



Introduction to Dalitz Plot Analysis

Tim Gershon University of Warwick

479.WE-Heraeus-Seminar Physics at LHCb 26 April 2011





What Is a Dalitz Plot?

- Visual representation of
 - the phase-space of a three-body decay
 - involving only spin-0 particles
 - (term often abused to refer to phase-space of any multibody decay)
 - Named after it's inventor, Richard Dalitz (1925–2006):
 - "On the analysis of tau-meson data and the nature of the tau-meson."
 - R.H. Dalitz, Phil. Mag. 44 (1953) 1068
 - (historical reminder: tau meson = charged kaon)
 - For scientific obituary, see
 - I.J.R. Aitchison, F.E. Close, A. Gal, D.J. Millener,
 - Nucl.Phys.A771:8-25,2006





In case anyone is wondering ...

- Previous plot shows $p\overline{p} \rightarrow \pi^0 \pi^0 \pi^0$ from Crystal Barrel experiment (LEAR)
 - C.Amsler et al., EPJC 23 (2002) 29
- Highly symmetrized because 3 identical bosons in the final state
 - symmetric under interchange of any 2 π^{0}







"Right-angled representation"

Removes some of the symmetry ... but easier for most software to handle (Most decays that we are interested in are not so symmetric anyway) Retains & heightens intuitive connection between visualisation and kinematics



Dalitz plots as visualiser of kinematics

- Illustration for $D \rightarrow K_s \pi^+ \pi^-$
 - green & blue: K*(892) (vector)
 - cyan & magenta : K₂*(1430) (tensor)
 - yellow : ρ(770) (vector)
 - red : f₀(980) (scalar)
- but main advantage of Dalitz plots is ability to exploit inference between different resonances

Image credit: Tom Latham





6

Some examples of Dalitz plots

 $D^0 \rightarrow K_s \pi^+ \pi^ D_s^+ \rightarrow \pi^+\pi^-\pi^+$ BaBar PRL 105 (2010) 081803 BaBar PRD 79 (2009) 032003 10³ 3 s₊ (GeV²/c⁴) a) K*, 3 (b) interference c^{\dagger} $m^2(\pi^+\pi^-)$ (GeV²/ 10² 2 10 0 3 2 0.5 2.5 3.5 1.5 2 3 0 s. (GeV²/c⁴) $m^{2}(\pi^{+}\pi^{-}) (GeV^{2}/c^{4})$

Note symmetries

Fim Gershor

ntroduction to Dalitz Plot Analysis

Some examples of Dalitz plots

 $B^+ \rightarrow D^- \pi^+ \pi^+$ Belle PRD 69 (2004) 112002

Tim Gershon

ntroduction to Dalitz Plot Analysis

 $B^+ \rightarrow K^+ K^+ K^-$ Belle PRD 71 (2005) 092003



Note symmetries

Why are we so interested in Dalitz plots?

- Condition for DCPV: |Ā/A|≠1
- Need \overline{A} and A to consist of (at least) two parts
 - with different weak (ϕ) and strong (δ) phases
- Often realised by "tree" and "penguin" diagrams

$$A = |T|e^{i(\delta_{T}-\phi_{T})} + |P|e^{i(\delta_{P}-\phi_{P})} \quad \overline{A} = |T|e^{i(\delta_{T}+\phi_{T})} + |P|e^{i(\delta_{P}+\phi_{P})}$$
$$A_{CP} = \frac{|\overline{A}|^{2} - |A|^{2}}{|\overline{A}|^{2} + |A|^{2}} = \frac{2|T||P|\sin(\delta_{T}-\delta_{P})\sin(\phi_{T}-\phi_{P})}{|T|^{2} + |P|^{2} + 2|T||P|\cos(\delta_{T}-\delta_{P})\cos(\phi_{T}-\phi_{P})}$$



Feynman tree (a) and penguin (b) diagrams for the $B_d^0 \to K^+ \pi^-$ decay

Why are we so interested in Dalitz plots?

