Overview of the CKM Matrix

Tim Gershon
University of Warwick & CERN

Lepton Photon 2011
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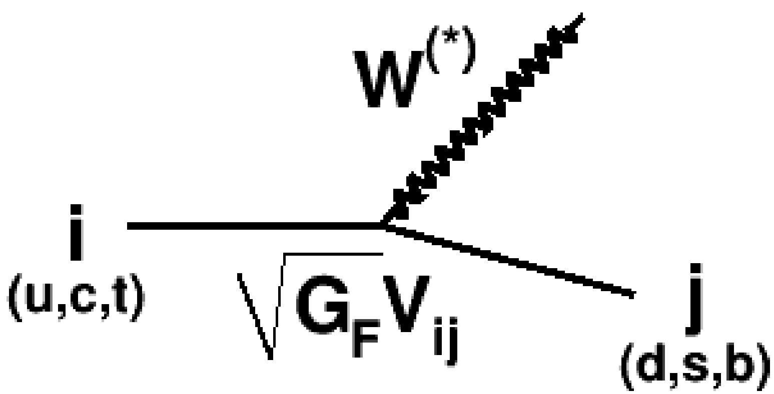
With thanks to numerous contributing experiments, theorists, fitting groups, and especially working group conveners from
CKM2010
http://ckm2010.warwick.ac.uk

The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix

Dirac medal 2010

Nobel prize 2008



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A 3x3 unitary matrix
- Described by 4 parameters – allows CP violation
  - PDG (Chau-Keung) parametrisation: θ_{12}, θ_{23}, θ_{13}, δ
  - Wolfenstein parametrisation: λ, A, ρ, η
- Highly predictive

Range of CKM phenomena

nuclear transitions

pion decays

kaons

hyperon decays

tau decays

neutrino interactions

charm

bottom

W decays

top

Range of CKM phenomena

nuclear transitions

PIBETA

pion decays

NA48, KTeV, KLOE, ISTRA

kaons

hyperon decays

CHORUS

tau decays

neutrino interactions

KEDR, FOCUS, CLEO, BES

charm

BABAR, BELLE, LHCb

bottom

ALEPH, DELPHI, L3, OPAL

W decays

CDF, D0, ATLAS, CMS

top

dispersion relations

hadronic matrix elements

chiral perturbation theory

lattice QCD

flavour symmetries

heavy quark effective theories

operator product expansion

perturbative QCD

apologies for omissions

Outline

- CKM phenomenology
- Measurements of magnitudes of CKM matrix elements through tree-level processes
  - $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$, $|V_{ub}|$
    - tree-level measurements of $|V_{tx}|$ covered in top session on Tuesday
    - loop-level level measurements covered in following talks
- Measurements of CP violation in the quark sector
  - Direct CP violation in D & B systems
  - Unitarity Triangle angles: α, β, γ
    - CP violation in D^0 and B_s^0 oscillations covered in followed talks
- Summary

apologies for omissions

CKM phenomenology

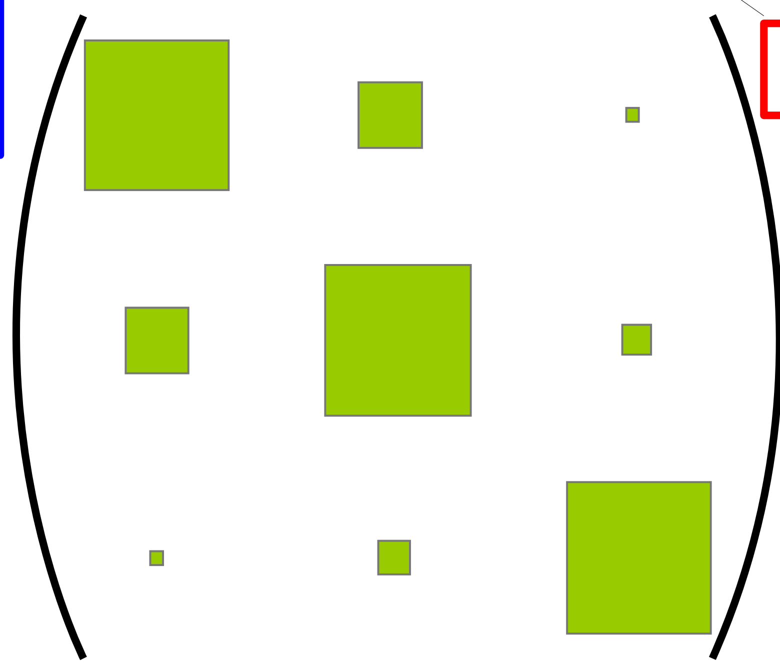
- CKM theory is highly predictive
  - huge range of phenomena over a massive energy scale predicted by only 4 independent parameters
- CKM matrix is hierarchical
  - theorised connections to quark mass hierarchies, or (dis-)similar patterns in the lepton sector
    - origin of CKM matrix from diagonalisation of Yukuwa (mass) matrices after electroweak symmetry breaking
  - distinctive flavour sector of Standard Model not necessarily replicated in extended theories → strong constraints on models
- CKM mechanism introduces CP violation
  - only source of CP violation in the Standard Model ($m_\nu = \theta_{QCD} = 0$)

Wolfenstein parametrisation

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

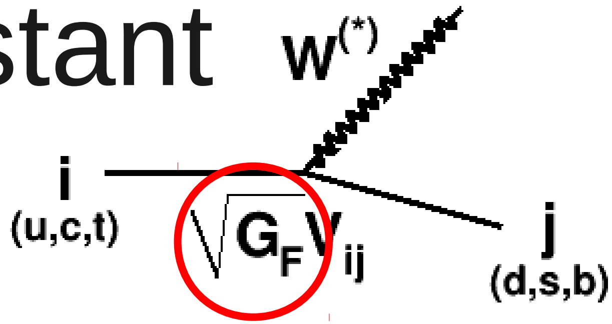
Expansion parameter
$\lambda = \sin(\theta_c) \sim 0.22$

Source of CP violation



Magnitudes of CKM matrix elements (starting with a digression)

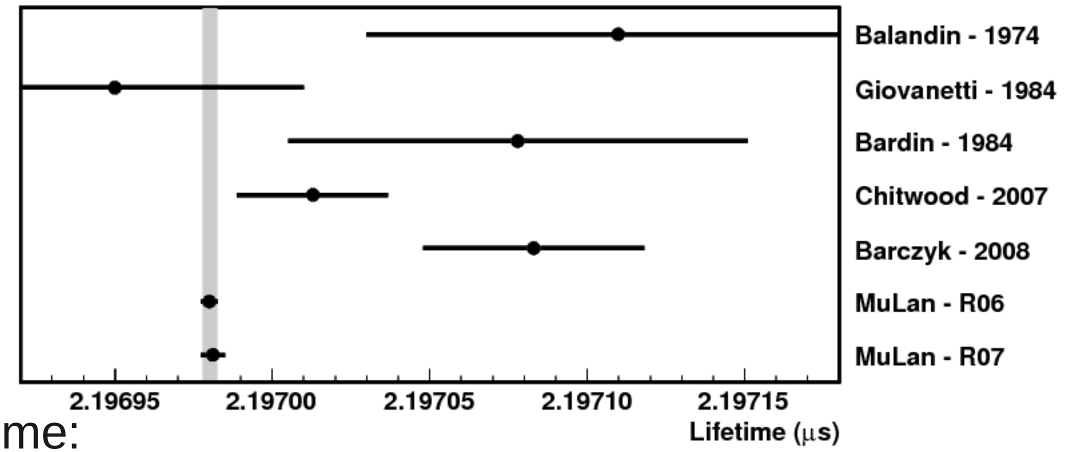
The Fermi constant



$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1+\Delta q)$$

phase-space, QED & hadronic radiative corrections

MuLan experiment
PRL 106 (2011) 079901



World's best measurement of the muon lifetime:

$$\tau_{\mu^+} = (2196980.3 \pm 2.2)\, ps$$

$$G_F = (1.1663788 \pm 7) \times 10^{-5} GeV^{-2}$$

< 1 part per million precision! (PDG 2010: 9 ppm)

$|V_{ud}|$ determination

J.C. Hardy, I.S. Towner, PRC 79 (2009) 055502

From $0^+ \rightarrow 0^+$ nuclear beta decays

Measure

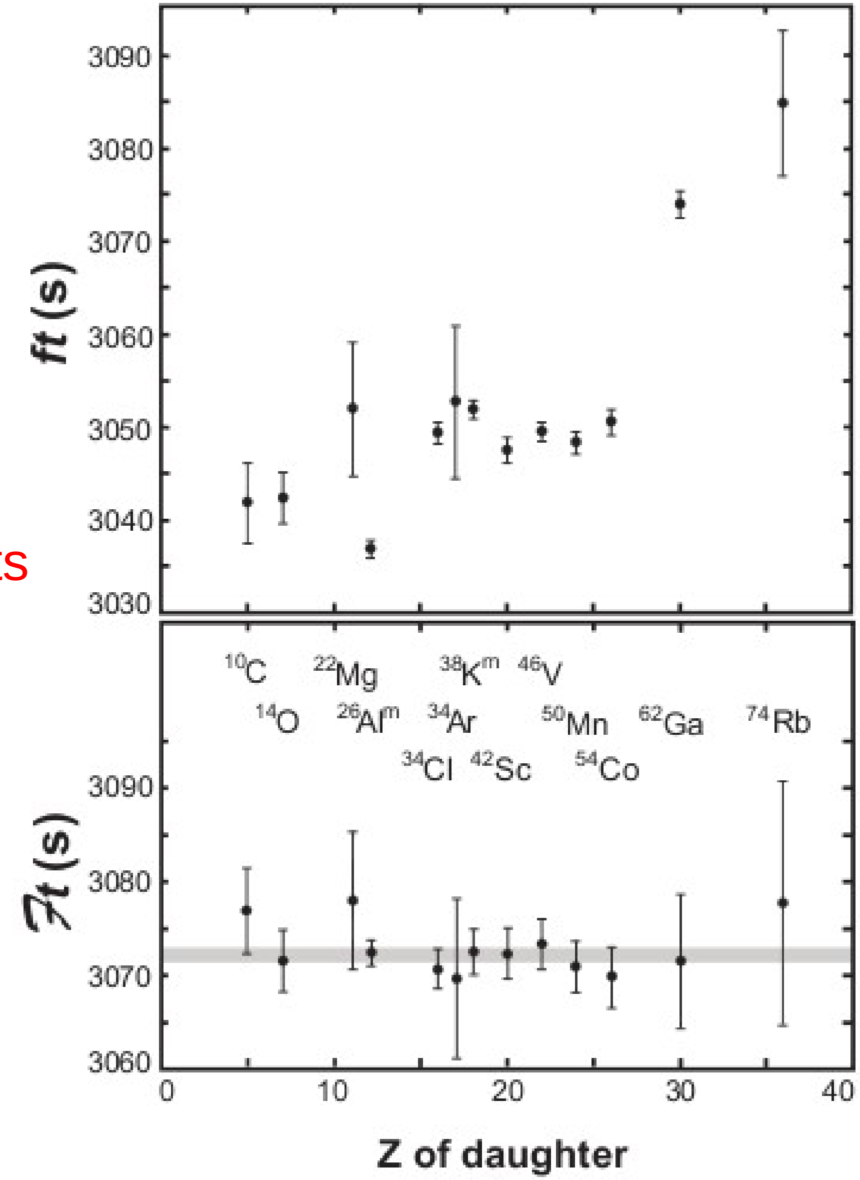
- energy gap Q → f
- half-life
- branching fraction } → t

$$ft = \frac{K}{2G_F^2 |V_{ud}|^2}$$

Correct for nuclear medium related effects

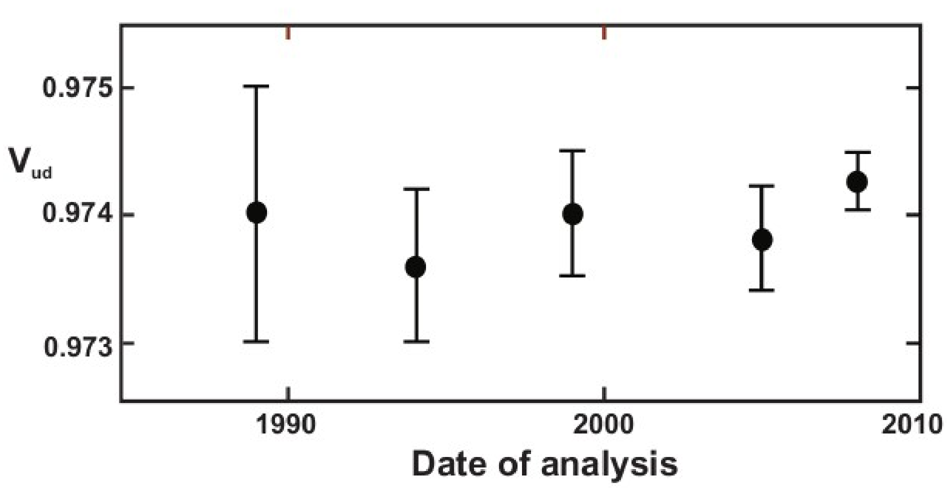
- radiative and isospin breaking corrections
  - → nucleus-independent quantity Ft
  - confirmed to be constant to 3 10^{-4}

