

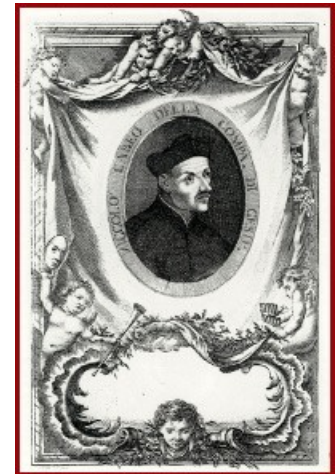
Studying CP Violation via Amplitude Analysis (i)

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**Ferrara International School Niccolò Cabeo
Hadronic spectroscopy**

24th May 2012



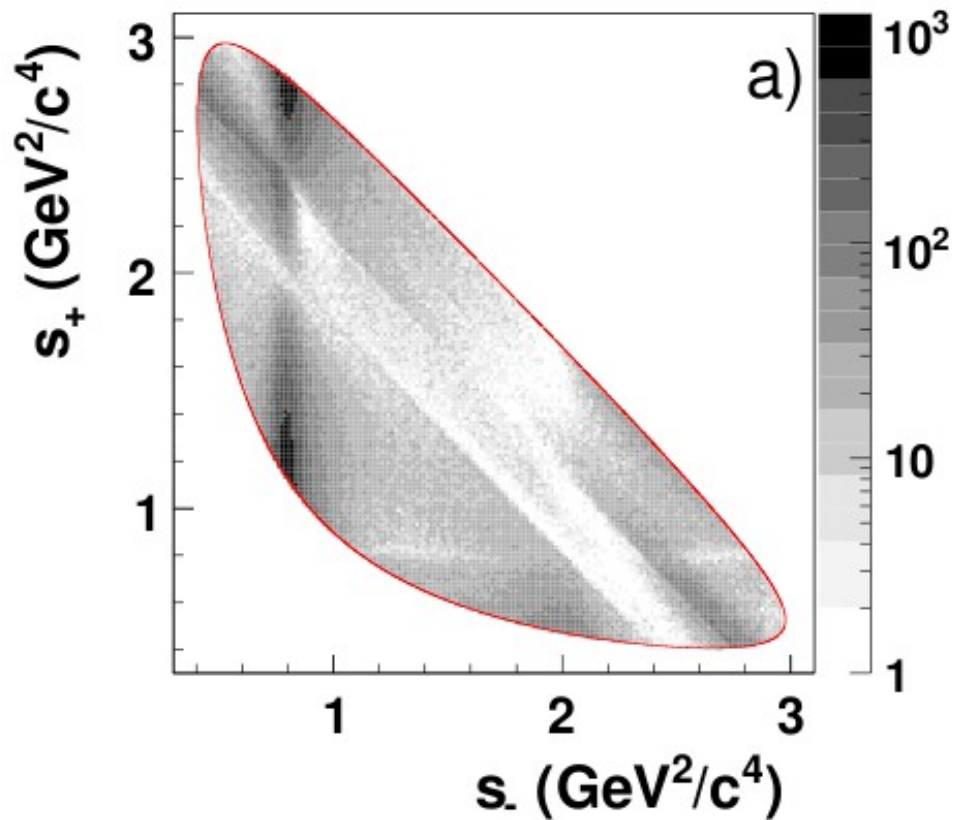
“Precision Experiments”

- I will focus on methods to search for CP violation beyond the Standard Model
 - Hadronic decays of heavy flavours (mainly D, B)
- Experiments in this field now reaching a “precision” era
 - Past: E791, FOCUS, CLEO, BESII, BaBar, Belle, ...
 - Current: CDF, D0, BESIII, LHCb
 - Future: Belle2, LHCb upgrade, SuperB
- “Precision” is relative – there are many higher precision experiments (at lower energies)
 - Studies of η , η' , K, ω , etc.

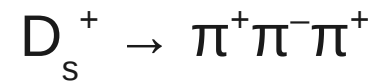
Example charm hadron decay samples in previous experiments