Condition for DCPV: |A/A|≠1

Problem with two-body decays:

- 2 observables (B, A_{CP})
- 4 unknowns (|*T*|, |*P*|, $\delta_{\tau} \delta_{\rho}$, $\varphi_{\tau} \varphi_{\rho}$)

$$A_{CP} = \frac{|\overline{A}|^2 - |A|^2}{|\overline{A}|^2 + |A|^2} = \frac{2|T||P|\sin(\delta_T - \delta_P)\sin(\phi_T - \phi_P)}{|T|^2 + |P|^2 + 2|T||P|\cos(\delta_T - \delta_P)\cos(\phi_T - \phi_P)}$$



Feynman tree (a) and penguin (b) diagrams for the $B^0_d \to K^+\pi^-$ decay

Direct CP violation in $B \to K \pi$

• Direct CP violation in $B \to K\pi$ sensitive to γ

too many hadronic parameters \Rightarrow need theory input

NB. interesting deviation from naïve expectation

Belle Nature 452 (2008) 332





Dalitz plot analysis

- Amplitude analysis to extract directly information related to the phase at each Dalitz plot position
- Most commonly performed in the "isobar model"
 - sum of interfering resonances
 - each described by Breit-Wigner (or similar) lineshapes, spin terms, etc.
 - fit can be unbinned, but has inherent model dependence
- Alternative approaches aiming to avoid model dependence usually involve binning
 - partial wave analysis



Pros and cons of Dalitz plots

- Pros
 - More observables ($B \& A_{CP}$ at each Dalitz plot point)
 - Using isobar formalism, can express total amplitude as coherent sum of quasi-two-body contributions

$$A(m_{12}^2, m_{23}^2) = \sum_r c_r F_r(m_{12}^2, m_{23}^2)$$

- where $c_r \& F_r$ contain the weak and strong physics, respectively
- n.b. each c_r is itself a sum of contributions from tree, penguin, etc.
- Interference provides additional sensitivity to CP violation
- Cons
 - Need to understand hadronic (F,) factors
 - lineshapes, angular terms, barrier factors, ...
 - Isobar formalism only an approximation
- Model dependence
 Tim Gershon
 Introduction to Dalitz Plot Analysis





 $B^+ \to K^+ \pi^+ \pi^-$

Model includes:

- K^{*0}(892)π+, K^{*0}₂(1430)π⁺
- $(K\pi)_{0}^{*}\pi^{+}$ (LASS lineshape)
- ρ⁰(770)K⁺, ω(782)K⁺, f₀(980)K⁺, f₂(1270)K⁺, χ₀K⁺
- $f_{V}(1300)K^{+}$, phase-space nonresonant



BaBar PRD 78 (2008) 012004 See also Belle PRL 96 (2006) 251803

$B^+ \rightarrow K^+ \pi^+ \pi^-$

TABLE II: Summary of measurements of branching fractions (averaged over charge conjugate states) and CP asymmetries. Note that these results are not corrected for secondary branching fractions. The first uncertainty is statistical, the second is systematic, and the third represents the model dependence. The final column is the statistical significance of direct CPviolation determined as described in the text.

