Principles of Modern Communications
Digital Communications II

based on 2011 lecture series by Dr. S. Waharte.
Department of Computer Science and Technology,
University of Bedfordshire.

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Outline

1. Frequency Modulation
2. Data Communications
   - Baseband transmission
   - Broadband transmission
   - Modulation
   - Transmission impairments
   - Nyquist Theorem
   - Shannon's Law
   - Performance

Frequency Modulation

Carrier wave equation

- Given a carrier wave whose equation is given by:

$$v(t) = V_c \cos\left(2\pi f_c t + \phi_c\right)$$

- where
  - V_c = Amplitude
  - f_c= carrier frequency and
  - ϕ_c = phase angle of the wave
- The modulating wave can be defined as above.

Modulating wave equation

- A modulating wave equation is given by:

$$e(t) = V_m \cos\left(2\pi f_m t + \phi_m\right)$$

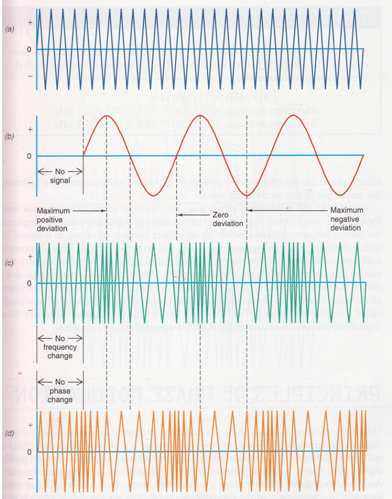
- where
  - V_m= Amplitude
  - f_m= modulating frequency and
  - ϕ_m = phase angle of the modulating wave
- The above equations can be used to define Amplitude Modulation (AM), Frequency Modulation (FM) and Phase Modulation (PM).

Defining AM, FM and PM

- In Amplitude Modulation, the modulating signal is used to modify the amplitude V_c of the carrier wave.
- In Frequency Modulation the modulating signal is used to modify the carrier frequency f_c.
- In Phase Modulation, the modulating signal is used to modify the phase ϕ_c of the carrier signal.

FM and PM analysis

- FM and PM signals
  - (a) Carrier signal
  - (b) Modulating signal
  - (c) FM signal
  - (d) PM signal



FM and PM analysis

- The complete mathematical analysis of FM and PM is beyond the scope this class.
- For our purposes, it will suffice to simply give solutions and discuss them.

FM signal

- An FM signal is given by the equation:

$$v(t) = A\cos\left(\omega_c t + m_f \sin \omega_m t\right)$$

- Where:
  - $A=$ amplitude of the carrier wave
  - ω_c, ω_m = angular frequencies for the carrier and modulating signals respectively.
  - m_f = modulation index for FM
  - $m_f = \frac{\delta}{f_m}$ where δ = max frequency shift caused by the modulating signal
- f_m : highest frequency component in the modulating signal

FM mathematical solution

- The equations for FM and PM are more complex than they look as they contain cosine of a sine.
- Therefore, to solve the frequency components of an FM wave we need use a mathematical tool known as Bessel functions.

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left(\omega_c + n\omega_m\right)t$$

- The Bessel functions gives us a way of calculating the sidebands of an FM signal when the modulating index is known.

Bessel functions

	n OR ORDER																
x (m_f)	(CARRIER) J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	−0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

Source: E. Cambi, *Bessel Functions,* Dover Publications, Inc., New York, 1948. Courtesy of the publisher.

Sidebands

- In FM and PM as in AM sum and difference sidebands are produced.
- Sidebands are determined in FM and PM in accordance with Bessel functions.
- FM and PM use a wider spectrum than in AM.
- All the modulations systems produce infinite number of sidebands but only the significant ones are relevant.

Example to illustrate use of Bessel chart

Determine a bandwidth required to transmit an FM signal with $f_i = 10$ kHz and a maximum deviation $\delta = 20$ kHz.

