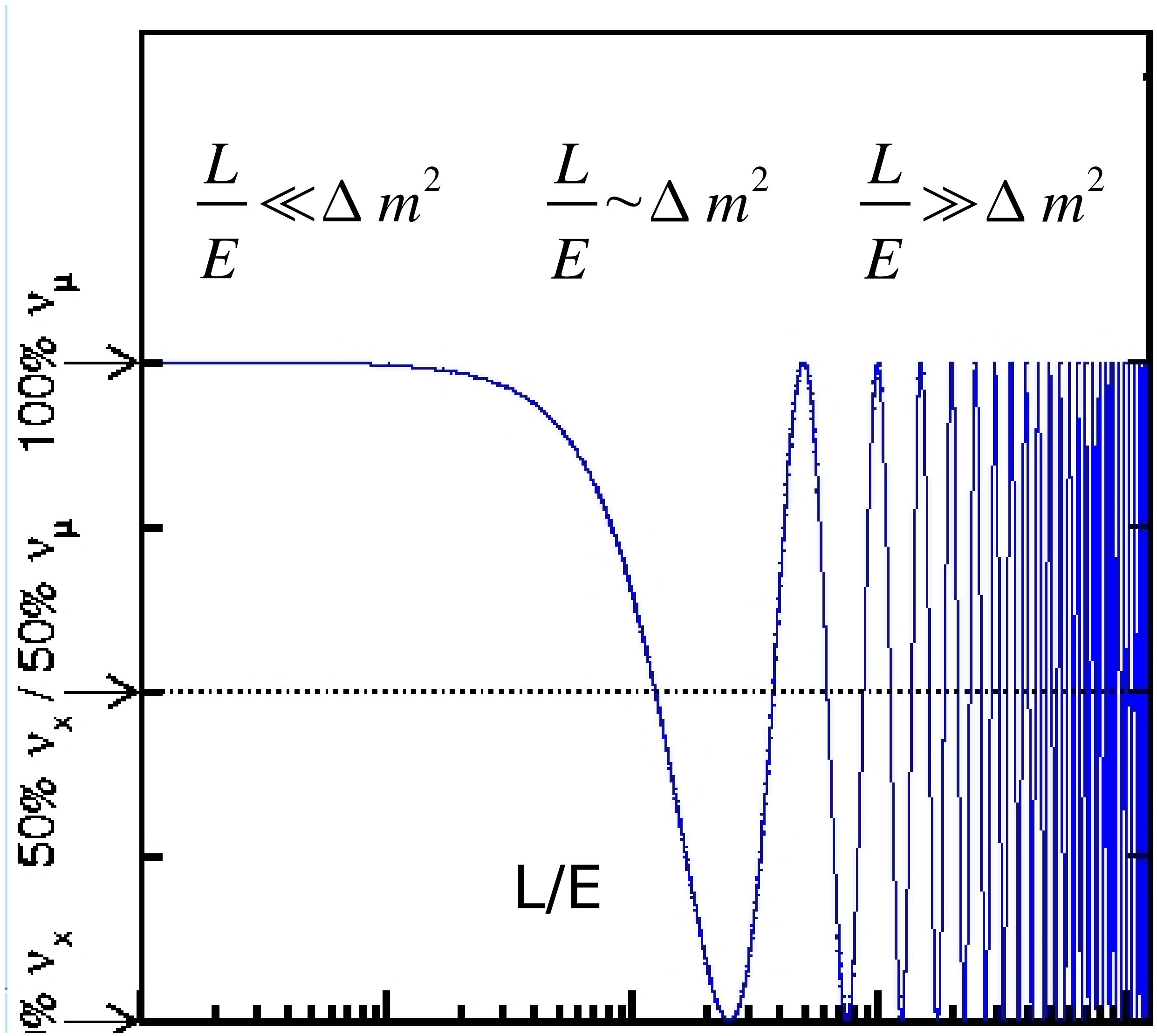
$$P(\nu_\alpha(0)\rightarrow\nu_\alpha(x))=1-\sin^2(2\theta)\sin^2\left(1.27\,\Delta m^2\,\frac{(L/km)}{(E/GeV)}\right)$$

