CIS009-2, Mechatronics

Mechanical Fundamentals: Forces and Equilibrium

David Goodwin

Department of Computer Science and Technology
University of Bedfordshire

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Outline

Introduction

Every footballer is a natural calculator. . .

- We have all matured to have an understanding of the laws of motion, it is even as simple as survival.
- If we kick a football towards the goal, we don't expect it to suddenly stop or change direction (unless there is an another force on the football, like a goalkeeper).
- If a ball is thrown in the air we expect it to come back down in the "reverse" of the motion with which it was thrown up.
- If we push a pedal on a bicycle we expect a forceful resistance to this motion, with the benefit that the bicycle will move forward.
- However, intuition and experience only go so far in predicting motion. We need something more accurate and less subjective to be able to design modern mechanisms.

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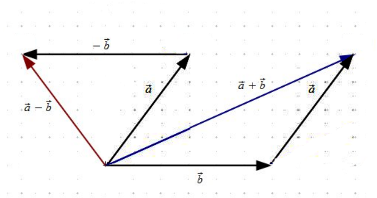
Scalars and vectors

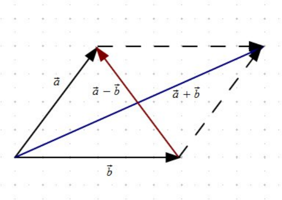
Scalars and Vectors

- There are two types of quantities in mechanics: those which have magnitude but no directional properties (called **scalar** quantities) and those which are associated with a direction as well as a magnitude (called **vector** quantities).
- Scalar quantities, e.g. mass and energy, can be added or subtracted by the ordinary mathematical rules for addition and subtraction while vector quantities, e.g. acceleration and force, cannot. The directions of vector quantities have to be taken into account.
- Coplanar vectors means that the vectors lie in the same plane while concurrent vectors means that the action lines of vectors pass through the same point.

Vector Addition and Subtraction

- There are two main way to add (subtract) vectors:
  - The "head to tail method" (top diagram)
  - The "parallelogram method" (bottom diagram)
- We note that the vector $-\mathbf{b}$ has the same size as $\mathbf{b}$ but the opposite direction. Therefore $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$.





Newton's Laws

Newton's Laws

Newton's Laws of Motion

1. A body at rest remains at rest and a body in motion continues to move at constant velocity unless acted upon by an external force.
2. A force acting on a body causes an acceleration which is in the direction of the force and has a magnitude inversely proportional to the mass of the body.
3. Whenever a body exerts a force on another body, the latter exerts a force of equal magnitude and opposite direction on the former.

Newton's first law - Law of Inertia

Also known as the law of inertial, The first law defines the condition for a body to be in what is termed equilibrium. Mathematical formulation: if the sum of all vector forces is zero; the change (differential in time) of the vector velocity is zero.

$$\sum F = 0 \rightarrow \frac{d\mathbf{v}}{dt} = 0$$

Newton's second law - Law of Inertia

Simply known as "newton's second law", or "$F = ma$". The second law explains what happens when there is no equilibrium i.e. there is a non-zero net vector force upon a body.

$$F = ma = \frac{d(m\mathbf{v})}{dt}$$

Newton's third law - action-reaction law

Also known as the law of reciprocal actions, implies the conservation of momentum. The third law defines the way two bodies interact.

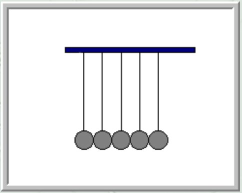
$$F_1 = -F_2$$

Example of Newton's Laws of Motion

Newton's Cradle

. . . Loading. . .

- 1^{st} law:
  - In equilibrium there is no movement, although there are forces.



Example of Newton's Laws of Motion

Newton's Cradle

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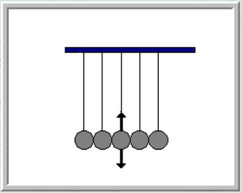
- 1^{st} law:
  - In equilibrium there is no movement, although there are forces.
  - The forces of gravity and the tension in the string are equal in magnitude but opposite in direction, and there is no net force.



