Heavy Flavour Physics
Lecture 1 of 2

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HCPSS 2010

Fifth CERN-Fermilab Hadron Collider Physics Summer School

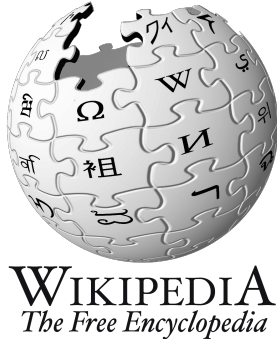17th August 2010

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Contents

- Today

  – What is "heavy flavour physics"?

  – Why is it interesting?

  – What do we know about it as of today?

- Thursday

  – What do we hope to learn from current and future heavy flavour experiments?

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What is flavour physics?

Flavour (particle physics)

From Wikipedia, the free encyclopedia

In particle physics, **flavour** or **flavor** is a quantum number of elementary particles. In quantum chromodynamics, flavour is a global symmetry. In the electroweak theory, on the other hand, this symmetry is broken, and flavour-changing processes exist, such as quark decay or neutrino oscillations.

WIKIPEDIA
The Free Encyclopedia

"The term flavor was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice cream has both color and flavor so do quarks."

RMP 81 (2009) 1887

Flavour in particle physics

Flavour quantum numbers:
- Baryon number: B
- Lepton number: L
- Strangeness: S
- Charm: C
- Bottomness: B'
- Topness: T
- Isospin: I or I_3
- Weak isospin: T or T_3
- Electric charge: Q
- X-charge: X

Combinations:
- Hypercharge: Y
  - $Y = (B + S + C + B' + T)$
  - $Y = 2 (Q - I_3)$
- Weak hypercharge: Y_W
  - $Y_W = 2 (Q - T_3)$
  - $X + 2Y_W = 5 (B - L)$

Flavour mixing
- CKM matrix
- PMNS matrix
- Flavour complementarity

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What is flavour physics?

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Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters
- 6 quark masses
- 3 quark mixing angles + 1 phase
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase)

( ) = with Dirac neutrino masses

Parameters of the Standard Model

- 3 gauge couplings

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CKM matrix

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PMNS matrix

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Parameters of the Standard Model

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( ) = with Dirac neutrino masses

CKM matrix

PMNS matrix

FLAVOUR PARAMETERS

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Mysteries of flavour physics

- Why are there so many different fermions?

- What is responsible for their organisation into generations / families?

- Why are there 3 generations / families each of quarks and leptons?

- Why are there flavour symmetries?

- What breaks the flavour symmetries?

- What causes matter–antimatter asymmetry?

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Mysteries of flavour physics

- Why are there so many different fermions?

- What is responsible for their organisation into generations / families?

- Why are there 3 generations / families each of quarks and leptons?

  Will not address these

- Why are there flavour symmetries?

- What breaks the flavour symmetries?

  Will briefly mention

- What causes matter–antimatter asymmetry?

  A "big science" question

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What is heavy flavour physics?

See PDG reviews for more detailed values

$$m_u \approx 3 \ MeV$$
$$m_d \approx 5 \ MeV$$
$$m_s \approx 100 \ MeV$$
$$m_c \approx 1300 \ MeV$$
$$m_b \approx 4200 \ MeV$$
$$m_t \approx 170000 \ MeV$$

$$m_{\nu_1} \lesssim 10^{-6} \ MeV$$
$$m_{\nu_2} \lesssim 10^{-6} \ MeV$$
$$m_{\nu_3} \lesssim 10^{-6} \ MeV$$
$$m_e \approx 0.5 \ MeV$$
$$m_\mu \approx 100 \ MeV$$
$$m_\tau \approx 1800 \ MeV$$

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What is heavy flavour physics?

$$m_u \approx 3 \ MeV$$
$$m_d \approx 5 \ MeV$$
$$m_s \approx 100 \ MeV$$
$$m_c \approx 1300 \ MeV$$
$$m_b \approx 4200 \ MeV$$
$$m_t \approx 170000 \ MeV$$

Light neutrinos

$$m_e \approx 0.5 \ MeV$$
$$m_\mu \approx 100 \ MeV$$
$$m_\tau \approx 1800 \ MeV$$

The neutrinos have their own phenomenology
(see talk by Siliva Pascoli)

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What is heavy flavour physics?

Light quarks
$(m \leq \Lambda_{QCD})$

Light neutrinos

$$m_c \approx 1300 \ MeV$$
$$m_b \approx 4200 \ MeV$$
$$m_t \approx 170000 \ MeV$$

$$m_e \approx 0.5 \ MeV$$
$$m_\mu \approx 100 \ MeV$$
$$m_\tau \approx 1800 \ MeV$$

Studies of the u and d quarks are the realm of nuclear physics
(does not mean uninteresting!)

Rare decays of kaons provide sensitive tests of the SM

What is heavy flavour physics?

Light quarks
$(m \leq \Lambda_{QCD})$

$$m_c \approx 1300 \ MeV$$
$$m_b \approx 4200 \ MeV$$
$$m_t \approx 170000 \ MeV$$

Light neutrinos

Light charged leptons

$$m_\tau \approx 1800 \ MeV$$

Studies of electric and magnetic dipole moments of the leptons test the Standard Model

Searches for lepton flavour violation are another hot topic

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What is heavy flavour physics?

Light quarks
$(m \leq \Lambda_{QCD})$

$$m_c \approx 1300 \ MeV$$
$$m_b \approx 4200 \ MeV$$

Not THAT heavy!
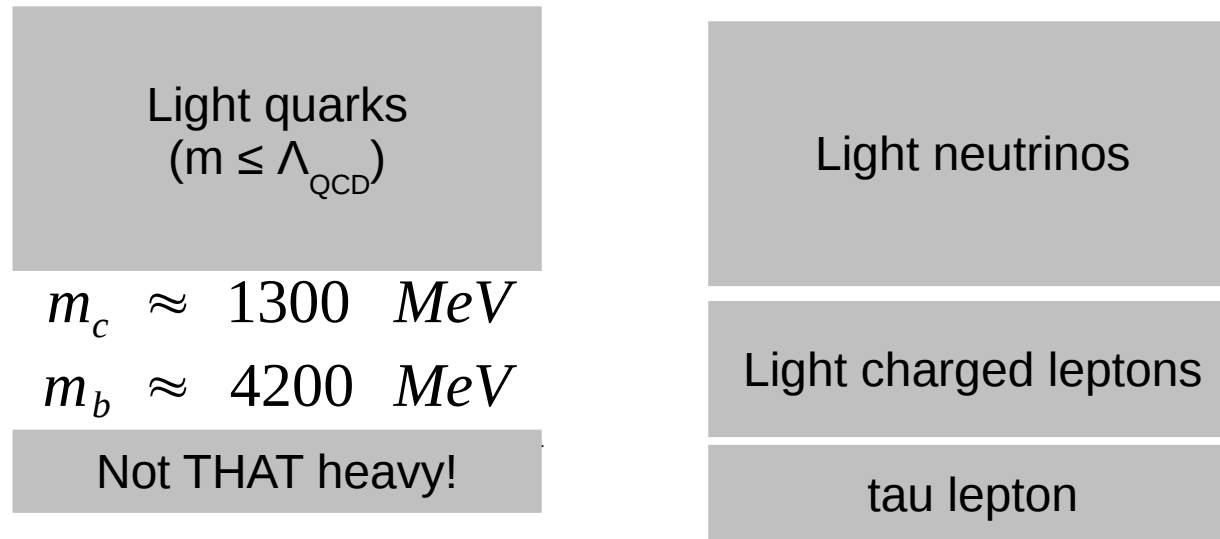
Light neutrinos

Light charged leptons

$$m_\tau \approx 1800 \ MeV$$

The top quark has its own phenomenology (since it does not hadronise)
(see other speakers)

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What is heavy flavour physics?

Light quarks
(m ≤ Λ_{QCD})

$$m_c \approx 1300 \;\; MeV$$

$$m_b \approx 4200 \;\; MeV$$

Not THAT heavy!

Light neutrinos

Light charged leptons

tau lepton

Focus in these lectures will be on
- flavour-changing interactions
- of charm and beauty quarks

These processes are well-suited for study at hadron colliders

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but I cannot resist a few slides about lepton flavour violation ...

Heavy quark flavour physics

- Focus in these lectures will be on
  - flavour-changing interactions of charm and beauty quarks
- But quarks feel the strong interaction and hence hadronise
  - various different charmed and beauty hadrons
  - many, many possible decays to different final states
- The hardest part of quark flavour physics is learning the names of all the damned hadrons!
- On the other hand, hadronisation greatly increases the observability of CP violation effects
  - the strong interaction can be seen either as the "unsung hero" or the "villain" in the story of quark flavour physics

I. Bigi, hep-ph/0509153

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Why is heavy flavour physics interesting?

- Hope to learn something about the mysteries of the flavour structure of the Standard Model

- CP violation and its connection to the matter–antimatter asymmetry of the Universe

- Discovery potential far beyond the energy frontier via searches for rare or SM forbidden processes

What breaks the flavour symmetries?

- In the Standard Model, the vacuum expectation value of the Higgs field breaks the electroweak symmetry

- Fermion masses arise from the Yukawa couplings of the quarks and charged leptons to the Higgs field (taking m_ν=0)

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks

- Consequently, the only flavour-changing interactions are the charged current weak interactions
    - no flavour-changing neutral currents (GIM mechanism)
    - not generically true in most extensions of the SM
    - flavour-changing processes provide sensitive tests

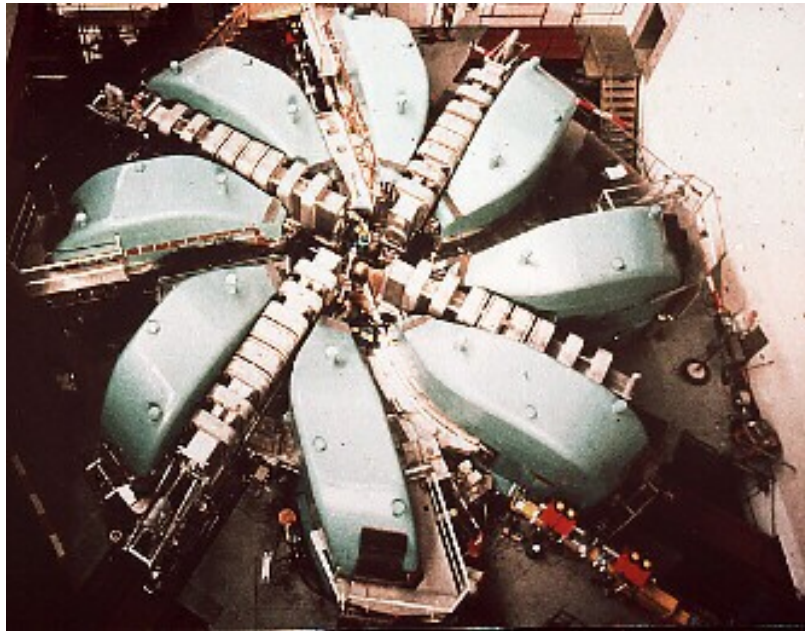Tim Gershon
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Lepton flavour violation

- Why do we not observe the decay $\mu \rightarrow e\gamma$?

  - exact (but accidental) lepton flavour conservation in the SM with $m_\nu = 0$

  - SM loop contributions suppressed by $(m_\nu / m_W)^4$

  - but new physics models tend to induce larger contributions

    - unsuppressed loop contributions
    - generic argument, also true in most common models

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The muon to electron gamma (MEG) experiment at PSI

$\mu^+ \to e^+ \gamma$

- positive muons → no muonic atoms
- continuous (DC) muon beam → minimise accidental coincidences

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First results published   NPB 834 (2010) 1
Expect improved limits (or discoveries) over the next few years
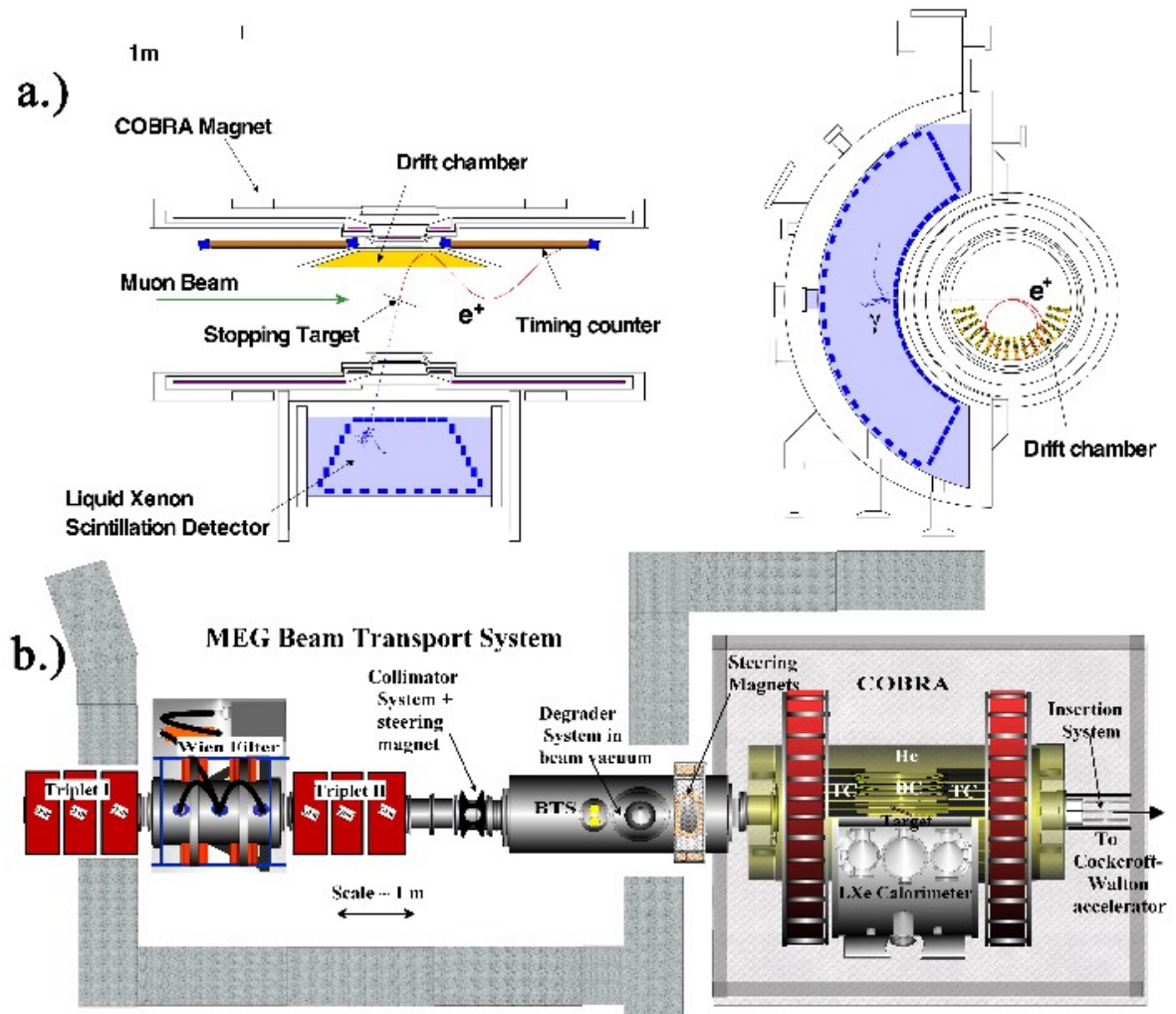
Prospects for Lepton Flavour Violation