Survival Probability

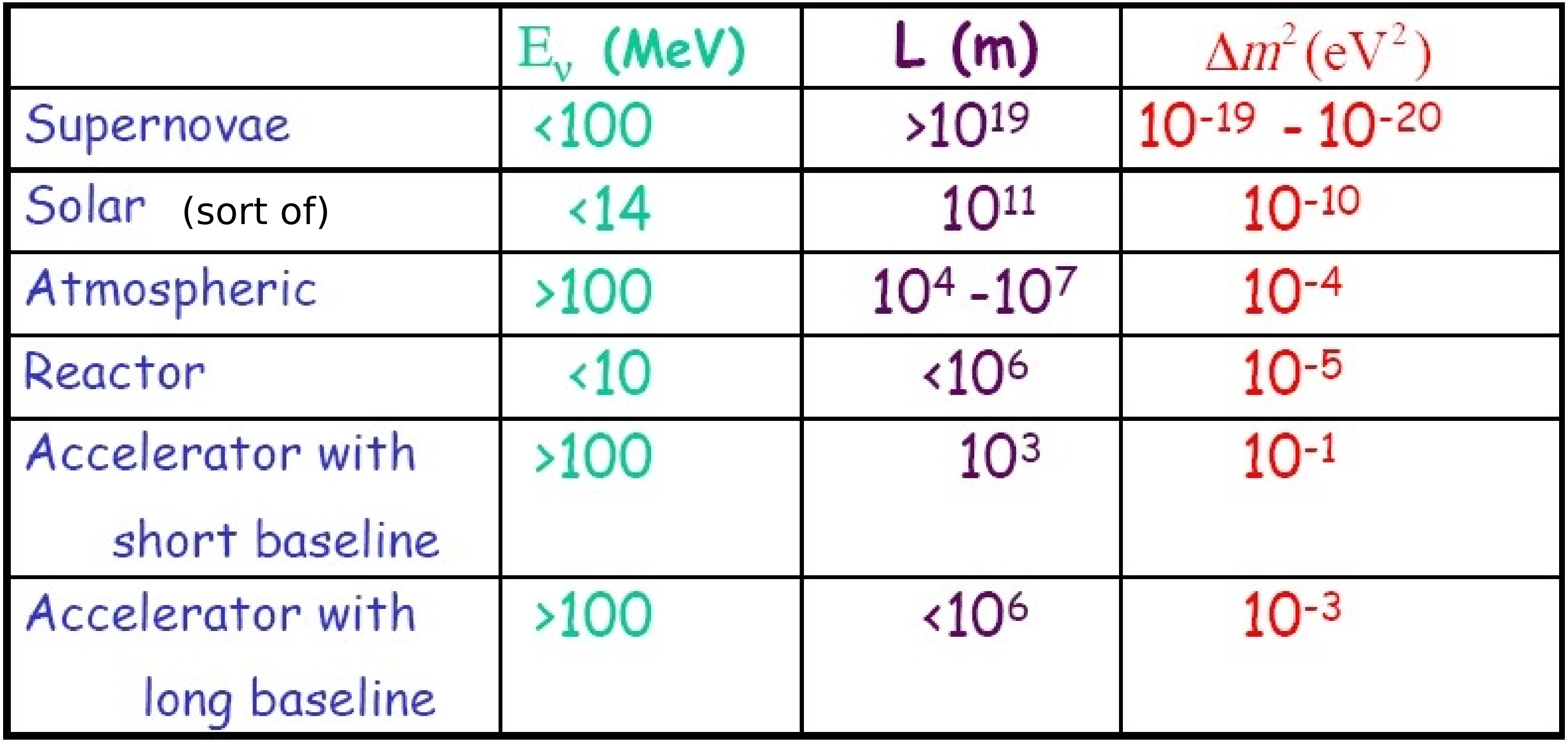


$\frac{L}{E}\ll\Delta m^2$

$\frac{L}{E}\sim\Delta m^2$

$\frac{L}{E}\gg\Delta m^2$

L/E



	E_ν (MeV)	L (m)	$\Delta m^2 (\mathrm{eV}^2)$
Supernovae	<100	$>10^{19}$	$10^{-19} - 10^{-20}$
Solar (sort of)	<14	10^{11}	10^{-10}
Atmospheric	>100	$10^{4} - 10^{7}$	10^{-4}
Reactor	<10	$<10^{6}$	10^{-5}
Accelerator with short baseline	>100	10^{3}	10^{-1}
Accelerator with long baseline	>100	$<10^{6}$	10^{-3}

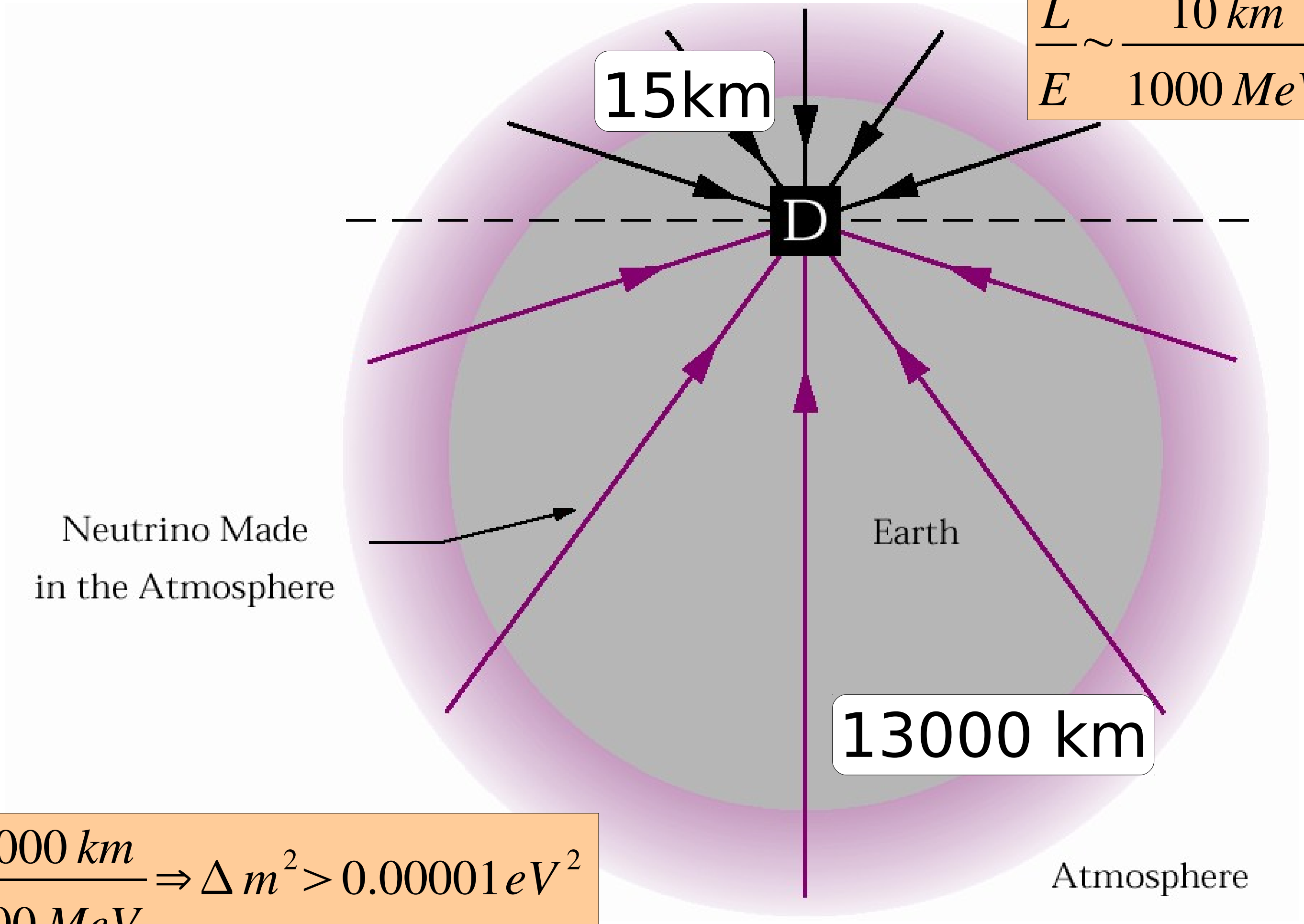
Explaining the atmospheric data

Cosmic Labs

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{2}\frac{L}{E}\right)$$

$\cos\theta_{zenith} = 1.0$

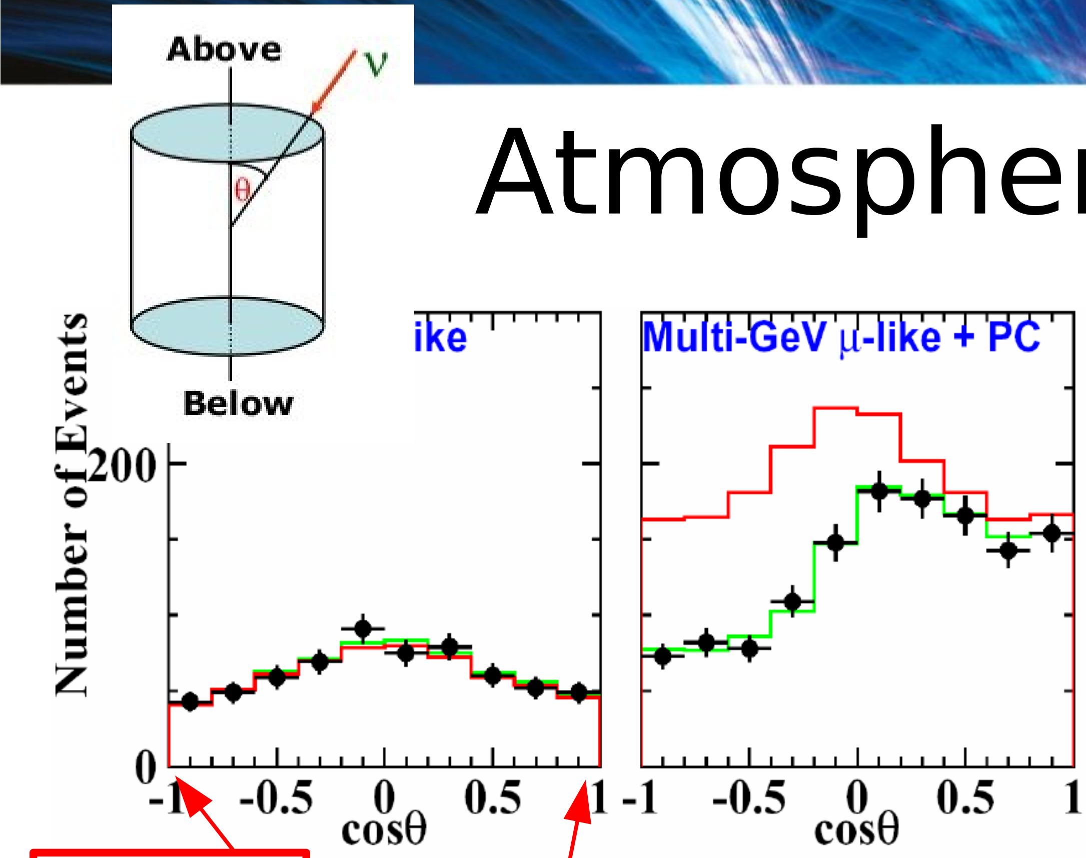
$$\frac{L}{E} \sim \frac{10\,km}{1000\,MeV} \Rightarrow \Delta m^2 > 0.01\,eV^2$$



