Last Lecture

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta)\sin^2\left(1.27(\Delta m^2/eV^2)\frac{L/km}{E_{nu}/GeV}\right)$$

Two-flavour oscillations

Vacuum mixing parameters

Atmospheric neutrinos : $\nu_\mu \rightarrow \nu_\tau$

$$\Delta m^2_{atmos} = \left|2.44^{+0.32}_{-0.31} \times 10^{-3}\right| eV^2 \qquad \sin^2(2\theta_{atmos}) > 0.96 (@90\ CL)$$

Solar neutrinos : $\nu_e \rightarrow \nu_\mu$

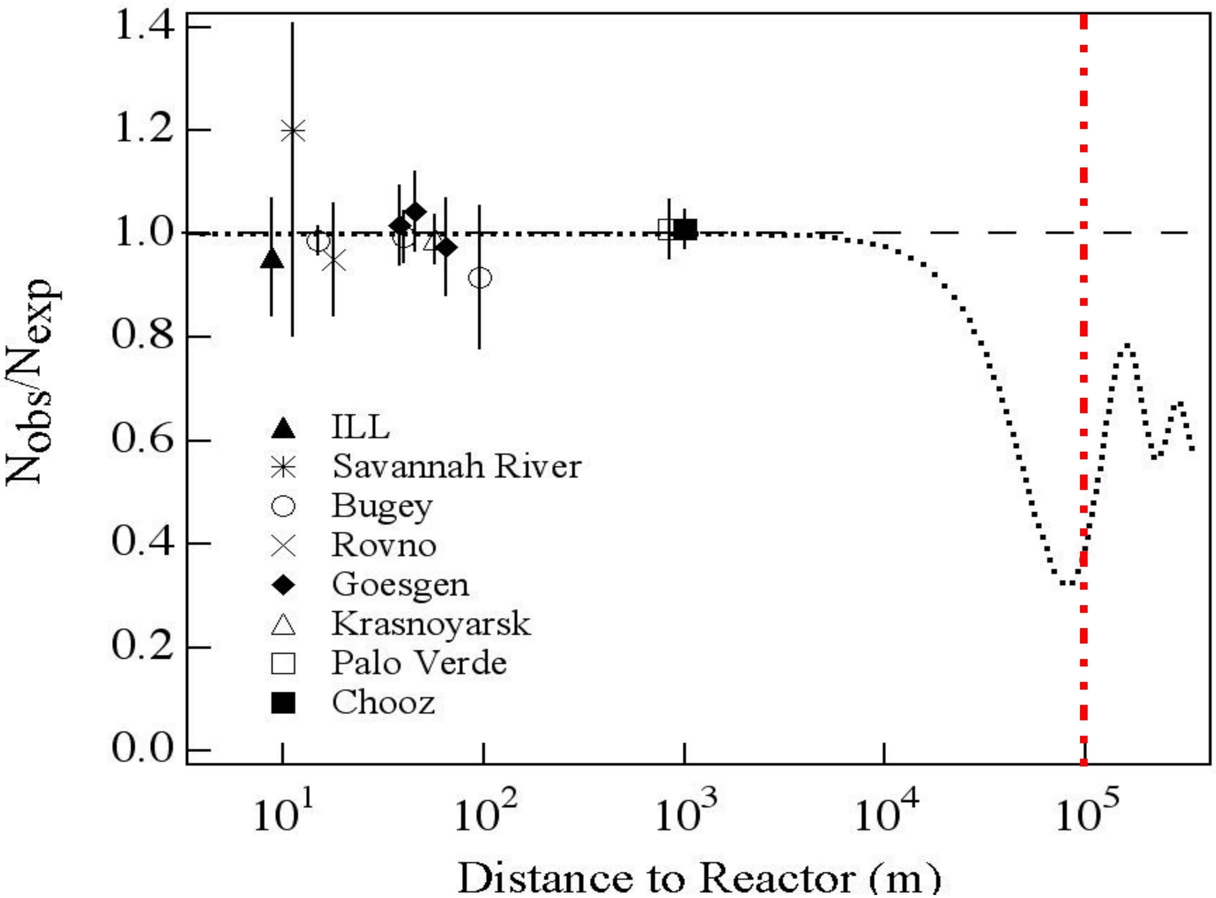
$$\Delta m^2_{sol} = +7.1\, x\, 10^{-5} eV^2 \qquad \sin^2(2\theta_{sol}) = 0.82 \pm 0.06$$

Sign known from matter effects

KAMLAND

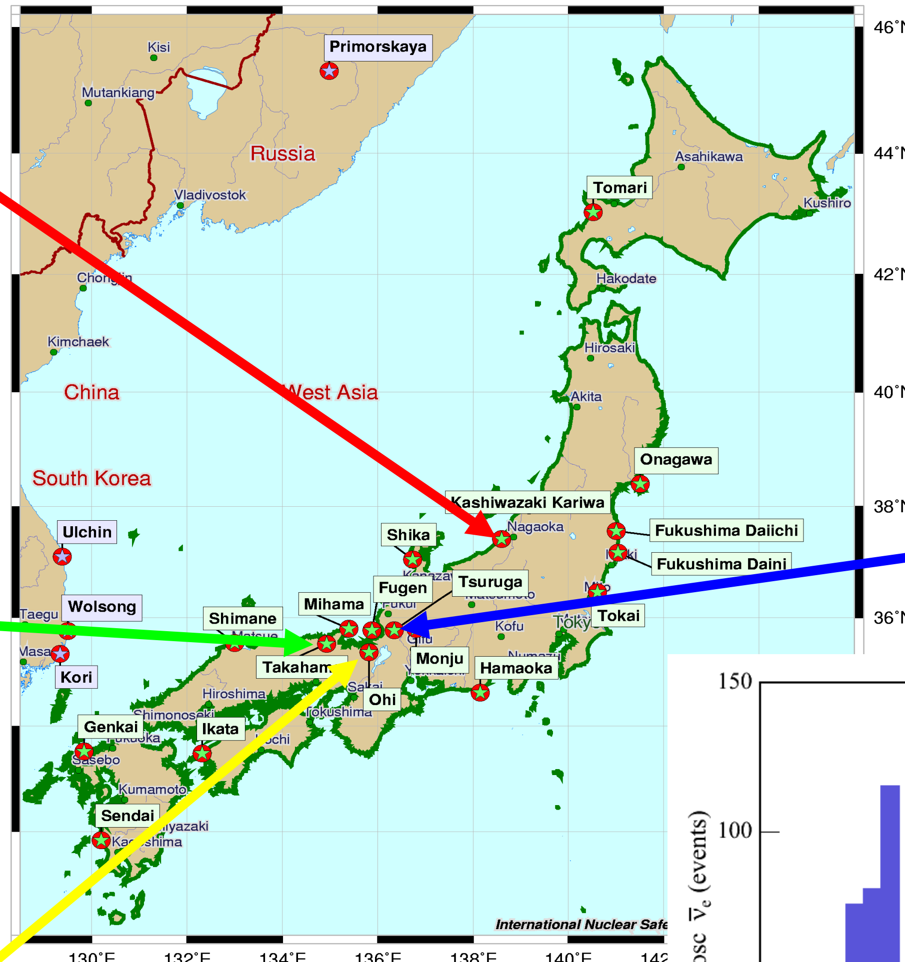
A test of the solar oscillation sector. KAMLAND baseline is too short for matter effects.

$$\frac{L}{E} \sim \frac{200}{0.002} = 1 \times 10^5 \Rightarrow \Delta m^2 = 1 \times 10^{-5} eV^2$$