- **MEG still taking data**
- **New generations of μ – e conversion experiments**
  - COMET at J-PARC, followed by PRISM/PRIME
  - mu2e at FNAL, followed by Project X
  - Potential improvements of $O(10^4) - O(10^6)$ in sensitivities!
- **τ LFV a priority for next generation e^+e^- flavour factories**
  - SuperKEKB/Belle2 at KEK & SuperB in Italy
  - $O(100)$ improvements in luminosity → $O(10) - O(100)$ improvements in sensitivity (depending on background)
  - LHC experiments have some potential to improve $\tau \to \mu\mu\mu$

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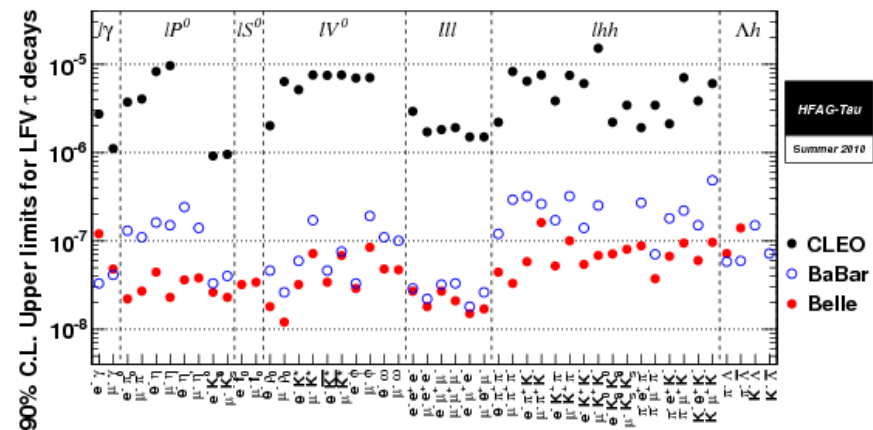
What causes the difference between matter and antimatter?

- **The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks**

$$V_{CKM} = U_u U_d^+$$

U matrices from diagonalisation of mass matrices

- It is a 3x3 complex unitary matrix
  - described by 9 (real) parameters
  - 5 can be absorbed as phase differences between the quark fields
  - 3 can be expressed as (Euler) mixing angles
  - the fourth makes the CKM matrix complex (i.e. gives it a phase)
    - weak interaction couplings differ for quarks and antiquarks
    - CP violation

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The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A 3x3 unitary matrix

- Described by 4 real parameters – allows CP violation
  - PDG (Chau-Keung) parametrisation: θ_{12}, θ_{23}, θ_{13}, δ
  - Wolfenstein parametrisation: λ, A, ρ, η

- Highly predictive

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A brief history of CP violation and Nobel Prizes

- 1964 – Discovery of CP violation in K^0 system

  PRL 13 (1964) 138

- 1973 – Kobayashi and Maskawa propose 3 generations

  Prog.Theor.Phys. 49 (1973) 652

- 1980 – Nobel Prize to Cronin and Fitch
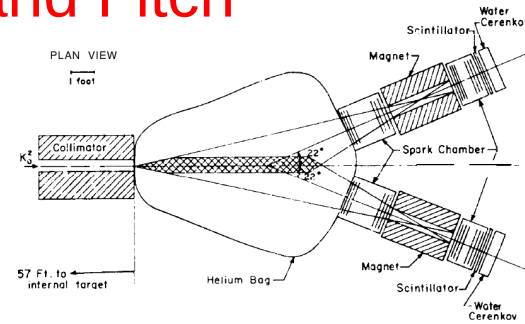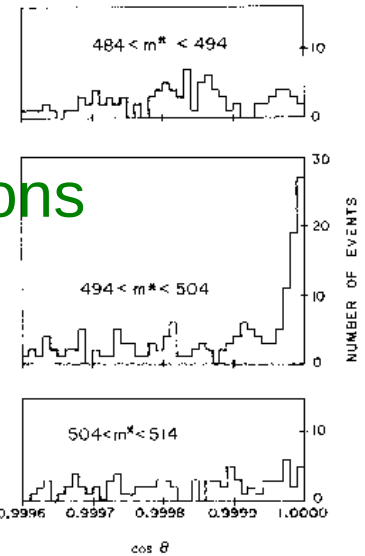
- 2001 – Discovery of CP violation in B_d system

- 2008 – Nobel Prize to Kobayashi and Maskawa

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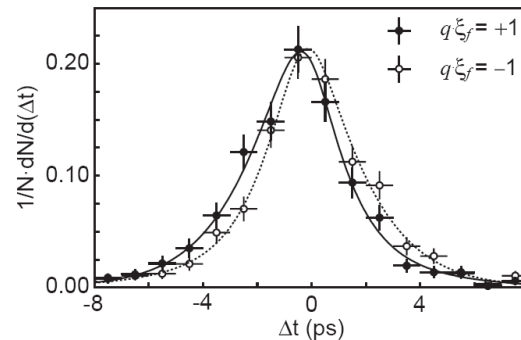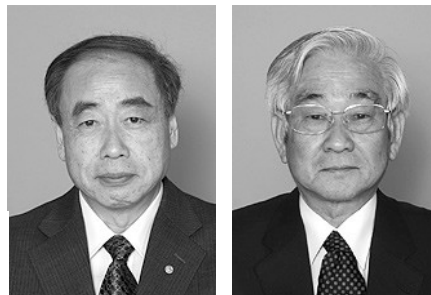
Belle PRL 87 (2001) 091802
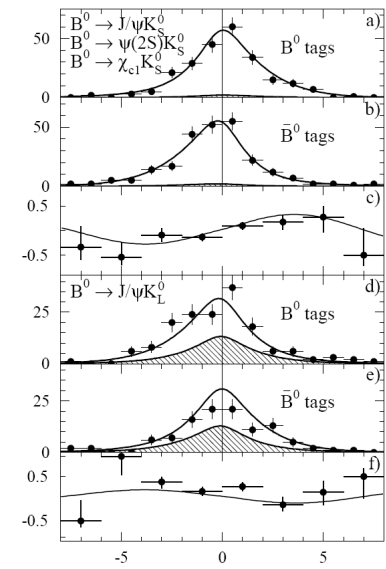
BABAR PRL 87 (2001) 091801

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Sakharov conditions

- Proposed by A.Sakharov, 1967

- Necessary for evolution of matter dominated universe, from symmetric initial state

  (1) baryon number violation

  (2) C & CP violation

  (3) thermal inequilibrium

- No significant amounts of antimatter observed

- $\Delta N_B / N_Y = (N(baryon) - N(antibaryon)) / N_Y \sim 10^{-10}$

Digression: Are there antimatter dominated regions of the Universe?

- Possible signals:

  - Photons produced by matter-antimatter annihilation at domain boundaries – not seen

    - Nearby anti-galaxies ruled out

  - Cosmic rays from anti-stars

    - Best prospect: Anti-^{4}He nuclei

    - Searches ongoing …

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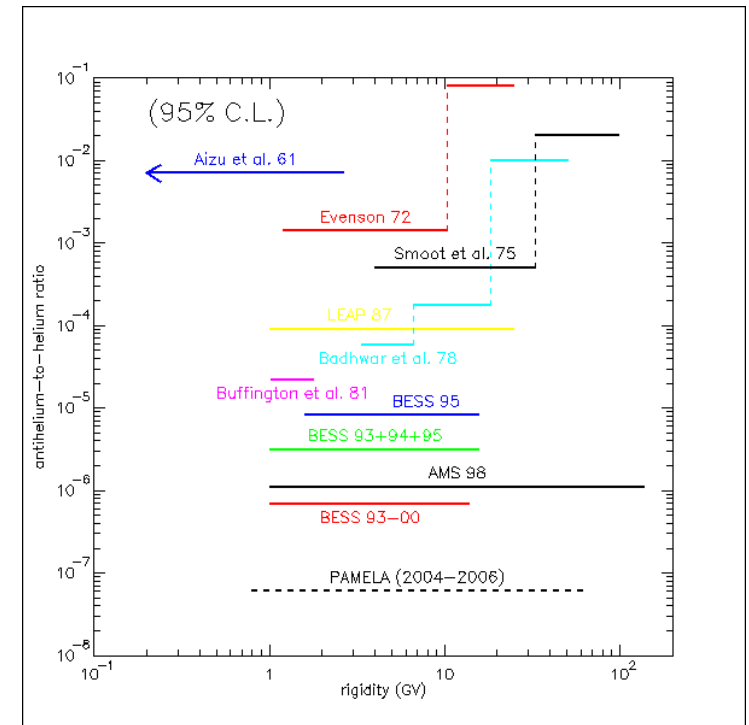
Searches for astrophysical antimatter
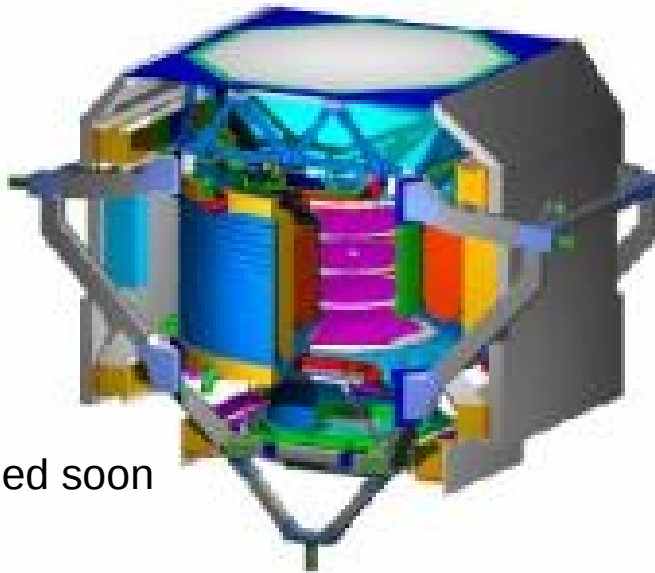
Alpha **M**agnetic **S**pectrometer Experiment on board the International Space Station
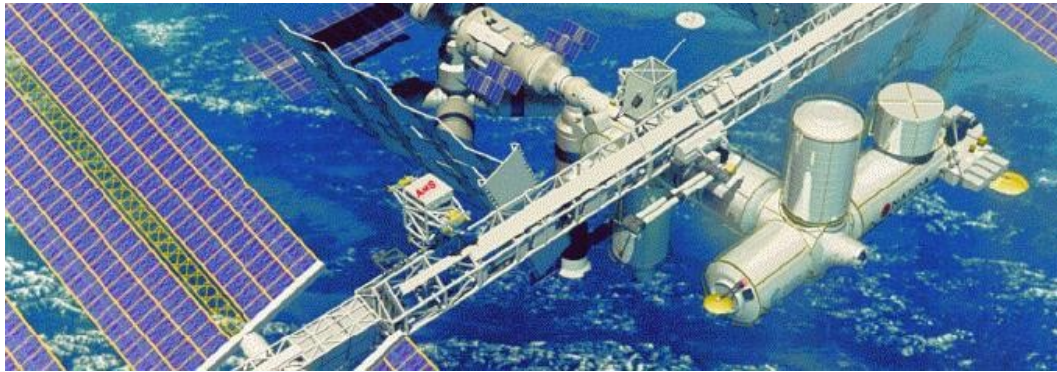
Payload for **A**nti**M**atter **E**xploration and **L**ight-nuclei **A**strophysics Experiment on board the Resurs-DK1 satellite

launch planned soon

launched 15th June 2006

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Dynamic generation of BAU

- Suppose equal amounts of matter (X) and antimatter ($\overline{\text{X}}$)
- X decays to
  - A (baryon number N_A) with probability p
  - B (baryon number N_B) with probability $(1-p)$
- $\overline{\text{X}}$ decays to
  - $\overline{\text{A}}$ (baryon number $-N_A$) with probability $\overline{p}$
  - $\overline{\text{B}}$ (baryon number $-N_B$) with probability $(1-\overline{p})$
- Generated baryon asymmetry:
  - $\Delta N_{TOT} = N_A p + N_B(1-p) - N_A \overline{p} - N_B(1-\overline{p}) = (p - \overline{p})(N_A - N_B)$
  - $\Delta N_{TOT} \neq 0$ requires $p \neq \overline{p}$ & $N_A \neq N_B$

CP violation and the BAU

- We can estimate the magnitude of the baryon asymmetry of the Universe caused by KM CP violation

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \sim \frac{J \times P_u \times P_d}{M^{12}}$$

N.B. Vanishes for degenerate masses

$$J = \cos(\theta_{12}) \cos(\theta_{23}) \cos^2(\theta_{13}) \sin(\theta_{12}) \sin(\theta_{23}) \sin(\theta_{13}) \sin(\delta)$$
$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

PRL 55 (1985) 1039

- The Jarlskog parameter J is a parametrization invariant measure of CP violation in the quark sector: $J \sim O(10^{-5})$
- The mass scale M can be taken to be the electroweak scale O(100 GeV)
- This gives an asymmetry $O(10^{-17})$
  - much much below the observed value of $O(10^{-10})$

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We need more CP violation!