$$\frac{L}{E} \sim \frac{10000\,km}{1000\,MeV} \Rightarrow \Delta m^2 > 0.00001\,eV^2$$

$\cos\theta_{zenith} = -1.0$

Atmospheric results



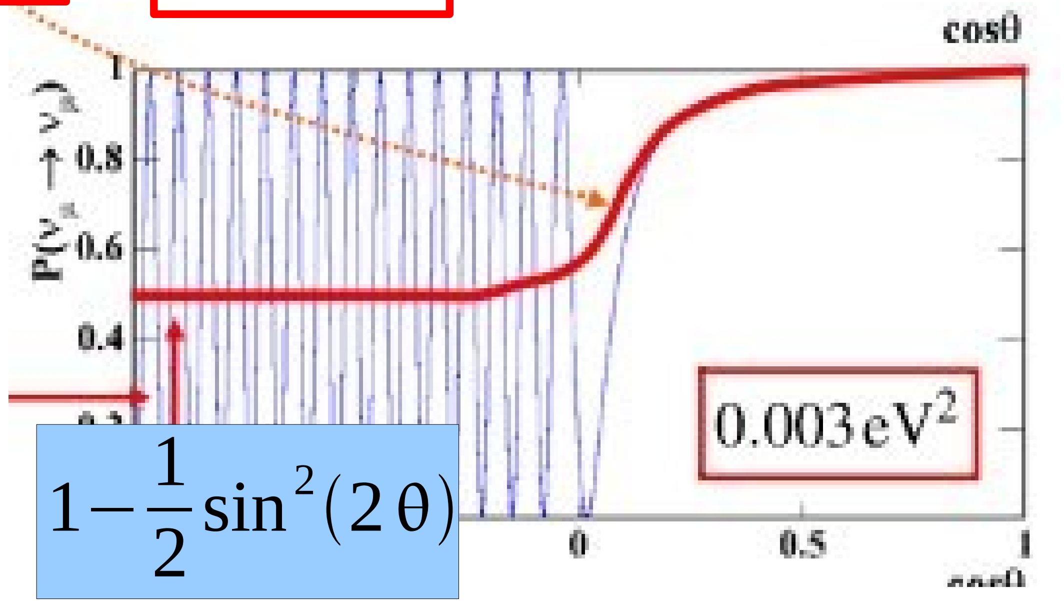
Upcoming

Downgoing

- Prediction for ν_e rate agrees with data.
- ν_μ disappear at large baseline consistent with $\nu_\mu \rightarrow \nu_\tau$
- Don't detect ν_τ as

-below τ mass threshold

-SuperK is awful at τ detection



$$1-\frac{1}{2}\sin^2(2\theta)$$

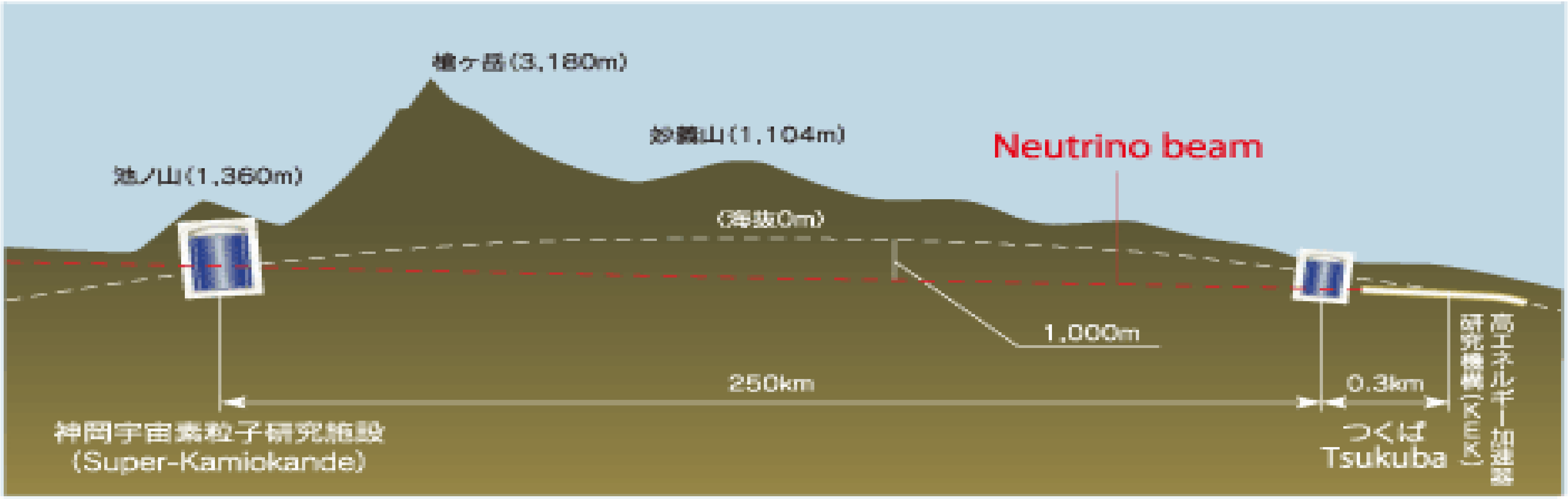
$$\left|\Delta m^2_{atmos}\right| \approx 0.0025\, eV^2$$