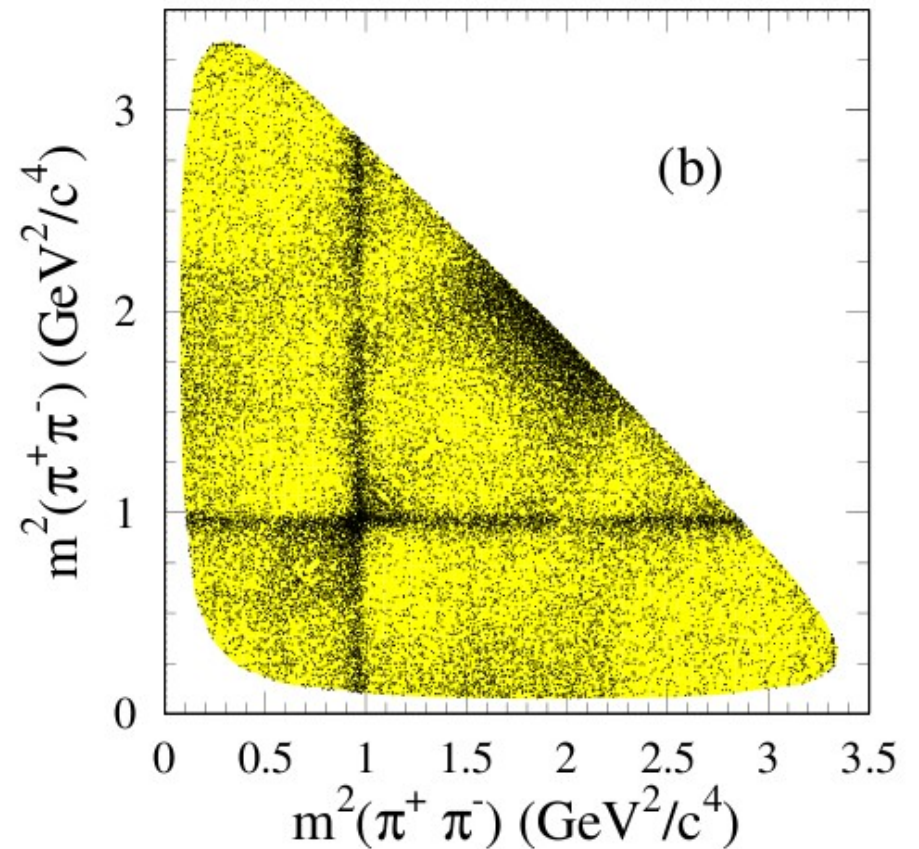
BaBar PRL 105 (2010) 081803



744 000 candidates



BaBar PRD 79 (2009) 032003

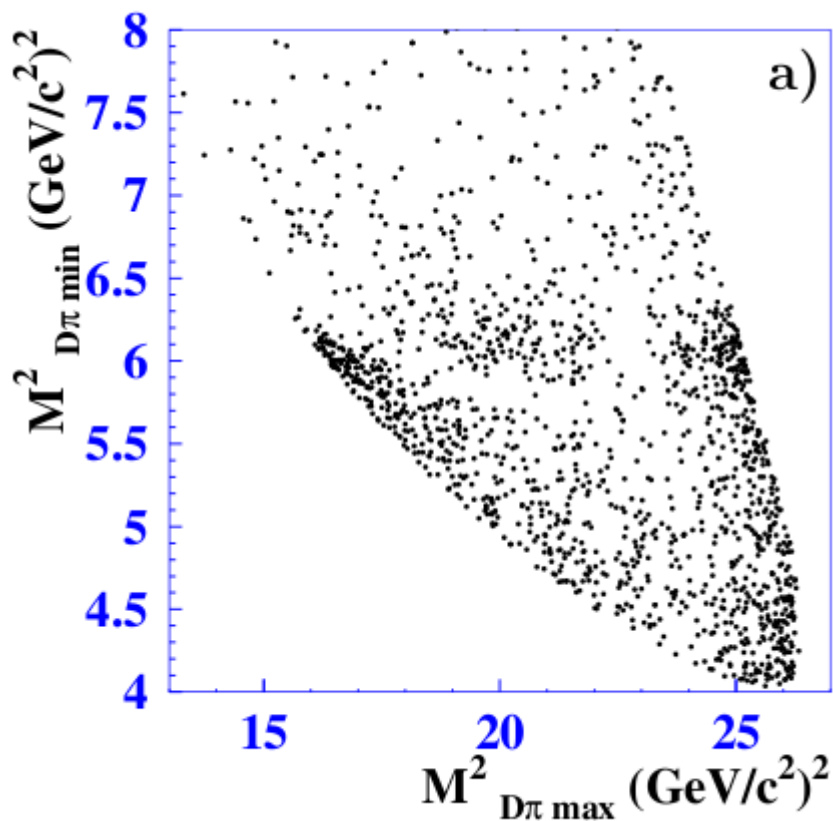


13 000 candidates

Example B hadron decay samples available in previous experiments



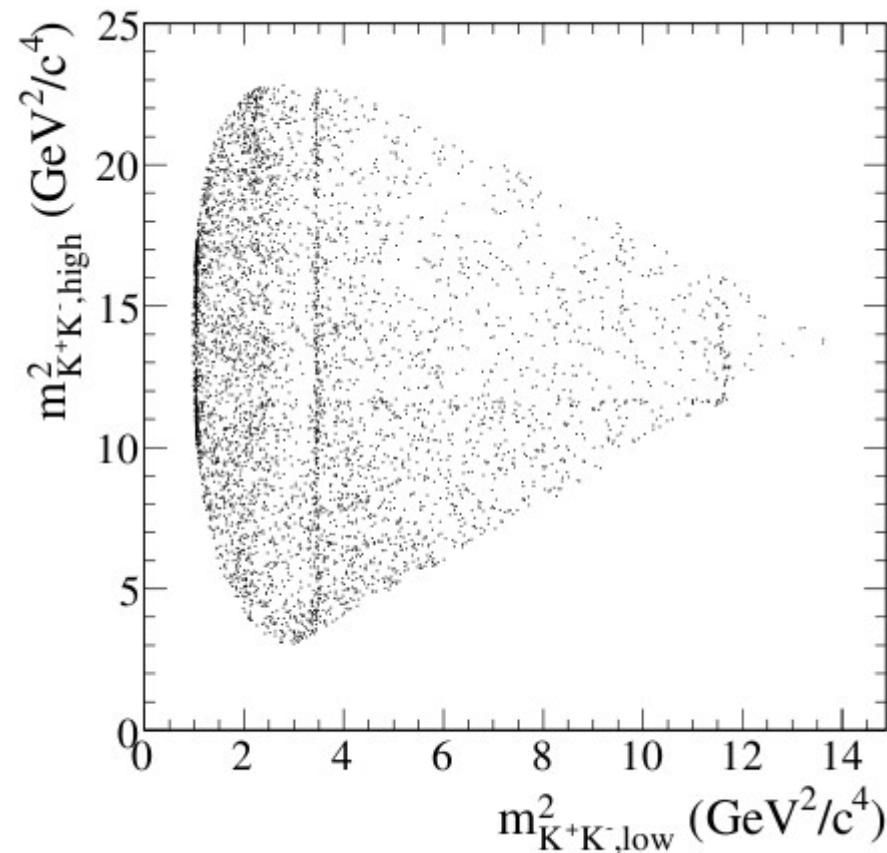
Belle PRD 69 (2004) 112002



1100 candidates



BaBar arXiv:1201.5897



5300 events

Content of the lectures

- Why do we believe that multibody hadronic decays of heavy flavours may provide a good laboratory to search for new sources of CP violation?
- Which decays in particular should we look at?
- What methods can we use to study them?
- What are the difficulties we encounter when trying to do the analysis?

Why do we believe that multibody hadronic decays of heavy flavours may provide a good laboratory to search for new sources of CP violation?

Heavy flavours & CP violation

- Studies of heavy flavours are ideal to study CP violation phenomena, since
 - In the SM, CP violation occurs only in flavour-changing weak interactions (the CKM matrix)
