The Nonlinear Dynamics of Flexural Ultrasonic Transducers

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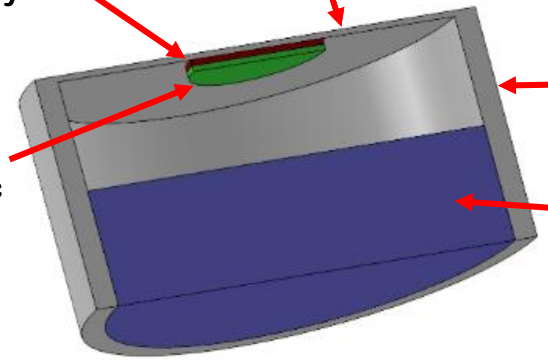
Research Overview

- The flexural ultrasonic transducer (FUT) is a unimorph for operation in different fluids such as air and water
- Piezoelectric or electromagnetic
- Proximity sensing and NDE
- Development of high-frequency FUTs for hostile environments



Section-view Schematic of a Classical Flexural Ultrasonic Transducer

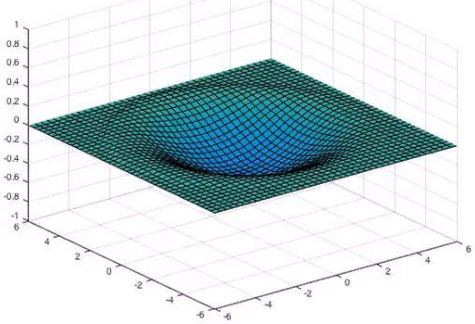
Application	Example Pressure (bar)
Residential gas meters	2
Domestic water meters	20
Industrial gas meters	300
Industrial flow meters	300+

Environment	Example Temperature (°C)
Oil production	120
District heating	250
Petrochemical	350-450
Power plants	560

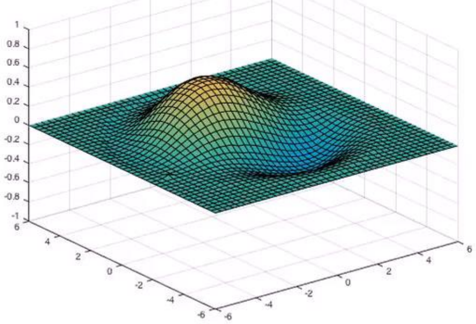
Objective: Examine the nonlinear dynamics of different FUTs