$$\sin^2(2\theta_{atmos}) \approx 1.0$$

Accelerator Cross-check

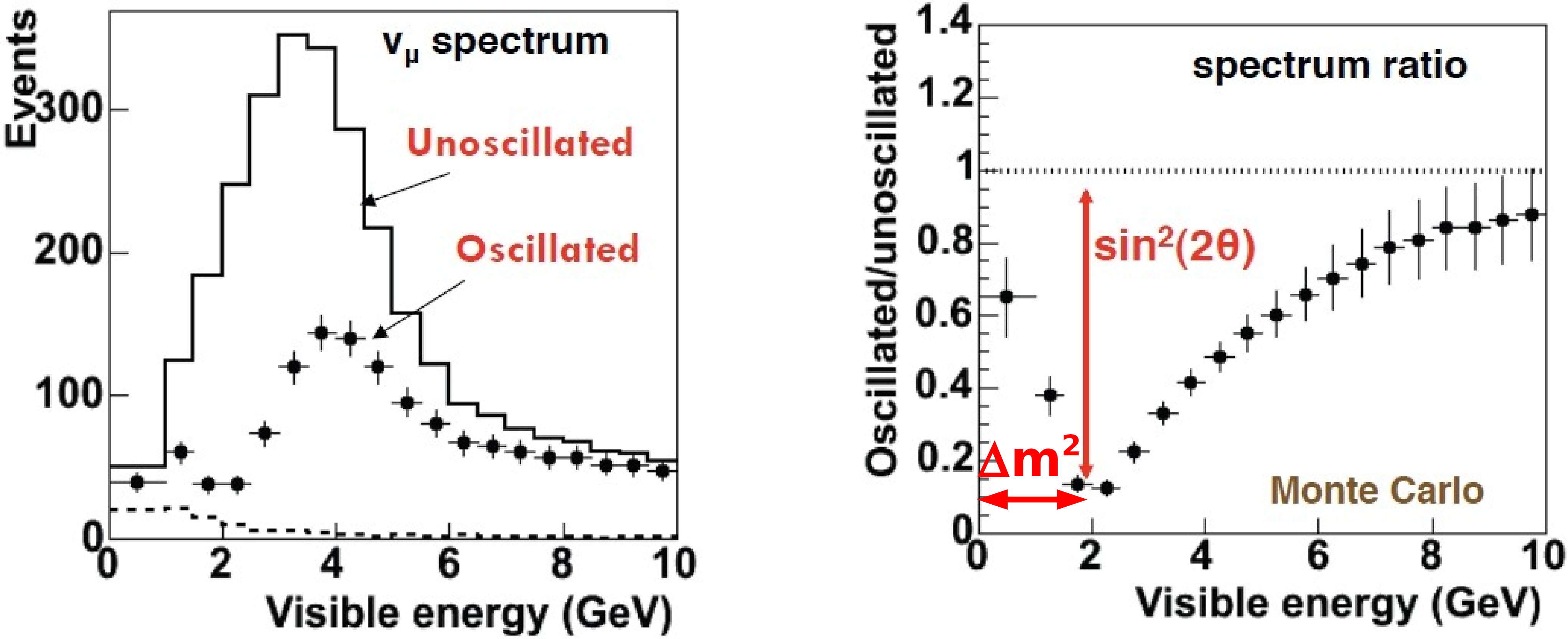
$$\Delta m^2_{atmos} \approx 3 \times 10^{-3}\, eV^2 \rightarrow L/E \approx 400\, km\, GeV^{-1}$$

$$L = 300\, km \rightarrow E_\nu \approx 0.8\, GeV$$



Beam events tagged using GPS at both near and far detector sites

Disappearance Experiments



$$P(\nu_\alpha \to \nu_\alpha) \to \frac{\Phi_\nu(@FD)}{\Phi_\nu(@ND)}$$

Φ_ν : Neutrino Flux

Use Near Detector to measure $\Phi_\nu(@ND)$

Appearance Experiments

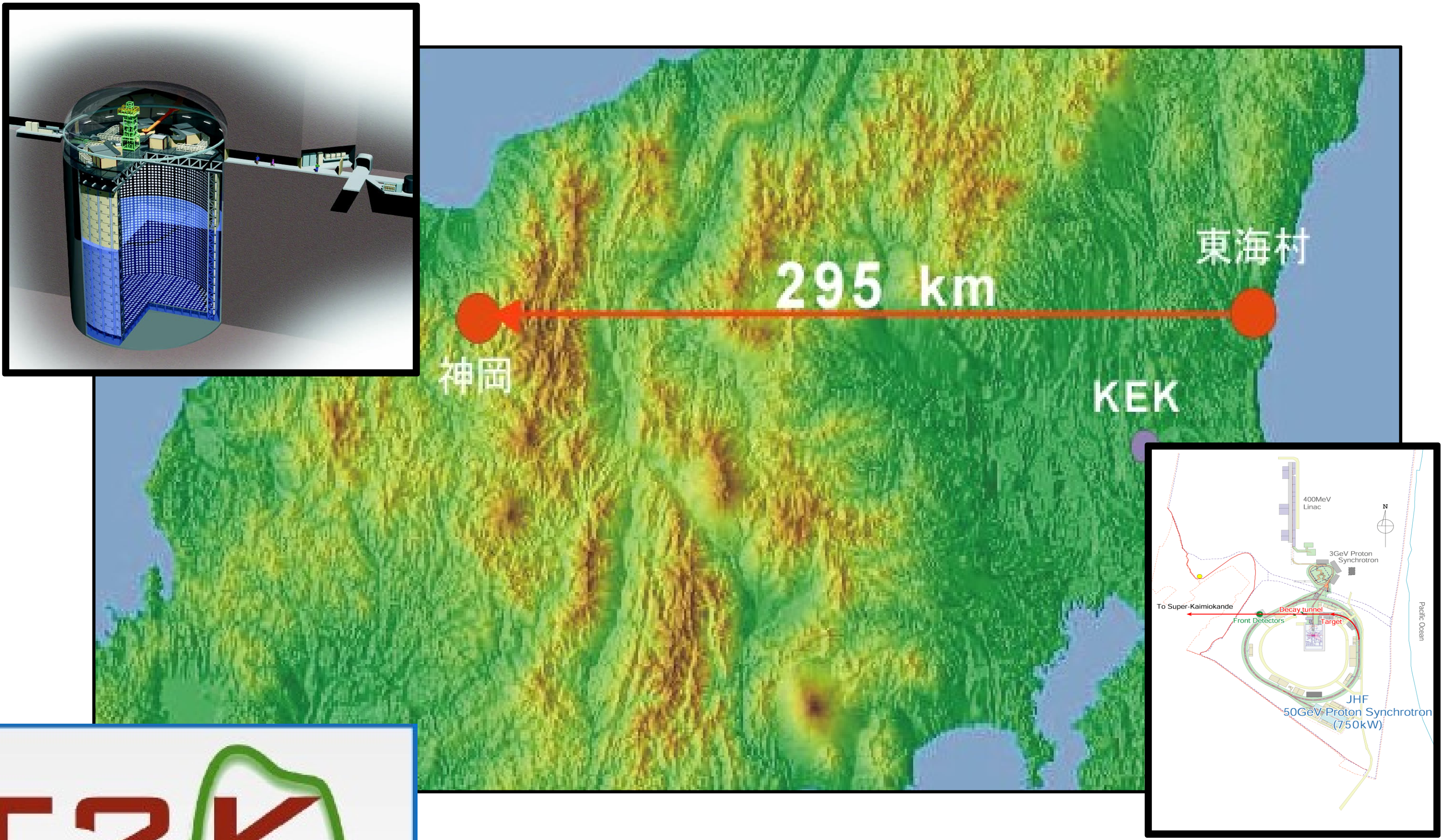
$$P\left(\nu_\alpha \rightarrow \nu_\beta\right)$$