SOLUTION

$$m_f = \frac{\delta}{f_i} = \frac{20 \text{ kHz}}{10 \text{ kHz}} = 2$$

with $m_f = 2$, the following significant components are obtained:

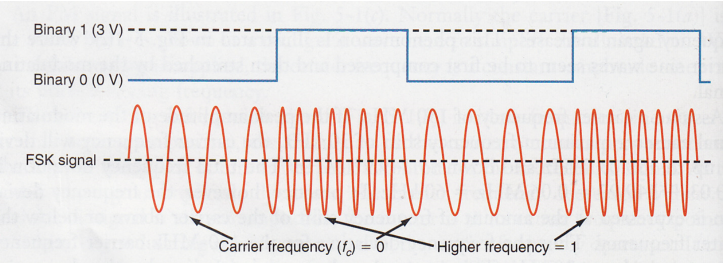
$$J_0, J_1, J_2, J_3, J_4$$

This means that besides the carrier, J_1 will exist ±10 kHz around the carrier, J_2 at ±20 kHz, J_3 at ±30 kHz, and J_4 at ±40 kHz. Therefore, the total required bandwidth is 2×40 kHz = 80 kHz.

Phase modulation

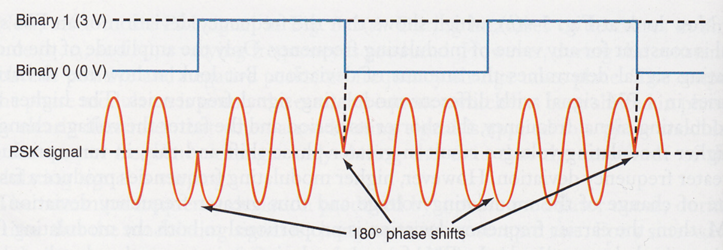
- Phase modulation is achieved when the amount of phase shift of a constant-frequency carrier is varied in accordance with a modulating signal.
- This can be achieved with a phase shifter that changes the phase of a carrier signal as the amplitude of the modulating signal changes.

Frequency-Shift Keying (FSK)



- Frequency modulating a carrier with binary data produces FSK

Phase-shift keying (PSK)



- Phase modulating a carrier with a binary data produces PSK

Directed Learning and Self Study

Example

1. Use the internet to find what Carson's rule is about and explain the formula that defines the rule.
2. What is the meaning of ASK? What is its relationship with FSK and PSK?

Frequency Modulation
Data Communications
Baseband transmission
Broadband transmission
Modulation
Transmission impairments
Nyquist Theorem
Shannon's Law
Performance

Data Communications

Overview

- Baseband transmission
- Broadband transmission
- Modulation
- Transmission impairment (attenuation, distortion, and noise)
- Nyquist Theorem
- Shannon's law
- Performance (Bandwidth, Throughput, Latency)

Baseband Transmission

- Sends digital signal over a channel.
- The signal is not changed to analogue signal.
- Require a channel with a low pass bandwidth. (i.e. 0 – infinity) Hz

Practical Baseband Bandwidth

- A low pass channel with a narrow bandwidth.
- A bandwidth from 0 - h Hz where h lies between 0 and infinity.

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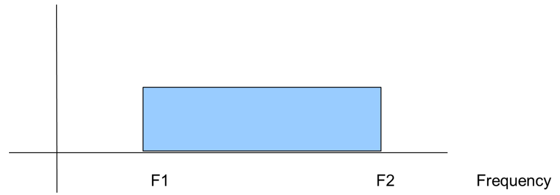
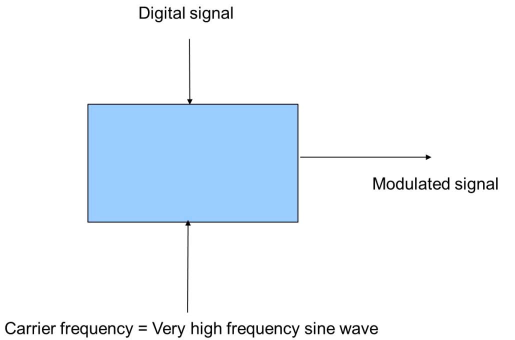
Broadband Transmission

- We change the digital signal to analogue signal by modulating the digital signal with a carrier frequency.
- This allows us to use a band pass channel.
- A band pass channel does not start from 0 Hz. E.g. F1



Modulation technique



Signal in transmission medium

- They suffer transmission impairments.
- For example:
- Attenuation (i.e. loss of energy).
- Distortion (due to transmission delays in the components of the composite signal and
- Noise (due to external and internal influences).