Dynamic Characteristics of the FUT

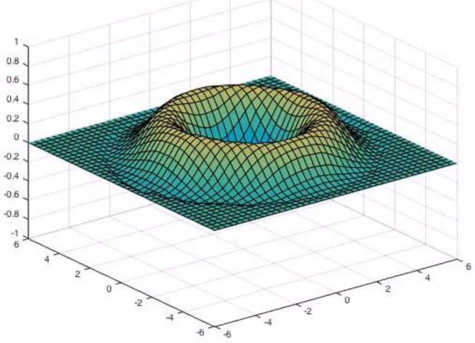
(0,0) Mode



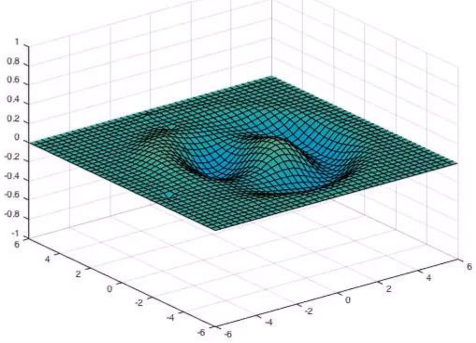
(0,1) Mode

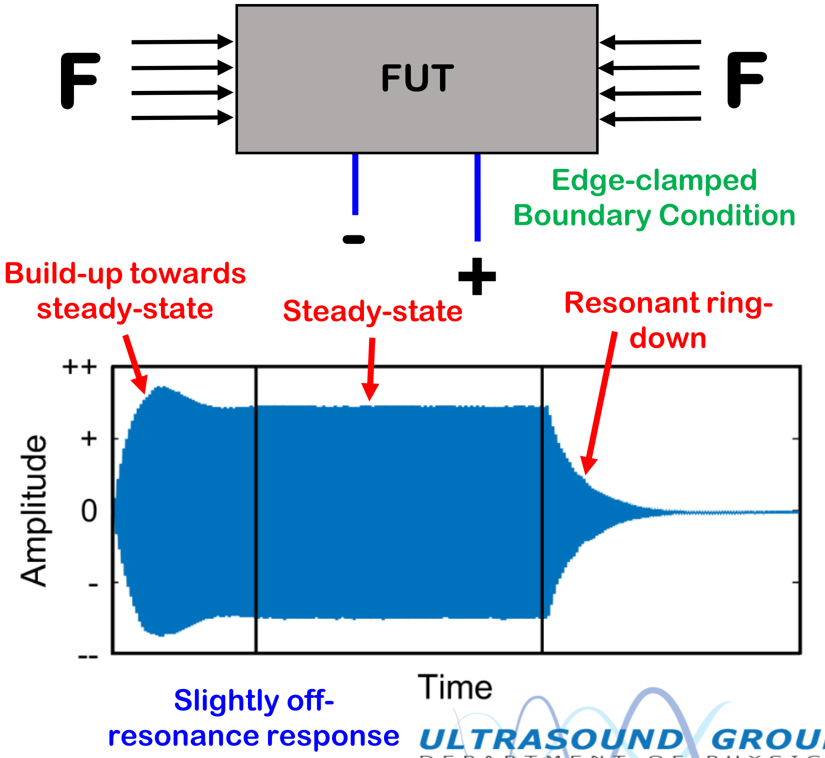


(1,0) Mode



(1,1) Mode





Dynamic Nonlinearity

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Governing Equation

$$M\ddot{x} + C\dot{x} + Kx = F(t) = Bcos\Omega t$$

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2\, x = Bcos\Omega t$$

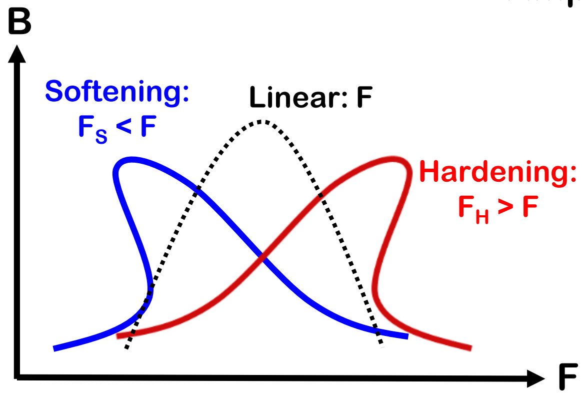
Damping Ratio (ζ); Resonance Frequency (ω_0); Amplitude (B)

Linear Case

$$\ddot{x} + 2\epsilon^2\mu\dot{x} + \epsilon\alpha_2 x^2 + \epsilon^2\alpha_3 x^3 + \omega_0^2\, x = \epsilon^2 Bcos\Omega t$$

Drive Frequency (Ω); Damping Term (μ); Stress Coefficients (α_2, α_3); Perturbation Parameter (ϵ)

Nonlinear Case (Duffing-type)



Causes of nonlinearity

- Geometric (boundary condition)
- Material (damping)
- Actuation mechanism
- Temperature/piezoelectric

Why is this important?