 - In several theories extending the SM, this remains true (to varying degrees) – weak interactions are a good place to look for new sources of CP violation
 - “New physics” can show up as deviations from precise CKM-based predictions, null or otherwise
- Aim is to make multiple, redundant measurements of the 4 independent parameters that define the CKM matrix and to find inconsistencies

The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

A 3x3 unitary matrix

Described by 4 real parameters – **allows CP violation**

PDG (Chau-Keung) parametrisation: $\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Wolfenstein parametrisation: λ, A, ρ, η

Highly predictive

Flavour oscillations, CP violation and Nobel Prizes

1964 – Discovery of CP violation in K^0 system

1980 – Nobel Prize to Cronin and Fitch



PRL 13 (1964) 138

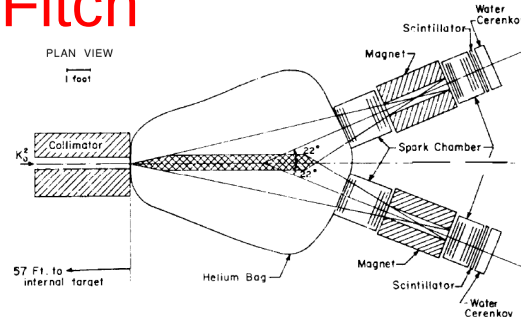


Fig. 1. Plan view of the apparatus as located at the A. G. S.

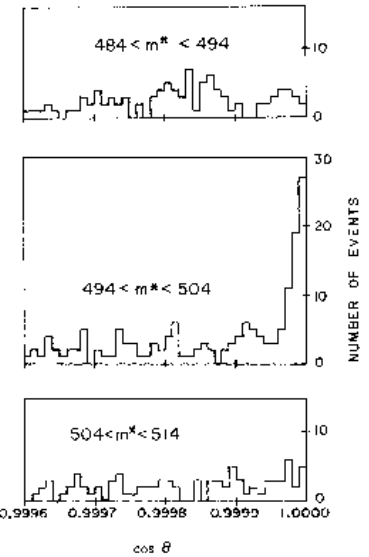
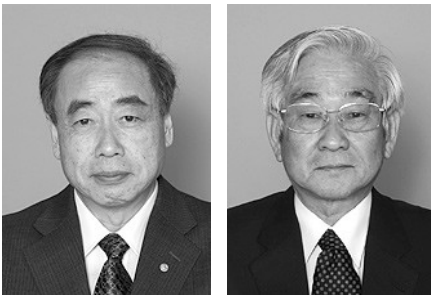