Attenuation

- Reduction in power when signals pass through a medium.
- If a signal with power P1 passes through a medium and the signal is measured as power P2 at the end of the medium then the gain of the signal is:

$$Gain = 10log_{10}\left(\frac{P2}{P1}\right)$$

- A reduction in the gain may require an amplifier to boost the signal up

Distortion

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- Different components in a composite signal reacts differently to the medium.
- Thus, signals arriving at a point may be out of phase and with different amplitudes.
- The cumulative effect is a distorted signal of the composite signal.

Noise (causes)

- Thermal noise: Random noise generated in the medium.
- Induced noise: Generated from motors and electrical appliances. These serve as transmitters and the medium serves as receivers.
- Cross-talk: This is the effect one wire on another.

Signal-to-noise Ratio

- This ratio helps us to estimate the effect noise on a signal.
- It is measured as:

$$SNR = 10log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right)$$

- This ratio is important as it is one of the determining factors of the channel capacity.

Exercise

Example

- Calculate the signal to noise ratio of a lossless channel

Solution

Example

- Solution:
- SNR=Signal Power/Noise Power
- Noise Power = 0 (for a lossless channel)
- Thus, SNR=Signal Power/0 = ∞
- Therefore, SNR=$1og_{10}(\infty) = \infty$

Data rate limits

- A very important consideration in data communications is how fast we can send data, in bits per second, over a channel. Data rate depends on three factors:
  1. The bandwidth available
  2. The level of the signals we use
  3. The quality of the channel (the level of noise)
- Noiseless Channel: Nyquist Bit Rate
- Noisy Channel: Shannon Capacity

Question

Example

- Increasing the levels of a signal increases the probability of an error occurring, in other words it reduces the reliability of the system. Why??

Capacity of a system

- The bit rate of a system increases with an increase in the number of signal levels we use to denote a symbol.
- A symbol can consist of a single bit or "n" bits.
- The number of signal levels = 2n.
- As the number of levels goes up, the spacing between level decreases → increasing the probability of an error occurring in the presence of transmission impairments.

Nyquist Lossless Channel

- Nyquist gives the upper bound for the bit rate of a transmission system by calculating the bit rate directly from the number of bits in a symbol (or signal levels) and the bandwidth of the system (assuming 2 symbols/per cycle and first harmonic).
- Nyquist theorem states that for a noiseless channel:

$$C = 2B log_2(2n)$$

  - C= capacity in bps
  - B = bandwidth in Hz

Transmitting over a lossless channel

- Let us suppose we want to transmit 500kbps over a noiseless channel with a 30Khz bandwidth. How many signal levels do we require?

Solution

Example

- Using Nyquist Theorem:
- Bit Rate $= 2 \times Bandwidth \times log_2(L)$
- $500 \times 1000 = 2 \times (30 \times 1000) \times log_2(L)$
- $Log_2(L) = 50/6 = 8.333$
- $L = Log_{10}(8.333)/Log_{10}(2)$

Shannon's Law

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- We can never have a lossless channel.
- Shannon(1944) proposed that the maximum bit rate (i.e. channel capacity) depends on the signal-to-noise ratio. i.e.

$$\text{Bit Rate} = \text{Bandwidth} \times log_2(1 + S/N)$$

Observations of Shannon's Law

- Shannon's Law does not mention signal levels.
- This means that no matter what signal levels you have, you cannot achieve a data rate higher than the channel capacity.

Problem

Example

- What is the channel capacity of a faint signal that is completely swamped by noise?

Answer

Example

- Using Shannon's Law:
- $C = BLog_2(1 + S/N)$
- Noise $>>$ S
- Therefore, S/N=0
- $Log_2(1 + S/N) = Log_2(1 + 0) = 0$
- Hence, $C = 0$
- This mean that the channel cannot transmit any signal.