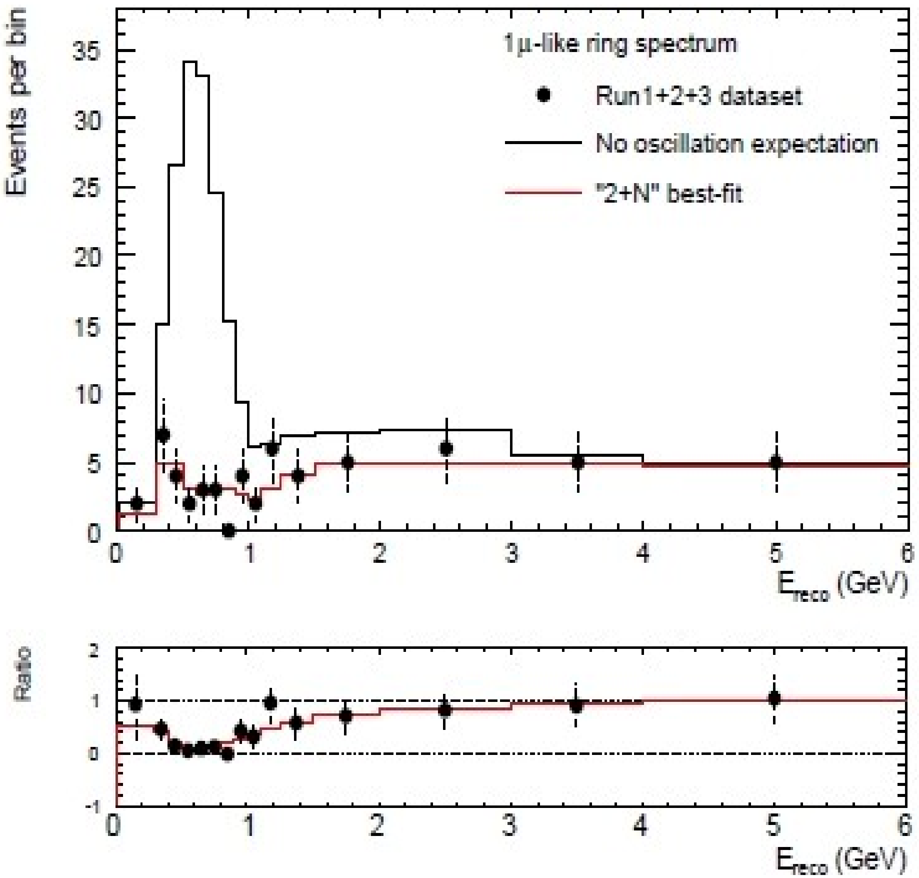
- Look for ν_β appearing in a beam of ν_α
- Usually one has backgrounds as well

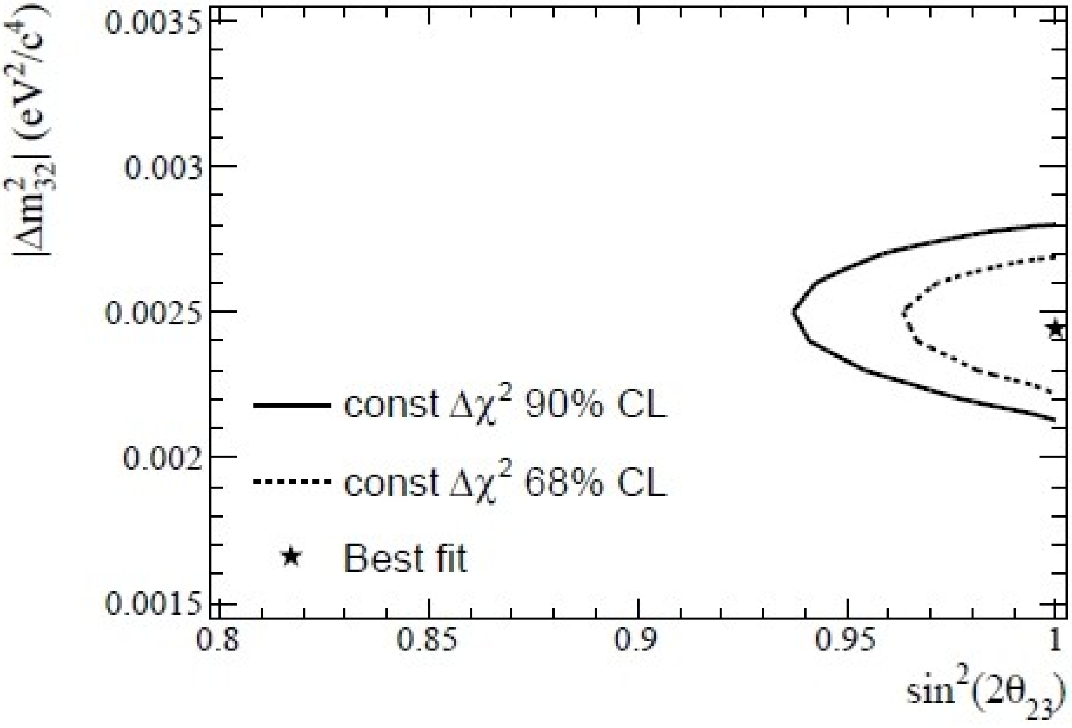
e.g. we know that ν_e can be generated in a ν_μ beam, which acts as a background to $\nu_\mu \rightarrow \nu_e$ searches

- Estimating these backgrounds is usually the difficult part of the experiment. We use a near detector to estimate the background before oscillations occur.

The T2K (Tokai-2-Kamioka) Experiment







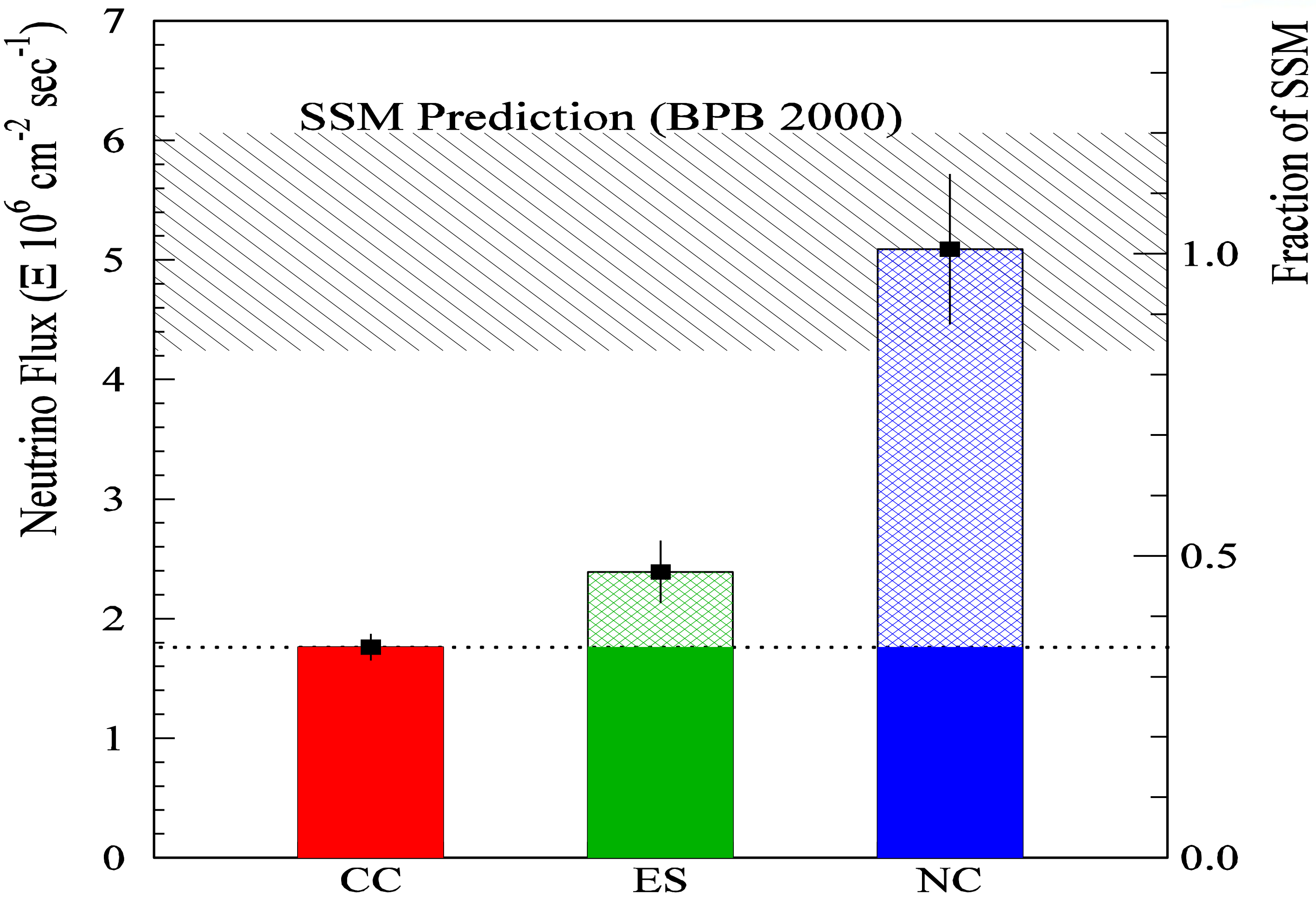
$$\frac{\#\,events\,observed}{\#\,events\,expected}=P(\nu_\mu\rightarrow\nu_\mu)=1-\sin^2(2\theta)\sin^2(\frac{\Delta m^2 L}{4\,E})$$