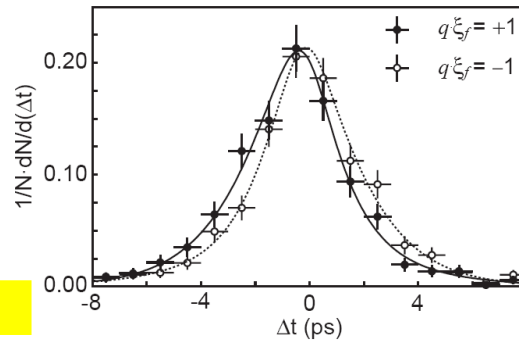
FIG. 3. Angular distribution in three mass ranges for events with $\cos\theta > 0.9995$.

2001 – Discovery of CP violation in B_d system

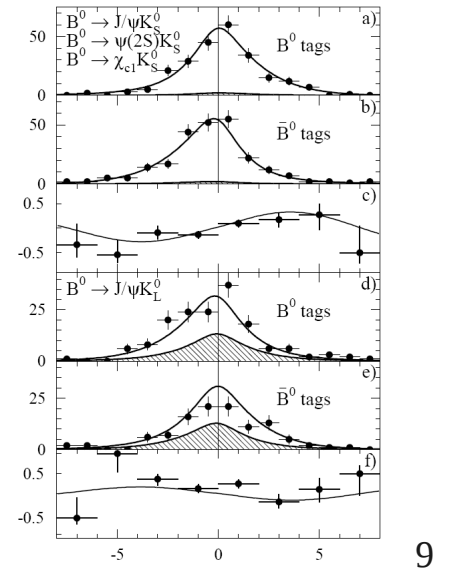
2008 – Nobel Prize to Kobayashi and Maskawa



Prog.Theor.Phys. 49 (1973) 652



Belle PRL 87 (2001) 091802

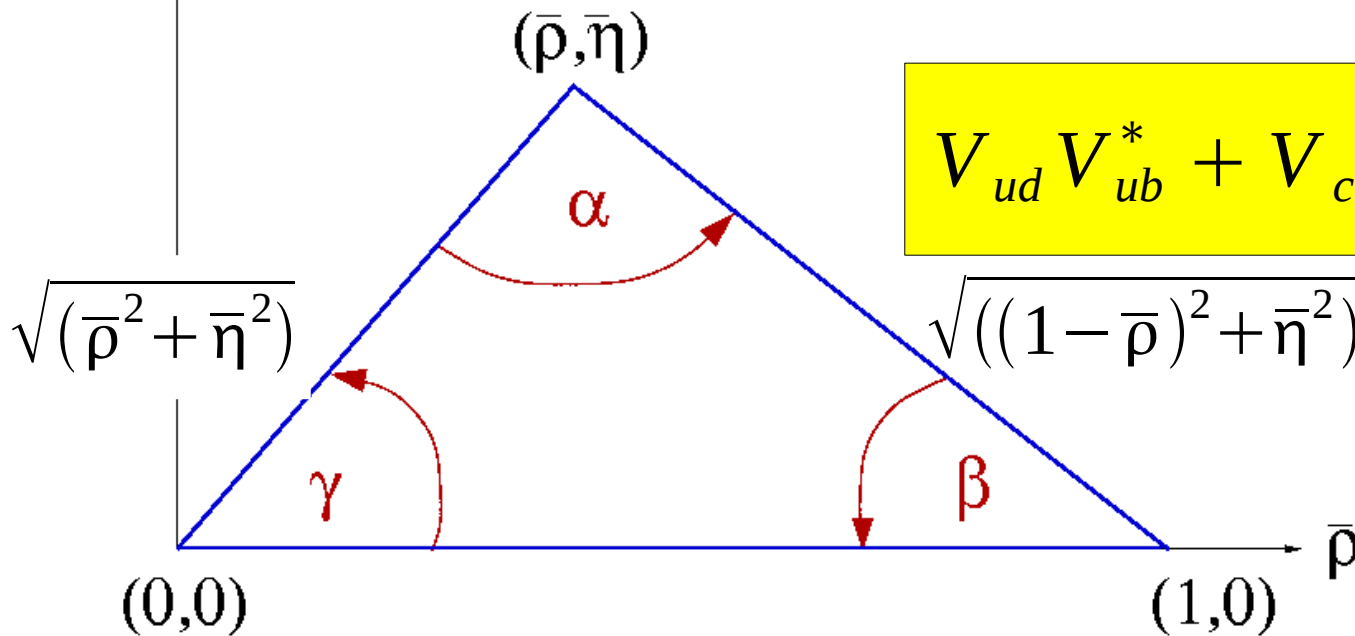


BABAR PRL 87 (2001) 091801

The Cabibbo-Kobayashi-Maskawa Matrix & The Unitarity Triangle

Quark couplings to W boson
described by 3x3 unitary matrix
(4 free parameters, inc. **1 phase**)