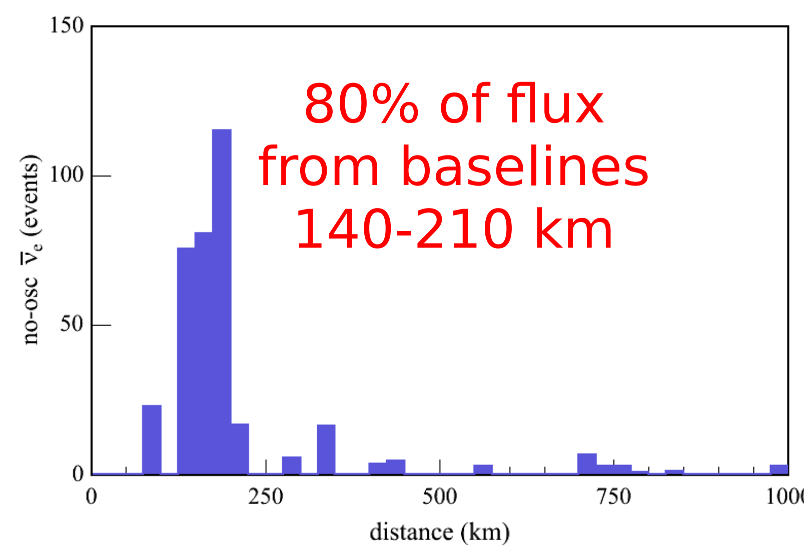
If one was 100km from reactor

KamLAND



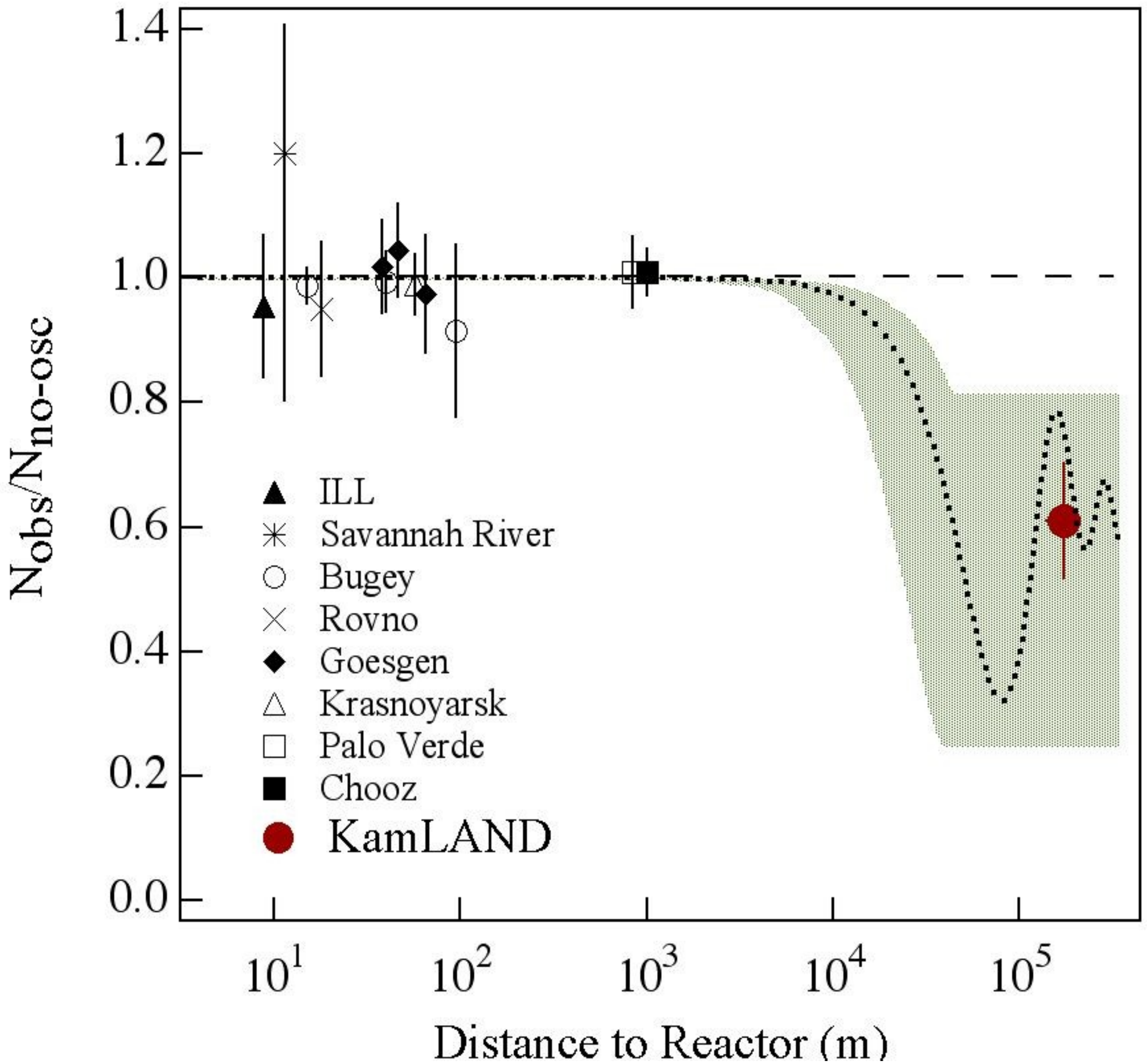
KamLAND uses the entire Japanese nuclear power industry as a longbaseline source

KamLAND @ Kamioka

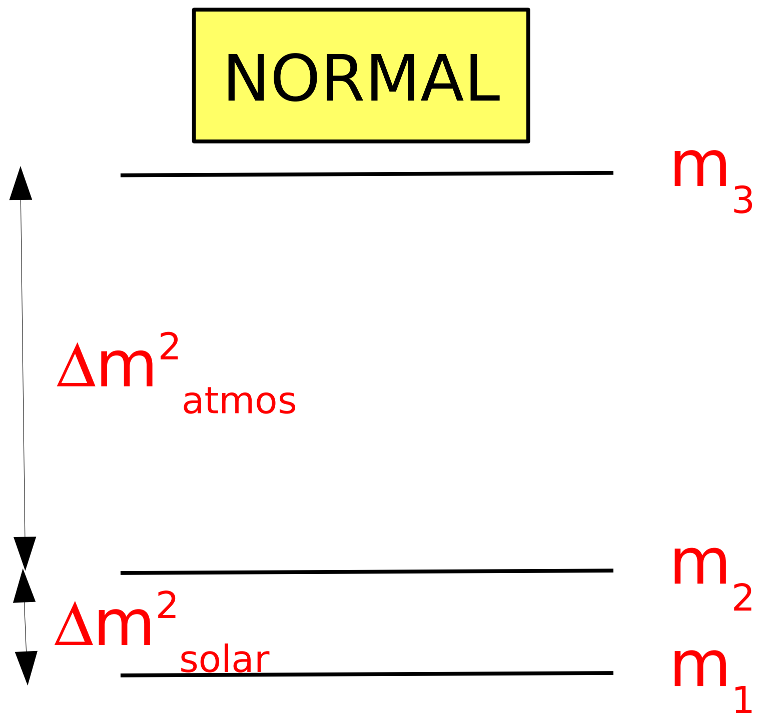


KamLAND

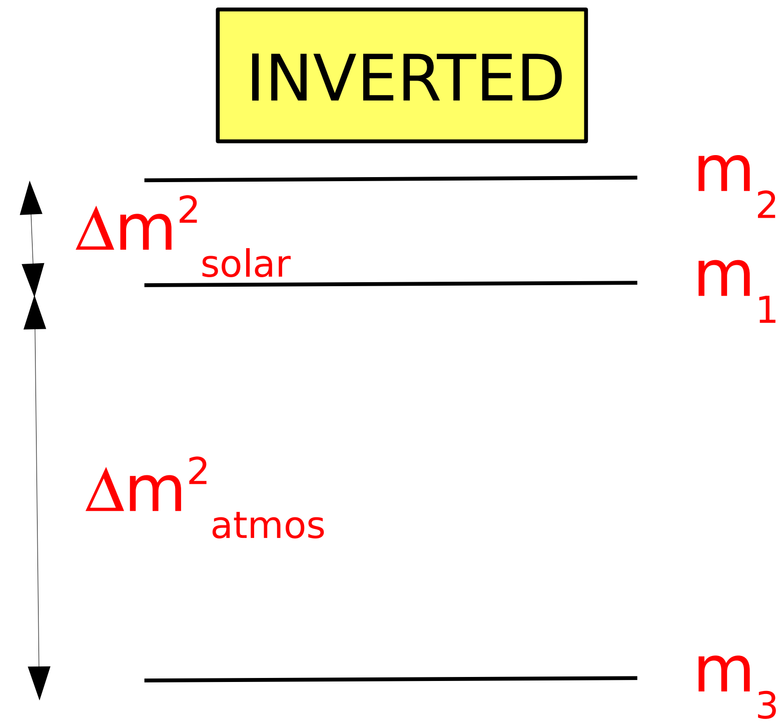
$$\Delta m^2_{solar} = +7.9 \pm 0.5 \times 10^{-5}\, eV^2 \qquad \tan^2(\theta) = 0.4 \pm 0.09$$



Possible mass hierarchies



1 heavy and 2 light states



2 heavy and 1 light state

3 Δm^2 but only two are independent → 3 massive neutrinos

There are actually 3 neutrinos….

3 Flavour Oscillations

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Leftrightarrow U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

U is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. In 3-dimensions, U can have complex parameters

$$Prob(\nu_\alpha \to \nu_\beta) = |\sum_{i=1}^{3} < \nu_\alpha | \nu_i > e^{-i\varphi_i} < \nu_i | \nu_\beta > |^2$$

3 Flavour Oscillations

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$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Leftrightarrow U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

U is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. In 3-dimensions, U can have complex parameters

$$Prob(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$
$$+ 2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

3-flavour oscillations

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$

2 independent Δm^2

$$Prob(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

3-flavour oscillations

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$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$

Three angles

$$Prob(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

3-flavour oscillations

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$

complex CP violating phase

$$Prob(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

Probabilities ($\delta=0$)

$$P(\nu_\alpha \to \nu_\beta)_{(\alpha \neq \beta)} = -4 \left\{ \underbrace{U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\beta 2}}_{c_{12}} \sin^2\left(1.27 \frac{\Delta m_{12}^2 L}{E}\right) + \underbrace{U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\beta 3}}_{c_{13}} \sin^2\left(1.27 \frac{\Delta m_{13}^2 L}{E}\right) + \underbrace{U_{\alpha 2} U_{\beta 2} U_{\alpha 2} U_{\beta 3}}_{c_{23}} \sin^2\left(1.27 \frac{\Delta m_{23}^2 L}{E}\right) \right\}$$

Now,

$$\Delta m_{13}^2 \approx \Delta m_{23}^2 = \Delta m^2$$

$2.5 \times 10^{-3}\ \mathrm{eV}^2$

"Large" mass splitting (atmos)
Small wavelength

$$\Delta m_{12}^2 = \delta m^2$$

$7.0 \times 10^{-5}\ \mathrm{eV}^2$

"small" mass splitting (solar)
Large wavelength

Probabilities

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For **large mass splitting (Δm^2)** and $\delta = 0$

$$P(\nu_\mu \to \nu_\tau) = \cos^4\theta_{13}\sin^2 2\theta_{23}\sin^2(1.27\Delta m_{23}^2 \frac{L}{E})$$