- To create a larger asymmetry, require
  - new sources of CP violation
  - that occur at high energy scales
- Where might we find it?
  - lepton sector: CP violation in neutrino oscillations
  - quark sector: discrepancies with KM predictions
  - gauge sector, extra dimensions, other new physics: precision measurements of flavour observables are generically sensitive to additions to the Standard Model

Flavour for new physics discoveries

A lesson from history

- **New physics shows up at precision frontier before energy frontier**
  - GIM mechanism before discovery of charm
  - CP violation / CKM before discovery of bottom & top
  - Neutral currents before discovery of Z

- **Particularly sensitive – loop processes**
  - Standard Model contributions suppressed / absent
  - flavour changing neutral currents (rare decays)
  - CP violation
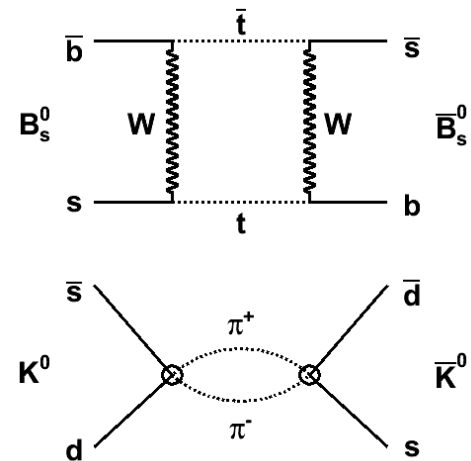 - lepton flavour / number violation / lepton universality

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Neutral meson oscillations

- ## We have flavour eigenstates M^0 and $\overline{M}^0$

  - M^0 can be K^0 ($\overline{s}d$), D^0 ($c\overline{u}$), B_d^0 ($\overline{b}d$) or B_s^0 ($\overline{b}s$)

- ## These can mix into each other

  - via short-distance or long-distance processes

- ## Time-dependent Schrödinger eqn.

$$i\frac{\partial}{\partial t}\begin{pmatrix} M^0 \\ \overline{M}^0 \end{pmatrix} = H\begin{pmatrix} M^0 \\ \overline{M}^0 \end{pmatrix} = \left(M - \frac{i}{2}\Gamma \right)\begin{pmatrix} M^0 \\ \overline{M}^0 \end{pmatrix}$$

  - H is Hamiltonian; M and Γ are 2x2 Hermitian matrices

- ## CPT theorem: $M_{11} = M_{22}$ & $\Gamma_{11} = \Gamma_{22}$

particle and antiparticle have equal masses and lifetimes

Solving the Schrödinger equation

- Physical states: eigenstates of effective Hamiltonian

$$M_{S,L} = p\, M^0 \pm q\, \overline{M}^0$$

p & q complex coefficients that satisfy $|p|^2 + |q|^2 = 1$

label as either S,L (short-, long-lived) or L,H (light, heavy) depending on values of Δm & $\Delta\Gamma$
(labels 1,2 usually reserved for CP eigenstates)

- – CP conserved if physical states = CP eigenstates ($|q/p| = 1$)

- Eigenvalues

$$\lambda_{S,L} = m_{S,L} - \tfrac{1}{2}i\Gamma_{S,L} = (M_{11} - \tfrac{1}{2}i\Gamma_{11}) \pm (q/p)(M_{12} - \tfrac{1}{2}i\Gamma_{12})$$

$$\Delta m = m_L - m_S \qquad \Delta\Gamma = \Gamma_S - \Gamma_L$$

$$(\Delta m)^2 - \tfrac{1}{4}(\Delta\Gamma)^2 = 4(|M_{12}|^2 + \tfrac{1}{4}|\Gamma_{12}|^2)$$

derivations left as exercise for the student!

$$\Delta m \Delta\Gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}^{*})$$

$$(q/p)^2 = (M_{12}^{*} - \tfrac{1}{2}i\Gamma_{12}^{*})/(M_{12} - \tfrac{1}{2}i\Gamma_{12})$$

Simplistic picture of mixing parameters
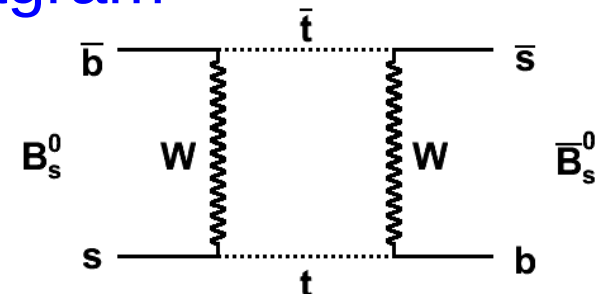
- **Δm: value depends on rate of mixing diagram**

  - together with various other constants ...

  $$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{tb}|^2 |V_{td}|^2$$
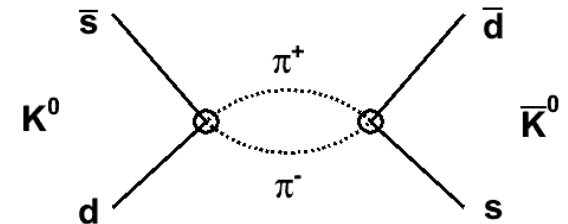
  - that can be made to cancel in ratios

  | remaining factors can be obtained |
  | from lattice QCD calculations |

  $$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{td}|^2}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s} |V_{ts}|^2}$$

- **$\Delta\Gamma$: value depends on widths of decays into common final states (CP-eigenstates)**

  - large for K^0, small for D^0 & B_d^0

- **$q/p \approx 1$ if $\arg(\Gamma_{12}/M_{12}) \approx 0$ ($|q/p| \approx 1$ if $M_{12} << \Gamma_{12}$ or $M_{12} >> \Gamma_{12}$)**

  - CP violation in mixing when $|q/p| \neq 1$

  $$\left(\epsilon = \frac{p-q}{p+q} \neq 0 \right)$$

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Calculations of M_{12} and Γ_{12}

- For B_q^0–$\bar{B}_q^0$ system

  - neglecting long-distance contribution from virtual intermediate states

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0(m_t^2/m_W^2)(V_{tq}^* V_{tb})^2 ,$$

$$\Gamma_{12} = \frac{G_F^2 m_b^2 \eta_B' m_{B_q} B_{B_q} f_{B_q}^2}{8\pi}$$

$$\times \left[(V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb}\, \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \right.$$

$$\left. + (V_{cq}^* V_{cb})^2\, \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right) \right] , \qquad \left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2}\frac{m_b^2}{m_W^2}\frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right)$$

Expect

- negligible CP violation in mixing
- $\Delta\Gamma << \Delta m$

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Constraints on NP from mixing

- All measurements of Δm & ΔΓ consistent with SM
  - K^0, D^0, B_d^0 and B_s^0

- This means $|A_{NP}| < |A_{SM}|$ where $\mathcal{A}_{SM}^{\Delta F=2} \approx \frac{G_F^2 m_t^2}{16\pi^2} (V_{ti}^* V_{tj})^2 \times \langle \overline{M} | (\overline{Q}_{Li}\gamma^\mu Q_{Lj})^2 | M \rangle \times F \left(\frac{M_W^2}{m_t^2} \right)$

- Express NP as perturbation to the SM Lagrangian
  - couplings c_i and scale $\Lambda > m_w$ $\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum \frac{c_i^{(d)}}{\Lambda^{(d-4)}} O_i^{(d)} (\text{SM fields})$

- For example, SM like (left-handed) operators $\Delta \mathcal{L}^{\Delta F=2} = \sum_{i \neq j} \frac{c_{ij}}{\Lambda^2} (\overline{Q}_{Li}\gamma^\mu Q_{Lj})^2$

arXiv:1002.0900

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} (Λ = 1 TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	$\Delta m_K; \epsilon_K$
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	$\Delta m_K; \epsilon_K$
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	$\Delta m_D; |q/p|, \phi_D$
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	$\Delta m_D; |q/p|, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	$\Delta m_{B_d}; S_{\psi K_S}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	$\Delta m_{B_d}; S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

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# New Physics Flavour Problem

- Limits on NP scale at least 100 TeV for generic couplings
  - model-independent argument, also for rare decays
- But we need NP at the TeV scale to solve the hierarchy problem (and to provide DM candidate, etc.)
- So we need NP flavour-changing couplings to be small
- Why?
  - minimal flavour violation?   NPB 645 (2002) 155
    - perfect alignment of flavour violation in NP and SM
  - some other approximate symmetry?
  - flavour structure tells us about physics at very high scales
- There are still important observables that are not yet well-tested

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Like-sign dimuon asymmetry

- Semileptonic decays are flavour-specific
- B mesons are produced in $B\overline{B}$ pairs
- Like-sign leptons arise if one of $B\overline{B}$ pair mixes before decaying
- If no CP violation in mixing N(++) = N(—)
- Inclusive measurement ↔ contributions from both B_d^0 and B_s^0
  - relative contributions from production rates, mixing probabilities & SL decay rates

arXiv:1005.2757 & arXiv:1007.0395

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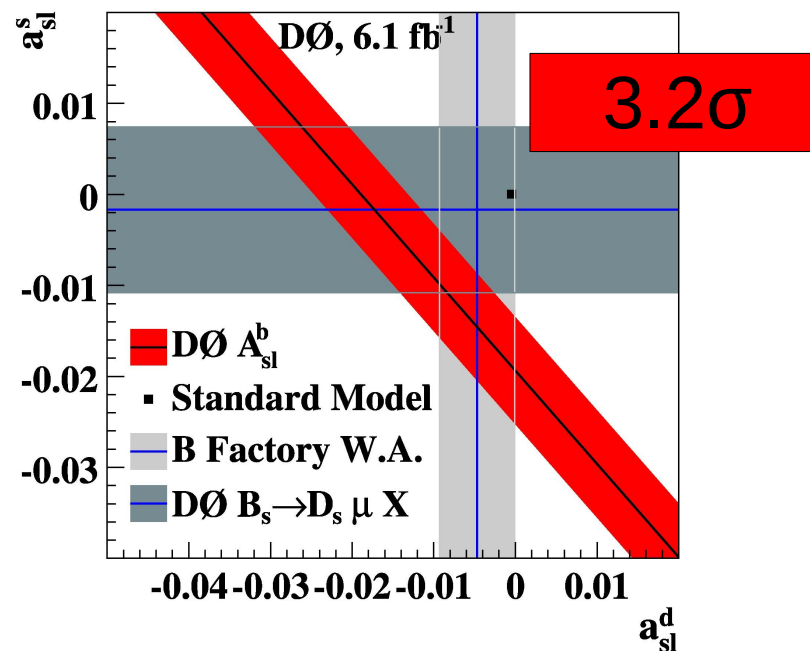
What do we know about heavy quark flavour physics as of today?

CKM Matrix : parametrizations

- Many different possible choices of 4 parameters

- PDG: 3 mixing angles and 1 phase

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Apparent hierarchy: $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{13} \sim 0.004$

    – Wolfenstein parametrization (expansion parameter $\lambda \sim \sin \theta_c \sim 0.22$)

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

- Other choices, eg. based on CP violating phases

Hierarchy in quark mixing

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$
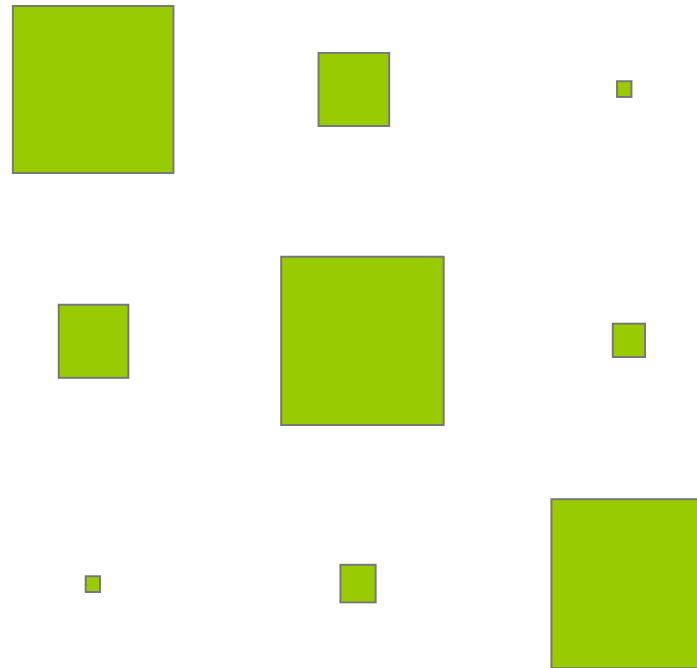
Very suggestive pattern

No known underlying reason

Situation for leptons (vs) is completely different

CKM matrix to $O(\lambda^5)$

$$V_{\text{CKM}} =$$

$$
\begin{pmatrix}
1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\
A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4
\end{pmatrix}
$$

imaginary part at $O(\lambda^3)$

imaginary part at $O(\lambda^4)$

imaginary part at $O(\lambda^5)$

Remember – only *relative* phases are observable

CKM matrix to O(λ^8)

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 - \frac{1}{16}\lambda^6\left(1 + 8A^2(\rho^2 + \eta^2)\right) - \frac{1}{128}\lambda^8\left(5 - 32A^2(\rho^2 + \eta^2)\right),$$

$$V_{us} = \lambda - \frac{1}{2}A^2\lambda^7(\rho^2 + \eta^2),$$

$$V_{ub} = A\lambda^3(\rho - i\eta),$$

$$V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5\left(1 - 2(\rho + i\eta)\right) + \frac{1}{2}A^2\lambda^7(\rho + i\eta),$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) - \frac{1}{16}\lambda^6\left(1 - 4A^2 + 16A^2(\rho + i\eta)\right) - \frac{1}{128}\lambda^8\left(5 - 8A^2 + 16A^4\right),$$

$$V_{cb} = A\lambda^2 - \frac{1}{2}A^3\lambda^8\left(\rho^2 + \eta^2\right),$$

$$V_{td} = A\lambda^3\left(1 - \rho - i\eta\right) + \frac{1}{2}A\lambda^5(\rho + i\eta) + \frac{1}{8}A\lambda^7(1 + 4A^2)(\rho + i\eta),$$

$$V_{ts} = -A\lambda^2 + \frac{1}{2}A\lambda^4\left(1 - 2(\rho + i\eta)\right) + \frac{1}{8}A\lambda^6 + \frac{1}{16}A\lambda^8\left(1 + 8A^2(\rho + i\eta)\right),$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4 - \frac{1}{2}A^2\lambda^6\left(\rho^2 + \eta^2\right) - \frac{1}{8}A^4\lambda^8.$$

derivations left as exercise for the student!