- Stability of dynamic response
- Reliability in industry application
- Optimised performance

A. H. Nayfeh and D. T. Mook, *Nonlinear Oscillations*. Hoboken, NJ, USA: Wiley, 2008.

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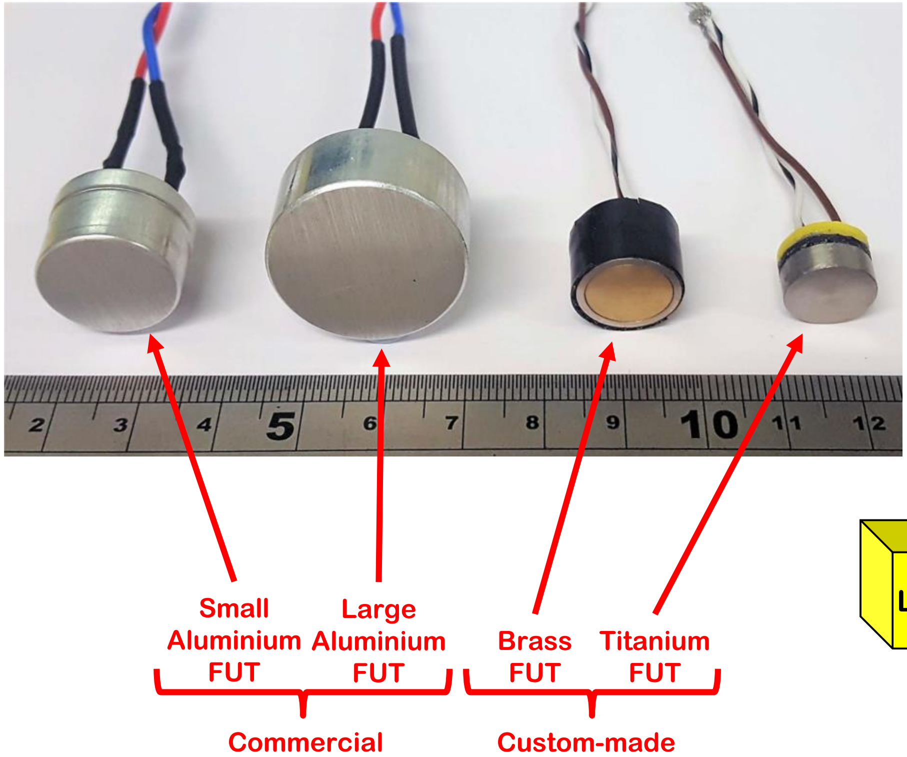
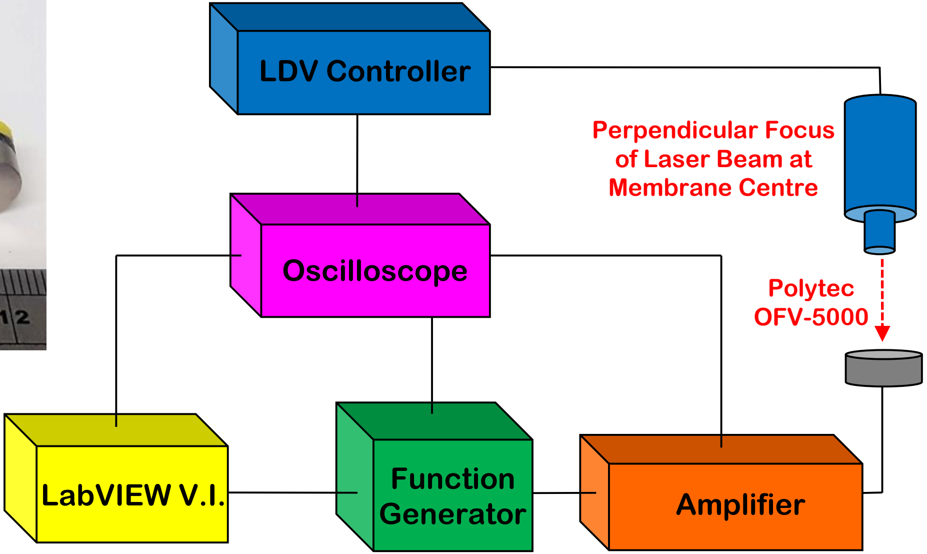
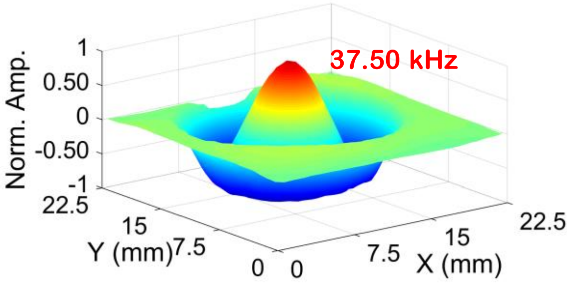
Experimental Method