Mode	Fit fraction $(\%)$	$\mathcal{B}(B^+ \to \mathrm{Mode})(10^{-6})$	$A_{C\!P}$ (%)	DCPV sig.	
$K^+\pi^-\pi^+$ total		$54.4 \pm 1.1 \pm 4.5 \pm 0.7$	$2.8 \pm 2.0 \pm 2.0 \pm 1.2$		
$K^{*0}(892)\pi^+; K^{*0}(892) \to K^+\pi^-$	$13.3\pm0.7\pm0.7{}^{+0.4}_{-0.9}$	$7.2\pm0.4\pm0.7{}^{+0.3}_{-0.5}$	$+3.2\pm5.2\pm1.1{}^{+1.2}_{-0.7}$	0.9σ	
$(K\pi)_0^{*0}\pi^+; (K\pi)_0^{*0} \to K^+\pi^-$	$45.0 \pm 1.4 \pm 1.2 {}^{+12.9}_{-0.2}$	$24.5 \pm 0.9 \pm 2.1 {}^{+7.0}_{-1.1}$	$+3.2\pm3.5\pm2.0{}^{+2.7}_{-1.9}$	1.2σ	
$\rho^0(770)K^+; \ \rho^0(770) \to \pi^+\pi^-$	$6.54 \pm 0.81 \pm 0.58 {}^{+0.69}_{-0.26}$	$3.56\pm0.45\pm0.43~^{+0.38}_{-0.15}$	$+44\pm10\pm4^{+5}_{-13}$	3.7σ	
$f_0(980)K^+; f_0(980) \to \pi^+\pi^-$	$18.9 \pm 0.9 \pm 1.7 {}^{+2.8}_{-0.6}$	$10.3 \pm 0.5 \pm 1.3 {}^{+1.5}_{-0.4}$	$-10.6\pm5.0\pm1.1{}^{+3.4}_{-1.0}$	1.8σ	
$\chi_{c0}K^+; \chi_{c0} \to \pi^+\pi^-$	$1.29 \pm 0.19 \pm 0.15 {}^{+0.12}_{-0.03}$	$0.70\pm0.10\pm0.10~^{+0.06}_{-0.02}$	$-14\pm15\pm3{}^{+1}_{-5}$	0.5σ	
$K^+\pi^-\pi^+$ nonresonant	$4.5\pm0.9\pm2.4{}^{+0.6}_{-1.5}$	$2.4\pm0.5\pm1.3{}^{+0.3}_{-0.8}$			
$K_2^{*0}(1430)\pi^+; K_2^{*0}(1430) \to K^+\pi^-$	$3.40 \pm 0.75 \pm 0.42 {}^{+0.99}_{-0.13}$	$1.85\pm0.41\pm0.28~^{+0.54}_{-0.08}$	$+5\pm23\pm4{}^{+18}_{-7}$	0.2σ	
$\omega(782)K^+; \ \omega(782) \to \pi^+\pi^-$	$0.17 \pm 0.24 \pm 0.03 {}^{+0.05}_{-0.08}$	$0.09\pm0.13\pm0.02{}^{+0.03}_{-0.04}$			
$f_2(1270)K^+; f_2(1270) \to \pi^+\pi^-$	$0.91 \pm 0.27 \pm 0.11 {}^{+0.24}_{-0.17}$	$0.50\pm0.15\pm0.07{}^{+0.13}_{-0.09}$	$-85\pm22\pm13^{+22}_{-2}$	3.5σ	
$f_{\rm X}(1300)K^+; f_{\rm X}(1300) \to \pi^+\pi^-$	$1.33 \pm 0.38 \pm 0.86 {}^{+0.04}_{-0.14}$	$0.73 \pm 0.21 \pm 0.47 {}^{+0.02}_{-0.08}$	$+28\pm26\pm13{}^{+7}_{-5}$	0.6σ	

• $f_{\chi}(1300)K^{+}$, phase-space nonresonant

Events / (0.024 GeV/c²) 00 01 00 05 00 05

50

0

Tim Gershon

Introduction to Dalitz Plot Analysis



Evidence for direct CP violation But significant model dependence

15

$B^+ \to K^+ \pi^+ \pi^-$



FIG. 4: Projection plots of the $\pi^+\pi^-$ invariant mass in the region of the $\rho^0(770)$ and $f_0(980)$ resonances. The left (right) plots are for $B^-^-(B^+)$ candidates. The top row shows all candidates, the middle row shows those where $\cos\theta_H > 0$, and the bottom row shows those where $\cos\theta_H < 0$. The data are the black points with statistical error bars, the lower solid (red/dark) histogram is the $q\bar{q}$ component, the middle solid (green/light) histogram is the $B\bar{B}$ background contribution, while the blue open histogram shows the total fit result.

THE -

BaBar PRD 78 (2008) 012004 See also Belle PRL 96 (2006) 251803

Sources of model dependence

- Lineshapes
 - coupled channels, threshold effects, etc.
- Isobar formalism
 - "sum of Breit-Wigners" model violates unitarity
 - problem most severe for broad, overlapping resonances
 - even talking about "mass" and "width" for such states is not strictly correct (process dependent) – can only be defined by pole position
- Nonresonant contributions
 - such terms are small for D decays, but are found to be large for some B decays (not well understood why)
 - interference with other (S-wave) terms can lead to unphysical phase variations



Are methods used for D decay Dalitz plots also valid for B decays?