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$\alpha \equiv \phi_2 = \arg \left[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right], \quad \beta \equiv \phi_1 = \arg \left[-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right], \quad \gamma \equiv \phi_3 = \arg \left[-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]$$

Global fit status at EPS2011

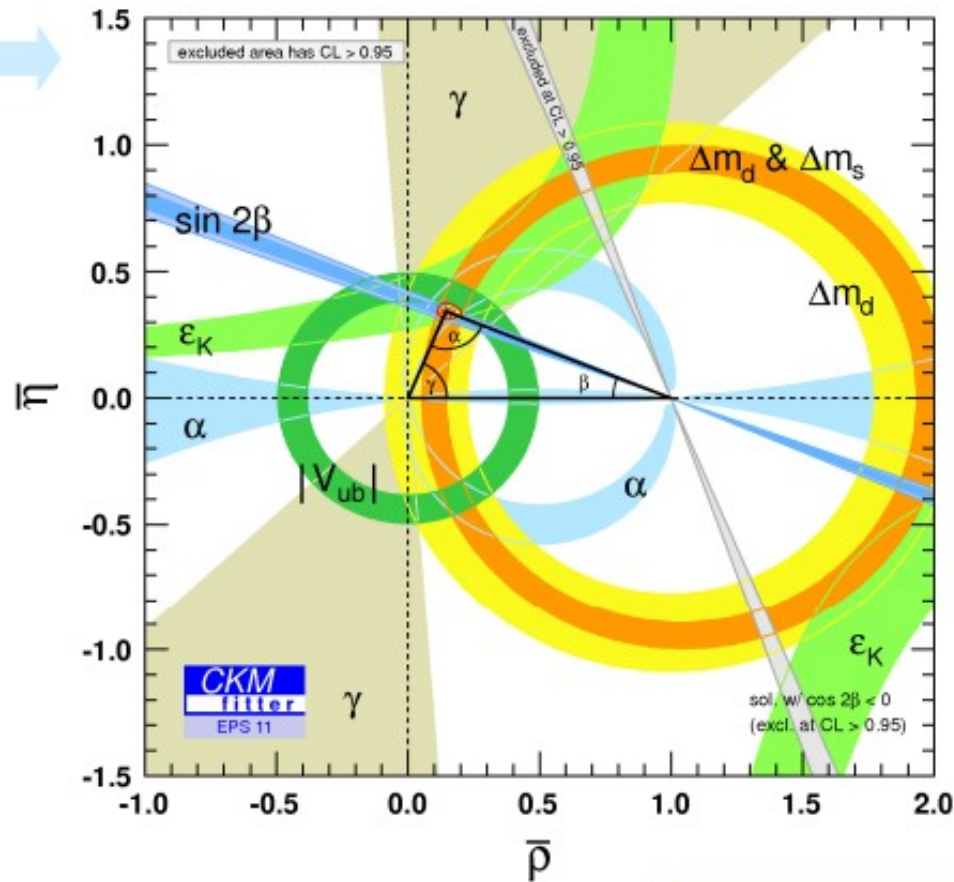
Update from CKMfitter collaboration (talk by V. Niess)

<http://indico.in2p3.fr/materialDisplay.py?contribId=392&sessionId=2&materialId=slides&confId=5116>

Global Fit of the UT

Observables

- $|V_{ud}|, |V_{us}|$
- $|V_{cb}|, |V_{ub}|$
- $\mathcal{B}[B \rightarrow \tau \nu]$
- $\Delta m_d, \Delta m_s$
- $|\varepsilon_K|$
- $\alpha, \sin(2\beta_{cc}), \gamma$



Fit of UT apex is dominated by $\sin(2\beta)$, $\Delta m_d/\Delta m_s$ and α . Excellent agreement between these 3 inputs.