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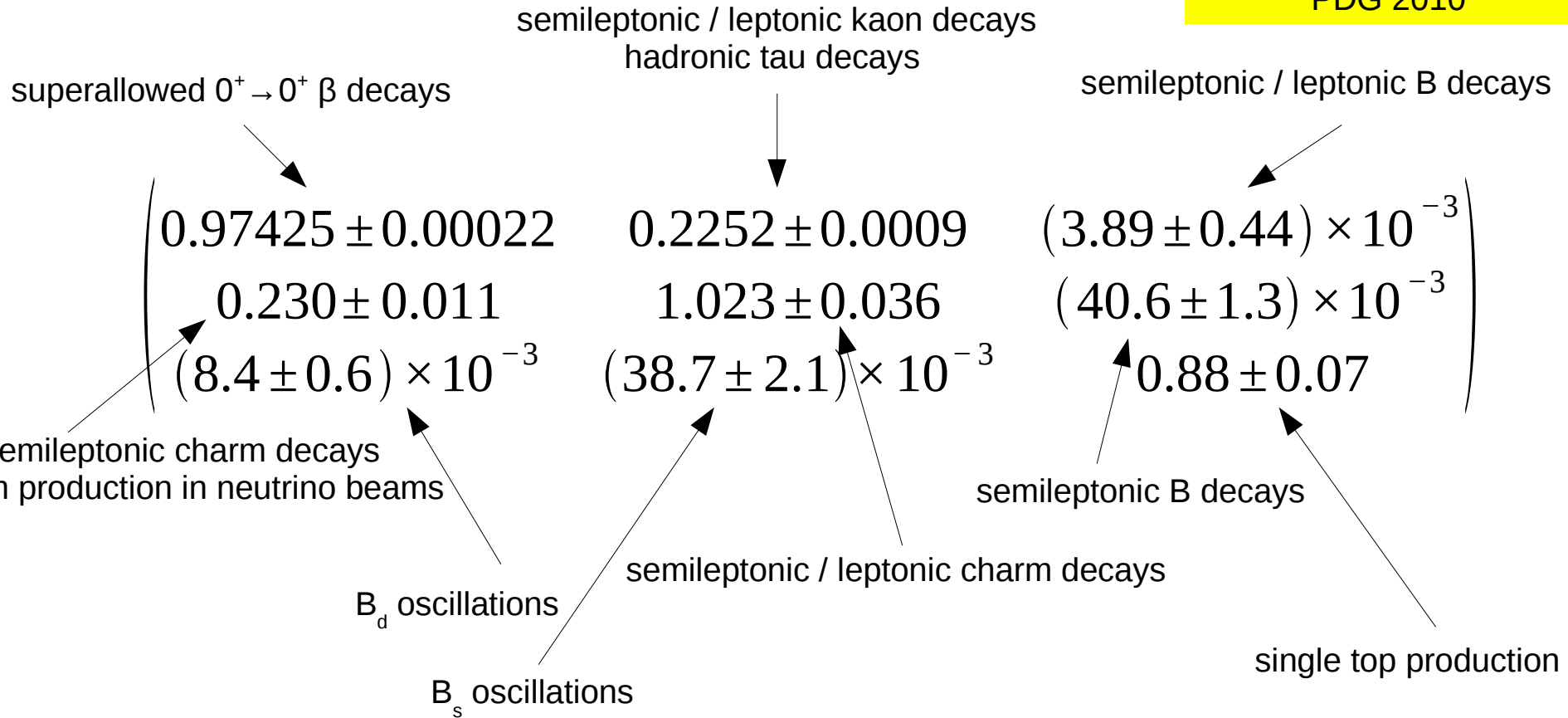
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Unitarity Tests

- The CKM matrix must be unitary

$$V_{CKM}^{+} V_{CKM} = V_{CKM} V_{CKM}^{+} = 1$$

- Provides numerous tests of constraints between independent observables, such as

$$\left| V_{ud} \right|^2 + \left| V_{us} \right|^2 + \left| V_{ub} \right|^2 = 1$$

$$V_{ud} V_{ub}^{*} + V_{cd} V_{cb}^{*} + V_{td} V_{tb}^{*} = 0$$

CKM Matrix – Magnitudes

semileptonic / leptonic kaon decays
hadronic tau decays

semileptonic / leptonic B decays

superallowed $0^+ \to 0^+$ β decays

$$\begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & (3.89 \pm 0.44) \times 10^{-3} \\ 0.230 \pm 0.011 & 1.023 \pm 0.036 & (40.6 \pm 1.3) \times 10^{-3} \\ (8.4 \pm 0.6) \times 10^{-3} & (38.7 \pm 2.1) \times 10^{-3} & 0.88 \pm 0.07 \end{pmatrix}$$

semileptonic charm decays
charm production in neutrino beams

semileptonic B decays

B_d oscillations

semileptonic / leptonic charm decays

B_s oscillations

single top production

theory inputs (eg., lattice calculations) required

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The Unitarity Triangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Three complex numbers add to zero
$\Rightarrow$ triangle in Argand plane

$$\alpha = \arg\left[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right] \quad \beta = \arg\left[-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right] \quad \gamma = \arg\left[-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right]$$

$$R_u = \left|\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right| \quad R_t = \left|\frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*}\right|$$

Axes are $\bar{\rho}$ and $\bar{\eta}$ where

$$\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\bar{\rho} + i\bar{\eta})}{\sqrt{1 - \lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}$$

imaginary $(\bar{\eta})$

R_u α R_t

γ β

$(0,0)$ $(1,0)$ real $(\bar{\rho})$
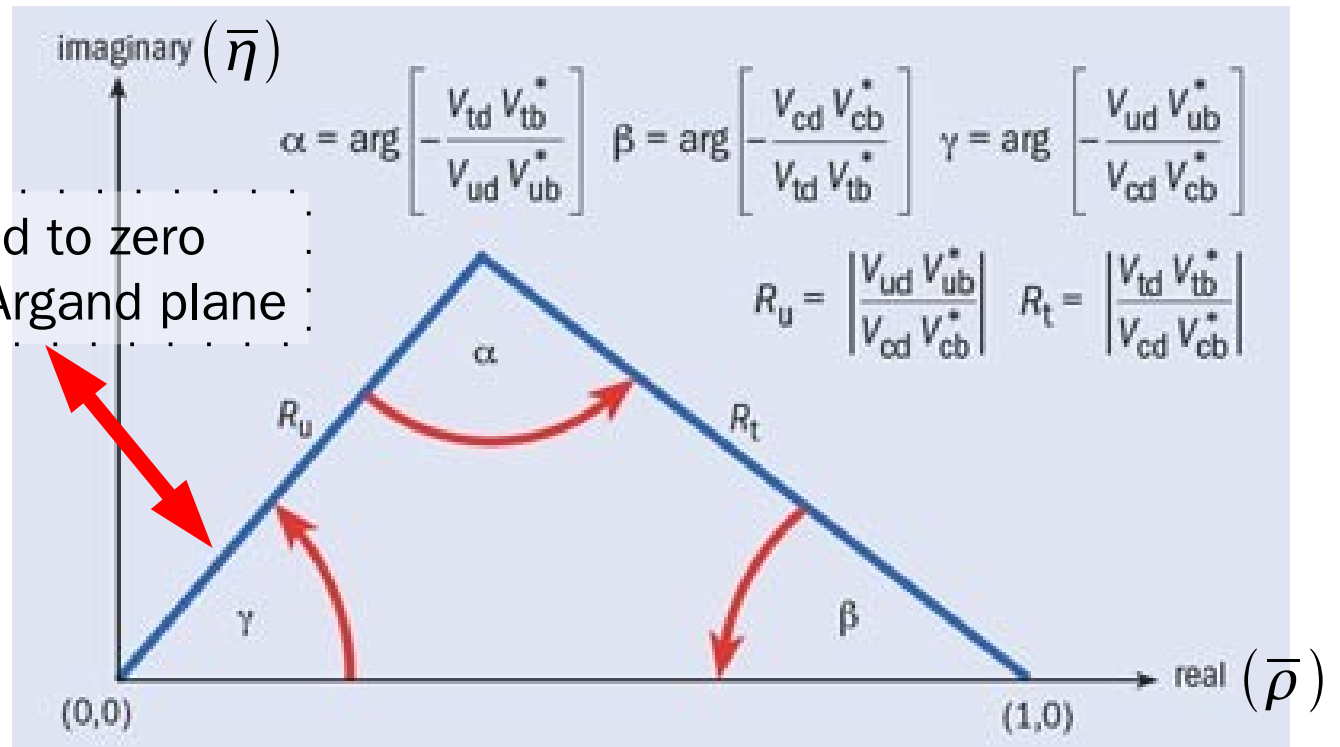
Still to come in today's lecture
β, α, R_t, R_u

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Predictive nature of KM mechanism

In the Standard Model the KM phase is the sole origin of CP violation

Hence:
   all measurements must agree on the position of the apex of the Unitarity Triangle

(Illustration shown assumes no experimental or theoretical uncertainties)

Area of (all of) the Unitarity Triangle(s) is given by the Jarlskog invariant
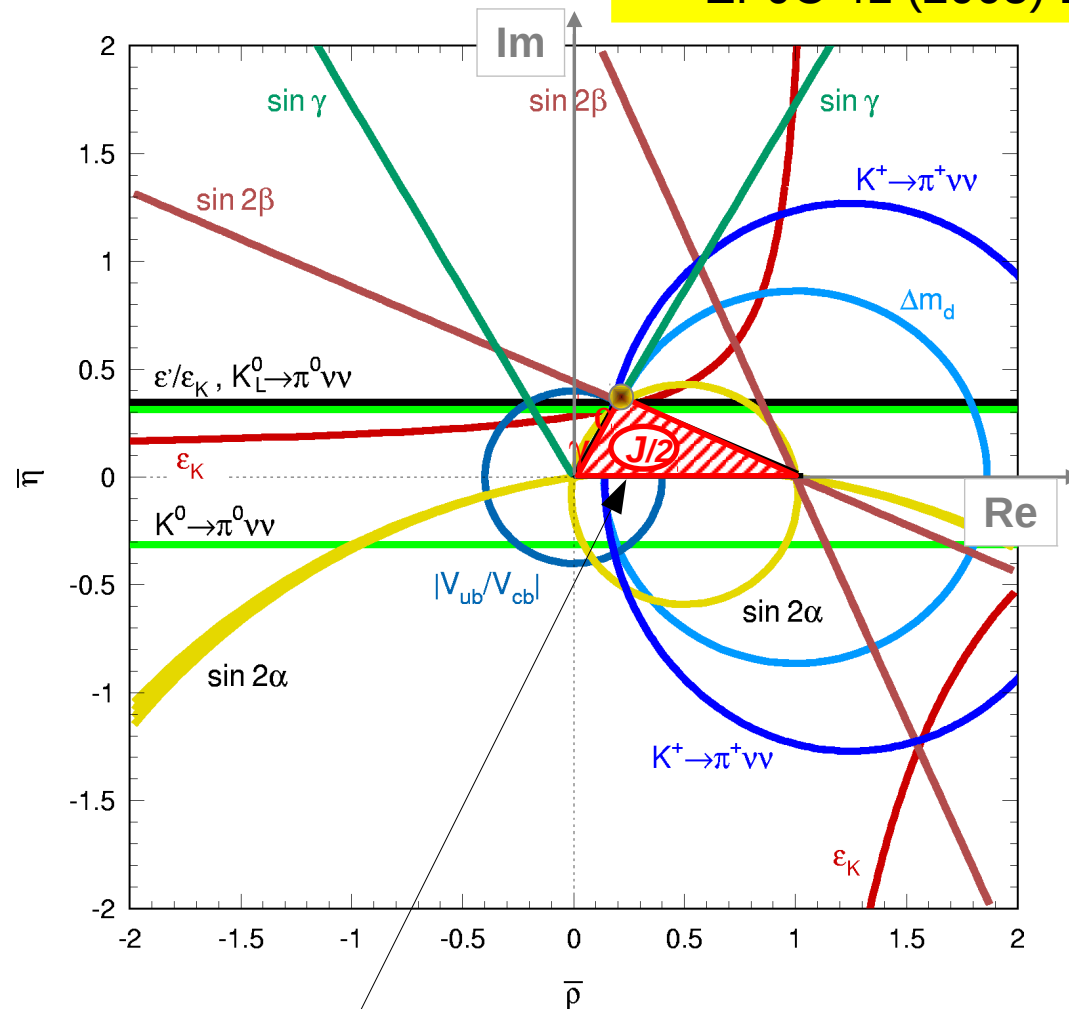
Time-Dependent CP Violation in the B^0–$\overline{B}^0$ System

- For a B meson known to be 1) B^0 or 2) $\overline{B}^0$ at time t=0, then at later time t:

$$\Gamma(B^0_{phys} \to f_{CP}(t)) \propto e^{-\Gamma t}\big(1-(S\sin(\Delta m t)-C\cos(\Delta m t))\big)$$

$$\Gamma(\overline{B}^0_{phys} \to f_{CP}(t)) \propto e^{-\Gamma t}\big(1+(S\sin(\Delta m t)-C\cos(\Delta m t))\big)$$

here assume ΔΓ negligible – will see full expressions tomorrow

B^0

$\frac{q}{p}$

A

Δm

$\overline{A}$

f_{CP}

$\overline{B}^0$

$$S=\frac{2\,\Im(\lambda_{CP})}{1+\left|\lambda^2_{CP}\right|} \qquad C=\frac{1-\left|\lambda^2_{CP}\right|}{1+\left|\lambda^2_{CP}\right|} \qquad \lambda_{CP}=\frac{q}{p}\frac{\overline{A}}{A}$$

For $B^0 \to$ J/ψ K$_S$, S = sin(2β), C=0
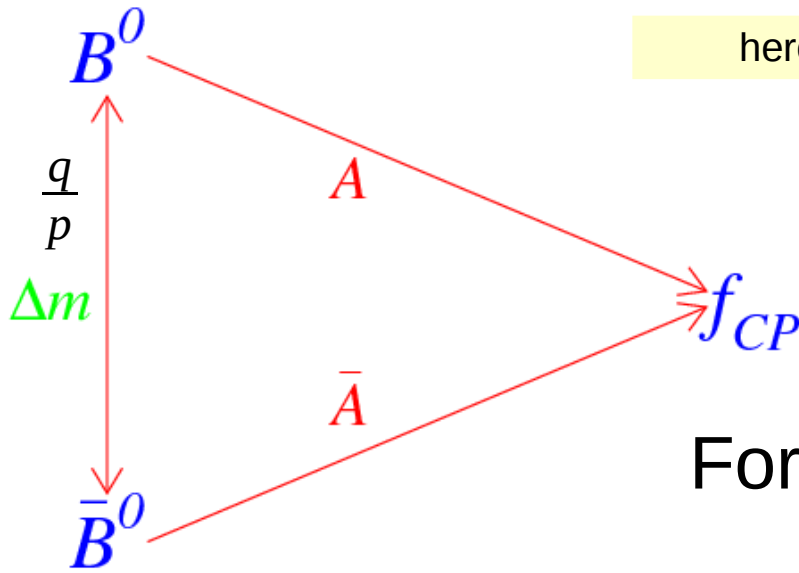
NPB 193 (1981) 85

Categories of CP violation

- Consider decay of neutral particle to a CP eigenstate

$$\lambda_{CP} = \frac{q}{p} \frac{\overline{A}}{A}$$
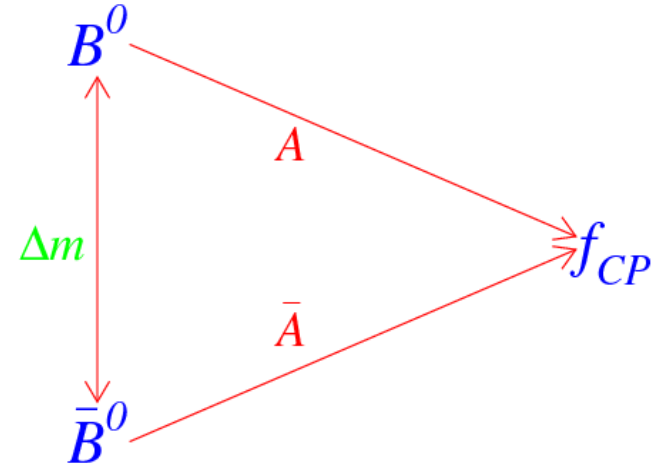
$$\left| \frac{q}{p} \right| \neq 1 \qquad \text{CP violation in mixing}$$

$$\left| \frac{\overline{A}}{A} \right| \neq 1 \qquad \text{CP violation in decay (direct CPV)}$$

$$\Im\left(\frac{q}{p} \frac{\overline{A}}{A} \right) \neq 0 \qquad \text{CP violation in interference between mixing and decay}$$

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Asymmetric B factory principle

To measure t require B meson to be moving
→ e^+e^- at threshold with asymmetric collisions (Oddone)
Other possibilities considered
→ fixed target production?
→ hadron collider?
→ e^+e^- at high energy?