Example of Newton's Laws of Motion

Newton's Cradle

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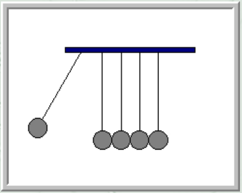
- 2^{nd} law:
  - An external influence can move the ball into a state with "potential energy".



Example of Newton's Laws of Motion

Newton's Cradle

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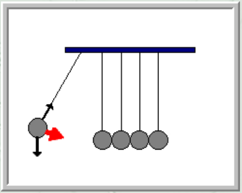
- 2^{nd} law:
  - When there is a net force on the ball, the body is not in equilibrium.
  - The direction of the net force vector shows the direction of acceleration.



Example of Newton's Laws of Motion

Newton's Cradle

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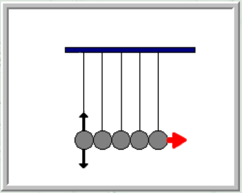
- 3^{rd} law:
  - The net force is transferred between the bodies.
  - Each ball stops the previous ball with an equal size but opposite direction force (from the surface tension of the balls).
  - The net force is transferred along the line of balls.



Example of Newton's Laws of Motion

Newton's Cradle

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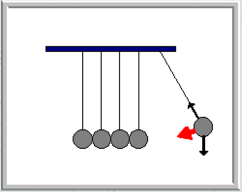
- Acceleration in the same direction as the net force.
- The acceleration is not nessesarily in the same direction as the velocity.
    - positive acceleration gives increasing velocity
    - negative acceleration gives decreasing velocity



Weight

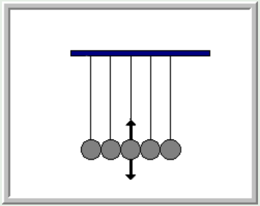
- Weight is the pull of the Earth's gravity, that is, weight is a force.
- As a vector quantity weight has a direction (downwards) and a magnitude.
- Units of weight are the same force i.e. newtons (N).
- According to Newton's 2^{nd} law: $\mathbf{F} = m\mathbf{a}$.
  - We approximate the acceleration due to gravity as constant (since in normal calculations, variations in height would be small compared to the radius of the earth). We denote this acceleration by the symbol $g = 9.81ms^{-1}$ (metres per second).
- Since the direction is always the same (downwards), we usually deal with the magnitude of the gravitational force called weight: $w = mg$.
  - This is why a person weighs differently on the moon (different gravity, but same mass).
  - Since g is usually considered constant, weight is sometimes confused with mass (measured in kilograms).

Weight

Example (1.1)

Consider on of the balls of Newton's cradle in equilibrium (figure below). What are the magnitudes of the two forces involved if the mass of a ball is 145 grams?

- Realise the the weight is $w = mg$ where we know the mass and $g = 9.81$ is a constant, so $w = 0.145 \times 9.81 = 1.42N$
- Also realise that the force from the tension in the string must equal the weight for the body to be in static equilibrium.



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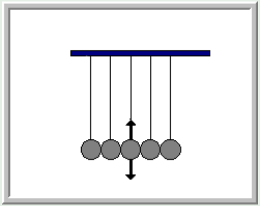


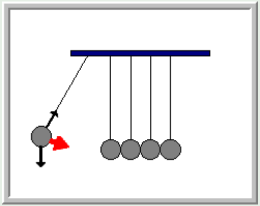
Force as a Vector

Weight as a Vector

Example (1.2)

Now consider the same forces acting on one of the balls not in equilibrium (figure below). What is the magnitude of the net force?

- Add the two vectors of force together to find the new direction of the net force, and the new magnitude:
-



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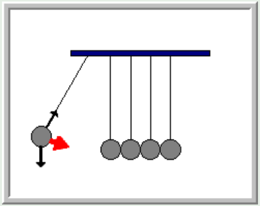

Newton's second law - action-reaction law

Stress and strain