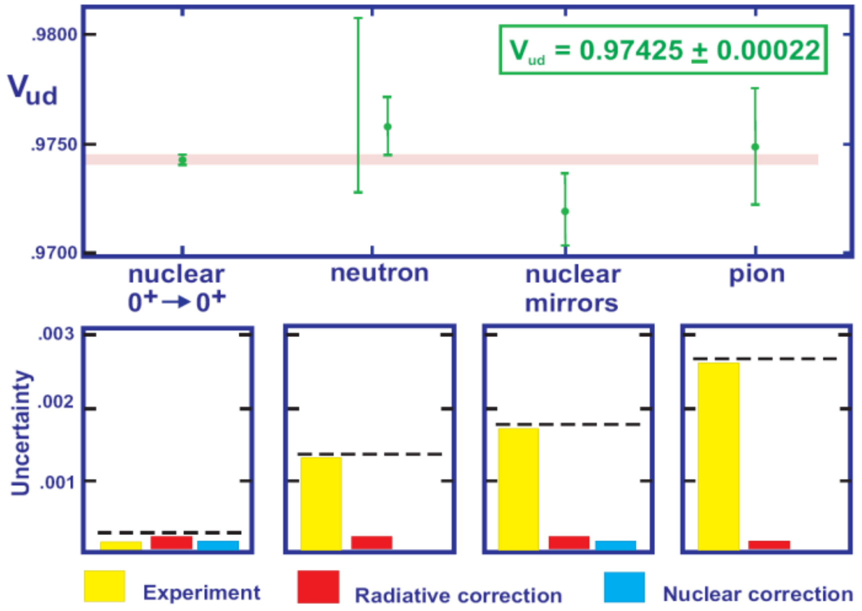
$$|V_{ud}| = 0.97425 \pm 0.00022$$



Alternative approaches to $|V_{ud}|$

- Can also measure $|V_{ud}|$ from
  - alternative nuclear decays ("nuclear mirrors")
  - neutron and pion β decay
    - do not require nucleus dependent or isospin breaking corrections
    - pion β decay is a pure vector transition (like $0^+ \rightarrow 0^+$)
      - potential for more precise future measurements

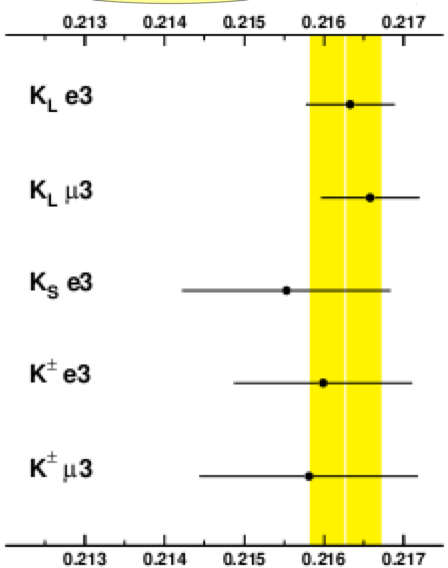




$|V_{us}|$ from semileptonic kaon decays

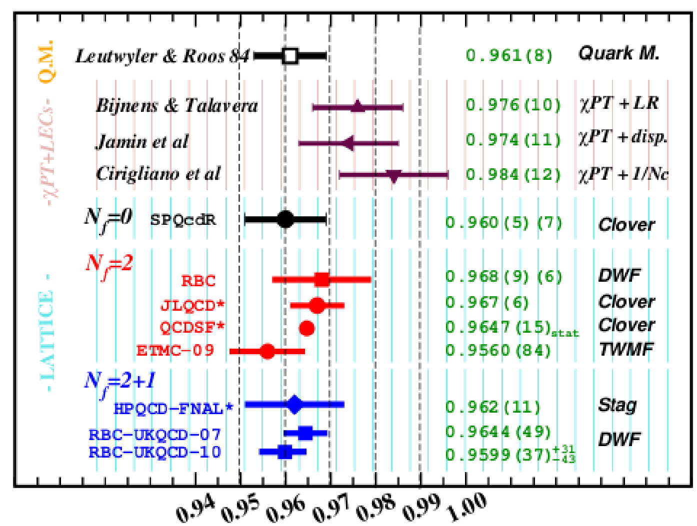
Latest NA48 preliminary results not included

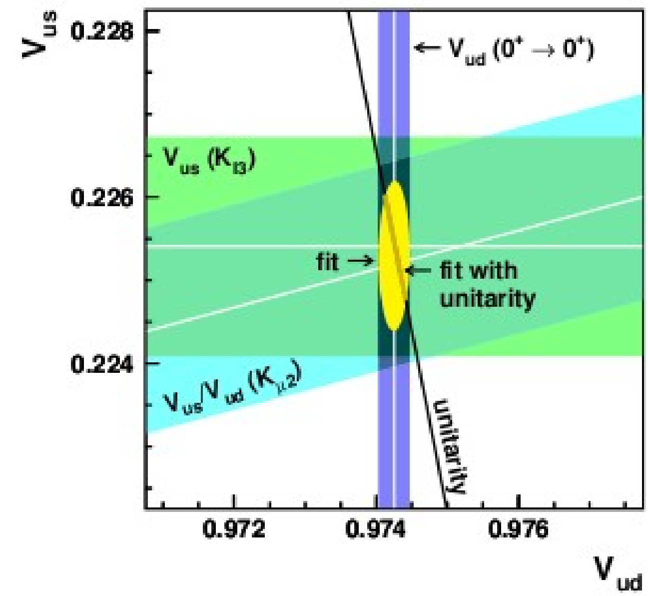
FlaviaNet Kaon Working Group
EPJC 69 (2010) 399



$f_+(0)|V_{us}|$

$f_+^{K^0\pi^+}(0)$





$$|V_{us}| = 0.2254 \pm 0.0013$$

Comparison with

- $|V_{us}|/|V_{ud}|$ from leptonic kaon and pion decays (using lattice input on f_K/f_π)
- $|V_{ud}|$

PLB 700 (2011) 7

Unitarity holds to better than 10^{-3}

Alternative approaches to $|V_{us}|$

- Can also measure $|V_{us}|$ from
  - hyperon decays
  - strange vs. non-strange hadronic tau branching fractions

A.Pich arXiv:1101.2107

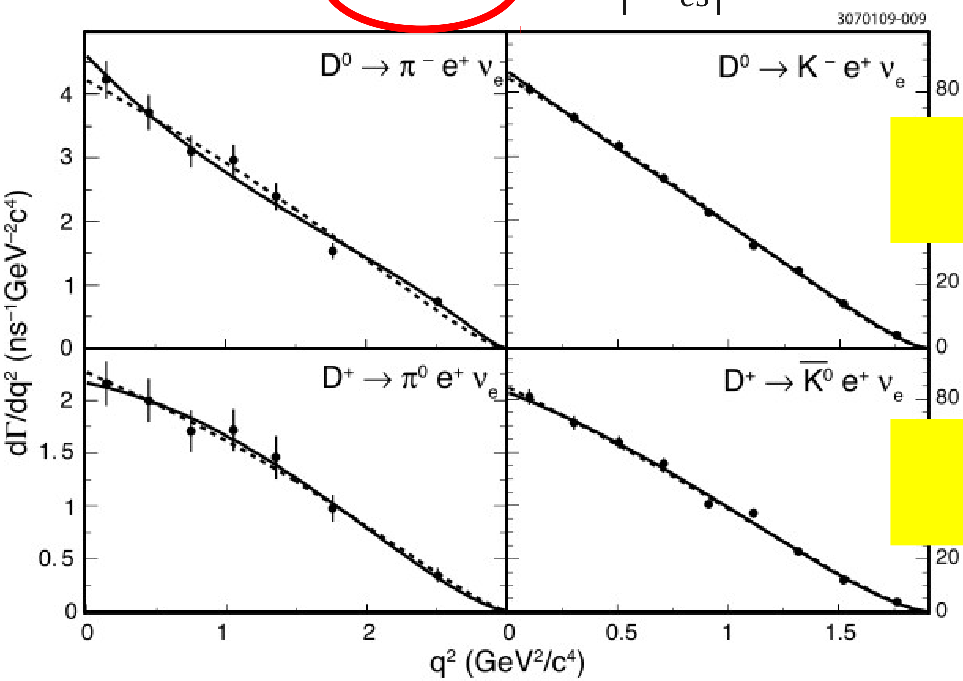
$$|V_{us}| = 0.2166 \pm 0.0019(\exp) \pm 0.0005(th)$$

  - discrepancy from $|V_{us}|$ from kaons: 3.7σ
    - also discrepant with $|V_{us}|$ from $B(\tau \to K\nu)/B(\tau \to \pi\nu)$ + f_K/f_π from lattice
    - several multibody tau decays not measured yet
  - improved measurements urgently needed

$|V_{cd}|$ and $|V_{cs}|$ from charm decays

- Benchmark measurement of $|V_{cd}|$ from charm production in nuclear interactions $|V_{cd}| = 0.230 \pm 0.011$
- Measurements from semileptonic charm decays suffer form-factor uncertainties
  - further improvement in lattice calculations needed

$$|V_{cd}| = 0.234 \pm 0.007 \pm 0.002 \pm 0.025 \qquad |V_{cs}| = 0.961 \pm 0.011 \pm 0.024$$



Lattice input from PRD 82 (2010) 114506

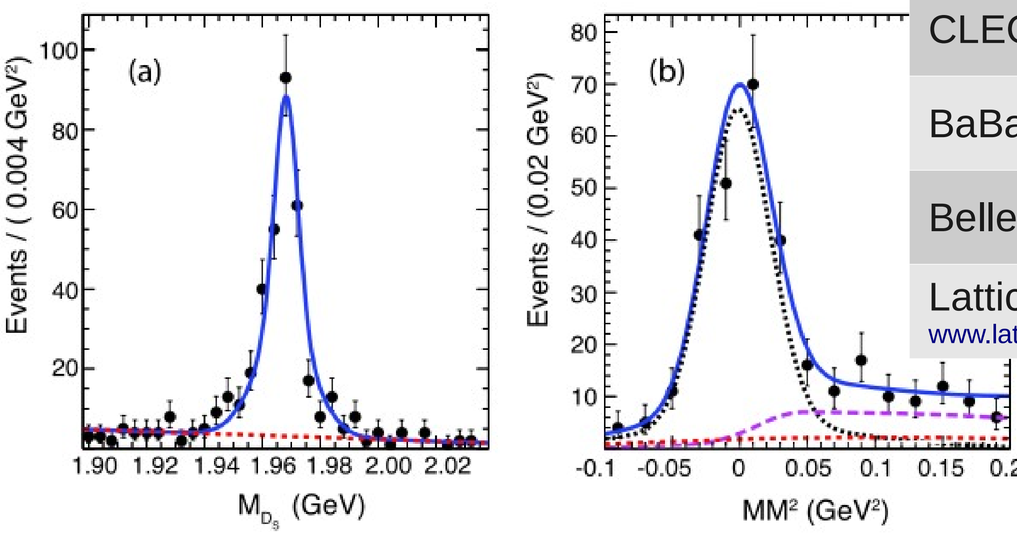
CLEOc experiment PRD 80 (2009) 032005

Alternative approaches to $|V_{cd}|$ and $|V_{cs}|$

- Leptonic D^+ and D_s^+ decays probe $f_D|V_{cx}|$, e.g.

$$\Gamma(D_s^+ \to l^+ \nu) = \frac{G_F^2}{8\pi} f_{D_s^+}^2 m_l^2 M_{D_s^+} \left(1 - \frac{m_l^2}{M_{D_s^+}^2}\right)^2 |V_{cs}|^2$$

CLEOc experiment
PRD 79 (2009) 052001



$D_s^+ \to \mu^+\nu$

	f_D (MeV)	f_{Ds} (MeV)
CLEOc	206.7 ± 8.5 ± 2.5	259.0 ± 6.2 ± 3.0
BaBar		275 ± 16 ± 12
Belle		258.6 ± 6.4 ± 7.5
Lattice average www.latticeaverages.org	213.9 ± 4.2	248.9 ± 3.9

$$|V_{cs}| = 1.005 \pm 0.026 \pm 0.016$$

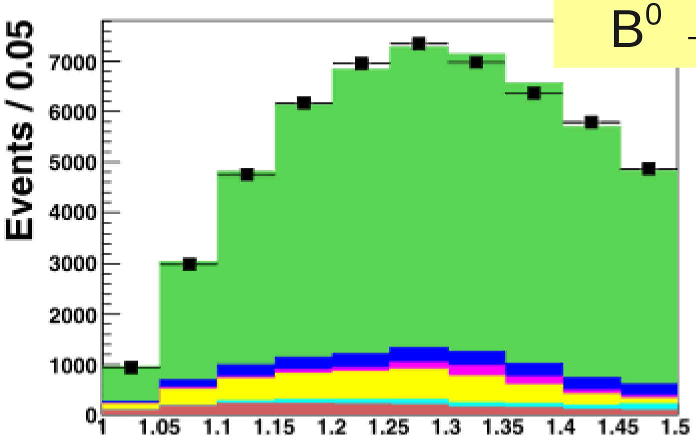
experiment (0.026) lattice (0.016)