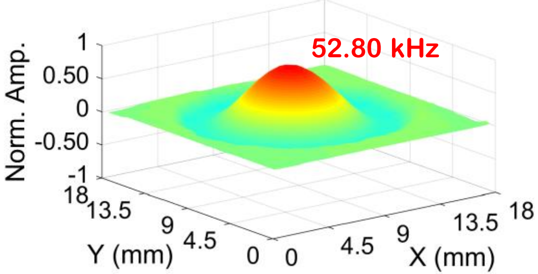




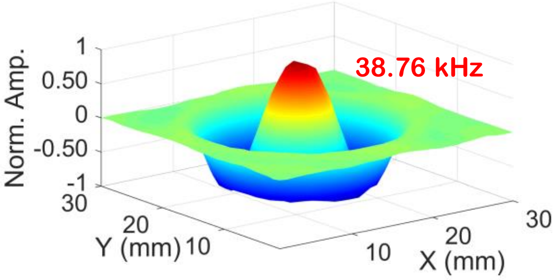
Mode Shape Measurements



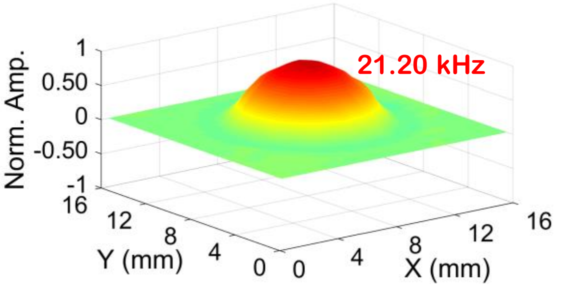
Small Aluminium FUT, (1,0) Mode



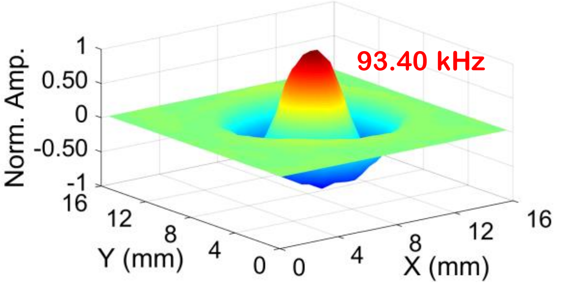
Brass FUT, (0,0) Mode



Large Aluminium FUT, (1,0) Mode



Titanium FUT, (0,0) Mode



Titanium FUT, (1,0) Mode

Primary Resonance Solution

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Displacement · Stress Coefficient · Drive Frequency · Excitation

$$x = b\cos(\Omega t - \gamma) + \frac{1}{2}\epsilon\alpha_2\omega_0^{-2}b^2\left[-1 + \frac{1}{3}\cos(2\Omega t - 2\gamma)\right] + O(\epsilon^2)$$

Amplitude · Phase · Resonance Frequency · Time · Perturbation Parameter

- Softening nonlinear response
- Asymmetry between maximum and minimum in response
- Phase shift as a function of excitation amplitude
- Linear relationship between vibration response and excitation amplitude