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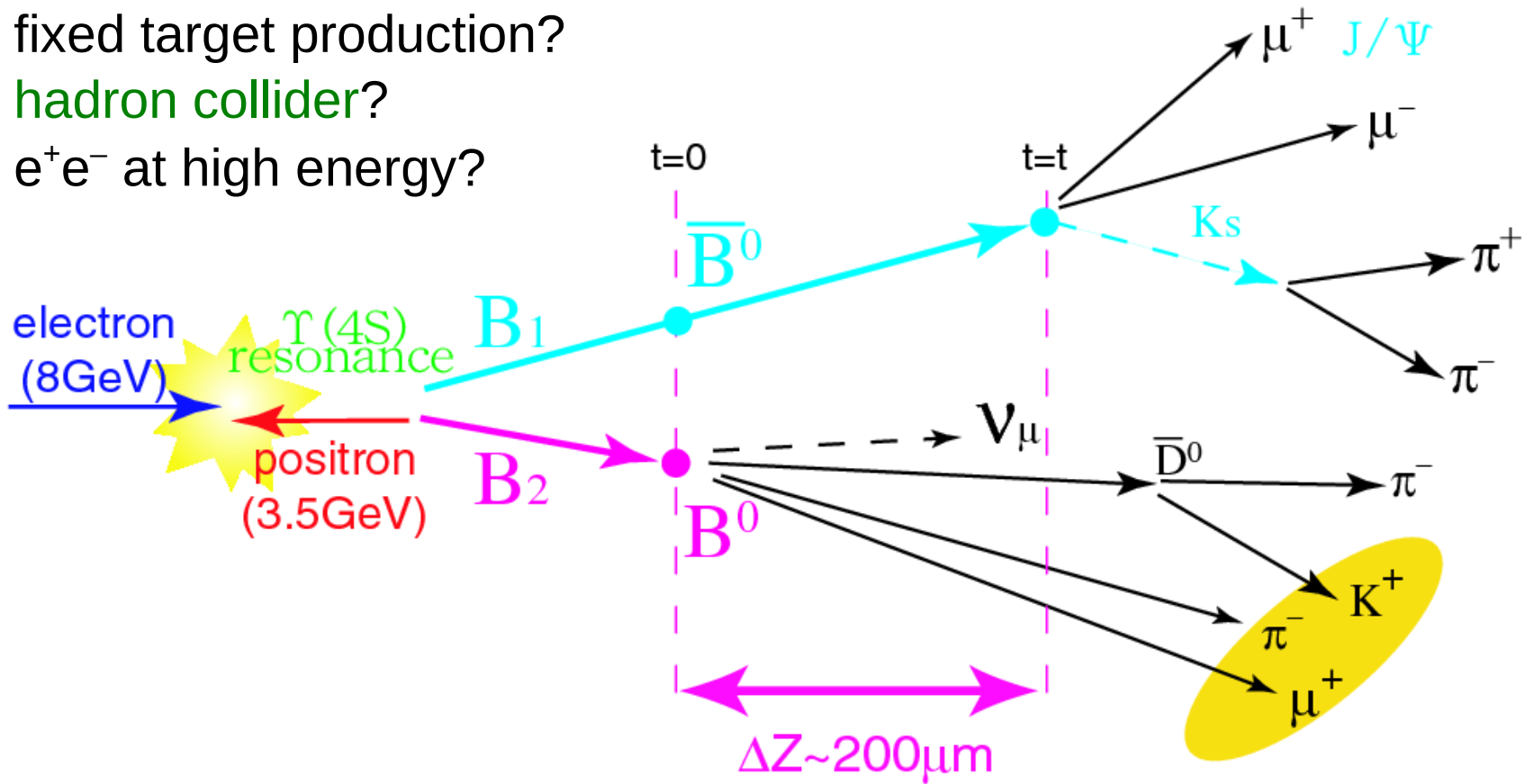
Asymmetric B Factories

PEPII at SLAC
9.0 GeV e$^-$ on 3.1 GeV e$^+$

KEKB at KEK
8.0 GeV e$^-$ on 3.5 GeV e$^+$
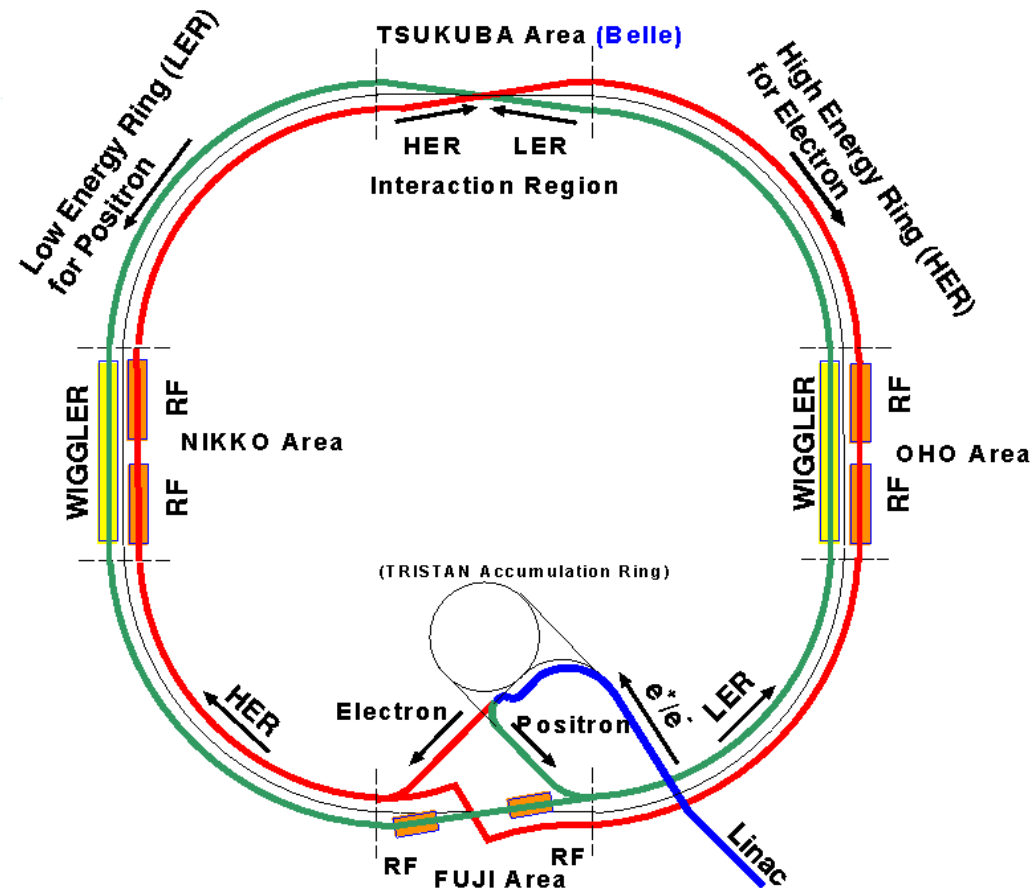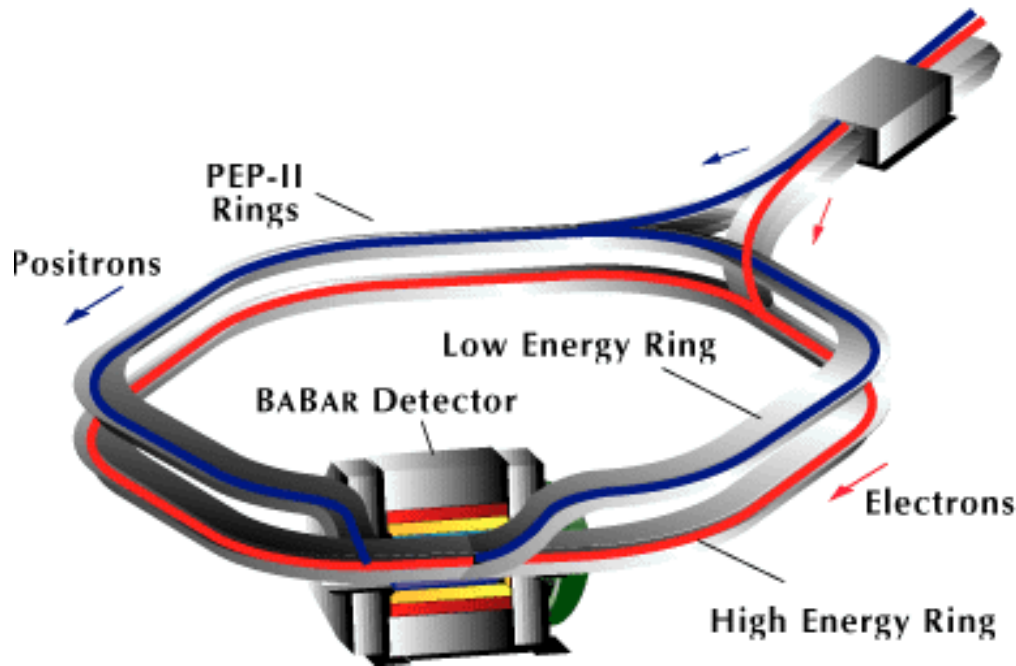
B factories – world record luminosities
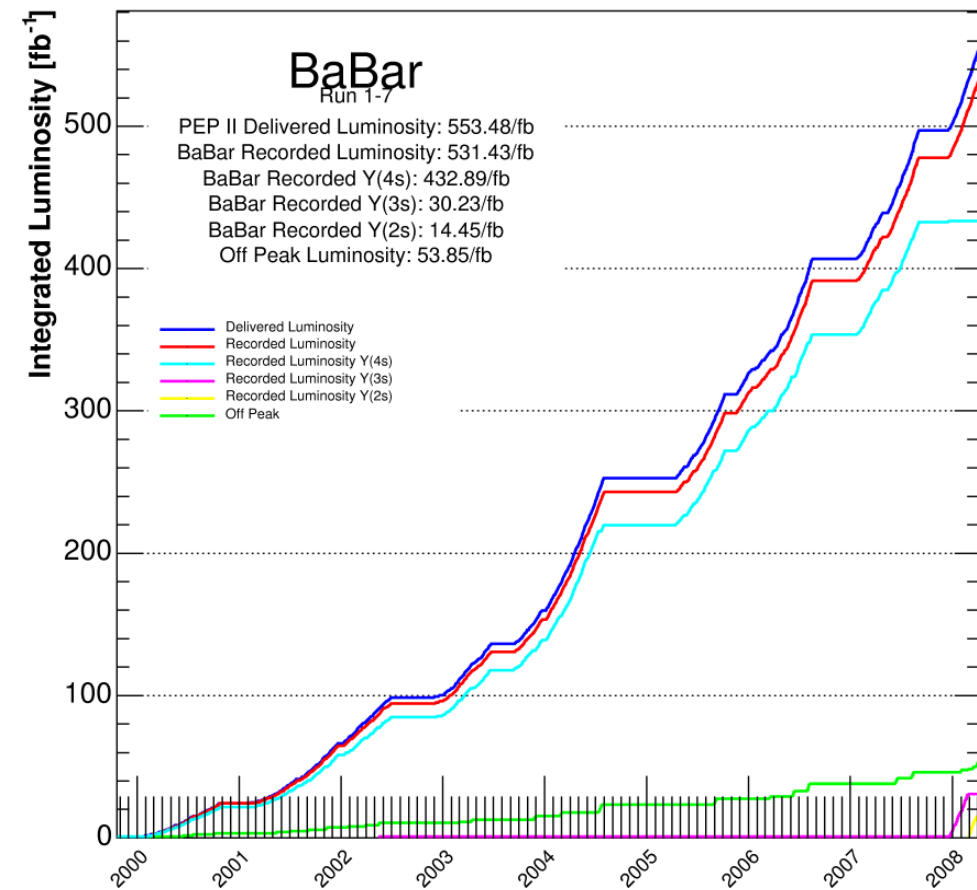
~ 433/fb on Y(4S)

~ 711/fb on Y(4S)

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Total over 10^9 B$\bar{\text{B}}$ pairs recorded
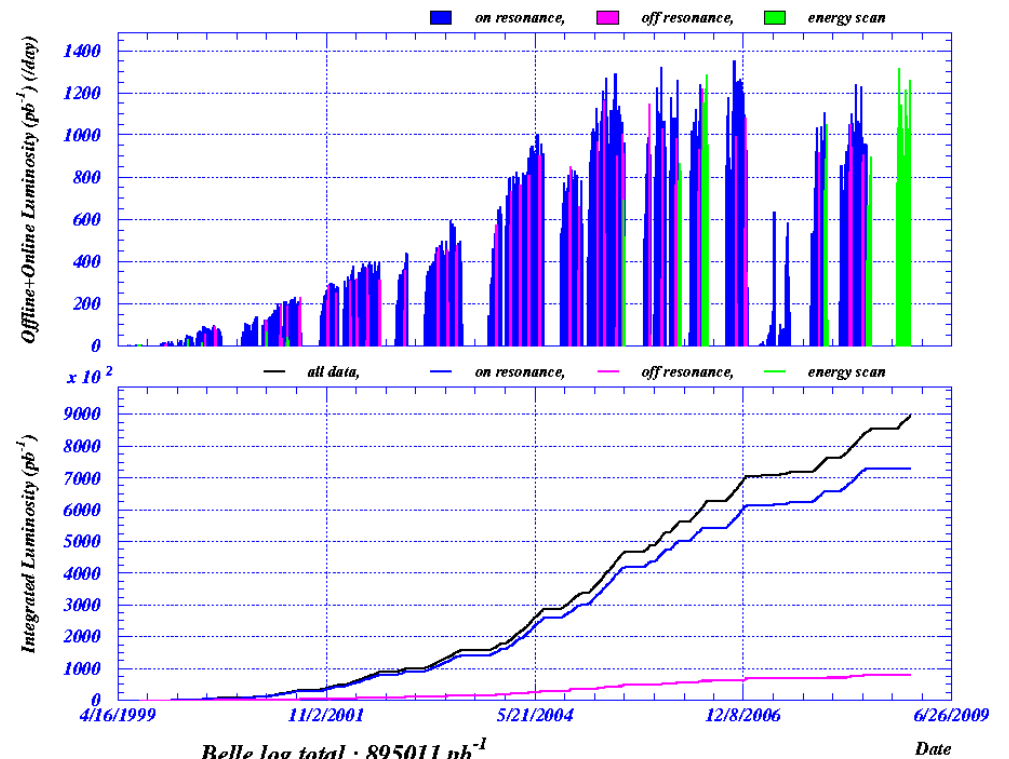
World record luminosities (2)