$|V_{cb}|$ from semileptonic B decays

- Both exclusive and inclusive approaches

Belle experiment
PRD 82 (2010) 112007

$B^0 \to D^{*-} l\nu$



$w \sim -q^2$

$$|V_{cb}| = (37.5 \pm 0.2 \pm 1.1 \pm 1.0) \times 10^{-3}$$

PDG 2010 quotes

$$|V_{cb}|(excl) = (38.7 \pm 1.1) \times 10^{-3}$$
$$|V_{cb}|(incl) = (41.5 \pm 0.7) \times 10^{-3}$$

2σ tension

lattice uncertainty – reduced to 0.7 in arXiv:1011.2166

tension reduced (~1.6σ)

Searches for charged Higgs in $B \to D^{(*)}\tau\nu$

Branching fraction ratio ($R^{(*)}$) relative to $B \to D^{(*)} l\nu$ predicted in the Standard Model with reduced form-factor uncertainty

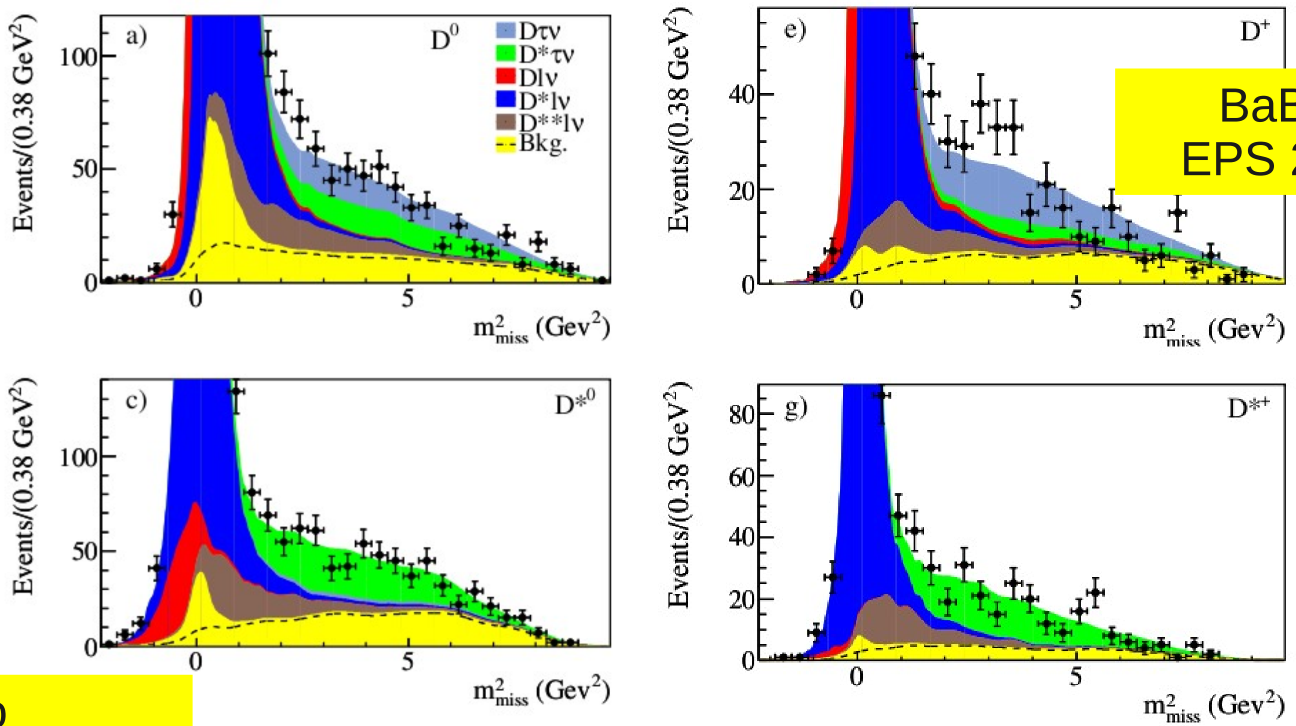
$B^- \to D^0 \tau^- \nu$

$\bar{B}^0 \to D^+ \tau^- \nu$

$B^- \to D^{*0} \tau^- \nu$

$\bar{B}^0 \to D^{*+} \tau^- \nu$

BaBar experiment
EPS 2011 preliminary



See also
Belle experiment
PRD 82 (2010) 072005

$$R(D) = 0.456 \pm 0.053 \pm 0.056 \qquad R^{SM}(D) = 0.31 \pm 0.02$$
$$R(D^*) = 0.325 \pm 0.023 \pm 0.027 \qquad R^{SM}(D^*) = 0.25 \pm 0.07$$

1.8σ excess over the Standard Model – more in Rare Decays talk

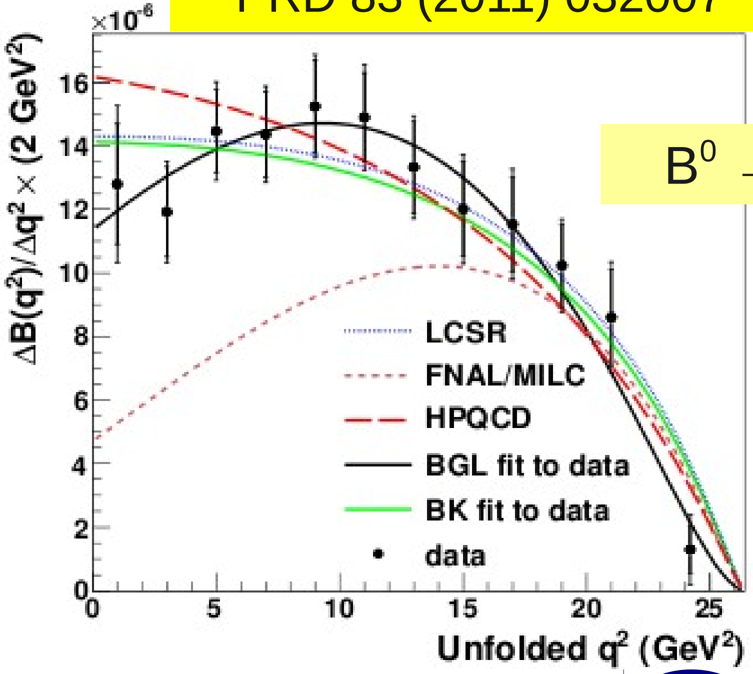
$|V_{ub}|$ from semileptonic B decays

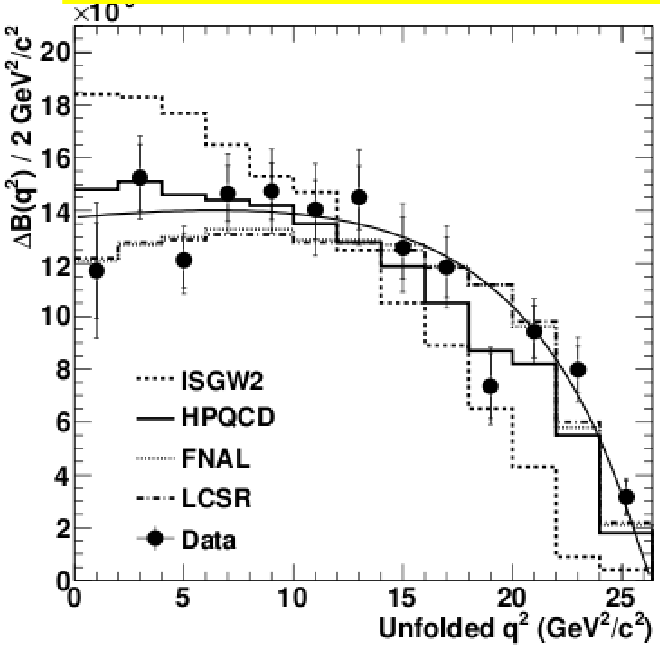
- Both exclusive and inclusive approaches

BaBar experiment
PRD 83 (2011) 052011
PRD 83 (2011) 032007

Belle experiment
PRD 83 (2011) 071101(R)

$B^0 \to \pi^- l \nu$



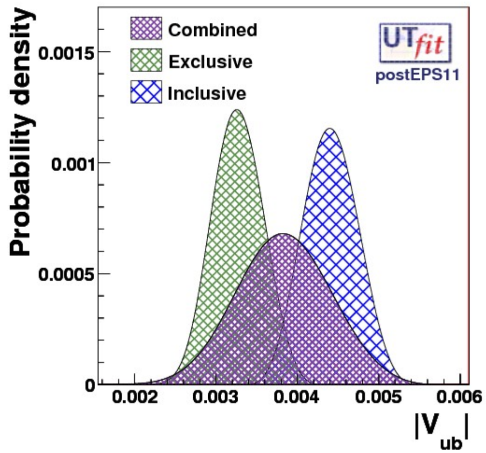


$$\left|V_{ub}\right| = \left(3.09 \pm 0.08 \pm 0.12 \left({}^{+0.35}_{-0.29}\right)\right) \times 10^{-3}$$

$$\left|V_{ub}\right| = \left(3.43 \pm 0.33\right) \times 10^{-3}$$

lattice uncertainty

$|V_{ub}|$ from semileptonic B decays



- Another tension between exclusive and inclusive
  - PDG2010 quotes

$$|V_{ub}|(excl) = (3.38 \pm 0.36) \times 10^{-3}$$
$$|V_{ub}|(incl) = (4.27 \pm 0.38) \times 10^{-3}$$

- A distinguished theorist recently said:

arXiv:1108.3514

"... this tension may be due to the fact that over the last 30 years hundreds of theory papers have been devoted to the determination of V_{ub} with each author claiming that his/her work led to a decrease of the theoretical error ..."

- In my view more, not less, theoretical attention is required
  - e.g. SIMBA collaboration to improve understanding of inclusive decays

arXiv:1101.3310

- N.B. $|V_{ub}|$ from leptonic decays covered in rare decays talk

CP violation

CP violation and the matter-antimatter asymmetry

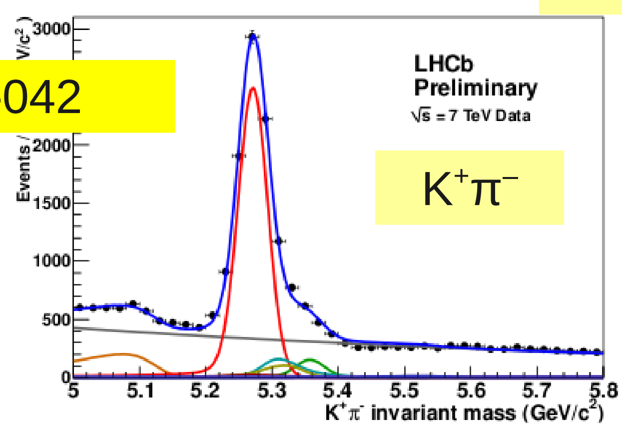
- Two widely known facts
  1) CP violation is one of 3 “Sakharov conditions” necessary for the evolution of a baryon asymmetry in the Universe
  2) The Standard Model (CKM) CP violation is not sufficient to explain the observed asymmetry
- Therefore, there must be more sources of CP violation in nature … but where?
  - extended quark sector, lepton sector (leptogenesis), supersymmetry, anomalous gauge couplings, extended Higgs sector, quark-gluon plasma, flavour-diagonal phases, …
- Testing the consistency of the CKM mechanism provides the best chance to find new sources of CP violation today