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_{13}\sin^2\theta_{23}\sin^2(1.27\Delta m_{23}^2 \frac{L}{E})$$

$$P(\nu_e \to \nu_\tau) = \sin^2 2\theta_{13}\cos^2\theta_{23}\sin^2(1.27\Delta m_{23}^2 \frac{L}{E})$$

For **small mass splitting (δm^2)** and $\delta = 0$

$$P(\nu_e \to \nu_{\mu,\tau}) = \cos^2\theta_{13}\sin^2 2\theta_{12}\sin^2(1.27\Delta m_{12}^2 \frac{L}{E}) + \frac{1}{2}\sin^2\theta_{13}$$

Probabilities : $\theta_{13} = 0$

For **large mass splitting (Δm^2)**, $\delta = 0$, $\theta_{13} = 0$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta_{23} \sin^2(1.27 \Delta m^2_{23} \frac{L}{E})$$

$$P(\nu_\mu \rightarrow \nu_e) = 0$$

$$P(\nu_e \rightarrow \nu_\tau) = 0$$

Atmospheric oscillations

For **small mass splitting (δm^2)** , $\delta = 0$, $\theta_{13} = 0$

$$P(\nu_e \rightarrow \nu_{\mu,\tau}) = \sin^2 2\theta_{12} \sin^2(1.27 \Delta m^2_{12} \frac{L}{E})$$

Solar oscillations

If $\delta, \theta_{13} = 0$, the PMNS matrix decouples into atmospheric (2-3) and a solar (1-2) sectors and we can treat oscillations at each mass splitting as effectively independent.

So how big is θ_{13}?

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$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Is it zero ?   This would be bad news since the CP violating phase always appears in the PMNS matrix *together* with θ_{13}.

Zero θ_{13} would make any CP violation in the light neutrino sector unobservable in principle.

Better try to measure it......

How do we measure θ_{13}?

$\nu_\mu \rightarrow \nu_e$ oscillations with atmospheric L/E

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 (1.27 \Delta m^2_{23} \frac{L}{E})$$

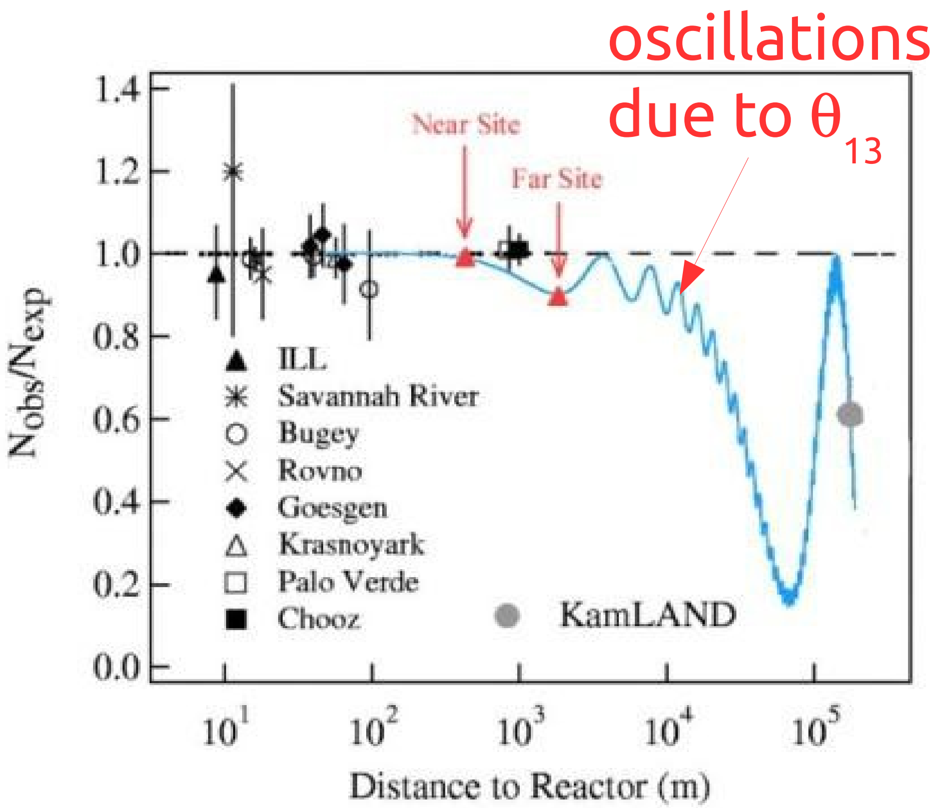
ν_e appearance in a ν_μ beam – ideal for *accelerator experiments*

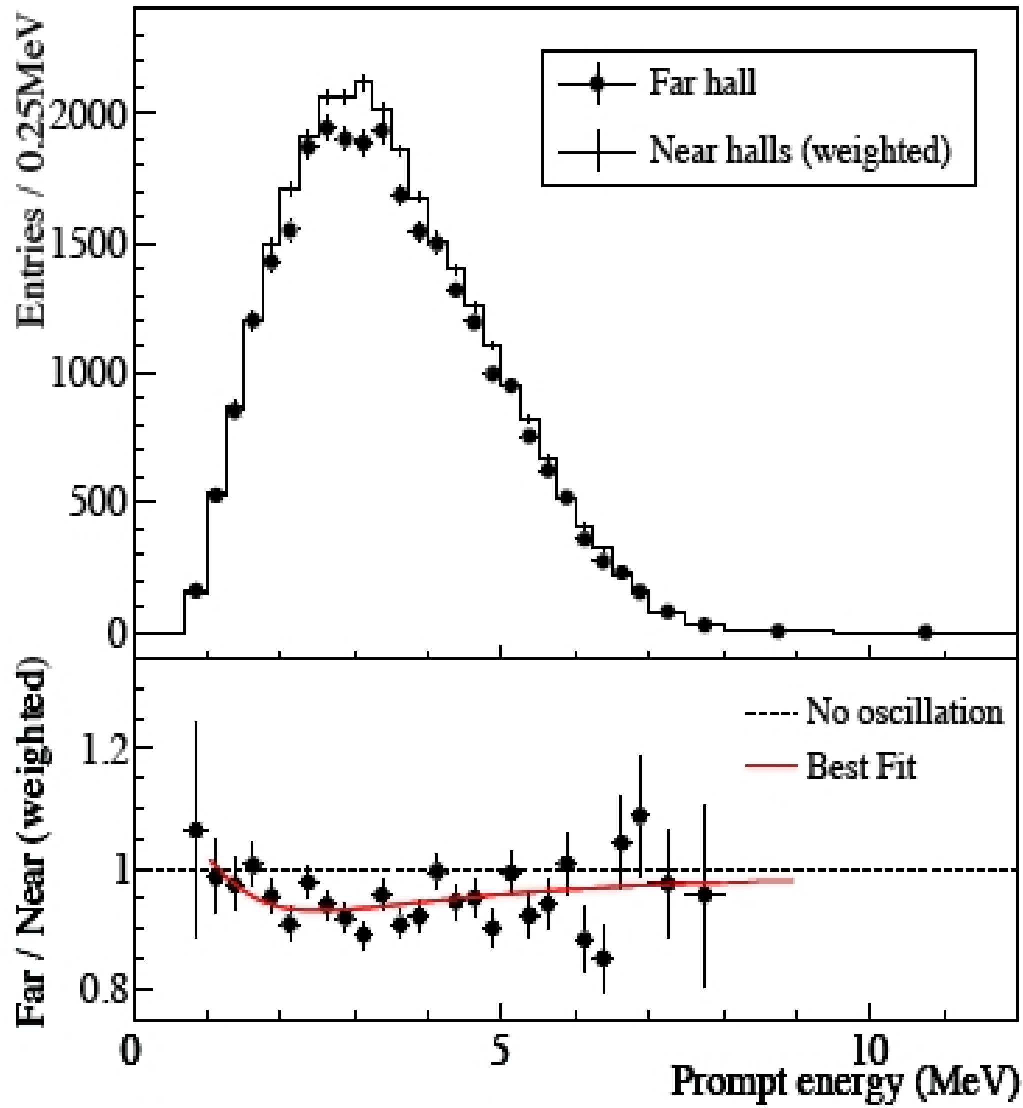
$\nu_e \rightarrow \nu_x$ disappearance oscillations with atmospheric L/E

$$p(\overline{\nu_e} \rightarrow \overline{\nu_x}) \overset{\hat{C}\hat{P}}{=} P(\nu_e \rightarrow \nu_x) = 1 - \sin^2(2\theta_{13}) \sin^2(1.27 \Delta m^2_{23} \frac{L}{E})$$