BaBar Detector

1.5 T solenoid

EMC
6580 CsI(Tl) crystals

e^+ (3.1 GeV)

DIRC (**PID**)
144 quartz bars
11000 PMs

e^- (9 GeV)

Drift Chamber
40 stereo layers

Instrumented Flux Return
iron / RPCs (muon / neutral hadrons)
2/6 replaced by LST in 2004
Rest of replacement in 2006

Silicon Vertex Tracker
5 layers, double sided strips

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Belle Detector

SC solenoid
1.5T

Aerogel Cherenkov cnt.
n=1.015~1.030

CsI(Tl)
$16X_0$

3.5 GeV e^+

TOF counter

8 GeV e^-

Central Drift Chamber
small cell +He/C_2H_6

Si vtx. det.
- 3 lyr. DSSD
- 4 lyr. since summer 2003

μ / K_L **detection**
14/15 lyr. RPC+Fe

Results for the golden mode

$$B^0 \rightarrow J/\psi K^0$$

BABAR

BELLE

$\eta_f = -1$

$\eta_f = +1$
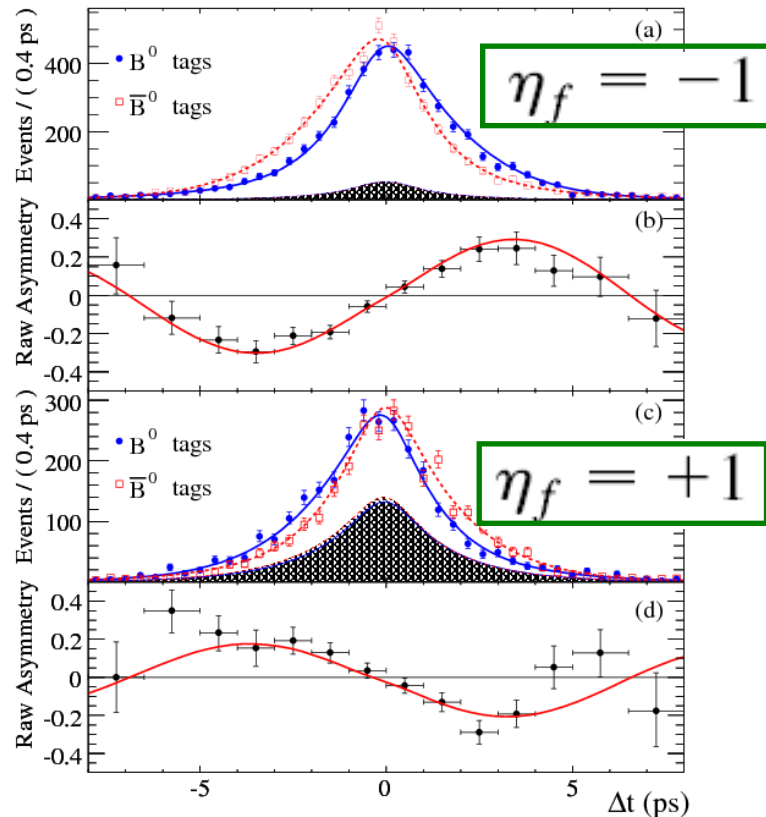
PRD 79 (2009) 072009
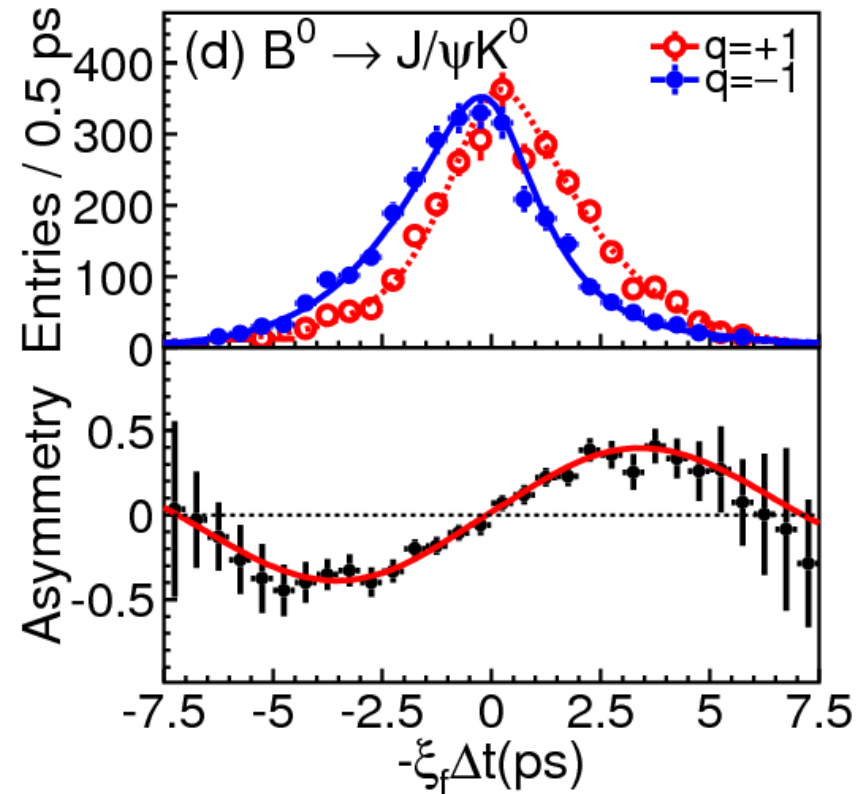
PRL 98 (2007) 031802

Compilation of results

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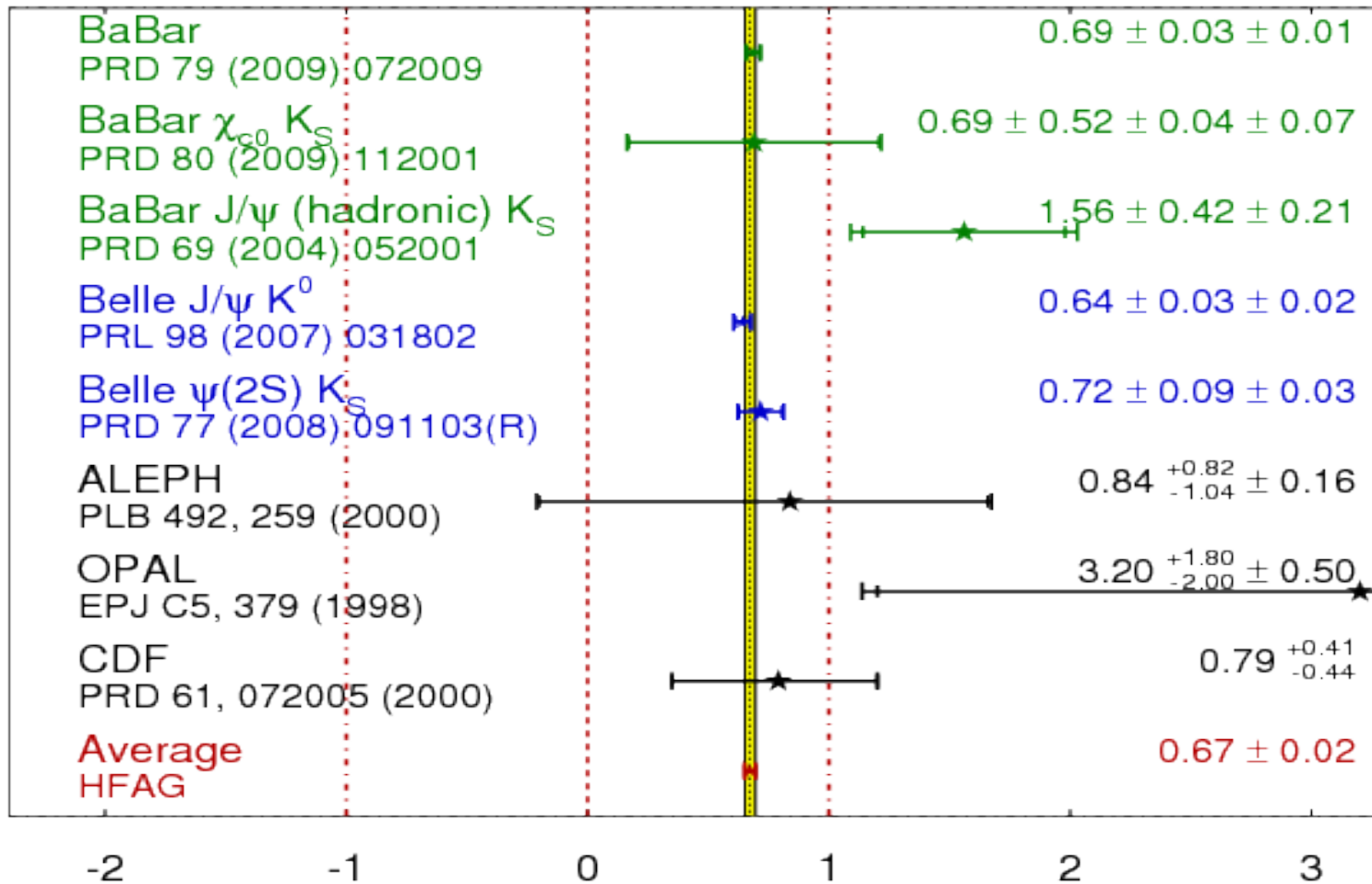
Measurement of α

- Similar analysis using b $\rightarrow$ u$\bar{\text{u}}$d decays (e.g. $B_d^0 \rightarrow \pi^+\pi^-$) probes π–(β+γ) = α

  - but b $\rightarrow$ du$\bar{\text{u}}$ penguin transitions contribute to same final states $\Rightarrow$ "penguin pollution"

  - C ≠ 0 ⇔ direct CP violation can occur

  - S ≠ +η_{CP} sin(2α)

- Two approaches   (optimal approach combines both)

  - try to use modes with small penguin contribution

  - correct for penguin effect (isospin analysis)

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Experimental Situation
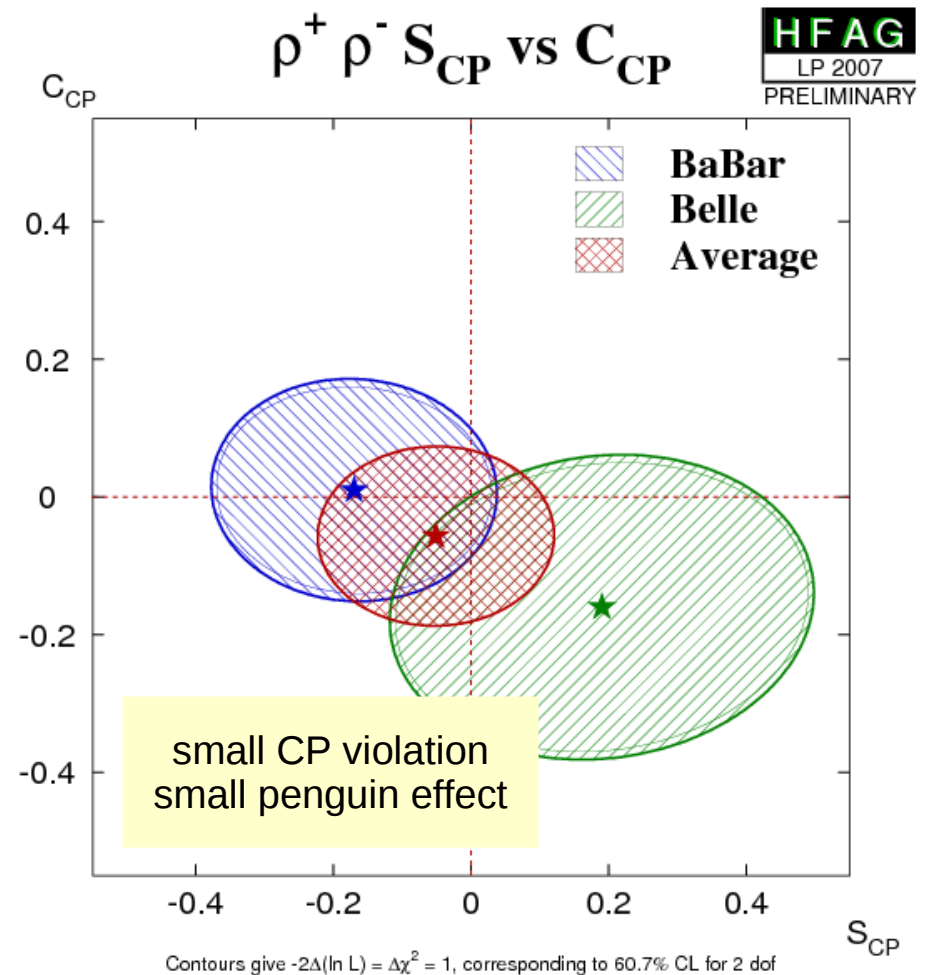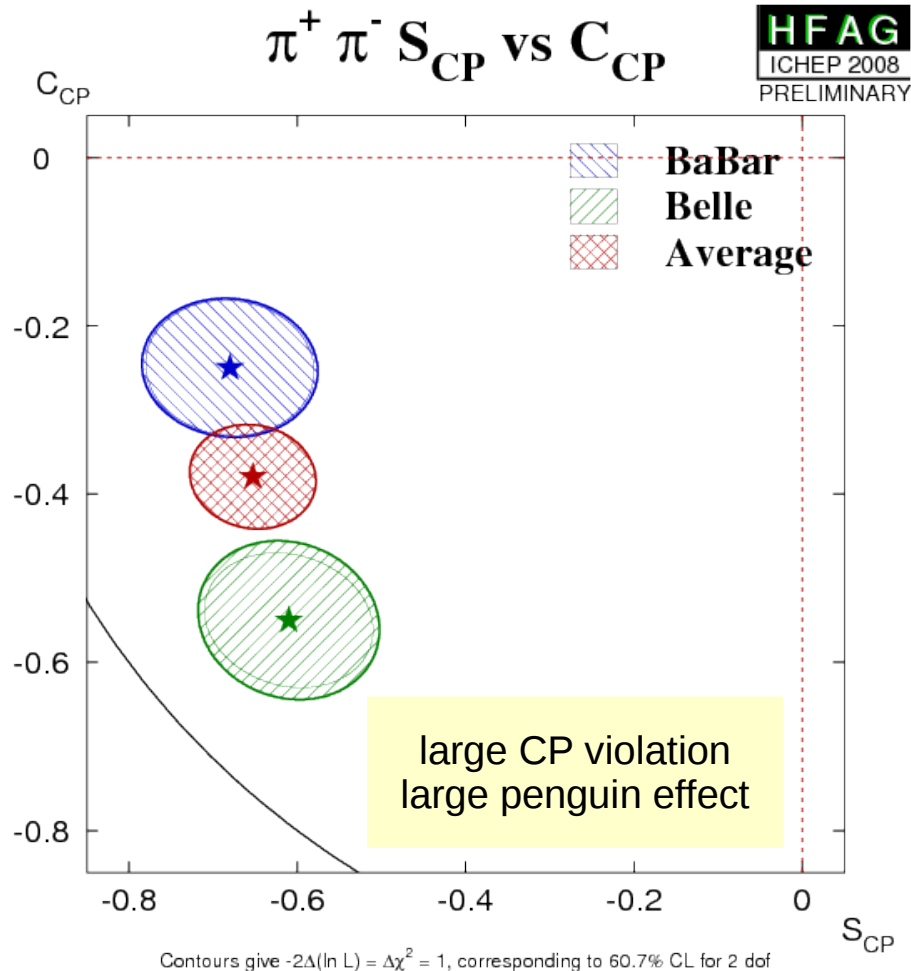
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Measurement of α

$$\alpha = (89.0\ ^{+4.4}_{-4.2})^\circ$$

CKM fit

no α meas. in the fit

Legend:
- B → ππ/ρρ/ρπ (BABAR)
- B → ππ/ρρ/ρπ (Belle)
- B → ππ/ρρ/ρπ (WA)