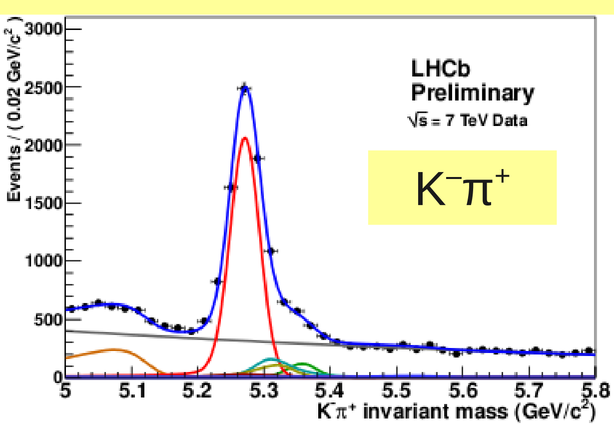
Observations of CP violation

- Still a rare phenomenon:
  - only seen (>5σ) in K^0 and B^0 systems
- In B system, only
  - $\sin(2\beta)$ in $B^0 \to J/\psi K_{S,L}$ (etc.) – BaBar & Belle
  - $S(B^0 \to \eta' K_{S,L})$ (etc.) – BaBar & Belle
  - $S(B^0 \to \pi^+\pi^-)$ – BaBar & Belle
  - $C(B^0 \to \pi^+\pi^-)$ – Belle
  - $A_{CP}(B^0 \to K^+\pi^-)$ – BaBar, Belle & LHCb

$$A_{CP}(B^0 \to K^+\pi^-) = -0.088 \pm 0.011 \pm 0.008$$

LHCB-CONF-2011-042





Unitarity Triangles

PLB 680 (2009) 328

Build matrix of phases between pairs of CKM matrix elements

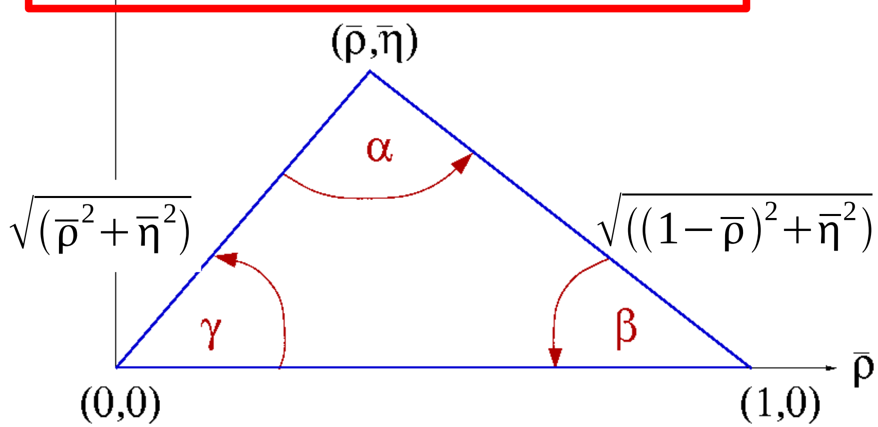
Φ_{ij} = phase between remaining elements when row i and column j removed

unitarity implies sum of phases in any row or column = 180° → 6 unitarity triangles

$$\Phi = \begin{matrix} \\ u \\ c \\ t \end{matrix} \begin{matrix} d & s & b \\ \end{matrix} \begin{pmatrix} \Phi_{ud} & \Phi_{us} & \Phi_{ub} \\ \Phi_{cd} & \Phi_{cs} & \Phi_{cb} \\ \Phi_{td} & \Phi_{ts} & \Phi_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1^o & 22^o & 157^o \\ 67^o & 90^o & 23^o \\ 112^o & 68^o & 0^o \end{pmatrix}$$

β_s (1°), $\beta \equiv \varphi_1$ (22°), $\alpha \equiv \varphi_2$ (90°), $\gamma \equiv \varphi_3$ (68°), $\varphi_D/2$ (0°)

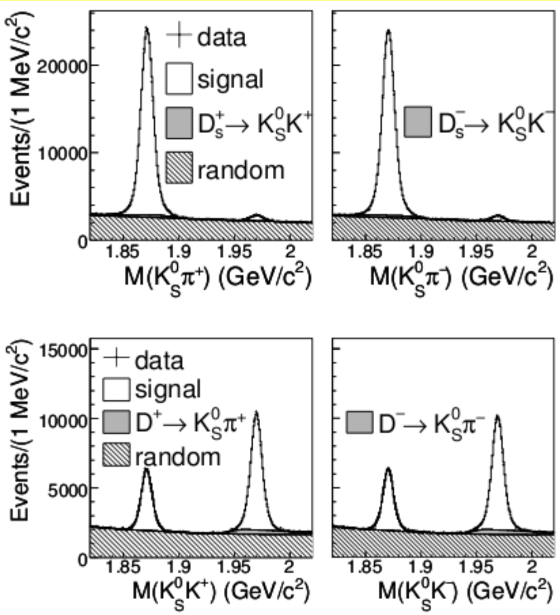
"The Unitarity Triangle"



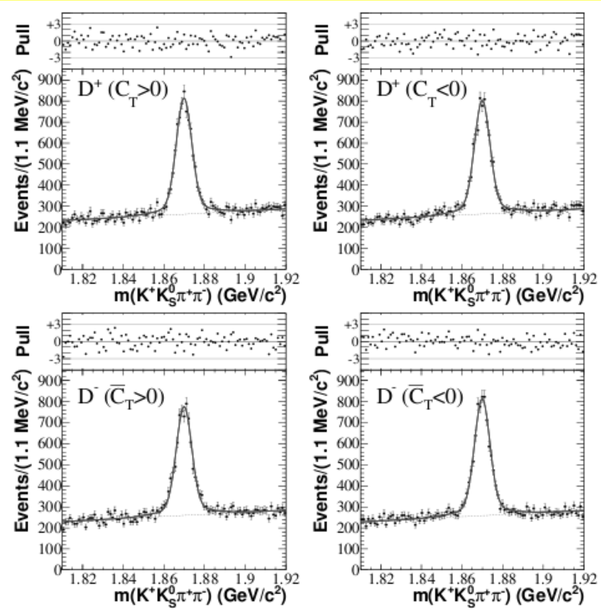
CP violation null tests: charm decays

- All (almost) CP violation effects in the charm system expected to be negligible
  - searches for direct CP violation (see also talk on mixing)

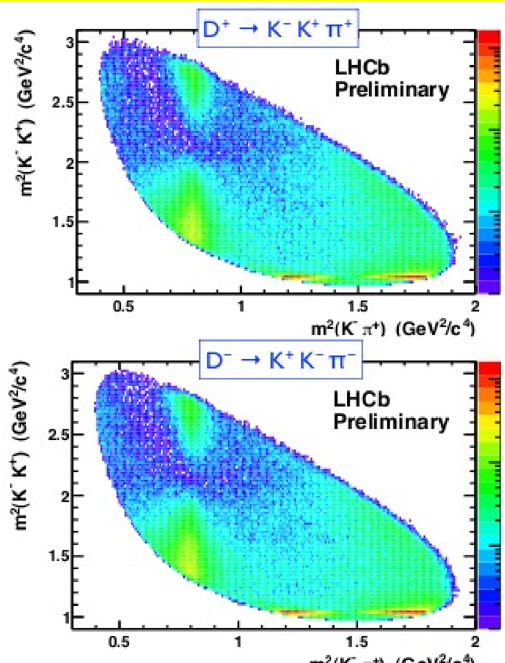
Belle experiment
PRL 104 (2010) 181602



BaBar experiment
arXiv:1105.4410 (PRD(R))



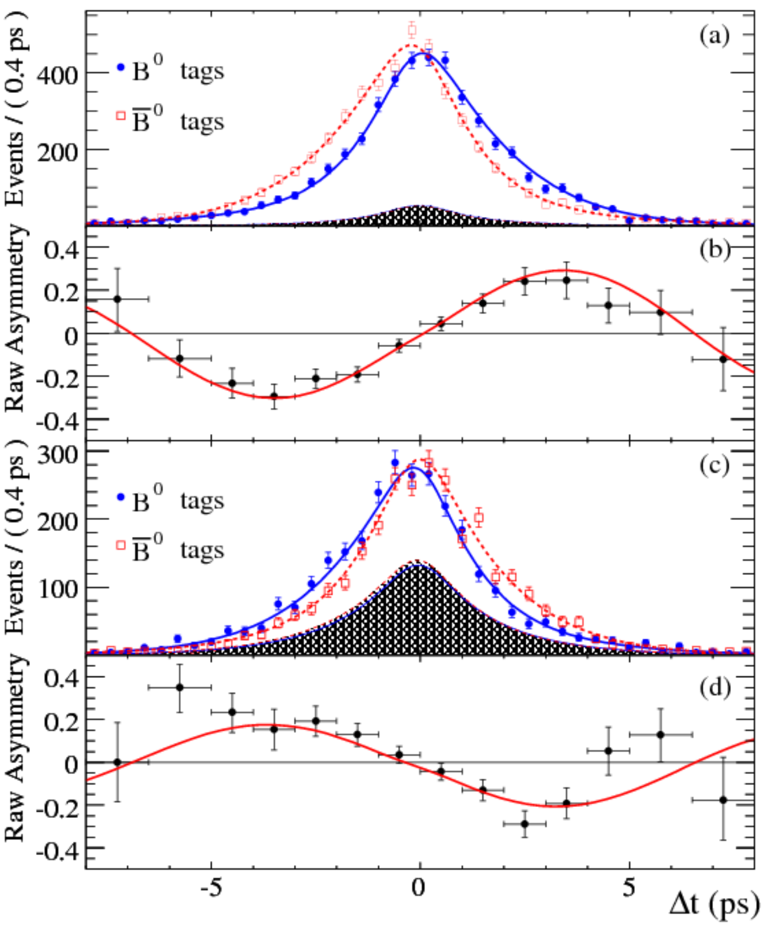
LHCb experiment
EPS 2011 preliminary



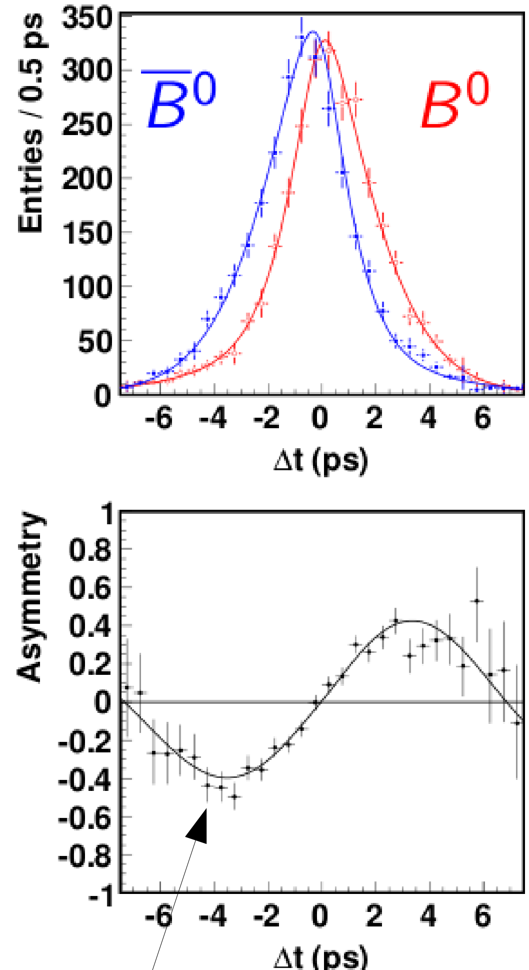
All consistent with zero and with the Standard Model

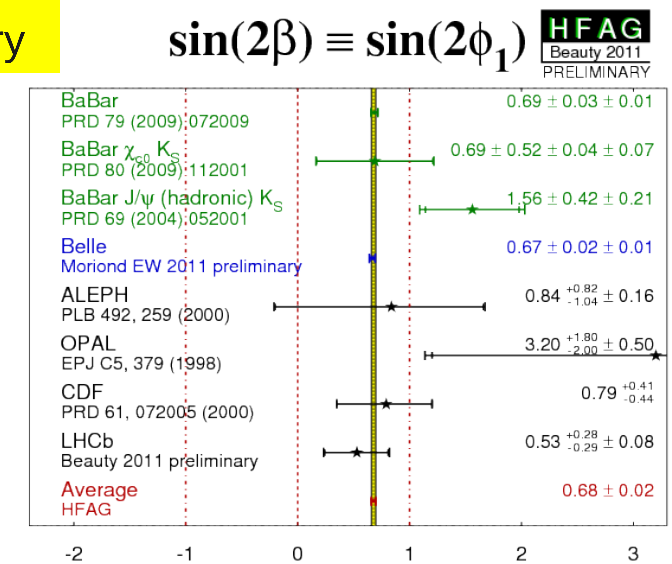
sin(2β) from $B^0 \to J/\psi\ K_{S,L}$ (etc.)

