot to place its end-effector in any position but not at any angle Adding mechanical wrist increases DOF which allows end-effector to be placed at any angle

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Link Rigid body between two neighbouring joints Position of a link – described using the following two parameters: Link length (a) – measured along a line that is mutually perpendicular to the axes of both joints Link twist (a) – angle of the projections of the two axes onto a plane whose normal is mutually perpendicular to the two axes

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Illustration

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Mechanical drawing – three views

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Calculation of link length and link twist from mechanical drawing

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Exercise 1 The arm with three degrees of freedom. Calculate a1, a1, and a2, a2

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Joint Connection of two neighbouring links Two parameters Link offset (d) – a measure about how far the two links are away from each other Joint angle (?) – a measure about difference between directions of the two links

Illustration

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Calculation of link offset and joint angle for intermediate links in a chain Link offset calculation: distance from one link to its neighbouring one along a common axis Joint angle calculation – rotation on the common axis between two neighbouring links Assumptions to the first and last links in the chain If Joint 1 is revolute, $d1 = 0$ and ?1 is arbitrary If Joint 1 is prismatic, $?1 = 0$ and d1 is arbitrary

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Exercise 2: Calculate d2 Two links given ion slide 2 are jointed together with Joint 2 d2=0

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Special cases Case1 – all joints are revolute Axes of all joints are in parallel a1 = L1 a1 = 0 d2 = 0 ?2 arbitrary

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Case2 – cylindrical joints involved Axis of Joint 1 intersects the axis of Joint 2 a1 = min(L1, L2), L1+L2 a1 = 90 d2 = 0 ?2 = 0

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Case3 – prismatic joint involved Axis of Joint 1 and axis of joint 2 are in perpendicular a1 = L1 a1 = 90 d2 = 0, Lj2 ?2 = 0

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Assigning frames

ASSIGNING FRAME

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For intermediate links in the chain Frame I defined for Joint i in the following way: Zi – coincident with the joint axis i Xi – pointing along link length ai Origin – the point where ai perpendicular intersects the joint axis i First and last links in the chain Conventions: Frame 0 coincides with frame 1 Frame N: Revolute: Xn aligns with Xn-1 so that ?n = 0 and origin chosen to let dn = 0 Prismatic: Xn chosen so that ?n = 0 and the origin chosen to let dn = 0

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Example 1: three-link planar arm

ASSIGNING FRAME

Frames

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Schematic representation of a robot

SCHEMATIC REPRESENTATION

Revolute joint

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SCHEMATIC REPRESENTATION

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Prismatic joint

SCHEMATIC REPRESENTATION

Example 2: A 3 DOF manipulator with two revolute and one prismatic joints

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Schematic diagram Assign frames

SCHEMATIC REPRESENTATION

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Example 3: a 3 DOF manipulator with three revolute joints

SCHEMATIC REPRESENTATION

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Schematic diagram Assign frames
Calculate parameters a1 = L1, a1 = 90o, d1 = 0, q1 = x a2 = L2, a2 = 0o, d2 = L1, q2 = 90o a3 = 0, a3 = 0o, d3 = 0, q3 = x

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Manipulator kinematics

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Position and Transformation Position of end-effector in different frames Transform from frame i to frame i-1 – representing joint i's position in frame i-1

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Transformation

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Three intermediate frames R, Q, and P Translating i di units along axis i derives P: Rotating P ?i along Zp yields Q: Translating Q ai-1 units along Xq gives R: Rotating R ai gives i-1:

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End-effector is a vector iP in frame i

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Case study: a PUMA robot

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6 DOFs Details of joints 3, 4, 5, and 6 are shown in the diagram

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Manipulator kinematics Representing the position of joint 6 in frame 0 Transformations

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Example 4 For a 2-link manipulator shown in the following figures, the link-transformation matrices were constructed

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The product is
The link-frame assignment used are indicated in figure (b). Note that the frame 0 is coincident with frame 1 when q1 ? 0. The length of the second link is l2. Find an expression for the vector 0Ptip which locates the tip of the arm relative to the 0 frame.

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Example 5 The figure in the following shows the schematic of a wrist which has three intersecting axes that are not orthogonal. Assign link frames to the wrist (as if it were a 3-DOF manipulator), and give the link parameters.

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SUMMARY

Manipulator Position of a link Description of connections Frame Schematic representation Assigning frames Static kinematics

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