$\overline{\nu}_e$ disappearance – ideal for *reactor experiments*

θ_{13} from reactors

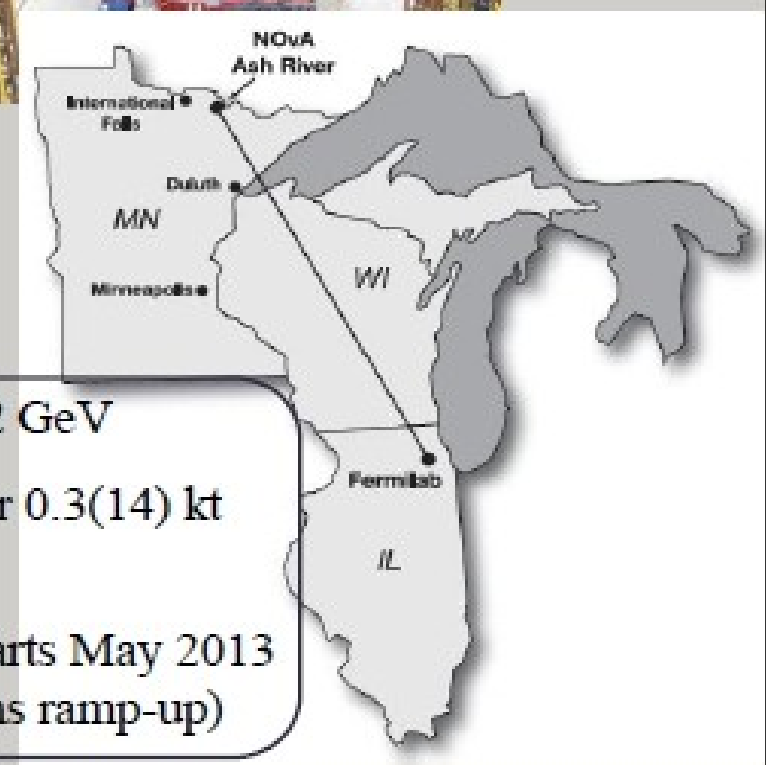




$$\sin^2 2\theta_{13} = 0.090 \pm 0.008$$

Current Experiments : ν_e appearance

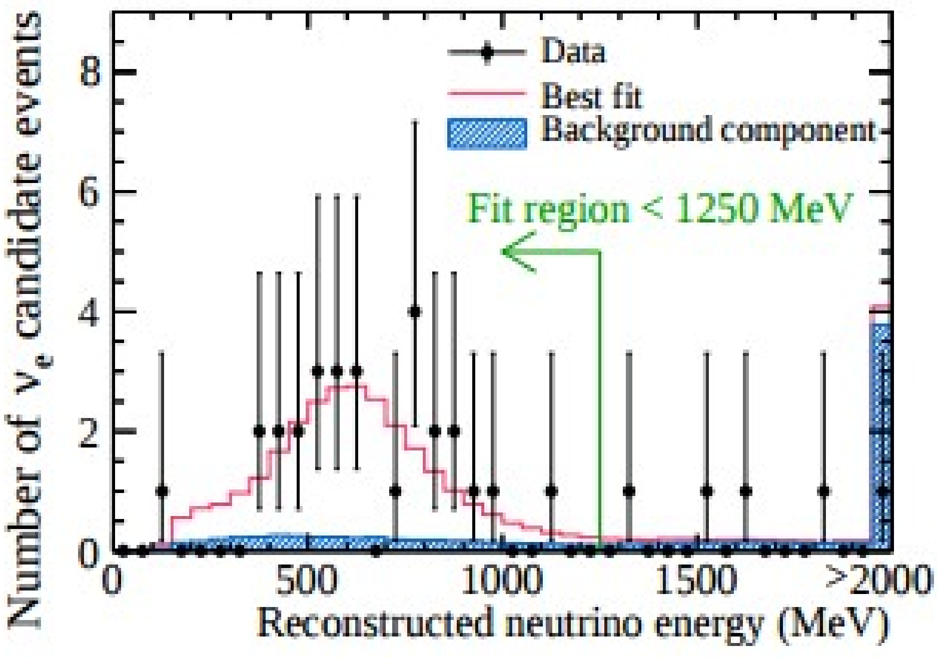
- L=295km, <E>=0.7GeV
- ND280 Near Detector, SuperK (22.5 kt) as Far Detector
- JPARC beam: currently 200kW ramping up to 700kW (<2019)



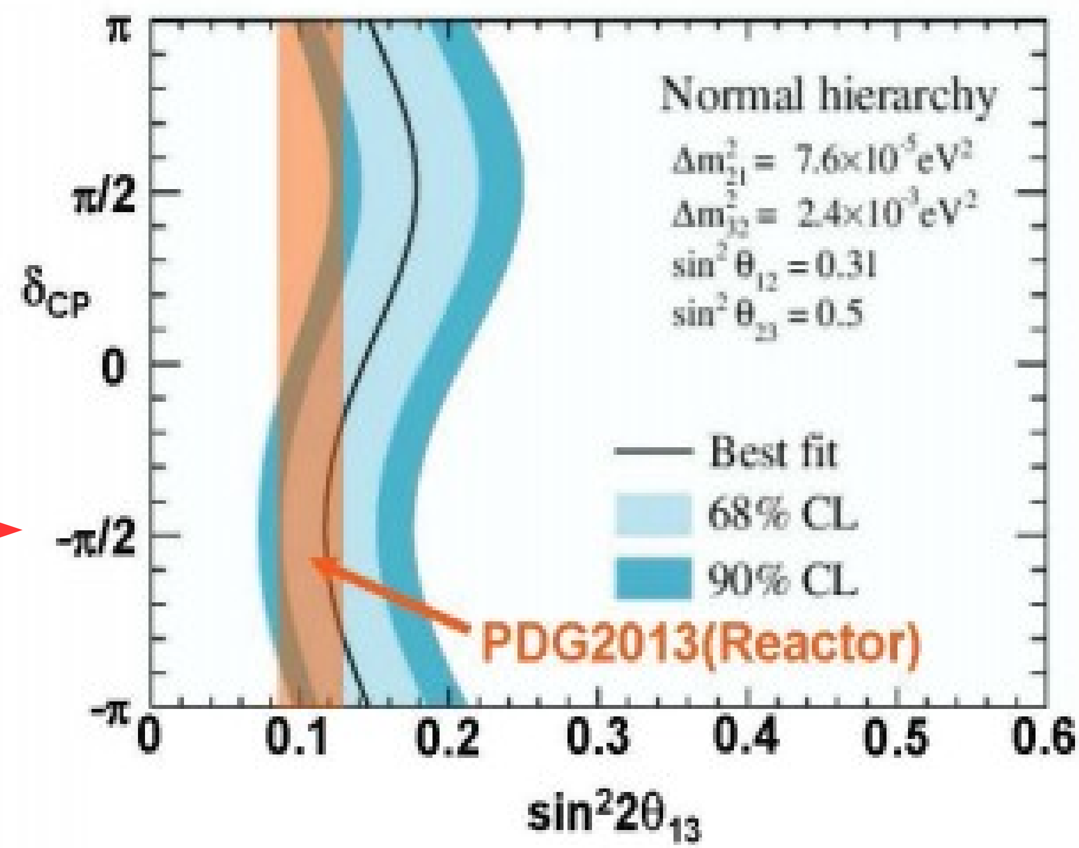
- L=810 km, <E>=2 GeV
- Near(Far) Detector 0.3(14) kt liquid scintillator
- NUMI beam re-starts May 2013 @ 700 kW (6 months ramp-up)

T2K Results

ν_e events observed in SuperK



Allowed region for (δ, θ_{13})



$$\sin^2(2\theta_{13}) = 0.14 \pm 0.036$$

$$\theta_{13} \approx 10.9^o$$

3-Neutrino Mixing

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric sector

$$\nu_\mu \rightarrow \nu_\tau$$

$$\theta_{e\mu} = 45.0^\circ \pm 2.4^\circ$$

$$\Delta m^2_{23} = |2.4 \times 10^{-3}| eV^2$$

13 Sector

$$\nu_e \rightarrow \nu_\mu$$

$$\theta_{13} = 10.6^\circ \pm 0.3^\circ$$

$$\Delta m^2_{23} = |2.4 \times 10^{-3}| eV^2$$

Solar sector

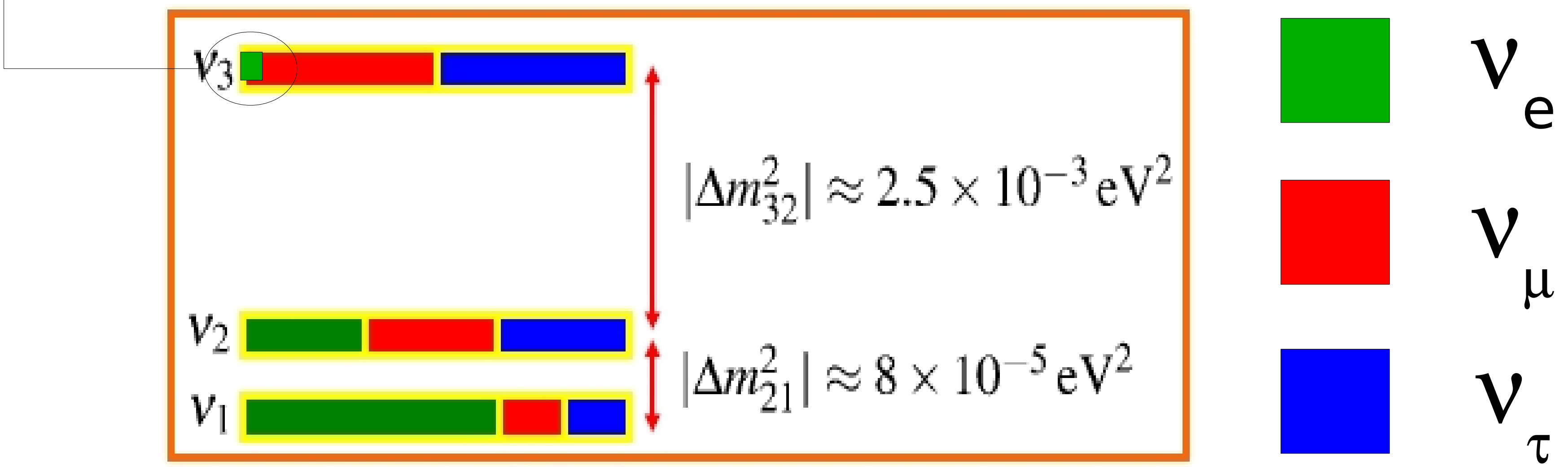
$$\nu_e \rightarrow \nu_\mu$$

$$\theta_{e\mu} = 32.5^\circ \pm 2.4^\circ$$

$$\Delta m^2_{12} = +7.5 \times 10^{-5} eV^2$$

Summary of Current Knowledge

θ_{13} : how much ν_e is in ν_3



$$U_{MNSP} = \begin{pmatrix} 0.8 & 0.5 & -0.15 \\ -0.4 & 0.7 & 0.6 \\ 0.4 & -0.5 & 0.7 \end{pmatrix}$$