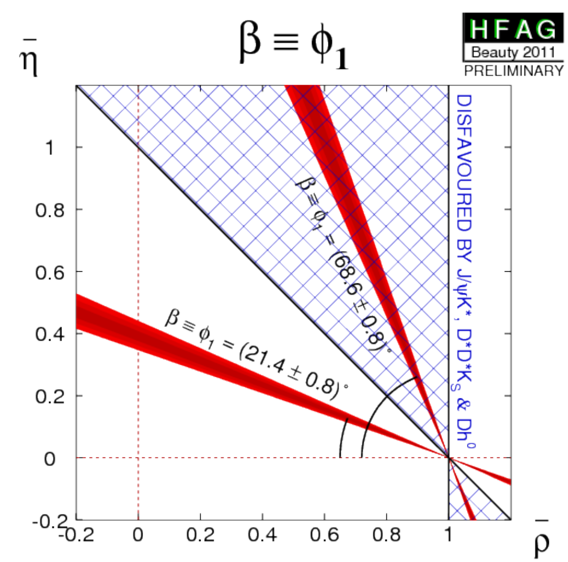
BaBar experiment
PRD 79 (2009) 072009



Belle experiment
Moriond EW 2011 preliminary





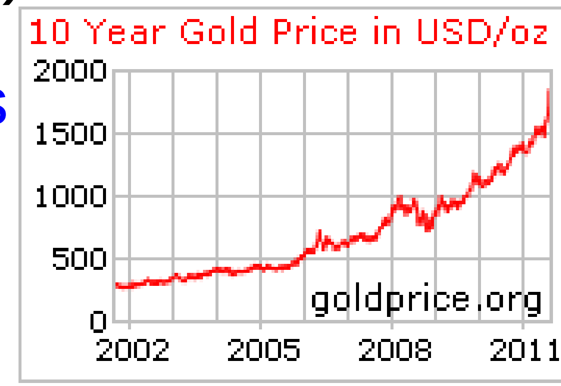


Final Belle dataset (772M $B\overline{B}$ pairs)
with reprocessed data

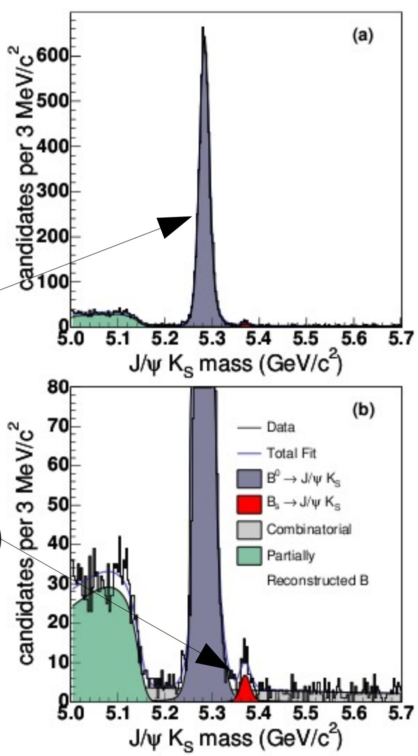
Tim Gershon
CKM Matrix Overview

Checking the quality of gold

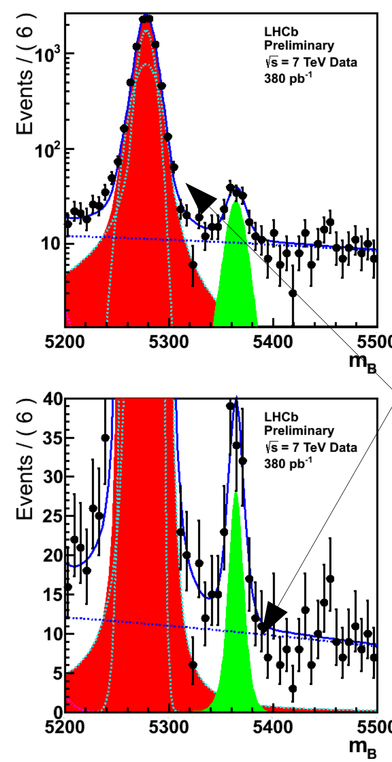
- $B^0 \to J/\psi\, K_S$ is a golden mode for sin(2β)
  - Can check purity using flavour symmetries
    - $B^0 \to J/\psi\, \pi^0$ (related by SU(3))
    - $B_s^0 \to J/\psi\, K_S$ (related by U spin)



CDF experiment
PRD 83 (2011) 052012



$$\frac{B\left(B_s^0 \to J/\psi\, K_S\right)}{B\left(B^0 \to J/\psi\, K_S\right)} = 0.041 \pm 0.007\,(stat) \pm 0.004\,(syst) \pm 0.005\,(f_s/f_d)$$



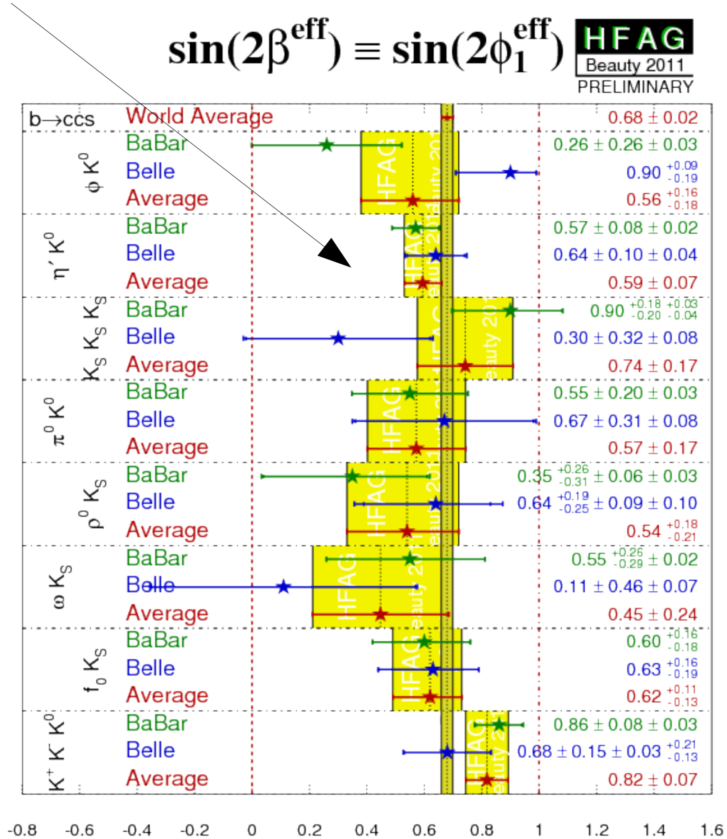
LHCb experiment
LHCb-CONF-2011-048

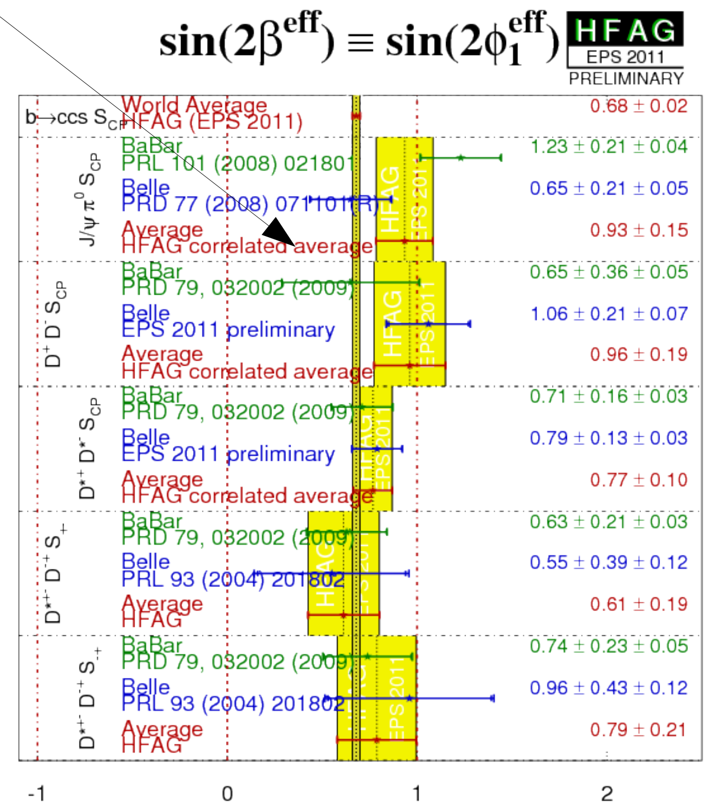
NEW

$$\frac{B\left(B_s^0 \to J/\psi\, K_S\right)}{B\left(B^0 \to J/\psi\, K_S\right)} = 0.0378 \pm 0.0058\,(stat) \pm 0.0020\,(syst) \pm 0.0030\,(f_s/f_d)$$

Other approaches to sin(2β)

- Compare b→cc̄s transitions (e.g. $B^0 \to J/\psi K_S$) with b→ss̄s (e.g. $B^0 \to \eta' K_S$), b→cc̄d (e.g. $B^0 \to D^+D^-$), or b→cūd (e.g. $B^0 \to D_{CP}\pi^0$)

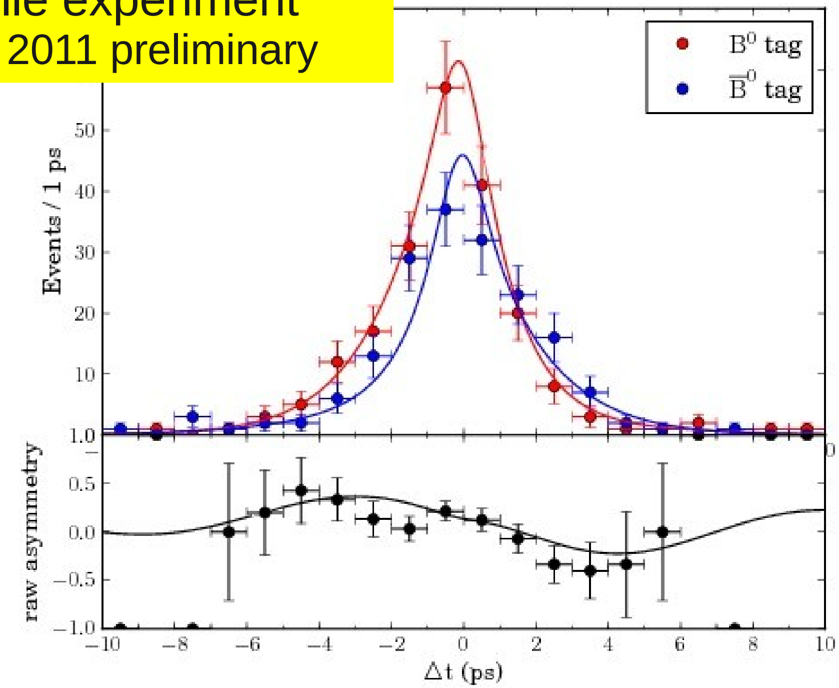




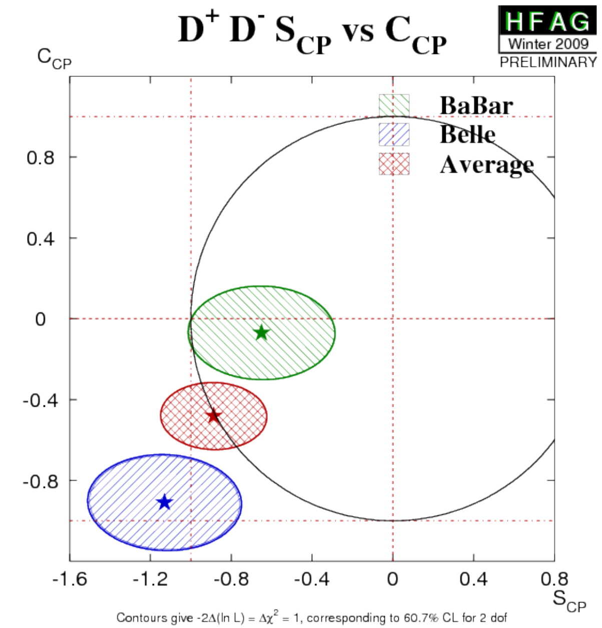
Hints of deviations in b→ss̄s diminished