CKM fitter
Moriond 09

1 - CL (vertical axis)

α (deg) (horizontal axis)

THESE SOLUTIONS RULED OUT BY OBSERVATION OF DIRECT CP VIOLATION IN $B^0 \to \pi^+\pi^-$

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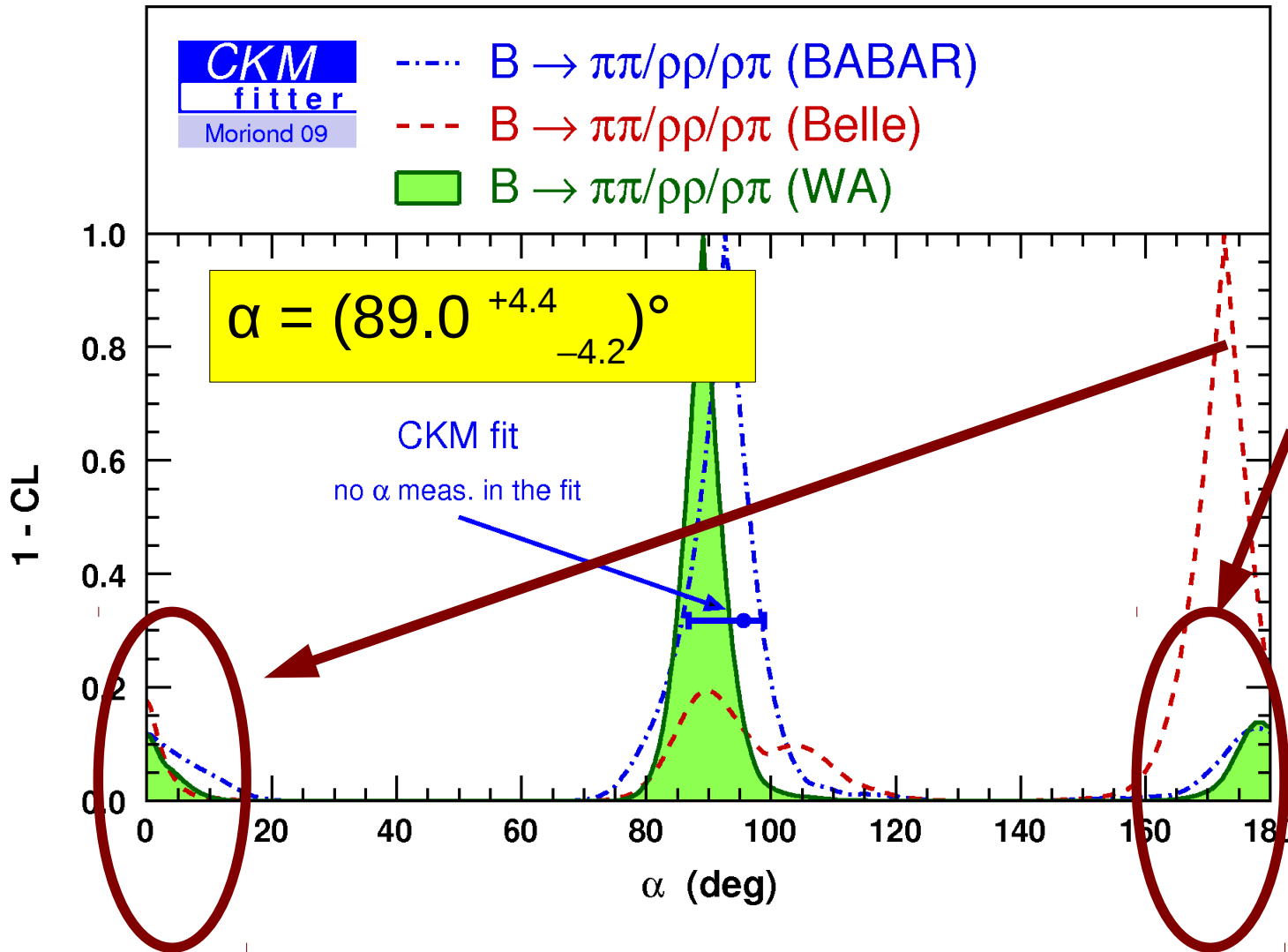
Is there any physical significance in the fact that α ≈ 90°?

R$_t$ side from B^0–B^0 mixing

$$R_t = \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right| \quad \& \quad \frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{td}|^2}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s} |V_{ts}|^2}$$

World average based on many measurements

$$P(\Delta t) = (1 \pm \cos(\Delta m \Delta t)) e^{-|\Delta t|}/2\tau$$
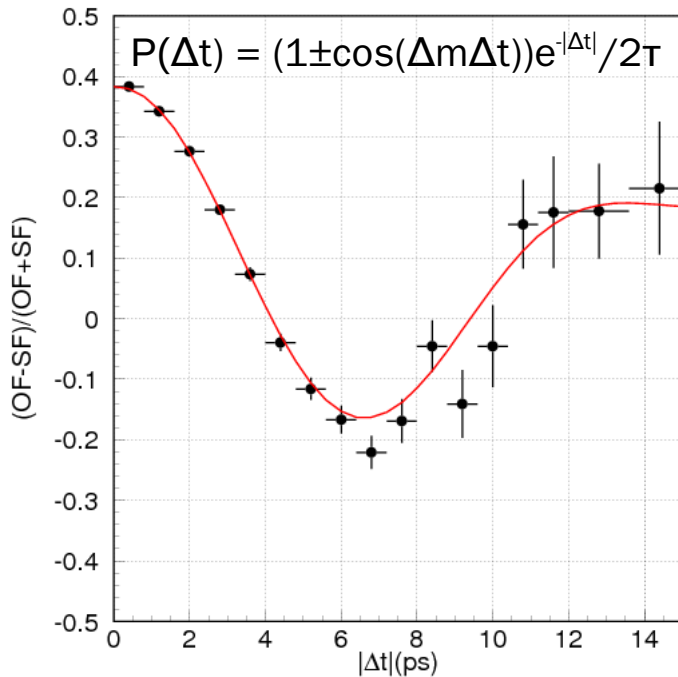
$\Delta m_d = (0.511 \pm 0.005 \pm 0.006)$ ps^{-1}

PRD 71, 072003 (2005)
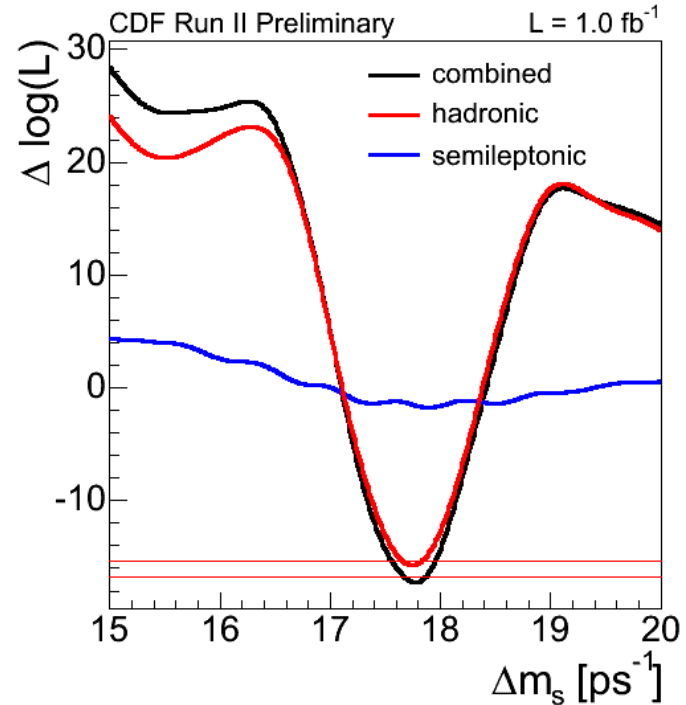
$\Delta m_s = (17.77 \pm 0.10 \pm 0.07)$ ps^{-1}

PRL 97, 242003 (2006)

$$\left| V_{td}/V_{ts} \right| = 0.211 \pm 0.001 \pm 0.005$$

experimental uncertainty   theoretical uncertainty

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R$_u$ side from semileptonic decays

$$R_u = \left| \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right|$$
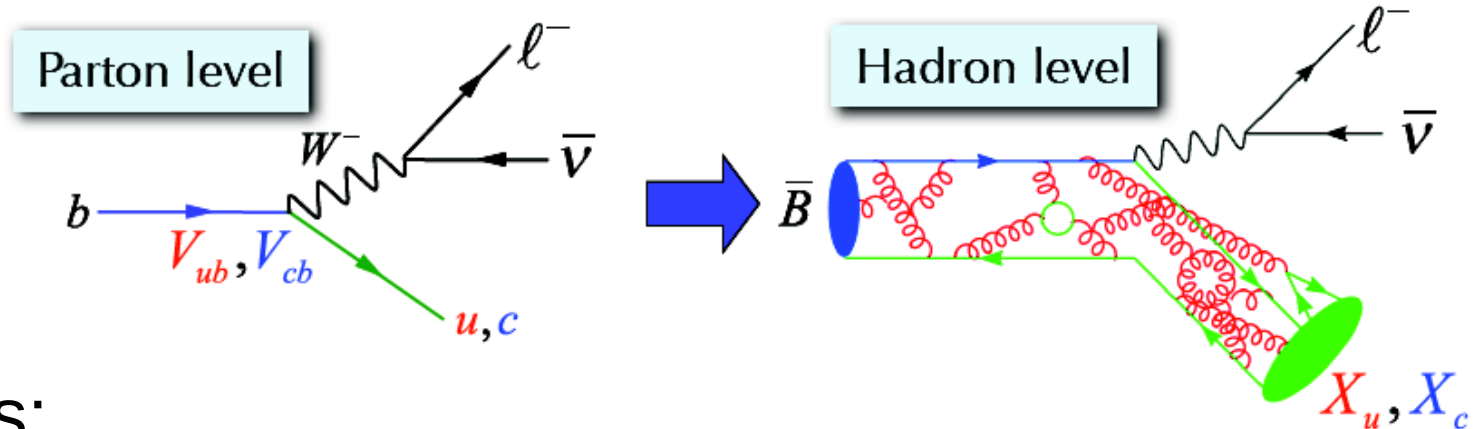
- Approaches:
  - exclusive semileptonic B decays, eg. $B^0 \rightarrow \pi^- e^+ \nu$
    - require knowledge of form factors
      - can be calculated in lattice QCD at kinematical limit
  - inclusive semileptonic B decays, eg. $B \rightarrow X_u e^+ \nu$
    - clean theory, based on Operator Product Expansion
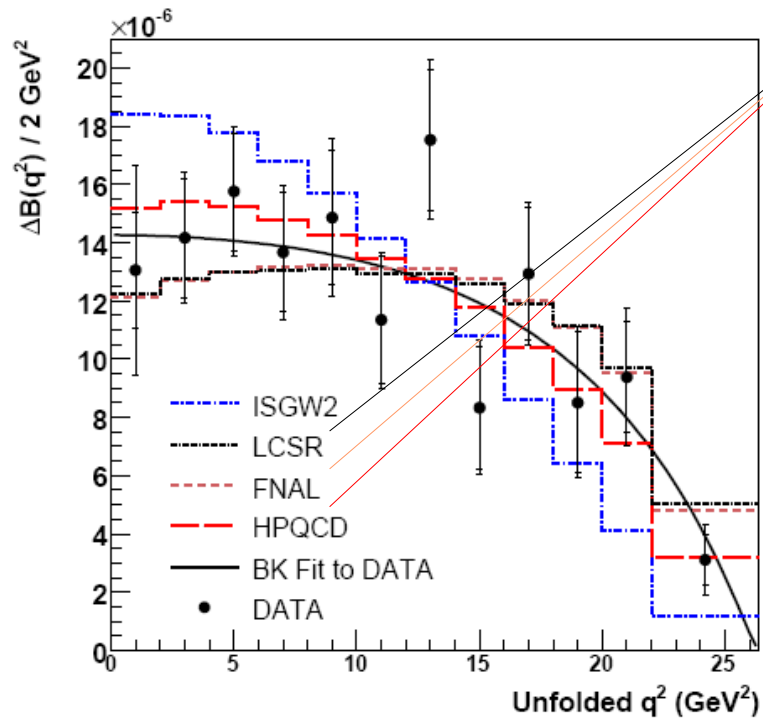   - experimentally challenging:
      - need to reject $b \rightarrow c$ background
      - cuts re-introduce theoretical uncertainties

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$|V_{ub}|$ from exclusive semileptonic decays