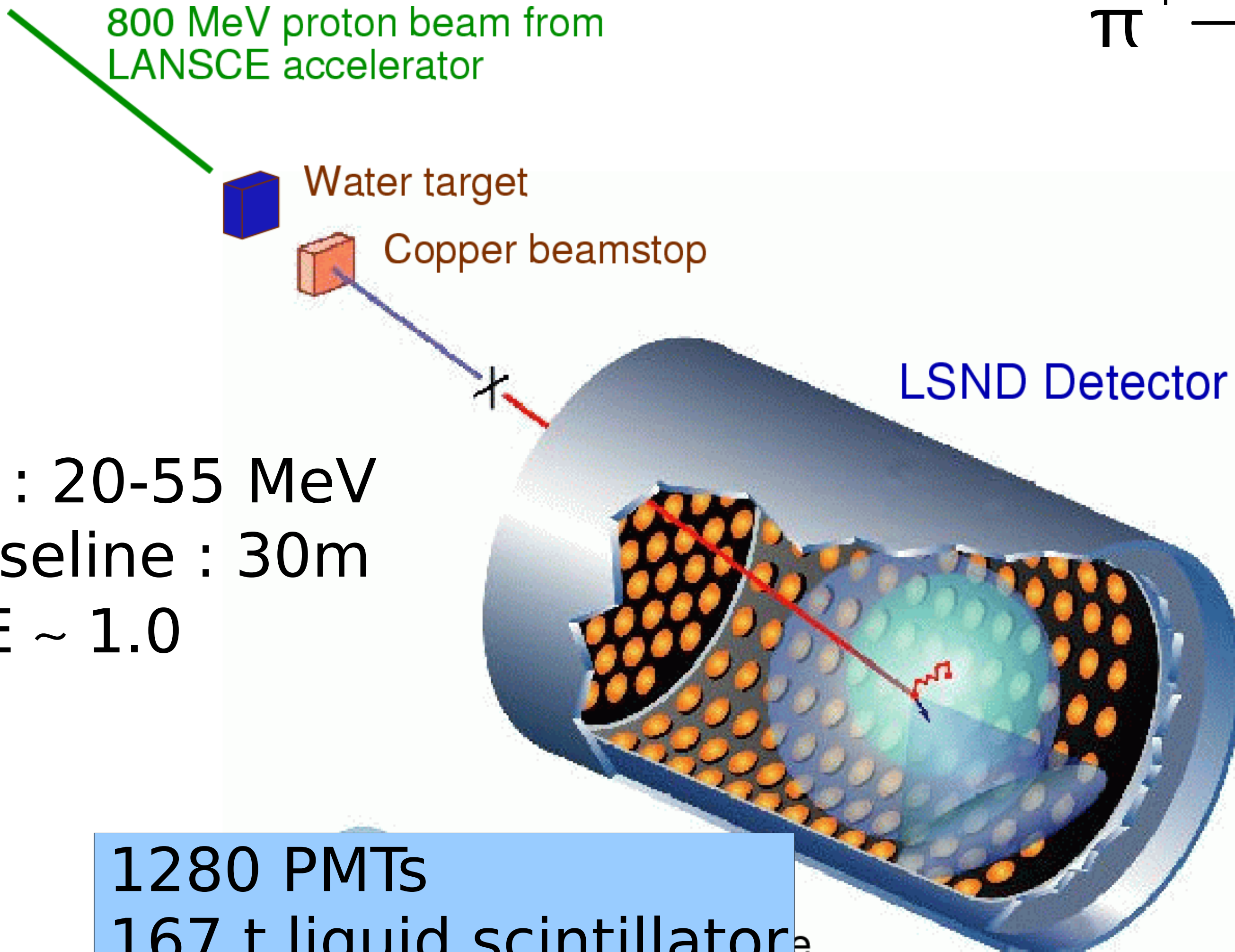
Some elements only known to 10-30%

Very very different from the quark CKM matrix

A bonfire of anomalies

The fly in the ointment

The LSND experiment was the first accelerator experiment to report a positive appearance signal



$E\nu$: 20-55 MeV
baseline : 30m
L/E ~ 1.0

1280 PMTs
167 t liquid scintillator

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\hookrightarrow e^+ \nu_e \bar{\nu}_\mu$$
$$\Updownarrow$$
$$\bar{\nu}_e$$

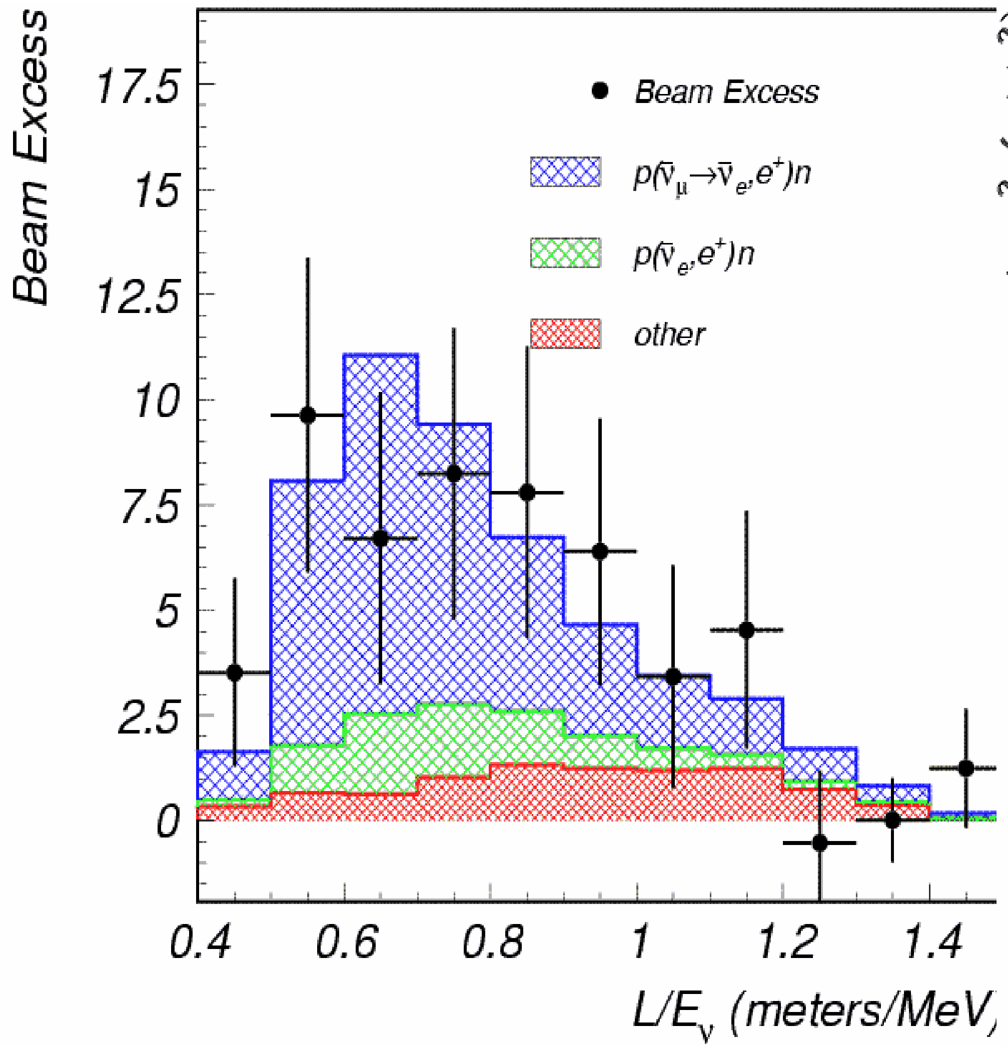
$$\bar{\nu}_e p \rightarrow e^+ n$$
20-60 MeV