Introduction to Dalitz Plot Analysis

18

How to address model dependence?

- Model independent methods
 - Dalitz plot anisotropy ("Miranda" method)
 - binned D Dalitz plots for measurement of γ in B→DK (and charm mixing & CP violation)
 - partial wave analysis
- More robust (but still model dependent) methods
 - K-matrix approach
 - use of scattering data to constrain phase variation (Watson's theorem) e.g. LASS shape for (K π) S-wave
 - input from theory (chiral symmetry, dispersion relations)

More details in talks of Christoph Hanhart and Leonard Lesniak in parallel session on Thursday



• Good model-independent way to identify CP violation

Tim Gershon

ntroduction to Dalitz Plot Analysis

- could be sufficient to identify non-SM physics in, e.g., charm decays
- Constant (DP independent) systematic asymmetries can be accounted for
- Can isolate region of the Dalitz plot where CP violation effects occur

But does not provide quantitative measure of weak phase

20

Example partial wave analysis: $D_{s}^{+} \rightarrow K^{+}K^{-}\pi^{+}$ (BaBar)



Example partial wave analysis: $D_{s}^{+} \rightarrow K^{+}K^{-}\pi^{+}$ (BaBar)



Quasi-model-independent partial wave analysis

- Pioneered by E791 (B.Meadows) in $D^+ \rightarrow K^- \pi^+ \pi^+$
- Describe S-wave by complex spline (many free parameters)
- Example: $D_{t}^{+} \rightarrow \pi^{-}\pi^{+}\pi^{+}$ from BaBar



ππ S-wave comparison



Data points from BaBar



ππ S-wave comparison



Extracting weak phases from Dalitz plots

- Many methods exist in the literature
 - some have been used by the B factories
 - most results are statistically limited
 - in some cases model uncertainties are an issue
 - still plenty of room for good new ideas
- Examples (there are many more!)
 - Snyder-Quinn method to measure α from B $\ \ \rightarrow \ \pi^+\pi^-\pi^0$
 - GGSZ/BP method to measure γ from B^{\pm} \rightarrow DK^{\pm} with D \rightarrow $K_{_S}\pi^{+}\pi^{-}$
 - Measurement of charm oscillation parameters using D \rightarrow $K_{_S}\pi^{\scriptscriptstyle +}\pi^{\scriptscriptstyle -}$
 - Various methods to measure γ from three-body charmless B decays (B_{u,d,s} $\rightarrow \pi\pi\pi$, K $\pi\pi$, KK π , KKK)
 - Penguin-free measurements of β & $\beta_{_{S}}$ from $D\pi^{\scriptscriptstyle +}\pi^{\scriptscriptstyle -}$ & DK^{\scriptscriptstyle +}K^{\scriptscriptstyle -}, respectively



$B \rightarrow \pi^+ \pi^- \pi^0 - B$ factory results

- Results from
 - Belle, 449 M BB pairs: PRL 98 (2007) 221602, PRD 77 (2008) 072001
 - BaBar, 375 M BB pairs: PRD 76 (2007) 012004

 $\rho^+\pi^-$



FIG. 10: Proper time distributions of good tag (r > 0.5) regions for $f_{\text{tag}} = B^0$ (upper) and $f_{\text{tag}} = \overline{B}^0$ (middle upper), in $\rho^+\pi^-$ (left), $\rho^-\pi^+$ (middle), $\rho^0\pi^0$ (right) enhanced regions, where solid (red), dotted, and dashed curves correspond to signal, continuum, and $B\overline{B}$ PDFs. The middle lower and lower plots show the background-subtracted asymmetries in the good tag (r > 0.5) and poor tag (r < 0.5) regions, respectively. The significant asymmetry in the $\rho^-\pi^+$ enhanced region (middle) corresponds to a non-zero value of U_{Δ}^- .