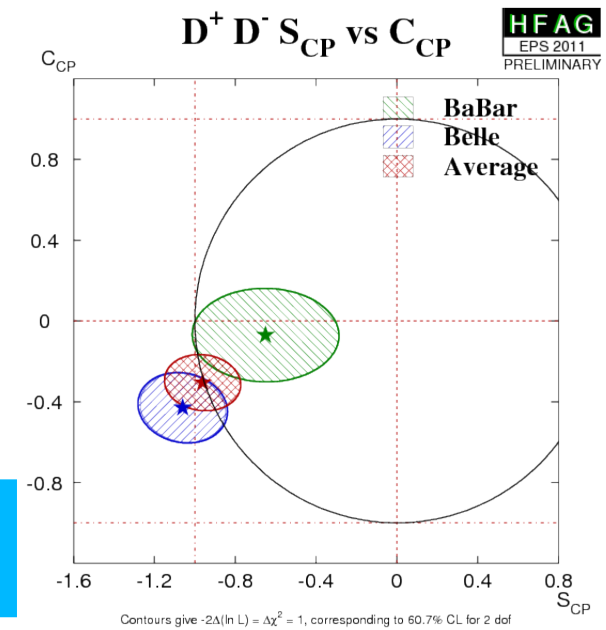
Belle update on $B^0 \to D^+D^-$

Belle experiment
EPS 2011 preliminary



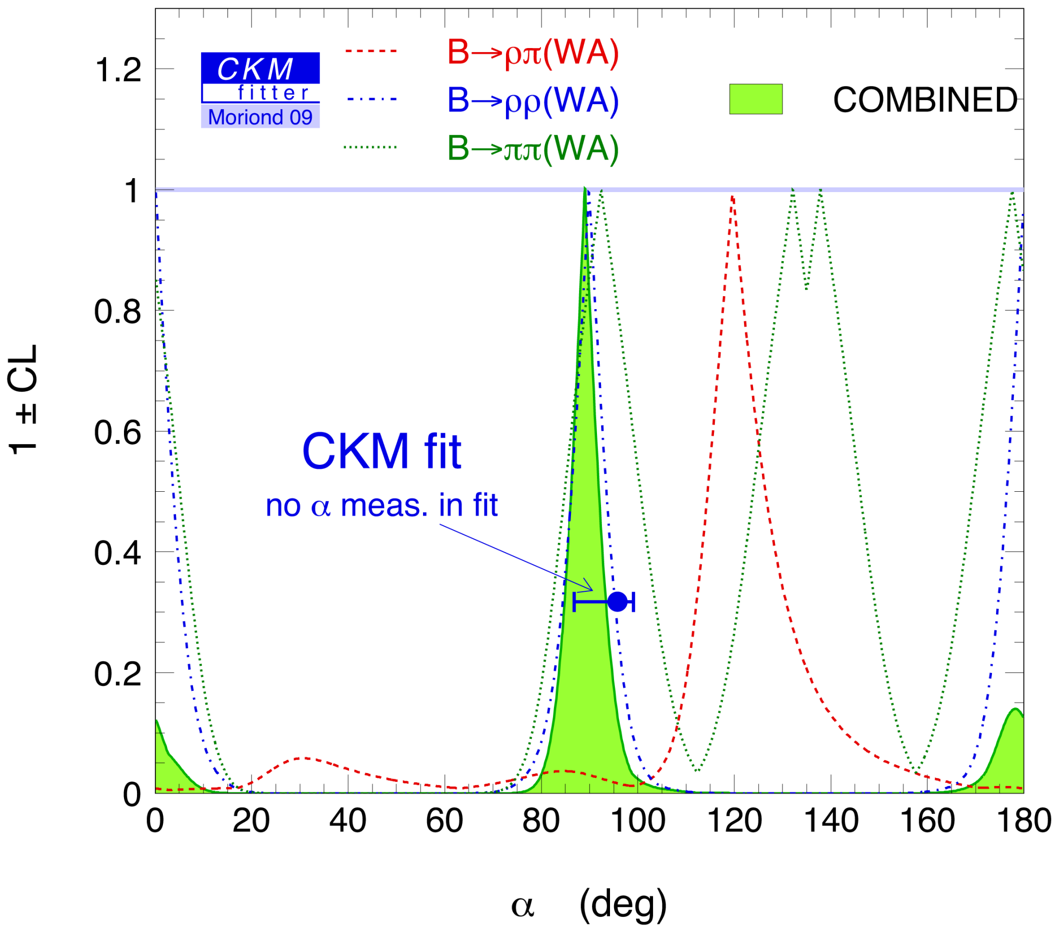
$$S(D^+D^-) = -1.06 \pm 0.21 \pm 0.07$$
$$C(D^+D^-) = -0.43 \pm 0.17 \pm 0.04$$





Belle – BaBar – Standard Model discrepancy diminished

α from $B \to \pi\pi$, $\rho\pi$, $\rho\rho$ systems



- Awaiting final results from both BaBar and Belle on
  - $B^0 \to \pi^+\pi^-$
  - $B^0 \to (\rho\pi)^0$
  - $B^0 \to \rho^+\rho^-$
- World average

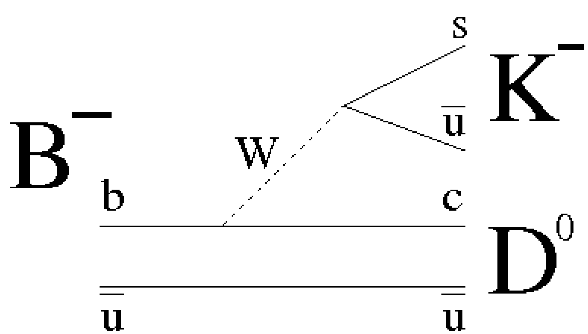
$$\alpha = \left(89.0\ ^{+4.4}_{-4.2}\right)^\circ$$

  - dominated by $B \to \rho\rho$
  - strong influence of single (BaBar) measurement of $B(B^+ \to \rho^+\rho^0)$
- Is $\alpha = 90°$?

γ from B → D$^{(*)}$K decays

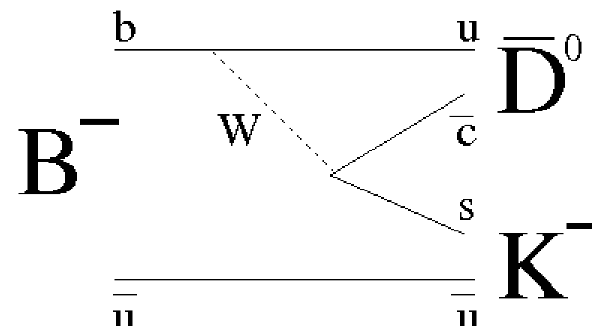
Tree-level determination of γ from interference of B → DK ($b \to c\bar{u}s$) and B → $\bar{D}$K ($b \to u\bar{c}s$) amplitudes

- need D and $\bar{D}$ to decay to common final state



$\propto V_{cb} V_{us}^*$

- colour allowed
- final state contains D^0



$\propto V_{ub} V_{cs}^*$

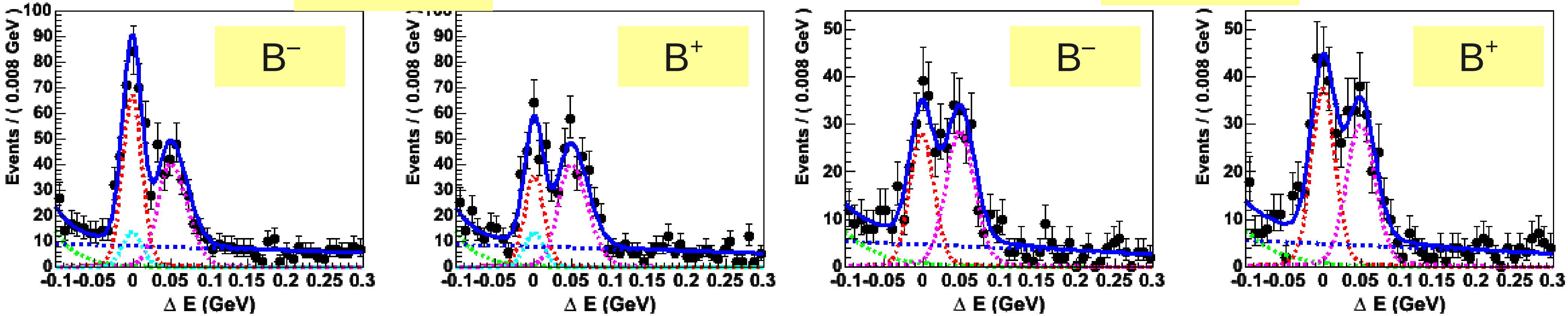
- colour suppressed
- final state contains $\overline{D}^0$

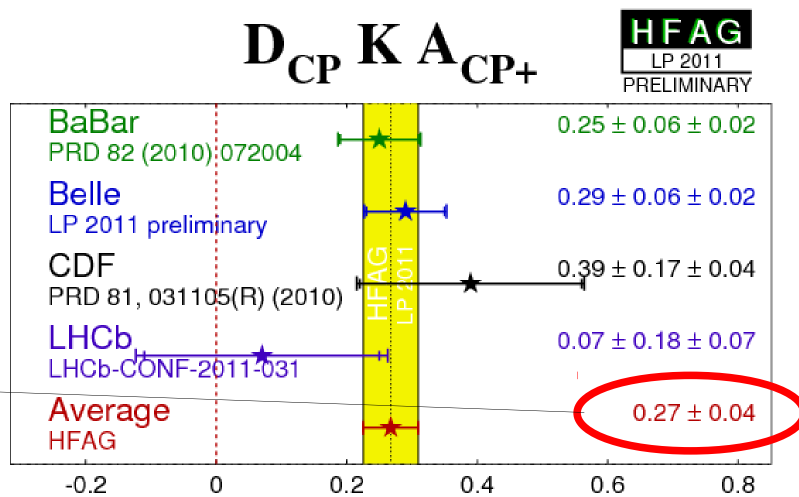
γ from B → DK, D → CP eigenstate (GLW)