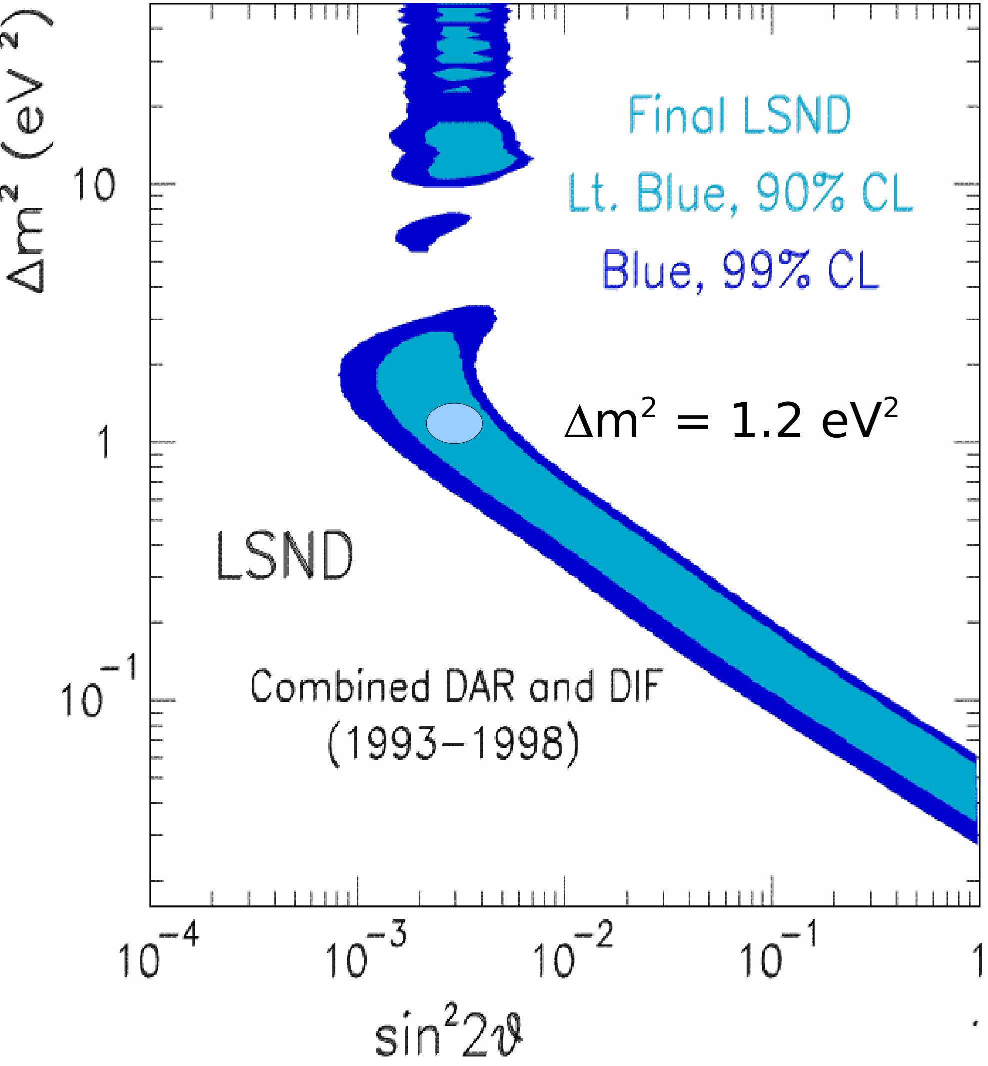
$$n p \rightarrow \gamma d$$
2.2 MeV

LSND Result (1997)

87.9 ± 22.4 ± 6 excess events from $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

3.3 σ evidence for oscillations





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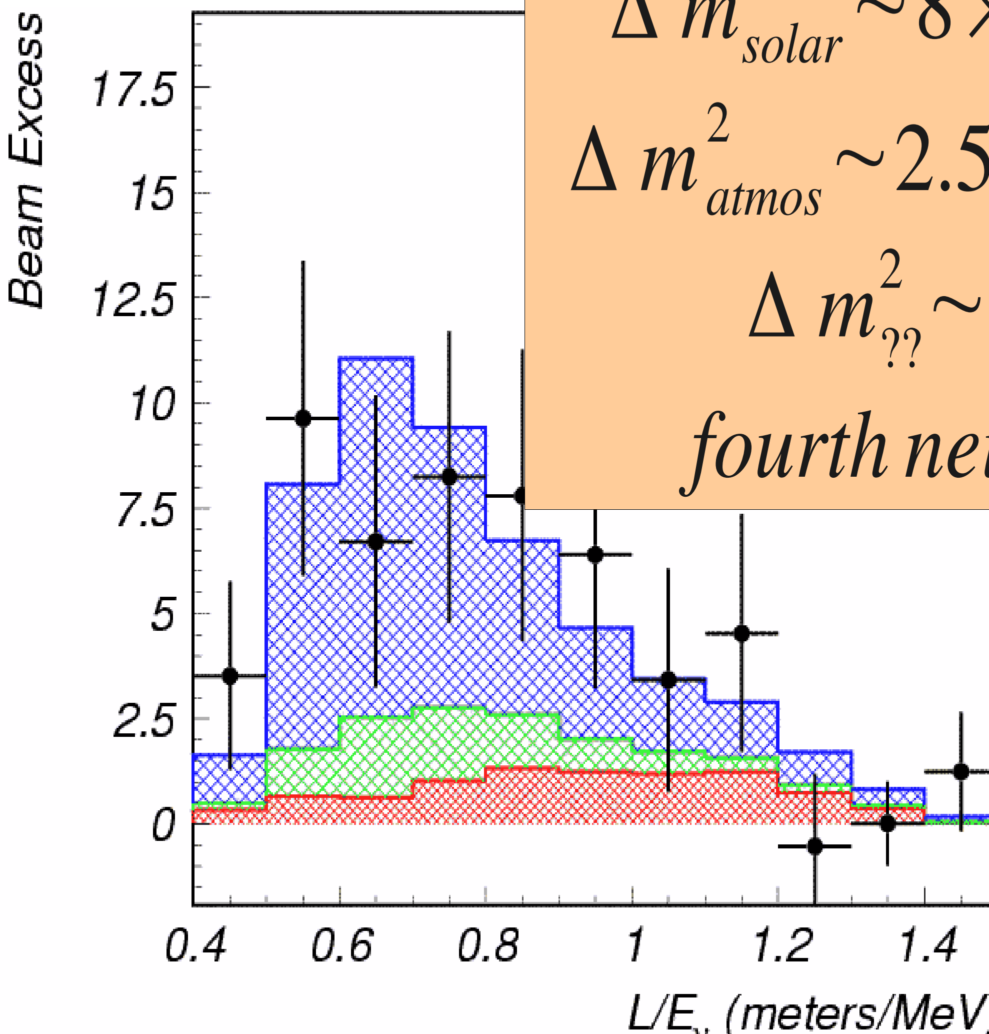
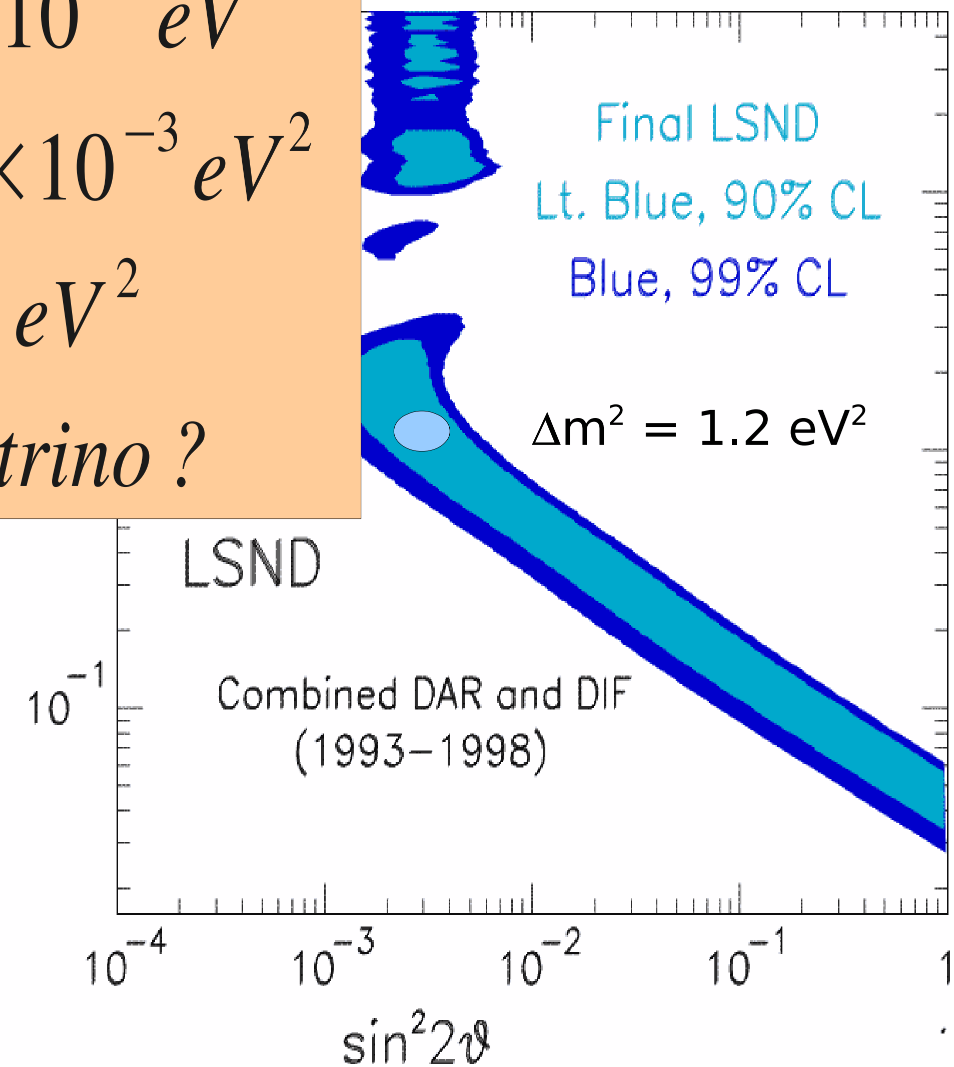