 $\rho^{-}\pi^{-}$

 $\rho^0 \pi^0$



$B \to \pi^+ \pi^- \pi^0 - B \ factory \ results$

- Results from
 - Belle, 449 M BB pairs: PRL 98 (2007) 221602, PRD 77 (2008) 072001
 - BaBar, 375 M BB pairs: PRD 76 (2007) 012004



$B^{\pm} \rightarrow DK^{\pm}$ with $D \rightarrow K_{s}\pi^{+}\pi^{-}$

Interference between $b \rightarrow u$ and $b \rightarrow c$ amplitudes when D is reconstructed in final state common to D^0 and \overline{D}^0 provides sensitivity to γ

$$|M_{\pm}(m_{+}^{2}, m_{-}^{2})|^{2} = |f_{D}(m_{+}^{2}, m_{-}^{2}) + re^{i\delta_{B} \pm i\phi_{3}}f_{D}(m_{-}^{2}, m_{+}^{2})|^{2}$$
$$= \left| \left| \int_{-\infty}^{\infty} + re^{i\delta_{B} \pm i\phi_{3}} \right|^{2} \right|^{2}$$

Model ($f_D(m_{+}^2, m_{-}^2)$) taken from measurements of $|f_D|^2$ using flavour tagging D* decays – model dependence

BaBar obtain γ = (68 ⁺¹⁵-14 ± 4 ± 3)° (from DK⁻, D*K⁻ & DK*⁻)

PRL 105 (2010) 121801

Belle obtain $\phi_3 = (78^{+11}_{-12} \pm 4 \pm 9)^{\circ}$ (from DK⁻ & D*K⁻) PRD 81 (2010) 112002

$B^{\pm} \rightarrow DK^{\pm}$ with $D \rightarrow K_{s}\pi^{+}\pi^{-}$

Solution – bin the Dalitz plot and use ψ(3770) → DD events (CLEOc, BES) to measure per-bin phases PRD 68, 054018 (2003), EPJ C 47, 347 (2006); EPJ C 55, 51 (2008) (unusual bin shapes to attempt to optimise sensitivity)



$$M_{i}^{\pm} = h\{K_{i} + r_{B}^{2}K_{-i} + 2\sqrt{K_{i}K_{-i}}(x_{\pm}c_{i} + y_{\pm}s_{i})\}$$
$$x_{\pm} = r_{B}\cos(\delta_{B} \pm \phi_{3}) \quad y_{\pm} = r_{B}\sin(\delta_{B} \pm \phi_{3})$$

 c_{i} , s_{i} measured by CLEO PRD 82, 112006 (2010)

First model independent measurement of y in this mode by Belle

THE TIM Gershon





Looking ahead

- I hope I have already convinced you that there is great physics potential in Dalitz plot analysis
- But there are also considerable challenges
 - theoretical (hadronic effects, etc.)
 - experimental
 - Dalitz plot dependent effects:
 - Efficiency
 - Background
 - Mass resolution
 - Misreconstruction effects ...
- I leave the details to the parallel session speakers, but ...



Motivation for "Les Nabis"

- From I. Bigi, "Hadronization the Unsung Hero rather than the alleged Villain in the Tale of CP Violation", proceedings of Hadron '05
- "We expect confidently that New Physics surfaces at the TeV scale. Yet we have to aim beyond 'merely' establishing the existence of New Physics our goal has to be to identify its salient features. The discovery potential in B decays is essential, not a luxury. Yet due to the past 'unlikely' success of the CKM description one cannot count on massive manifestations of New Physics. Therefore we need high accuracy both on the experimental and theoretical side in heavy flavour studies. This requires a better quantitative understanding of hadronization to exhaust the discovery potential in B decays."
- "The expertise required to attain an essential goal namely to exhaust the discovery potential in heavy flavour transitions by harnessing low-energy hadronization does exist or can be acquired with no need for a breakthrough. However it tends to reside in a community all too often disjoint from the heavy flavour community this has to change!"



Erm, What is "Les Nabis"?