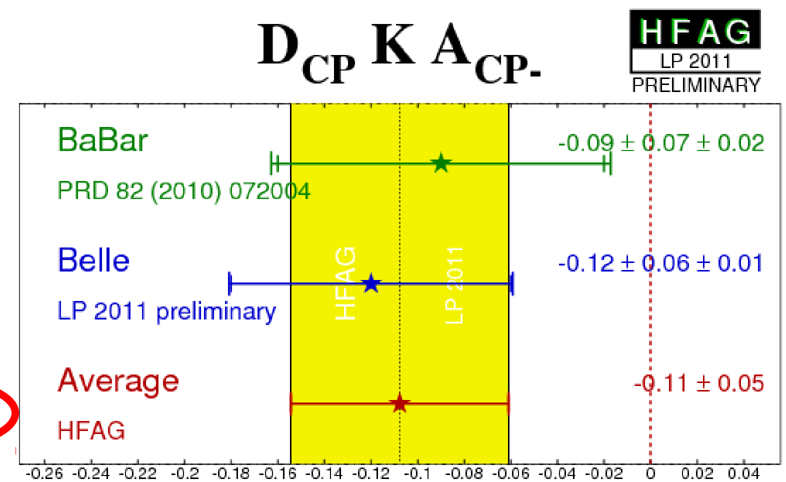
Belle experiment
BELLE-CONF-1112

NEW

CP+ CP–



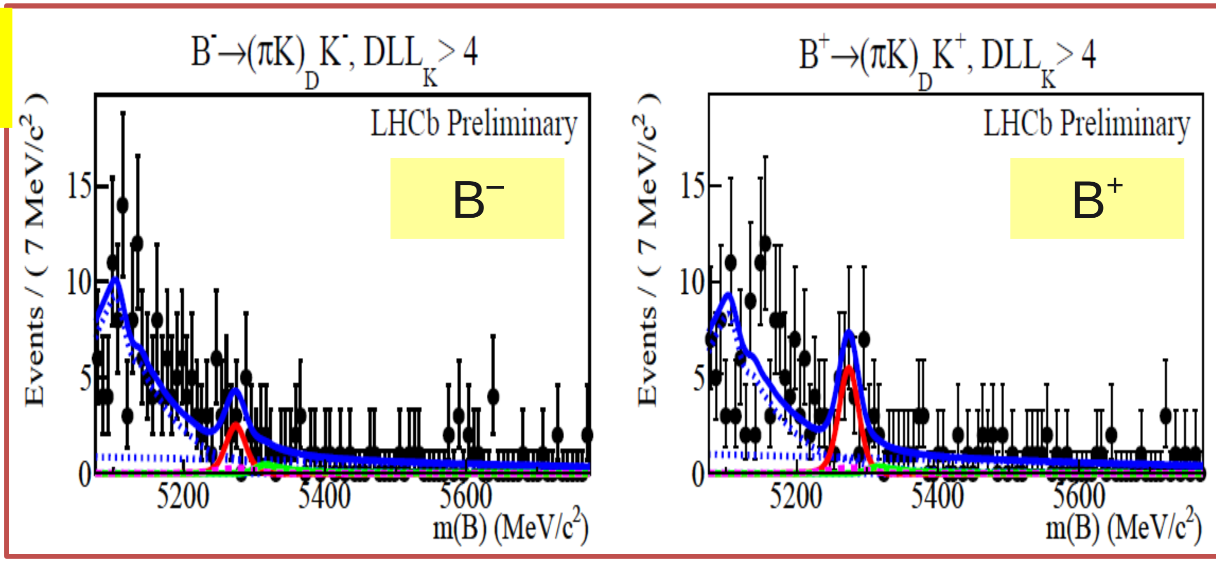


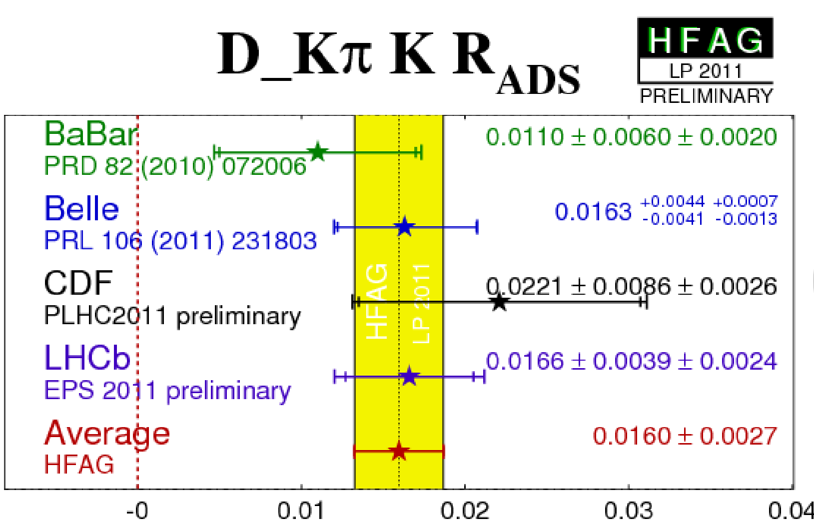


CP violation clearly established

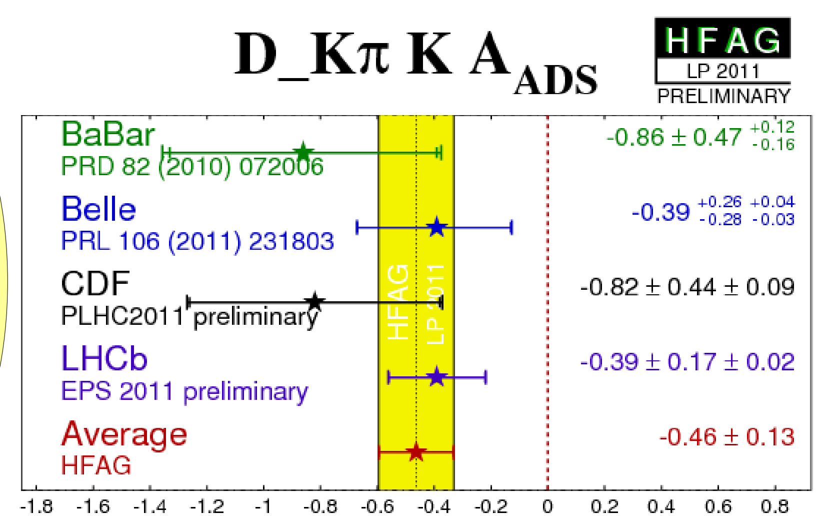
γ from B→DK, D→suppressed states (ADS)

LHCb experiment
LHCb-CONF-2011-044





All new results in last 2 years

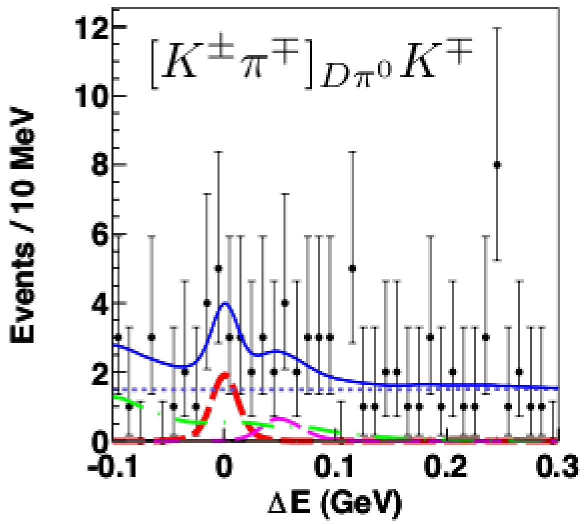


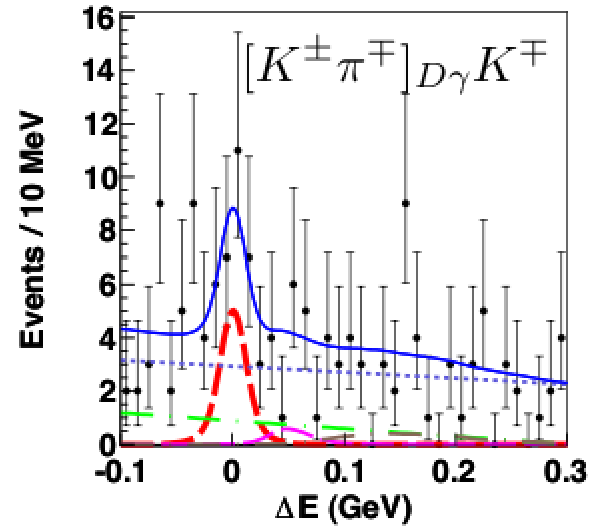
ADS suppressed mode now clearly established ...
… very promising for γ determination

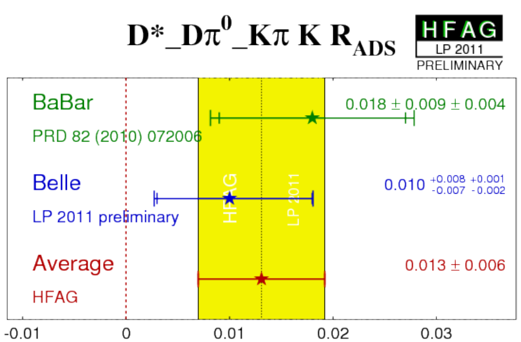
γ from $B \to D^{*}K$, $D \to$ suppressed states (ADS)

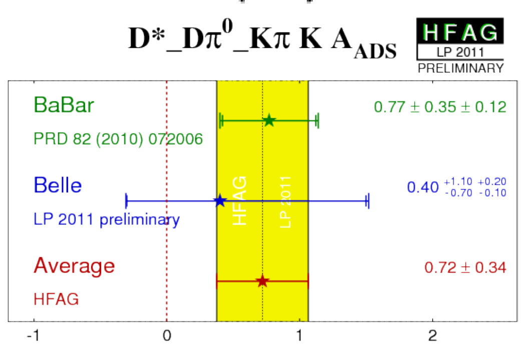
Belle experiment
BELLE-CONF-1112

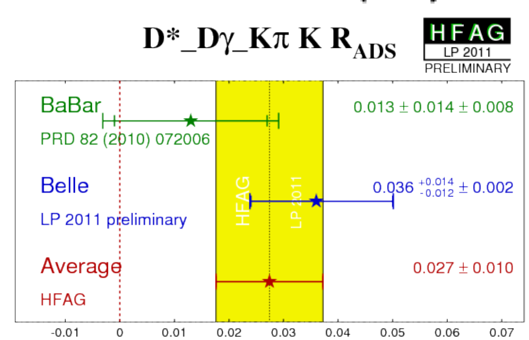
NEW

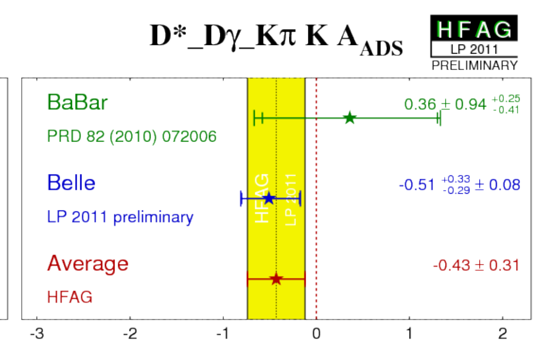












Suppressed modes also appearing in D*K?

γ from B→DK, D→multibody states (GGSZ)

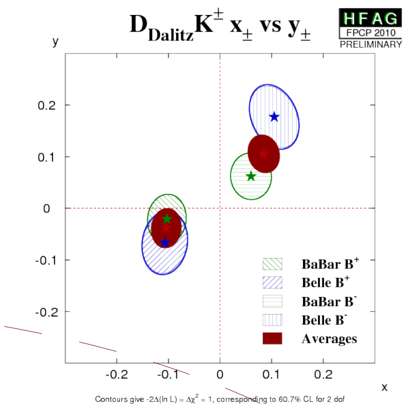
Study of $D \to K_S \pi^+ \pi^-$ Dalitz plot distribution provides good statistical sensitivity to γ but with model dependence

BaBar experiment
PRL 105 (2010) 121801

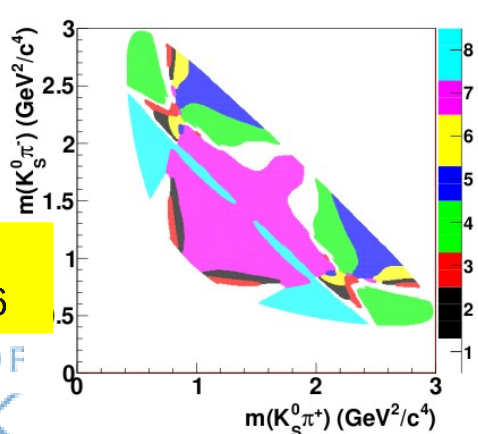
$$\gamma = \left(68 \,^{+15}_{-14} \pm 4 \pm 3\right)^\circ$$

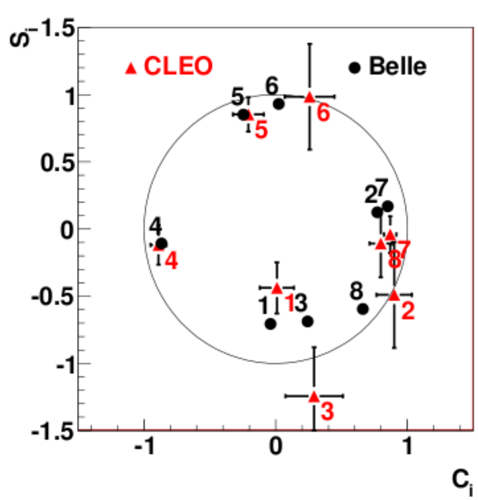
Belle experiment
PRD 81 (2010) 112002

$$\gamma = \left(78 \,^{+11}_{-12} \pm 4 \pm 9\right)^\circ$$



Model independent (binned) approach exploiting $\Psi(3770) \to D\overline{D}$ data





CLEOc experiment
PRD 82 (2010) 112006

Belle experiment
arXiv:1106.4046

$$\gamma = \left(77 + 15 \pm 4 \pm 4\right)^\circ$$

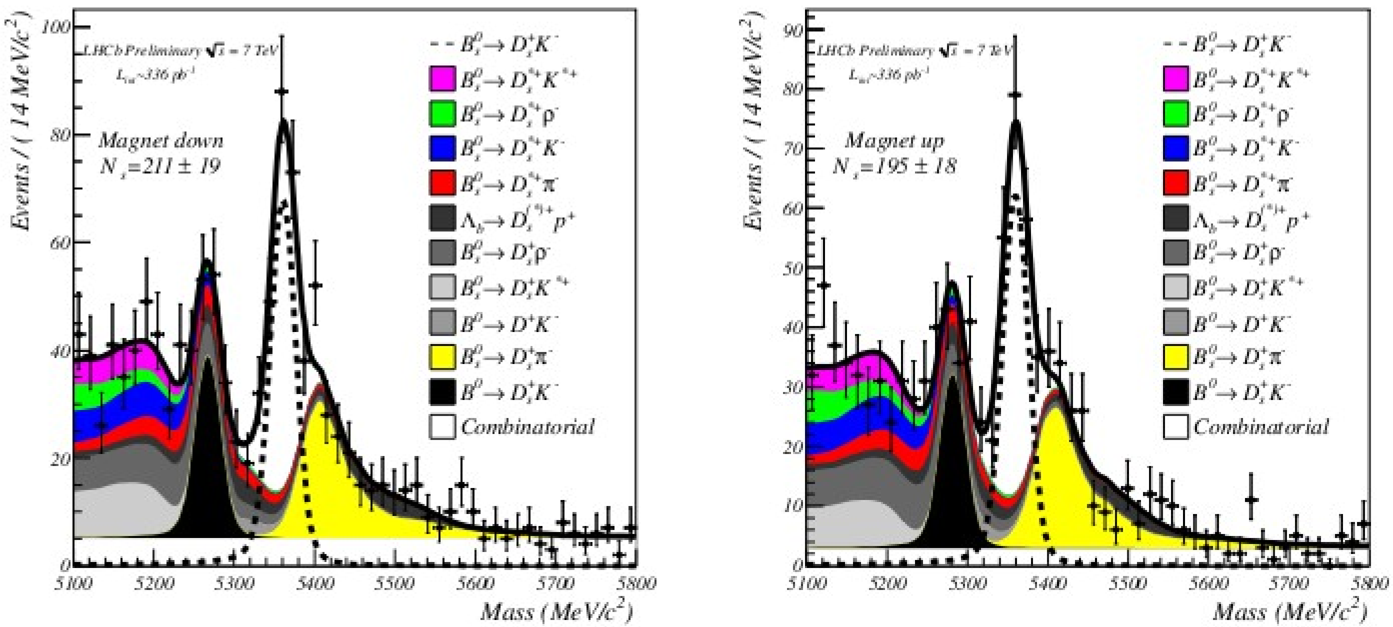
γ from $B_s \to D_s K$

LHCb experiment
LHCb-CONF-2011-057

NEW

γ can be extracted from time-evolution of $B_s \to D_s K$ decays

first stage: establish signals & measure branching fraction
yields split by magnet polarity