Overall consistent picture

The KM mechanism is the dominant source of CP in B's

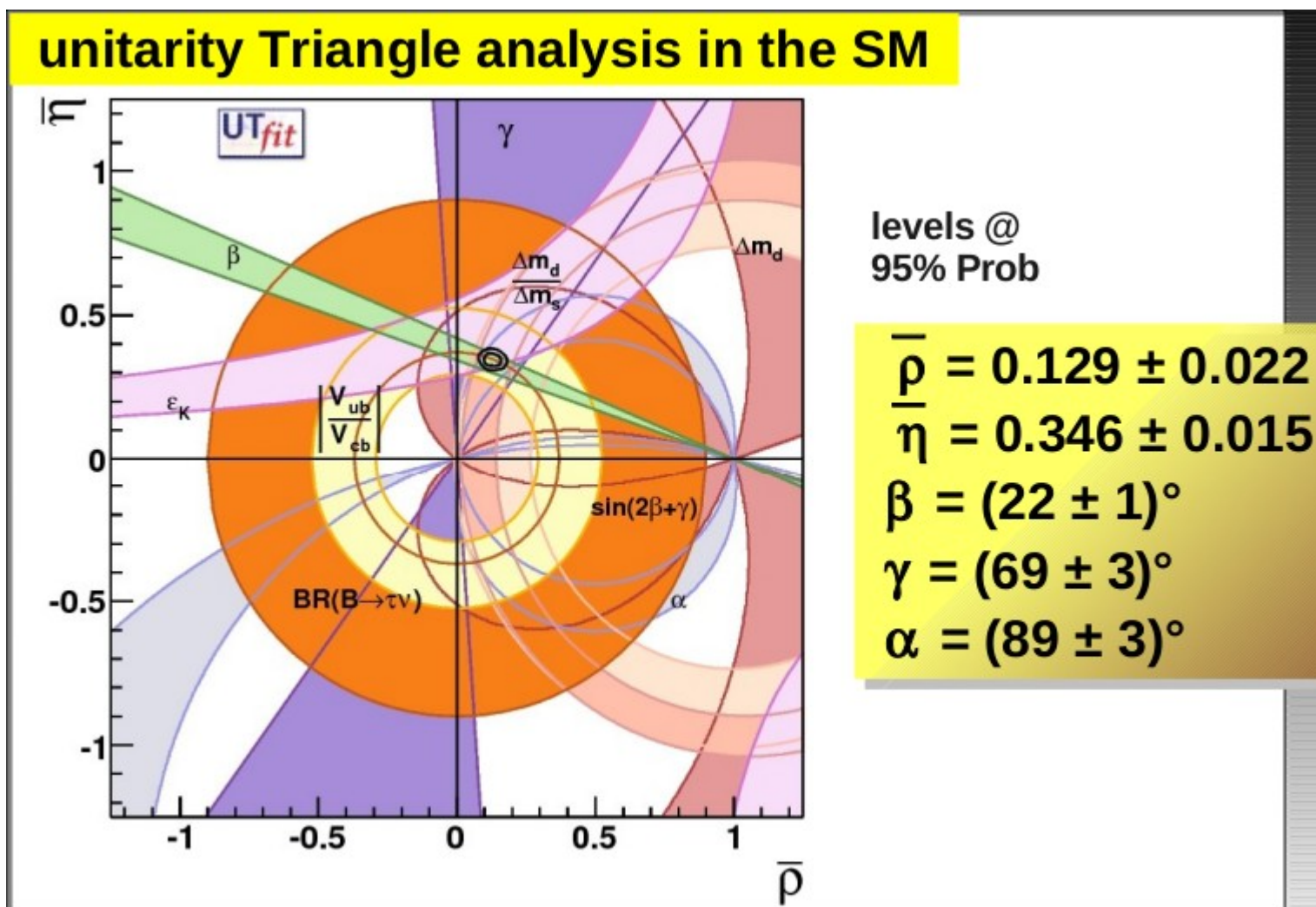
$\bar{\rho} = 0.144^{+0.027}_{-0.018}, \quad \bar{\eta} = 0.343^{+0.014}_{-0.014}$

(1 σ interval)

Global fit status at EPS2011

Update from UTfit collaboration (talk by M. Bona)

<http://indico.in2p3.fr/materialDisplay.py?contribId=424&sessionId=2&materialId=slides&confId=5116>



OK, but why do we believe that *multibody* hadronic decays of heavy flavours may provide a good laboratory to search for new sources of CP violation?

Direct CP violation in $B \rightarrow K\pi$

- Direct CP violation in $B \rightarrow K\pi$ sensitive to γ
too many hadronic parameters \Rightarrow need theory input
- NB. interesting deviation from naïve expectation