$$\Delta m^2=2.44^{+0.32}_{-0.31}\times10^{-3}\,eV^2$$

$$\sin^2(2\theta)>0.96\,(@\,90\;CL)$$

Explaining the solar data

SNO Results



5.3 σ appearance of $\nu_{\mu\tau}$ in a ν_e beam

Roughly 70% of ν_e oscillates away

Naively...

First instinct is to assume that neutrinos leave the sun as ν_e and oscillate on their way to the earth. Assuming this

$$L \sim 10^8\, km,\, E_\nu < 10\, MeV \rightarrow \Delta m^2 \sim 3\times 10^{-10}\, eV^2$$

Oscillations come from phase difference between mass states. In a vacuum the phase diff comes from free particle Hamiltonian. In a material there are interaction potentials as well

$$-i\hbar\frac{\partial\psi}{\partial t} = \boxed{E\psi} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \rightarrow -i\hbar\frac{\partial\psi}{\partial t} = \boxed{(E+V)}\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

$$E^2 - p^2 = m_{vac}^2 \rightarrow (E+V)^2 - p^2 = m_{mat}^2 \rightarrow \boxed{m_{mat} \approx \sqrt{m_{vac}^2 + 2EV}}$$

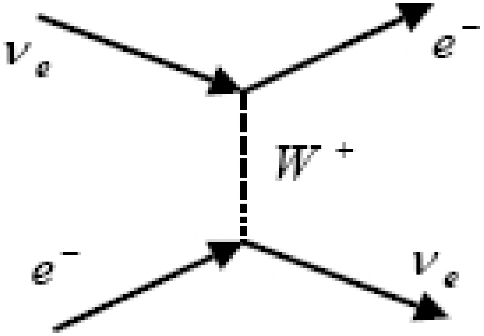
c.f. effective mass of an electron in a semiconductor or light in glass

Oscillations in Matter

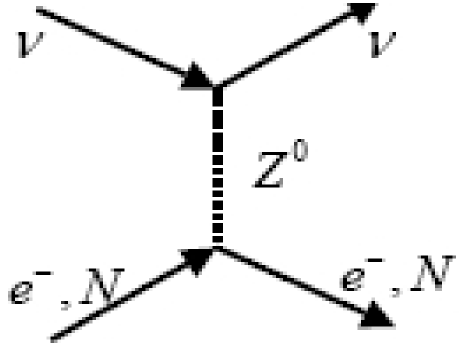
Electrons exist in standard matter – μ/τ do not. Electron neutrinos travelling in matter can experience an extra charged current interaction that other flavours cannot.

$$V_W = \sqrt{2}\, G_F N_e$$

Interaction Potential



ν_e



ν_e, ν_μ, ν_τ

$$V_Z = -\frac{\sqrt{2}}{2} G_F N_n$$

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

Oscillation probability modified by matter effects

$$\boxed{\begin{gathered}\Delta m_M^2 = \Delta m_V^2 \sqrt{\sin^2(2\theta) + (\cos 2\theta - \zeta)^2} \\ \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2}\end{gathered}}$$

$$\zeta = \frac{2\sqrt{2}\, G_F N_e E}{\Delta m_V^2}$$

Implications

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta_V}{\sin^2 2\theta_V + (\cos 2\theta_V - \zeta)^2} \qquad \zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2_{Vac}}$$

- At high densities $\zeta \rightarrow \infty$: $\sin^2(2\theta_M) \rightarrow 0$ for any θ_V
- At low densities $\zeta \rightarrow 0$: $\sin^2(2\theta_M) \rightarrow \sin^2(2\theta_V)$

- No effect if $\theta_V = 0$

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2} = \cos 2\theta_V \;\Rightarrow\; \sin^2 2\theta_M = 1$$

- Even if the vacuum mixing angle is tiny, there is a density for which the matter mixing is large

Mass heirarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \qquad \zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m_V^2}$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - sgn(\Delta m^2)|\zeta|)^2}$$

The matter effect is sensitive to the sign of Δm^2

This is the only means we have to determine the order of the mass states.

In the sun

- ν_e born in high density conditions in the solar core
- Density is too high to support oscillations
- As they propagate outwards they hit a region of density that supports the resonance condition. They oscillation to ν_μ here.

- Only some do this – very low energy neutrinos (PP) are too low in energy to oscillate in matter, but will in the vacuum.

- Matter enhanced oscillation predominantly affect the Be7 flux.

Solar neutrinos

SNO/SuperK/other experimental data show that the solar neutrino oscillations mostly arise from matter effects.

The neutrinos have oscillated by the time they get to the solar surface

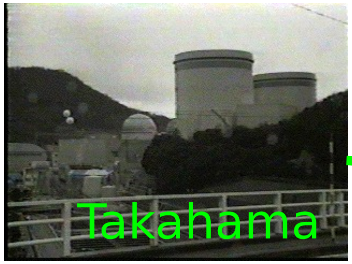
Transition is mostly : $\nu_e \rightarrow \nu_\mu$