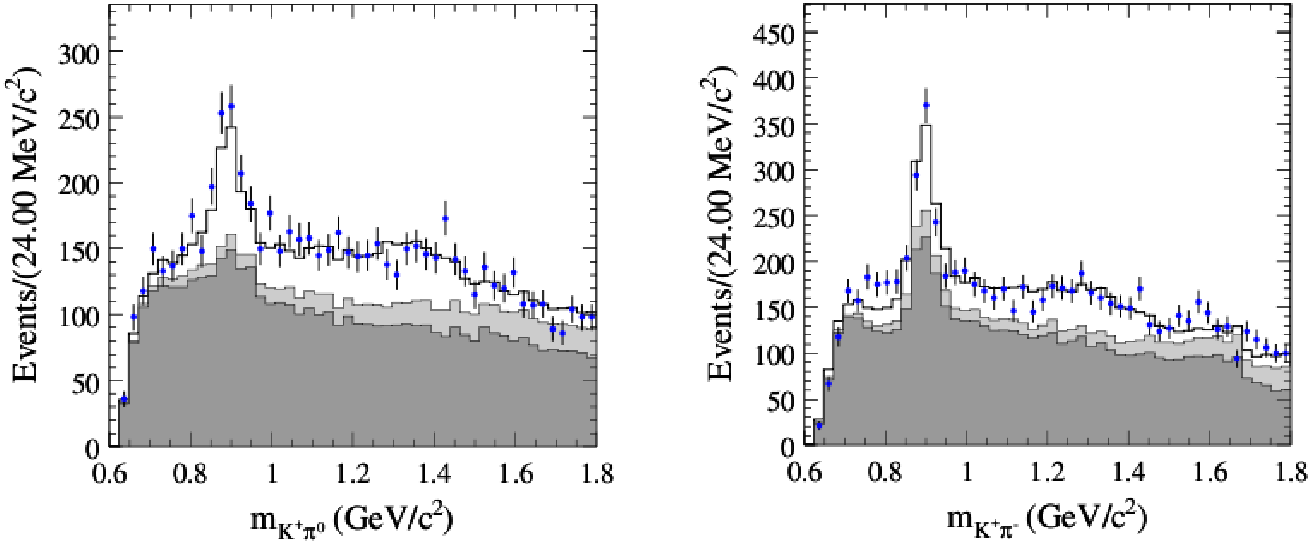
$$B\left(B_s \to D_s^{\mp} K^{\pm}\right) = \left(1.97 \pm 0.18\,(stat)\,^{+0.19}_{-0.20}\,(syst)\,^{+0.11}_{-0.10}\,(f_s/f_d)\right) \times 10^{-4}$$

Promising for future γ measurement

Alternative ways to measure γ

- Test Standard Model by comparing γ from tree-level processes to γ from loop-dominated amplitudes
  - various approaches exploiting flavour symmetries
    - $B^0 \to K^+\pi^-$ (see rare decays talk)
    - $B_s^0 \to K^+K^-$ & $B^0 \to \pi^+\pi^-$ (see LHCb talk)
    - $B^0 \to K_S\pi^+\pi^-$ & $B^0 \to K^+\pi^-\pi^0$

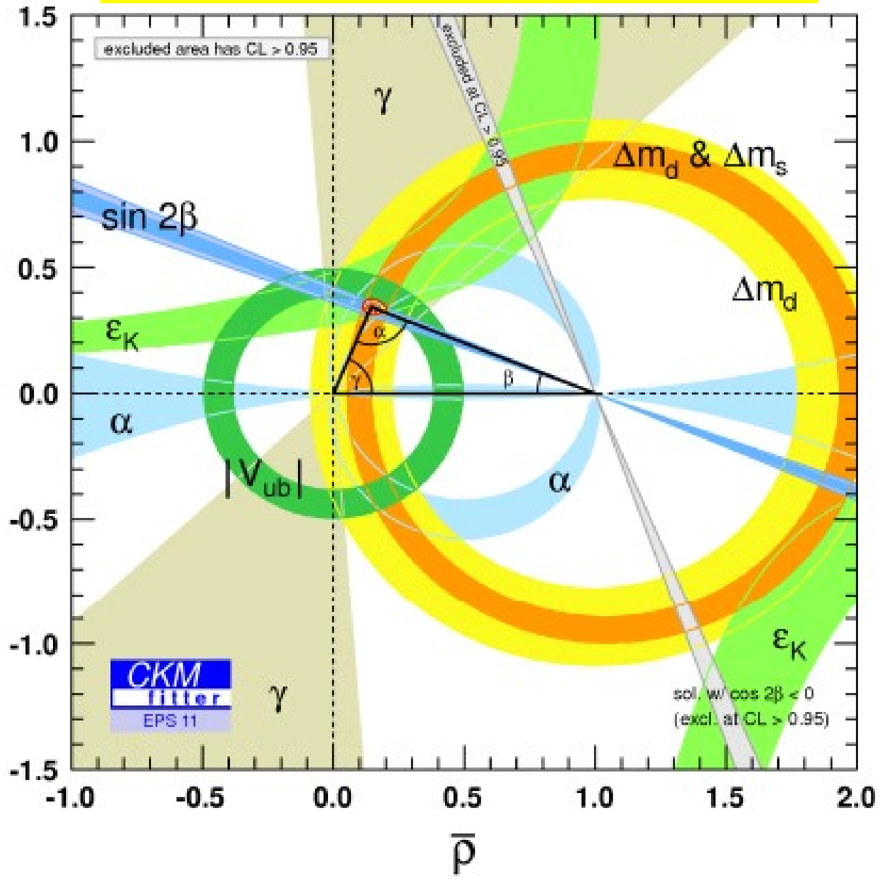
BaBar experiment
PRD 83 (2011) 112010



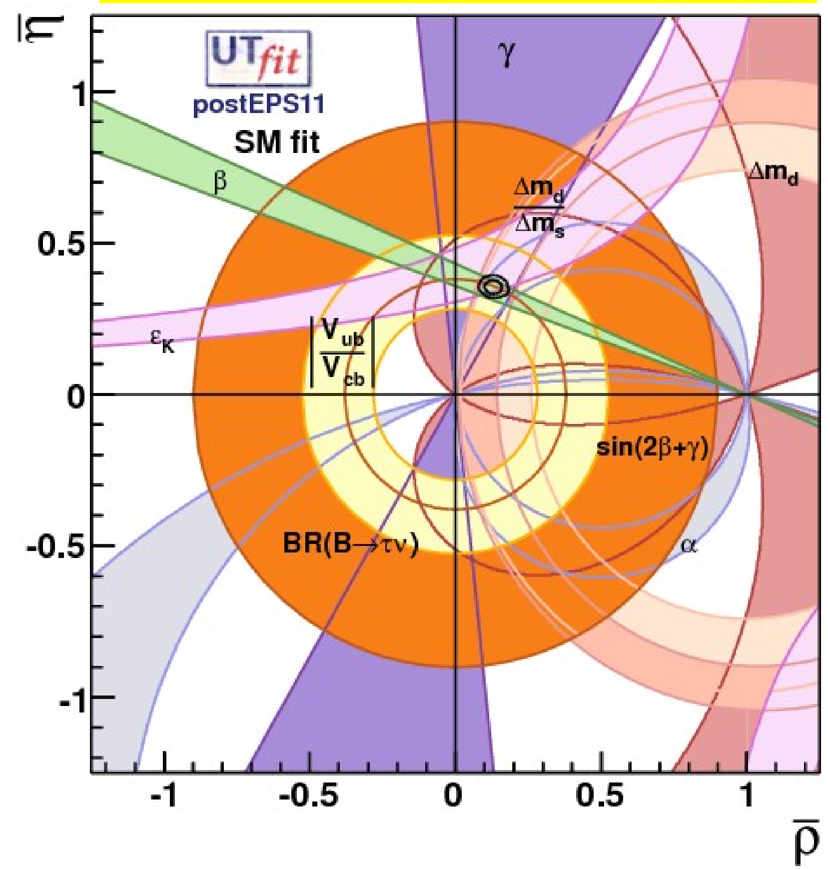
Global CKM fits

Does not include new results on γ shown today

http://ckmfitter.in2p3.fr



http://www.utfit.org

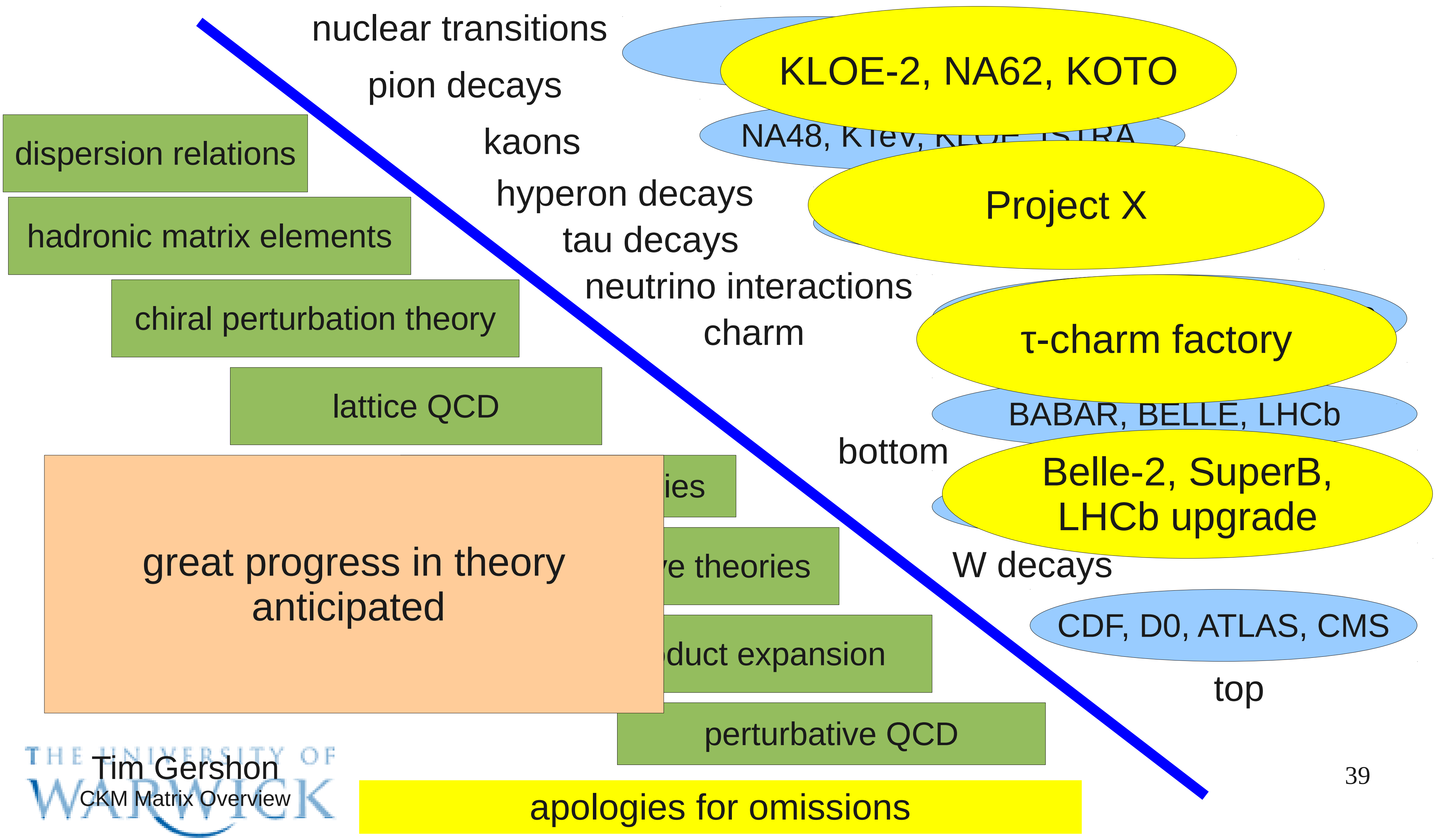


$$\bar{\rho} = 0.144\,^{+0.027}_{-0.018}\ (CKMfitter) = 0.132 \pm 0.020\ (UTfit)$$

$$\bar{\eta} = 0.343 \pm 0.014\ (CKMfitter) = 0.353 \pm 0.014\ (UTfit)$$

Different statistical approaches – similar results
Overall good consistency with the Standard Model

Future projects



Summary

- CKM paradigm continues its unreasonable success
- Current and future projects promise significant improvements
  - short term: BESIII, LHCb, lattice
- Look forward to discovering the destiny of our hopes and hints
  - one certainty: new sources of CP violation exist, somewhere

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- Will we be top of the world … ?

… or do we have to wait for the historic achievement?