- A group of experimentalists and theorists, drawn from both particle/heavy flavour and nuclear/hadronic physics communities, set up to address issues in Dalitz plot analysis with the aim of obtaining both qualitative and quantitative information on weak phases and CP violation
- Theory:
 - I. Bigi, S. Gardner, C. Hanhart, B. Kubis, T. Mannel, U.-G. Meißner, J. Pelaez, M. Pennington
- Experiment:
 - I. Bediaga, A. Bondar, A. Denig, T. Gershon, W. Gradl, B. Meadows, K. Peters, U. Wiedner, G. Wilkinson

(This is the list on the Nabis web page – apologies for any omissions)
 Tim Gershon OF
 Malysis

OK, but why is it called "Les Nabis"?

• From the manifesto (yes, really):

ction to Dalitz Plot Analy

 "Les Nabis" means "the Prophets" in Hebrew, and this name had been adopted by a group of post-impressionist artists who set the pace for paintings and graphics in France in the 1890s. With some of us harbouring a similar sense of modesty as those artists, we found a reincarnation of "Les Nabis" an appropriate motto for our group and its efforts. This is further strengthened by the fact that one work by a central member of the original "Les Nabis" seems to show a (somewhat artistically enhanced) image of a Dalitz plot – it is "The Talisman" by Paul Sérusier. We have chosen it as our logo.



"The Talisman", Paul Sérusier



Summer school

- School on Amplitude Analysis in Modern Physics: from hadron spectroscopy to CP phases
- August 1-5, 2011 at Physikzentrum Bad Honnef, Germany
 - http://www2.fz-juelich.de/ikp/workshops/LesNabis/index.shtml



preliminary program August 1-5, 2011

	09:00 - 10:30	11:00 - 12:30	14:00 - 15:30	16:00 - 17:30
Mo	Introduction	Theory I	Experiment I	Discussion I
Tu	Theory II	Experiment II	Discussion II	Supplementary I
We	Theory I	Experiment I	Discussion III	Supplementary II
Th	Theory II	Experiment II	Discussion IV	Supplementary III
Fr	Experiment I	Theory I	Discussion V	

Introduction (NN)

Theory I (Jose Pelaez): **Dispersion Theory** Theory II (Thomas Mannel): **CP violation in the Standard Model** Experiment I (Klaus Peters): **Phenomenological Tools** Experiment II (Timothy Gershon): **Precision Experiments / Data Analysis** Supplementary II: Regge Theory Supplementary II: Chiral Perturbation Theory Supplementary III: Experimental Issues

Summary

- Dalitz plot analyses provide promising methods to measure weak phases and CP violation
- Many attractive features ...
- ... but significant complications due to model dependence
- Need progress on several fronts
 - Understand better ($\pi\pi$), (K π), (KK), (D π), (DK) systems
 - "Nonresonant" contributions and 3-body unitarity
 - Methods to combat model-dependence
 - Nabis initiative set up to try to address this
- Many new possibilities opening up with LHCb



For more details, please come to the parallel session

Thursday 28 April 2011

Parallel Session: Dalitz Analysis chairperson: Tim Gershon

14:00 - 14:35	Christoph Hanhart (Juelich)	Theoretical issues in Dalitz plot analysis	pdf
14:35 - 15:05	Marco Pappagallo (Bari)	Charm Dalitz plot analyses	pdf
15:05 - 15:30	Carla Gobel (Rio)	Model independent approaches to DP analysis	pdf
15:30 - 15:50	Hamish Gordon (Oxford)	Study of CP violation in D+ -> K+K-pi+ at LHCb	pdf
15:50 - 16:20	Coffee		
16:20 - 16:50	Jussara Miranda (Rio)	Dalitz plot analyses of charmless B decays	pdf
16:50 - 17:25	Leonard Lesniak (Krakow)	Model for charmless B decay Dalitz plots	pdf
17:25 - 17:50	Anton Poluektov (Warwick)	B->Dhh' Dalitz plot analyses	pdf
17:50 - 18:10	Daniel Johnson (Oxford)	Use of binned Dalitz plots to measure gamma at LHCb	pdf
18:10 - 18:30	Paras Naik (Bristol)	Four-body Amplitude Analyses at LHCb	pdf