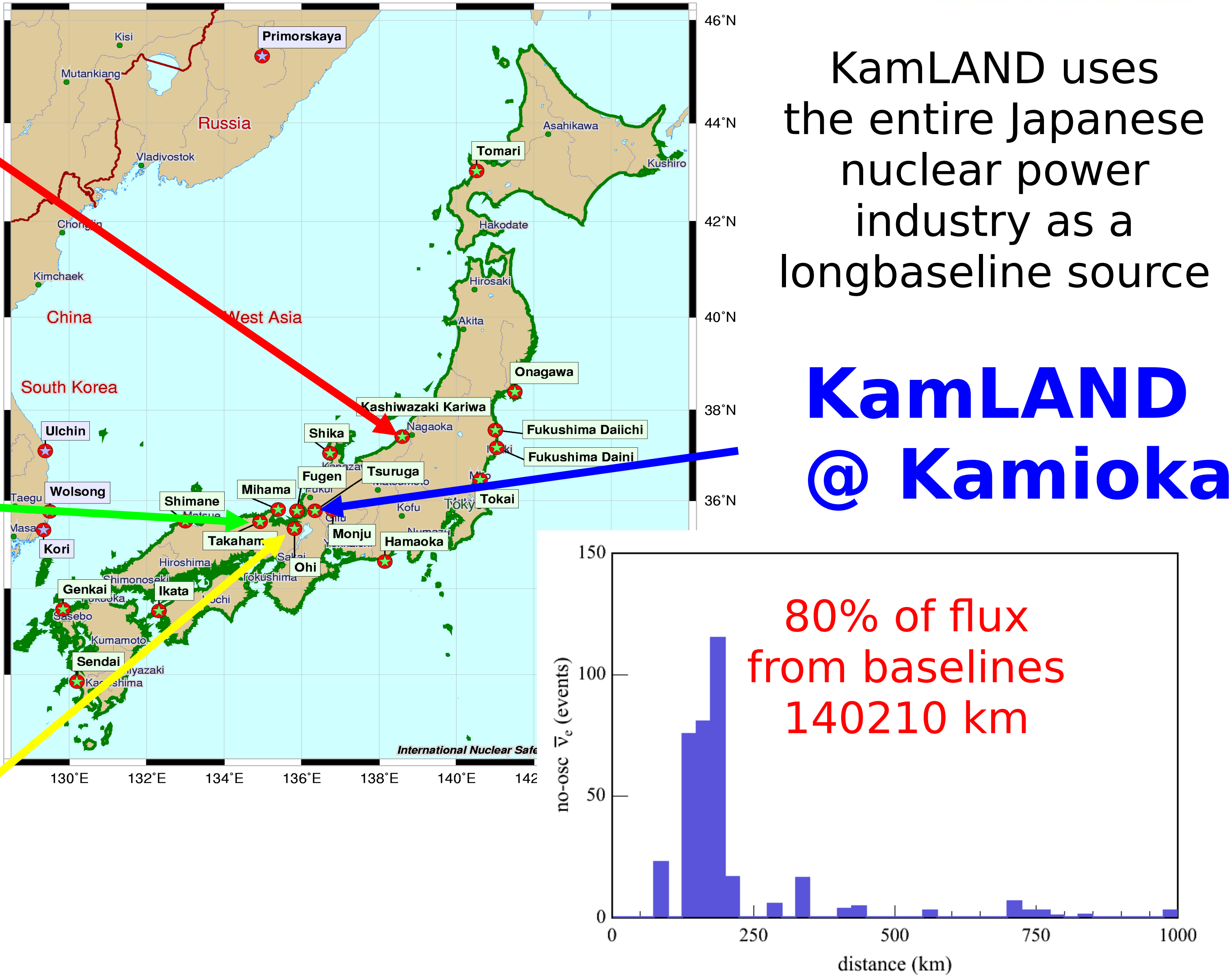
$$\theta_{e\mu} = 32.5^o \pm 2.4^o$$

$$\Delta m_{12}^2 = +7.1\, x\, 10^{-5}\, eV^2$$

we know the sign of this one

KamLAND





80% of flux from baselines 140210 km

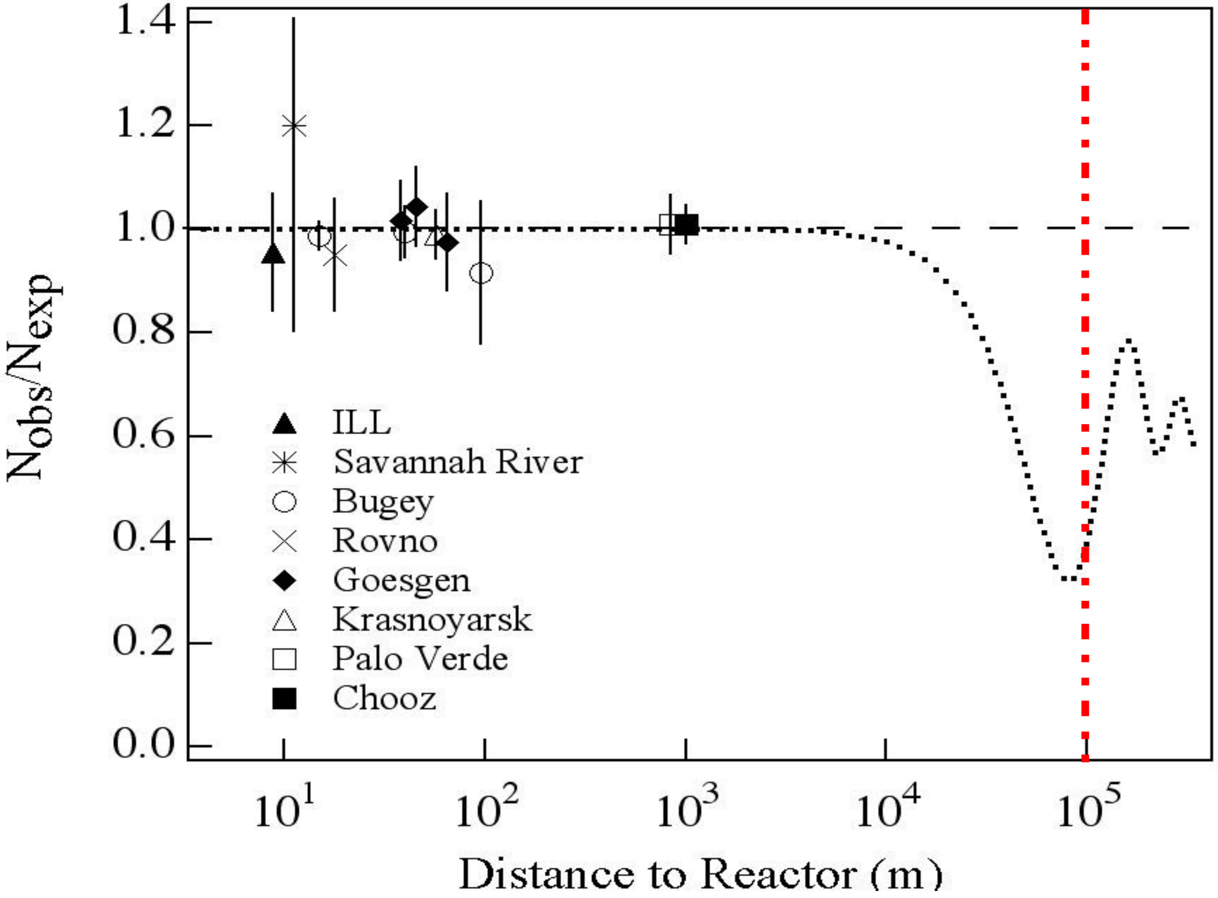
KamLAND uses the entire Japanese nuclear power industry as a longbaseline source

KamLAND @ Kamioka

KAMLAND

A test of the solar oscillation sector. KAMLAND baseline is too short for matter effects.