Performance metrics of a communication system

- Bandwidth: Defines the channel or the bit rate
- Throughput: Refers to the actual bit rate measured at the end of a transmission
- Latency (Delay): Refers to delays caused during the transmission of a signal.
- These metrics are important in communication systems.

Bandwidth (Hz)

- This means a range of channel frequencies that allows a signal to pass.
- E.g. The bandwidth of a telephone line is 4KHz.

Bandwidth (Bits Per Second)

- This is the number of bits per second that a link, channel or a network can transmit.
- E.g. A fast Ethernet network can transmit at 100 Mbps (i.e. Mega bits per second)

Bandwidth in Hz Versus Bandwidth in Bps

- Bandwidth in Hz refer to a range of frequencies.
- Bandwidth in Bps refers to the speed of transmission in a channel or a link.
- The relationship between them is that, the bandwidth in Bps is proportional to bandwidth in Hz

Throughput

- A measure of how fast we actually send data through a network.
- Suppose you have a link designated as C bps but you may be able to measure the signal through the link as T bps where T<<C.
- This is because intermediary devices connected to the channel have slower bit rates affecting the transmission.

Example

Example

- A network with a bandwidth of 12 Mbps can only pass an average 15,000 frames per minute. If each frame carries an average of 10,000 bits, calculate the throughput of the network

Solution

Example

- Throughput is that actual bit rate measured.
- It is independent of the channel speed but depends on devices located on the channel.
- Therefore,
  - Throughput$= 15,000 \times 10,000/60$=???
- Answer is less than 12 Mbps channel speed

Latency (Delays)

- This defined as the time time it takes for a message to arrive at the destination i.e. from the time it was first sent.
- Latency = Propagation Time + Transmission Time + Queuing Time + Processing Delay.

Propagation Time

- This is a measure of the time it takes a bit to travel from the source to the destination.
- Propagation Time = Distance/Propagation Speed
- This depends on the transmission medium (in a vacuum electro magnetic waves travels at 3×108 m/s but in a coaxial cable it is far less.

Transmission Time

- The Transmission Time is defined as: Transmission Time = Message Size/Bandwidth
- Note that the Bandwidth is in Bps.

Exercise

Example

- Calculate:
  - The propagation time and Transmission time, for a 2.5 kbyte email message if the bandwidth is 1.0 Gbps. Assume the distance between sender and receiver is 12,000km and that light travels at 2.4×108 m/s.

Solution

Example

- Propagation Time $= 12,000 \times 1000/2.4 \times 108$ =50 ms
- Transmission Time =$2500 \times 8/109$ =0.020 ms
- Note that the dominant factor is the propagation time and not transmission time.

Queuing Time

- This is the time required for devices (i.e. intermediate or end devices) to hold data before it can be processed.
- QT changes with load on the network.

Bandwidth-Delay Product

- An important metric in data transmission.
- It is used by network engineers to define the number of bits that can fill a link.

Jitter

- This is the delay caused by different packets arriving at a destination at different times e.g. time sensitive data such as audio and video.
- This can cause serious effects on the received signal.

Summary (1)

- Data communication uses periodic analogue and non-periodic digital signals.
- Fourier analysis shows that a composite signal is a combination of simple sine waves with different frequencies, amplitudes an phases.
- A digital signal is a composite analogue signal with infinite bandwidth.
- Baseband transmission transmits digital signals through a low pass channel with an infinite bandwidth.
- Nyquist Theorem defines the theoretical maximum bit rate a digital signal.
- Signal impairment are: attenuation, distortion and noise.

Directed Reading and Self Study

Example

- Given a list of frequencies shown below calculate their corresponding periods.
    - a) 24 Hz, b) 8 MHz, c) 140 GHz, d) 100 GHz
- What are the corresponding frequencies of following periods:
    - a) 5s, b) 12 μs, c) 200 ms.
- A device is sending out data at the rate of 1000 bps. How long does it take to send:
    - a) 10 bits, b) 1 single character, c) a thesis containing 20,000 words? The average word length is 6 characters.