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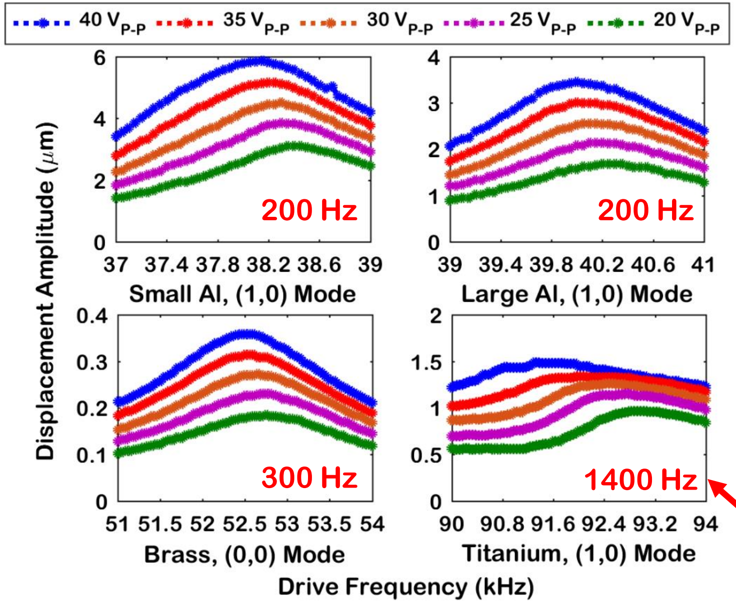
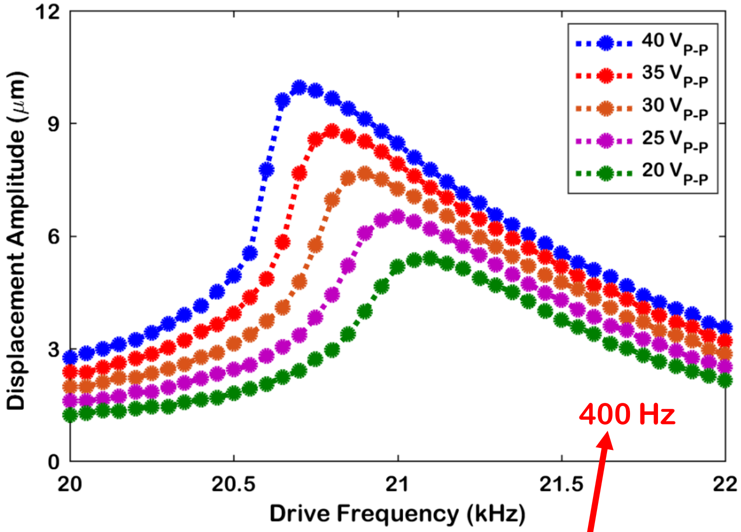
FUTs in Continuous Mode





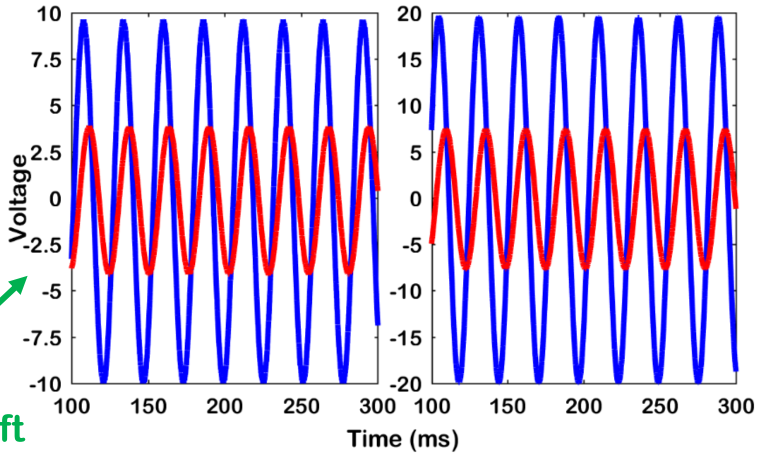
$$b = \frac{1}{\sigma}\left(\frac{9\alpha_3\omega_0^2 - 10\alpha_2^2}{24\omega_0^3}a^3 - \frac{B}{2\omega_0}cos\gamma\right)$$