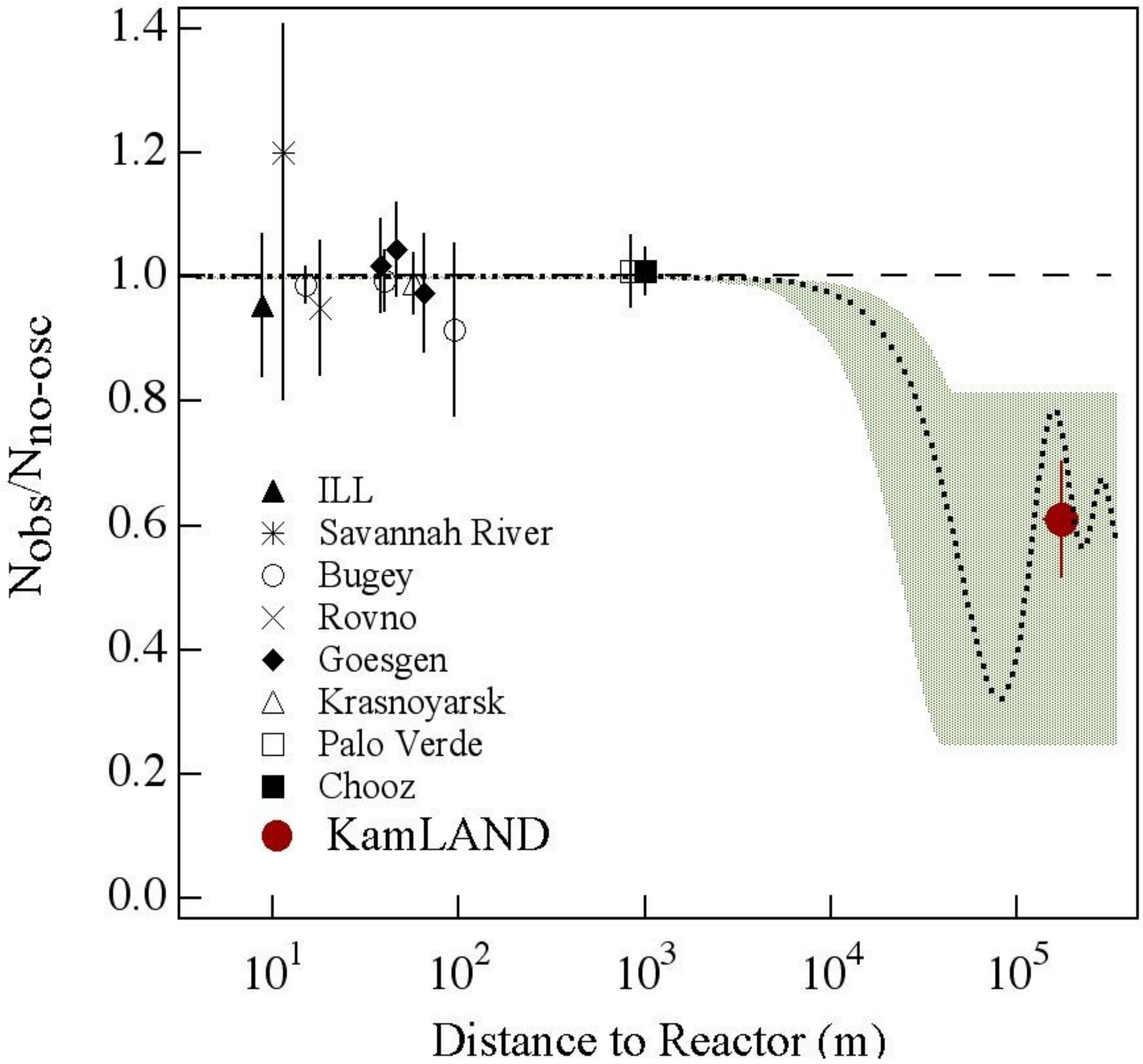
$$\frac{L}{E} \sim \frac{200}{0.002} = 1\times10^{5} \Rightarrow \Delta m^{2} = 1\times10^{-5}\, eV^{2}$$



If one was 100km from reactor

KamLAND

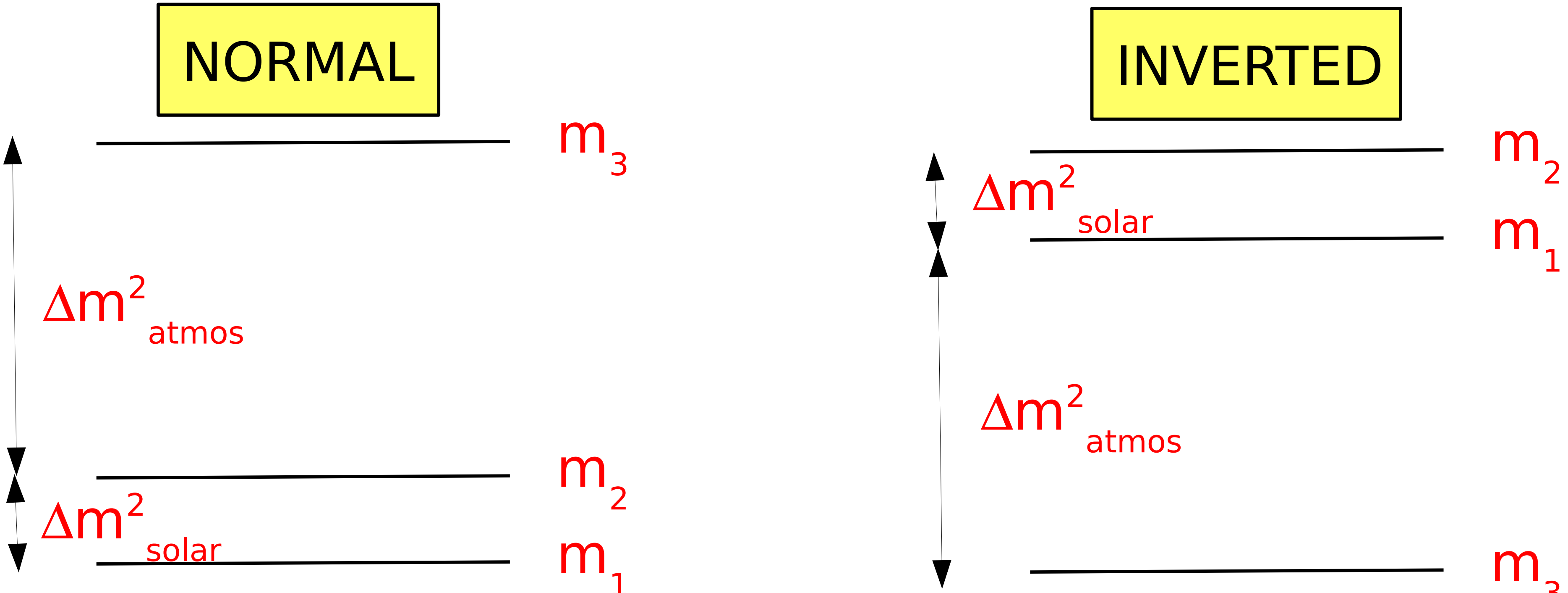
$$\Delta m^2_{solar} = +7.9 \pm 0.5 \times 10^{-5}\, eV^2 \qquad \tan^2(\theta) = 0.4 \pm 0.09$$



Summary so far

Atmospheric mass scale : $\left|\Delta m^2_{atmos}\right| = 2.35 \times 10^{-3}\, eV^2$

Solar mass scale : $\Delta m^2_{solar} = +\,7.9 \times 10^{-5}\, eV^2$



3 Δm^2 but only two are independent → 3 massive neutrinos

There are actually 3 neutrinos….