$$\Delta m^2_{solar} \sim 8 \times 10^{-5} eV^2$$

$$\Delta m^2_{atmos} \sim 2.5 \times 10^{-3} eV^2$$

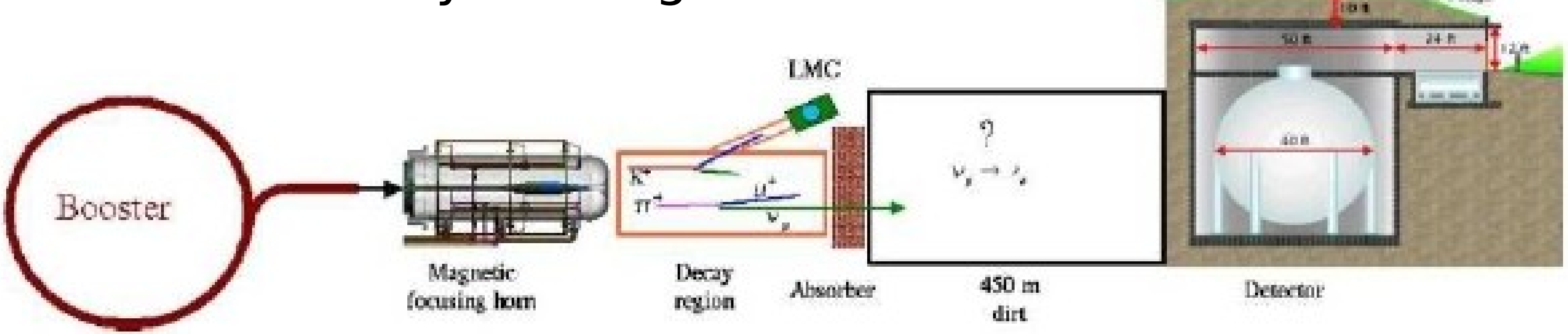
$$\Delta m^2_{??} \sim 1 eV^2$$

fourth neutrino ?



MiniBooNE

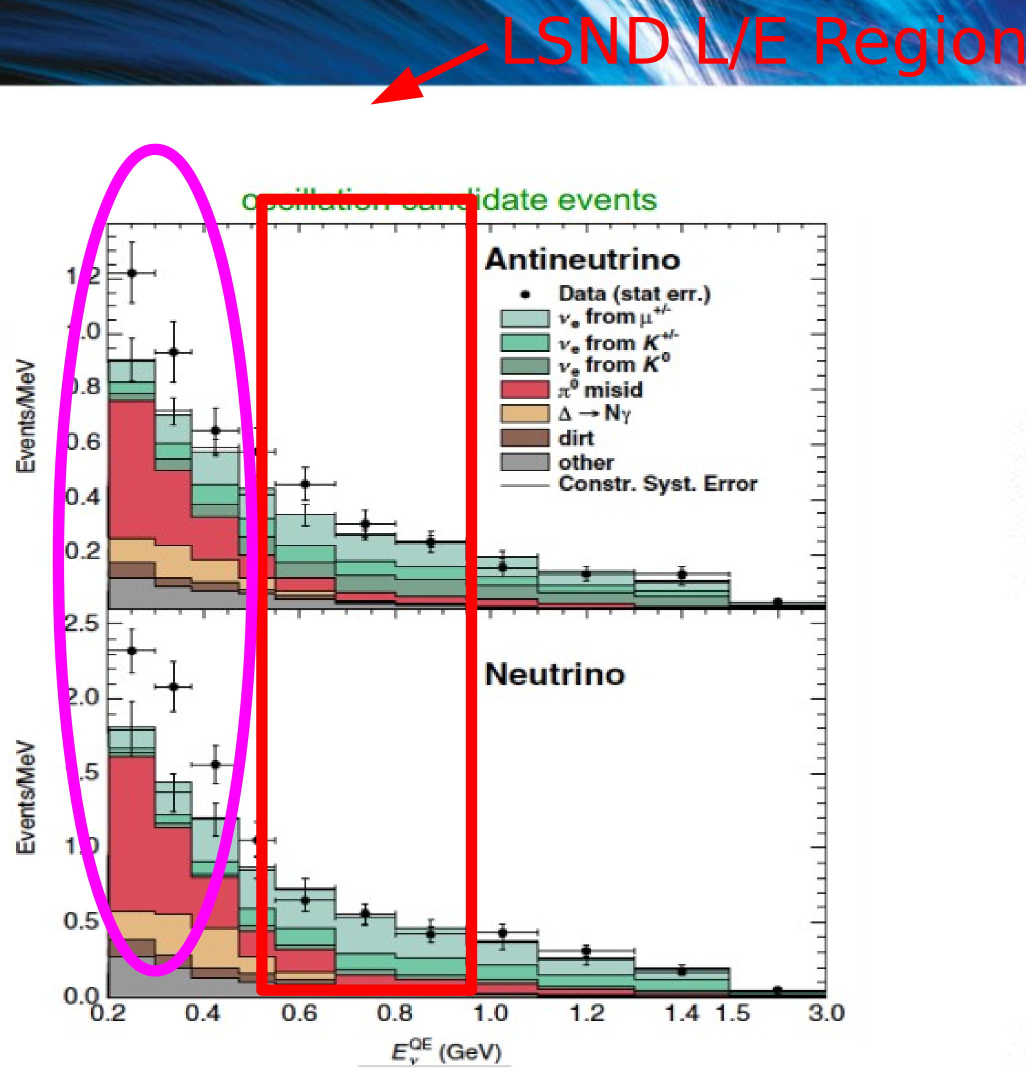
Currently running since 2002 at Fermilab

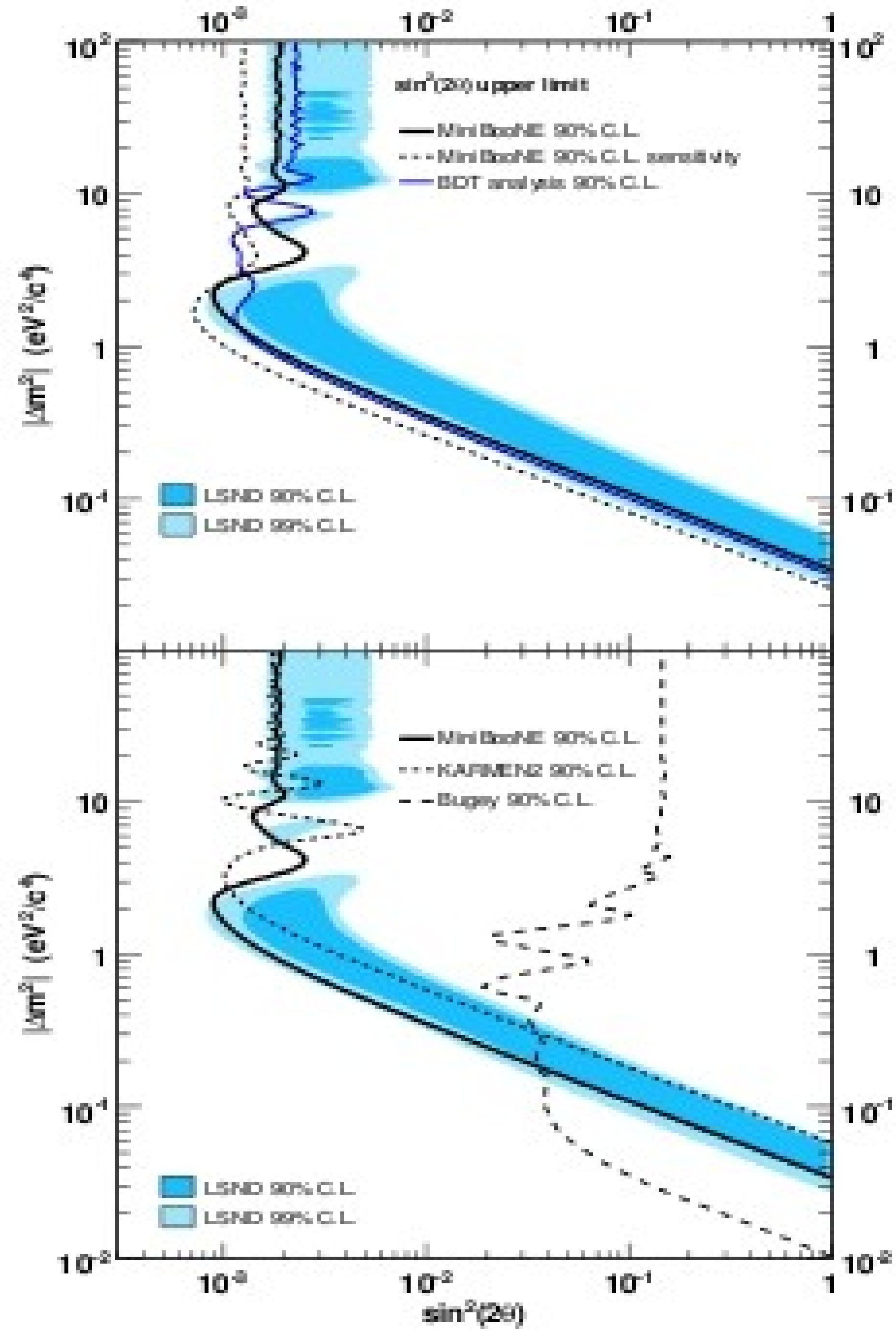


- Average neutrino energy ≈ 1 GeV
- L/E the same as LSND
- Same technology as LSND
- Different energy = different event types = different systematics