Softening identified: $9\alpha_3\omega_0^2 < 10\alpha_2^2$

Resonance frequency reductions shown

FUTs in Continuous Mode

Small Al, (1,0) Mode



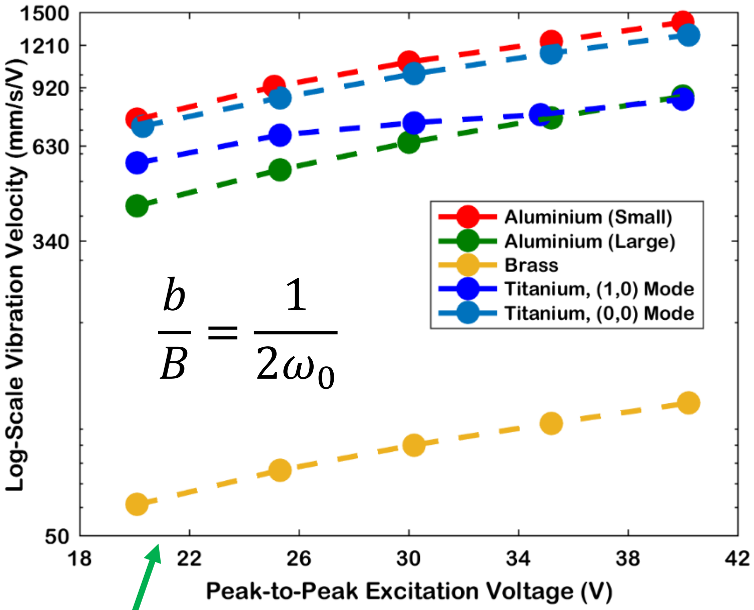
Drive
Response

Asymmetry and phase shift

$\gamma = 56.6°$ → $\gamma = 64.5°$

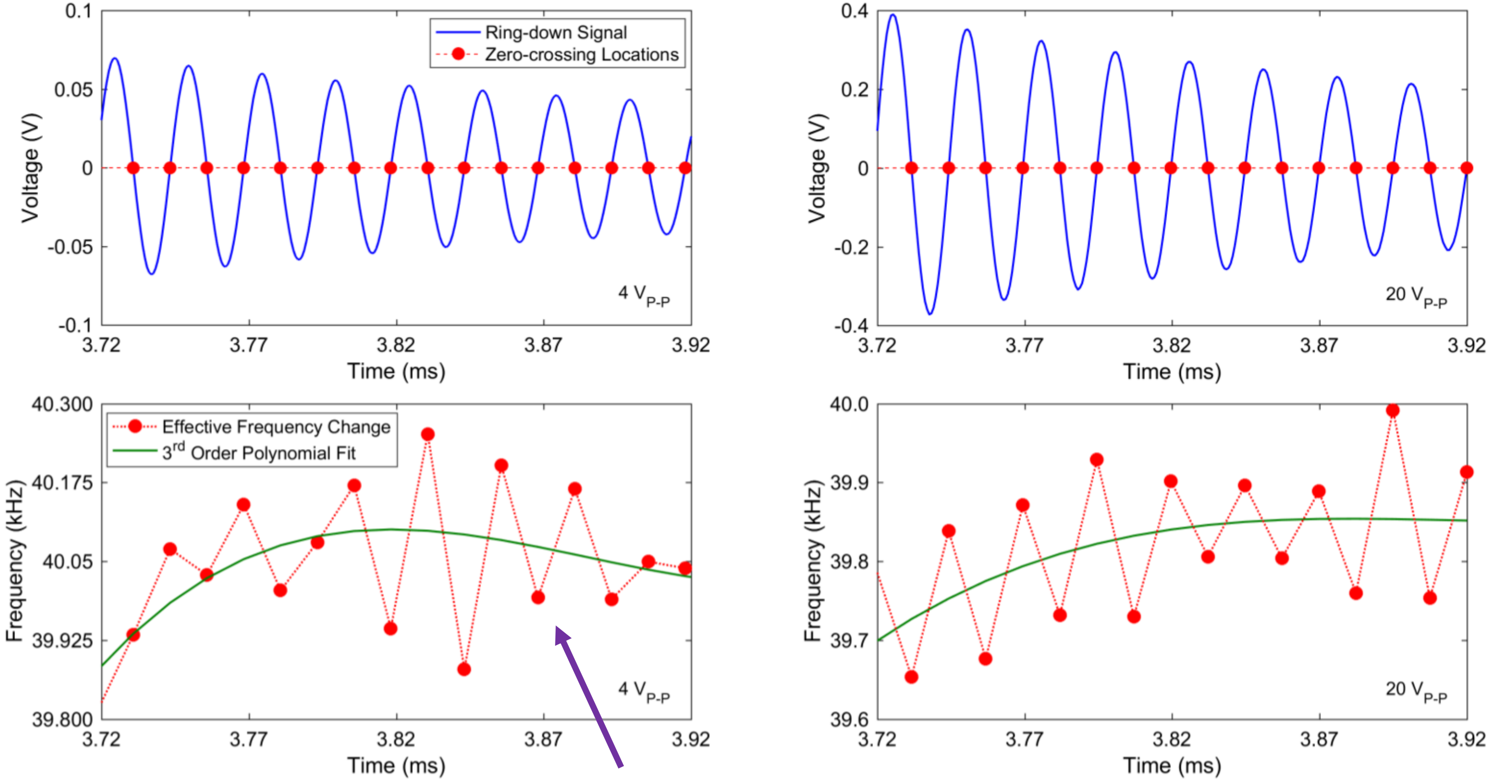
$$\mu b = \left(\frac{B}{2\omega_0}\right) sin\gamma$$