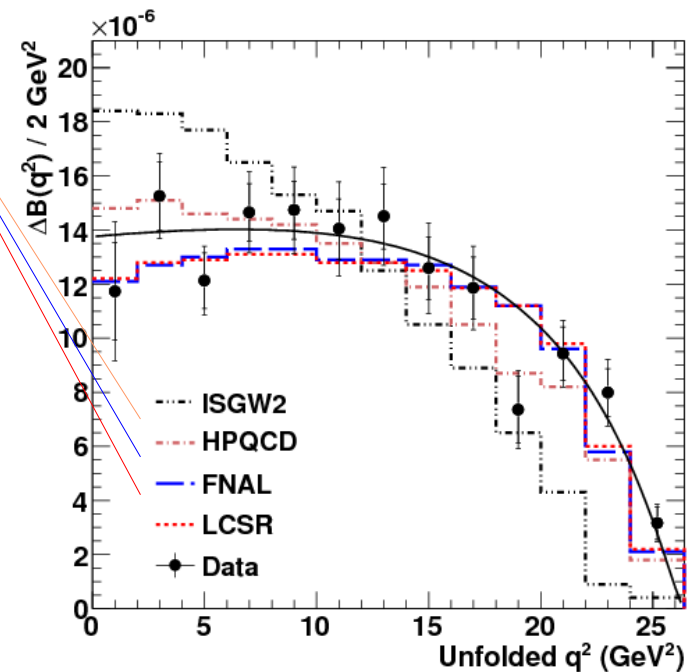## Current best measurements use $B^0 \rightarrow \pi^- l^+ \nu$

Different lattice calculations

$BF(B^0 \rightarrow \pi^- l^+ \nu) = (1.46 \pm 0.07 \pm 0.08) \times 10^{-4}$
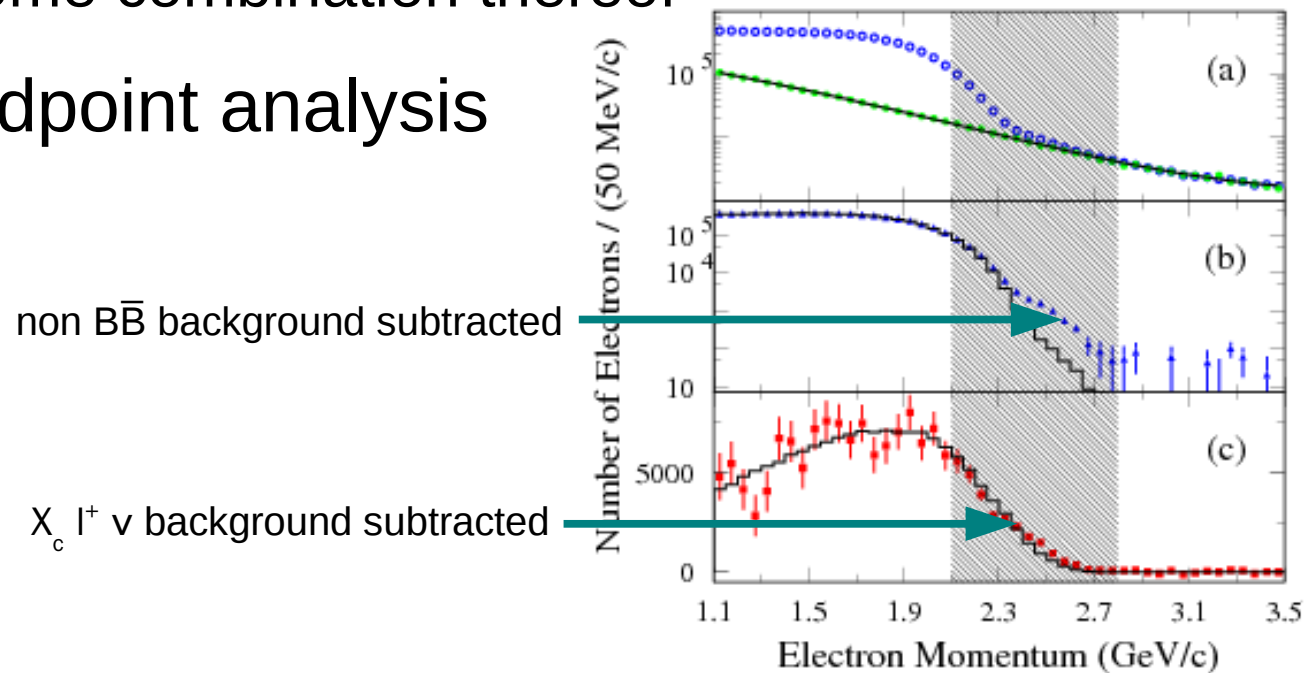
BaBar: PRL 98 (2007) 091801

$BF(B^0 \rightarrow \pi^- l^+ \nu) = (1.49 \pm 0.04 \pm 0.07) \times 10^{-4}$
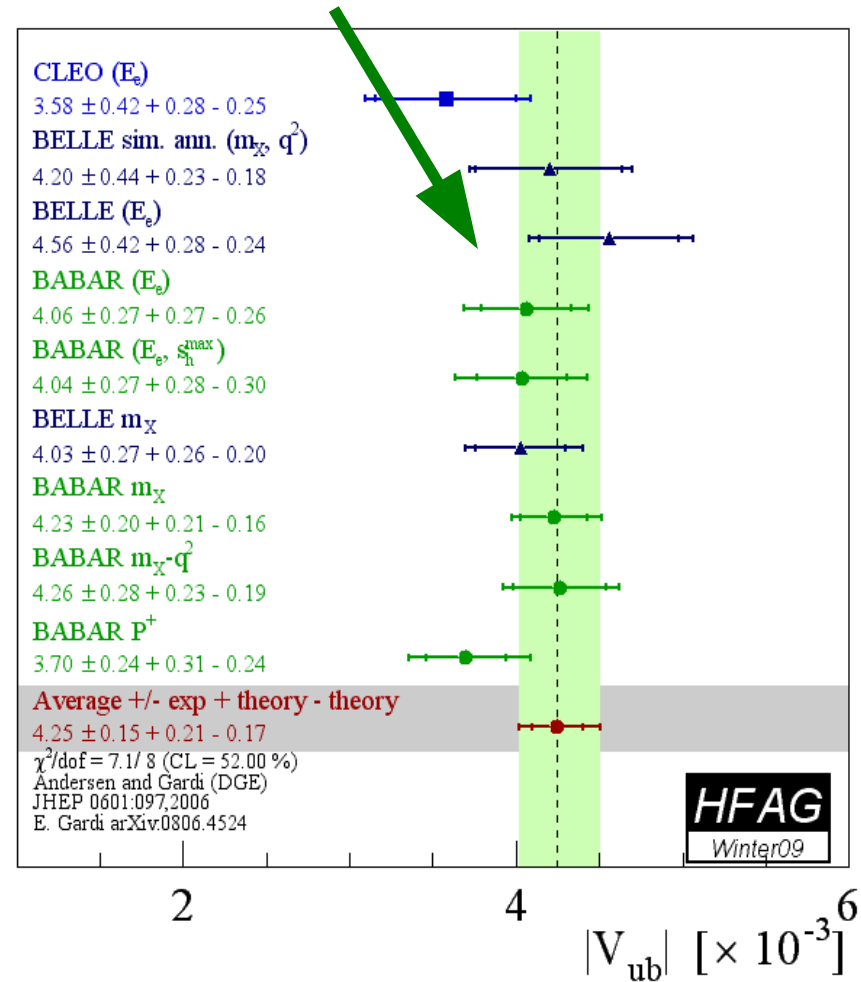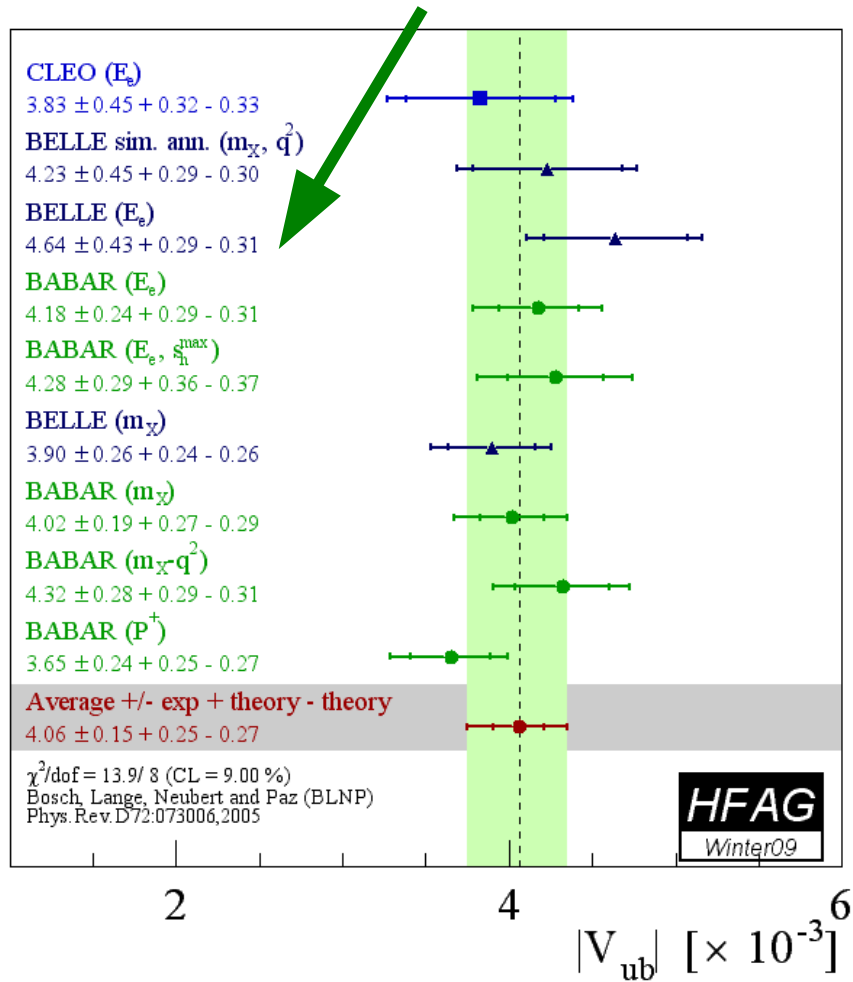
Belle: ICHEP2010 preliminary

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$|V_{ub}|$ from inclusive semileptonic decays

- **Main difficulty to measure inclusive** $B \rightarrow X_u \, l^+ \, \nu$

  – background from $B \rightarrow X_c \, l^+ \, \nu$

- **Approaches**

  – cut on E_l (lepton endpoint), q^2 ($l\nu$ invariant mass squared), $M(X_u)$, or some combination thereof

- **Example: endpoint analysis**

non B$\overline{\text{B}}$ background subtracted

$X_c \, l^+ \, \nu$ background subtracted

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$|V_{ub}|$ inclusive - compilation

Different theoretical approaches (2 of 4 used by HFAG)

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$|V_{ub}|$ average

- Averages on $|V_{ub}|$ from both exclusive and inclusive approaches
  - exclusive: $|V_{ub}| = (3.38 \pm 0.36) \times 10^{-3}$
  - inclusive: $|V_{ub}| = (4.27 \pm 0.38) \times 10^{-3}$
  - slight tension between these results
  - in both cases theoretical errors are dominant
    - but some "theory" errors can be improved with more data
  - PDG2010 does naïve average rescaling due to inconsistency to obtain $|V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}$
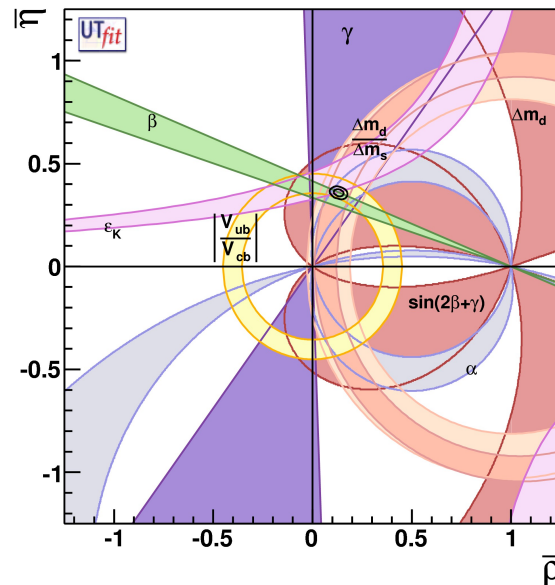
Summary for today

$$\sin(2\beta) \qquad \alpha \qquad |\Delta m_d / \Delta m_s| \qquad |V_{ub}/V_{cb}|$$
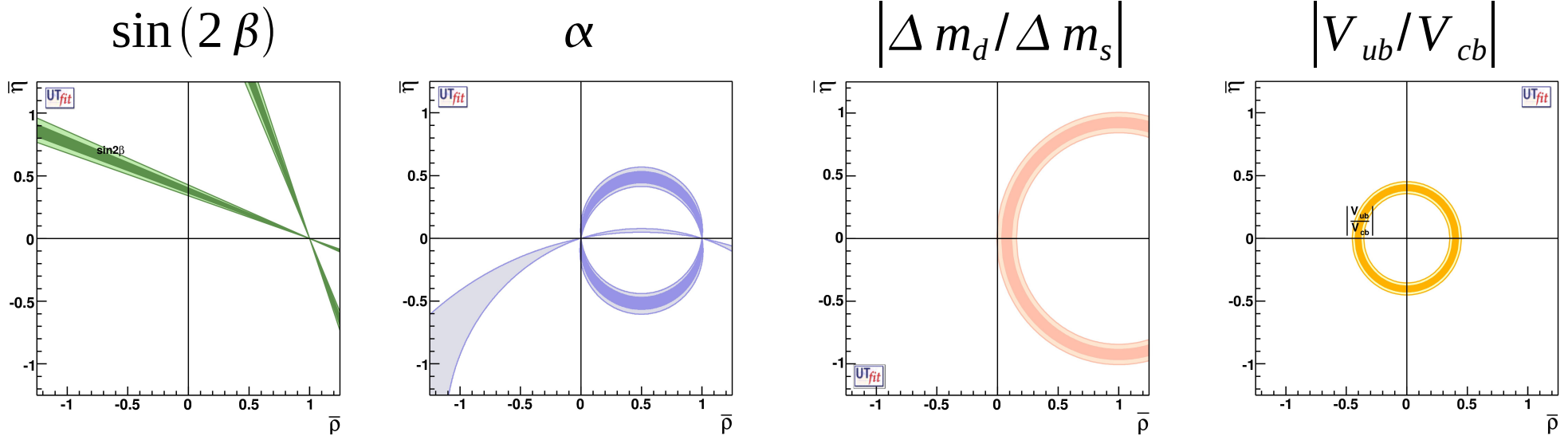
Adding a few other constraints we find

$$\overline{\rho} = 0.132 \pm 0.020$$
$$\overline{\eta} = 0.358 \pm 0.012$$

Consistent with Standard Model fit

- some "tensions"

Still plenty of room for new physics