Neutrino mode : $\overline{\nu}_{\mu} \rightarrow \overline{\nu}_{e}$ oscillation (CPT transform of LSND)

Antineutrino mode : $\nu_{\mu} \rightarrow \nu_{e}$ oscillation (identical to LSND)





- 2013 analysis
- No excess of ν_e events in signal region (E>450 MeV)
- Unknown excess of events at low energy

The Gallium Anomaly

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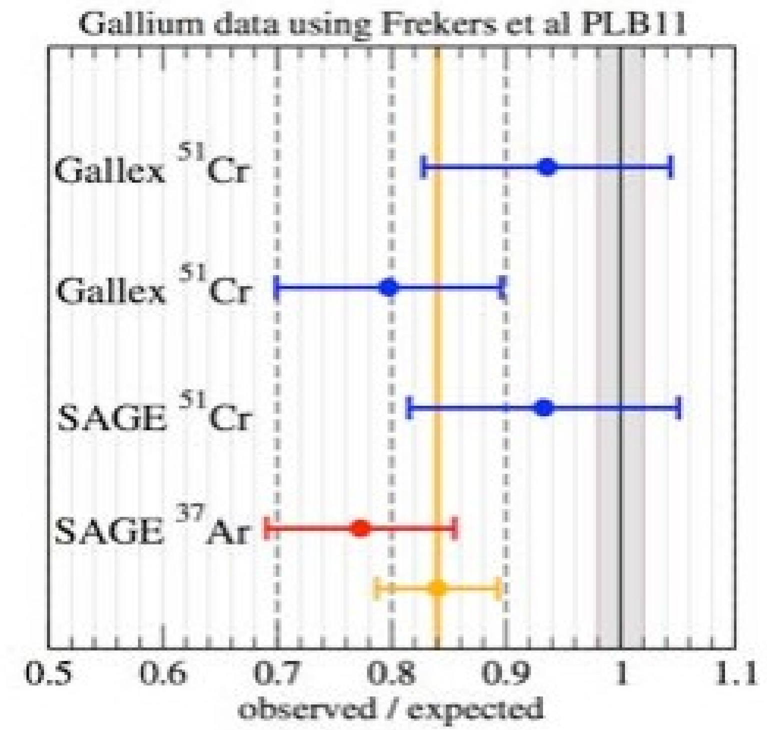
In early 2000's the response of the Gallex experiment (remember that?) was being tested using radioactive sources.

Sources emitted ν_e which were then observed using the standard Ge signature

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}\mathrm{Ge} + e^-$$

They reported a lower observed rate than expected – significant at 3 σ



$$L/E \approx 0.1\,m/0.1\,MeV \rightarrow \Delta m^2 \approx 1\,eV^2$$

(or is it our understanding of the inverse beta decay cross section?)

Reactor Anomaly

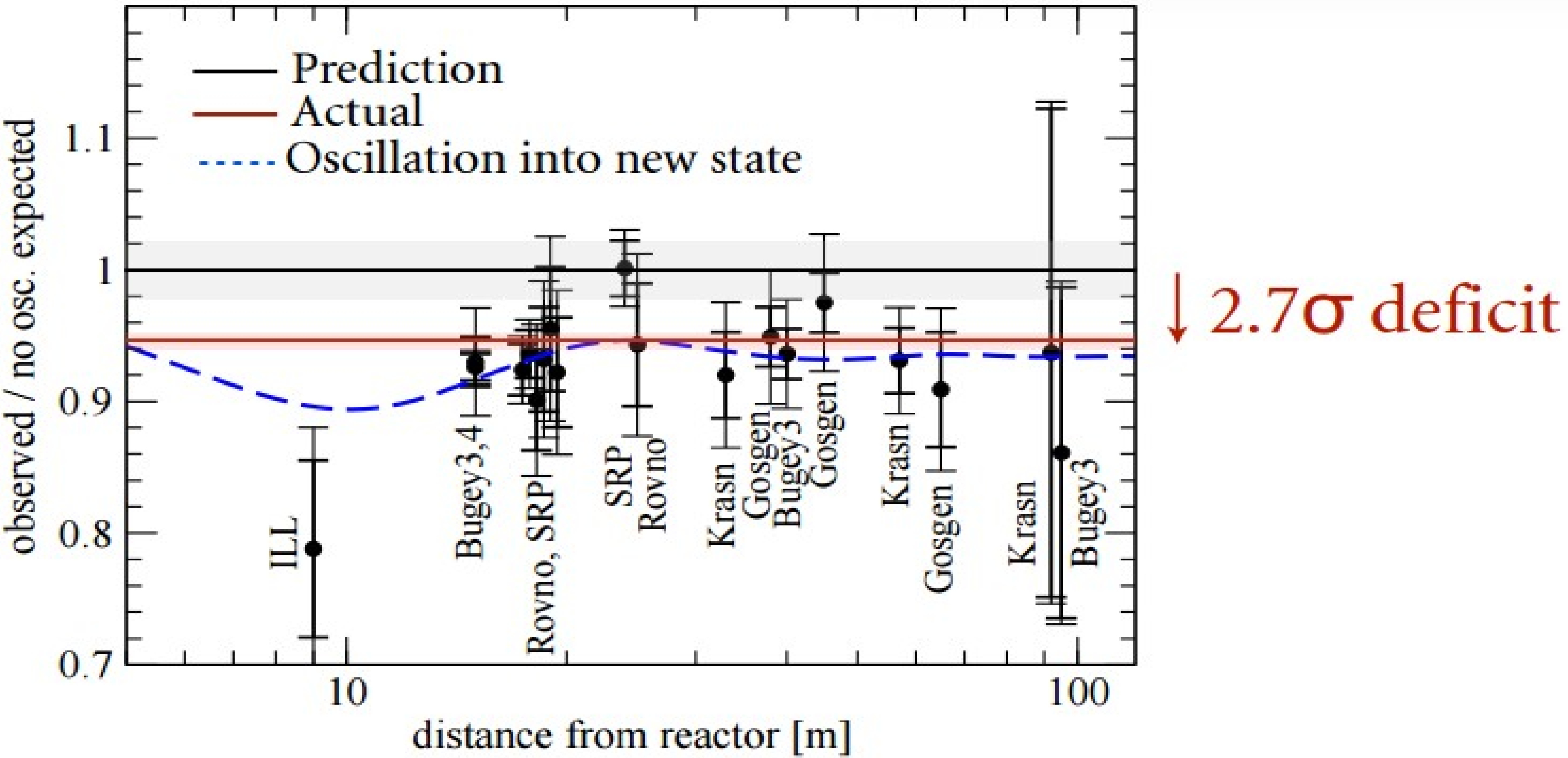
Over the years there have been lots of reactor experiments who measured the electron antineutrino flux from reactors and found that observed rates matched expected rates.

In 2011, new techniques in modelling nuclear reactions led to a re-evaluation of the expected electron antineutrino flux. The new estimate was about 6% **higher** than the old.

Suddenly all the experiments now observed a general **deficit** of electron antineutrinos being emitted from reactors

Could this be (i) the new flux estimate is just a bit dodgy or (ii) we have short baseline neutrino oscillations to a sterile state?

Reactor Anomaly



Deficit consistent with a sterile state with $\Delta m^2 \sim 1.5\ eV^2$

Decaying sterile neutrinos?

CPT Violation?

3+1 sterile?
3+2 ?
3+n ?

Lorentz violation?

Extra dimensions?

Experimental problems?

No bleedin' idea

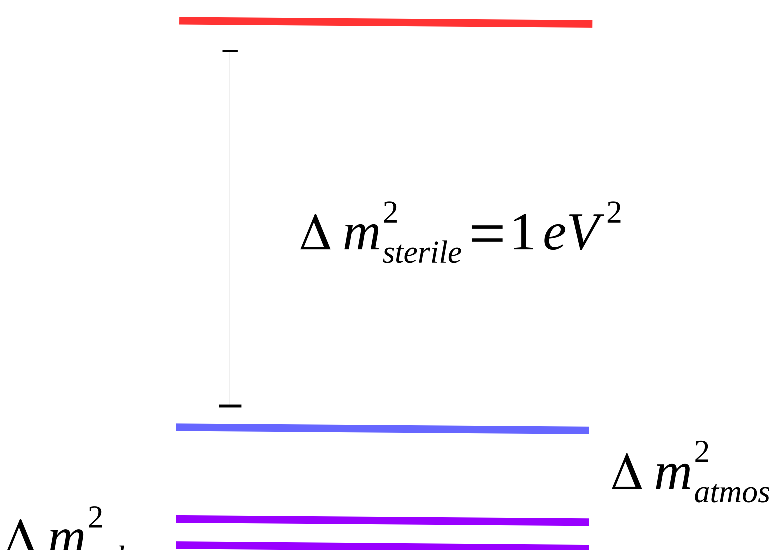
Wait for more data

Summary of sterile hints

There are odd hints, each at the level of 2-3 σ, that they may be at least one other light sterile state floating around with $\Delta m^2 \sim 1\ eV^2$. This is not very easy to fit into the standard model.

It is very hard to find an oscillation model, including steriles, which is consistent with *all* of the data

Current “best model” is a 3+1 model but it doesn't fit very well and it could all be a conspiracy of systematic uncertainties



Many new experiments being proposed to search for signs of steriles in neutrino oscillations

Story is certainly not over.....watch this space