$$b = \frac{1}{\sigma}\left(\frac{9\alpha_3\omega_0^2 - 10\alpha_2^2}{24\omega_0^3} b^3 - \frac{B}{2\omega_0} cos\gamma\right)$$



Linearity between b and B

Nonlinearity in Resonant Decay



Aluminium FUT, Nominal 40 kHz Resonance in the (0,0) Mode

Fluctuations are likely signal processing artifacts

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Analysis on a Set of FUTs

- Aluminium FUT, (0,0) mode
- Five transducers, nominally identical

	Electrical Impedance Analysis			Laser Doppler Vibrometry	
FUT	Coupling Coefficient k^2	Quality Factor Q_M	Resonance Frequency f_r (kHz)	f_r, nom. 4 V_{p-p} (kHz)	$f_N - f_S$ (Hz) nom. 4 to 20 V_{p-p}
1	0.33	71.01	39.51	40.00	300
2	0.32	56.13	40.64	41.00	200
3	0.33	56.71	39.97	40.40	200
4	0.31	54.17	38.23	37.90	200
5	0.32	49.75	39.72	40.10	200
Mean	0.322	57.55	39.59	39.66	220
Standard Deviation	0.007	7.16	0.88	0.90	40

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Summary and Future Research

- Nonlinear behaviour can manifest from boundary condition, operating mode, excitation amplitude, FUT cap material, piezoelectricity
- Although individual causes of nonlinearity are difficult to isolate, it is closely related to stiffness properties
- Experimental data consistent with mathematical theory
- Investigate alternative FUTs in future, such as electromagnetic-driven devices

Transducer and Associated Operating Mode	Frequency Reduction (Hz) 20 V_{P-P} to 40 V_{P-P}
Small Aluminium FUT: (1,0) Mode	200
Large Aluminium FUT: (1,0) Mode	200
Brass FUT: (0,0) Mode	300
Titanium FUT: (0,0) Mode	400
Titanium FUT: (1,0) Mode	1400

Acknowledgement

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