“ $K\pi$ puzzle”

$$A_{\text{CP}}(K^-\pi^+) = -0.087 \pm 0.008$$

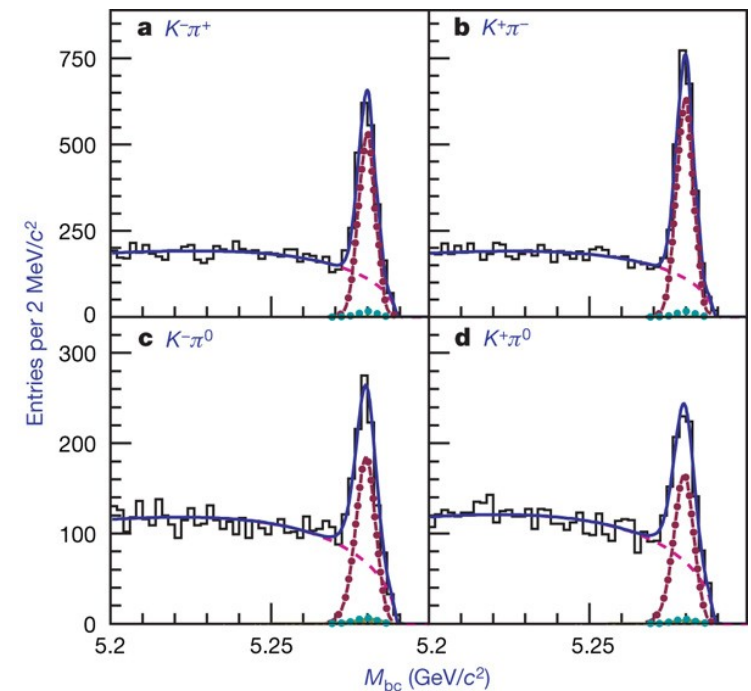
$$A_{\text{CP}}(K^-\pi^0) = +0.037 \pm 0.021$$

HFAG averages

Could be a sign of new physics ...

... but need to rule out possibility of larger than expected QCD corrections

Belle Nature 452 (2008) 332



We have excellent data, in clear disagreement with “the naïve Standard Model prediction” on $B \rightarrow K\pi$...

... but can't be sure that corrections to the SM prediction aren't larger than expected ...

... need methods that provide more observables to help control uncertainties

Why are we so interested in Dalitz plots?

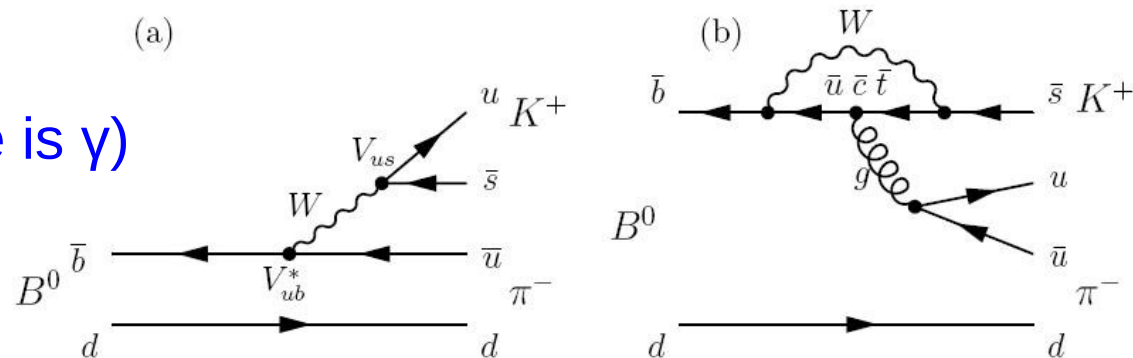
- Condition for DCPV: $|\bar{A}/A| \neq 1$
- Need \bar{A} and A to consist of (at least) two parts
 - with different weak (φ) and strong (δ) phases
- Often realised by “tree” and “penguin” diagrams

$$A = |T|e^{i(\delta_T - \phi_T)} + |P|e^{i(\delta_P - \phi_P)} \quad \bar{A} = |T|e^{i(\delta_T + \phi_T)} + |P|e^{i(\delta_P + \phi_P)}$$

$$A_{CP} = \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} = \frac{2|T||P|\sin(\delta_T - \delta_P)\sin(\phi_T - \phi_P)}{|T|^2 + |P|^2 + 2|T||P|\cos(\delta_T - \delta_P)\cos(\phi_T - \phi_P)}$$

Example: $B \rightarrow K\pi$

(weak phase difference is γ)



Feynman tree (a) and penguin (b) diagrams for the $B_d^0 \rightarrow K^+\pi^-$ decay

Why are we so interested in Dalitz plots?

- Condition for DCPV: $|\bar{A}/A| \neq 1$

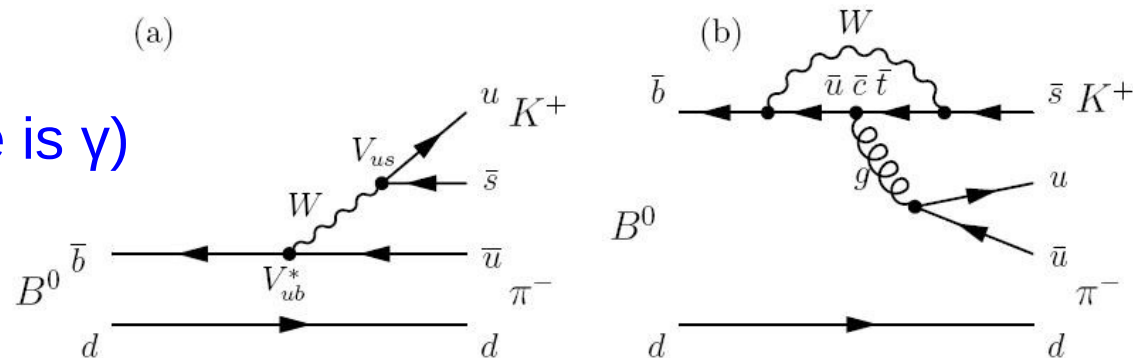
Problem with two-body decays:

- 2 observables (B, A_{CP})
- 4 unknowns ($|T|, |P|, \delta_T - \delta_P, \varphi_T - \varphi_P$)

$$A_{CP} = \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} = \frac{2 |T| |P| \sin(\delta_T - \delta_P) \sin(\phi_T - \phi_P)}{|T|^2 + |P|^2 + 2 |T| |P| \cos(\delta_T - \delta_P) \cos(\phi_T - \phi_P)}$$

Example: $B \rightarrow K\pi$

(weak phase difference is γ)



Feynman tree (a) and penguin (b) diagrams for the $B_d^0 \rightarrow K^+ \pi^-$ decay

What Is a Dalitz Plot?

- Visual representation of
 - the phase-space of a three-body decay
 - involving only spin-0 particles
 - (term often abused to refer to phase-space of any multibody decay)
 - Named after it's inventor, Richard Dalitz (1925–2006):
 - “On the analysis of tau-meson data and the nature of the tau-meson.”
 - R.H. Dalitz, Phil. Mag. 44 (1953) 1068
 - (historical reminder: tau meson = charged kaon)
 - For scientific obituary, see
 - I.J.R. Aitchison, F.E. Close, A. Gal, D.J. Millener,
 - Nucl.Phys.A771:8-25,2006

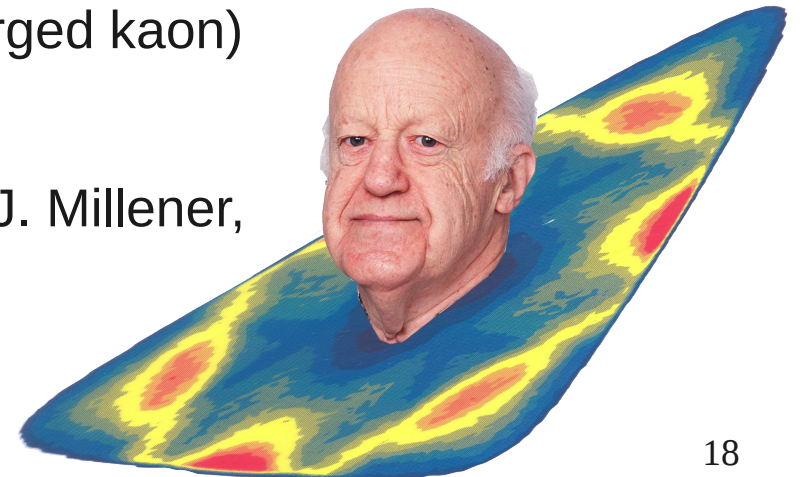
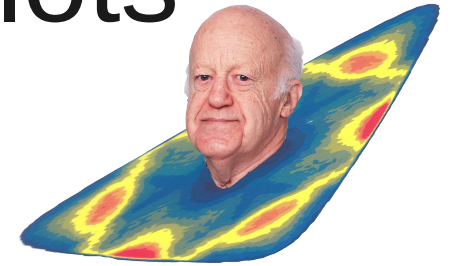


Image credit: Mike Pennington

Dalitz plot analysis

- **Amplitude analysis** to extract directly information related to the phase at each Dalitz plot position
- Most commonly performed in the “**isobar model**”
 - sum of interfering resonances
 - each described by Breit-Wigner (or similar) lineshapes, spin terms, etc.
 - **fit can be unbinned, but has inherent model dependence**
- Alternative approaches aiming to avoid model dependence usually involve binning

Pros and cons of Dalitz plots



- Pros

- More observables (B & A_{CP} at each Dalitz plot point)
- Using isobar formalism, can express total amplitude as coherent sum of quasi-two-body contributions

$$A(m_{12}^2, m_{23}^2) = \sum_r c_r F_r(m_{12}^2, m_{23}^2)$$

- where c_r & F_r contain the weak and strong physics, respectively
- n.b. each c_r is itself a sum of contributions from tree, penguin, etc.

- **Interference provides additional sensitivity to CP violation**

- Cons

- Need to understand hadronic (F_r) factors
 - lineshapes, angular terms, barrier factors, ...
- Isobar formalism only an approximation
- **Model dependence**

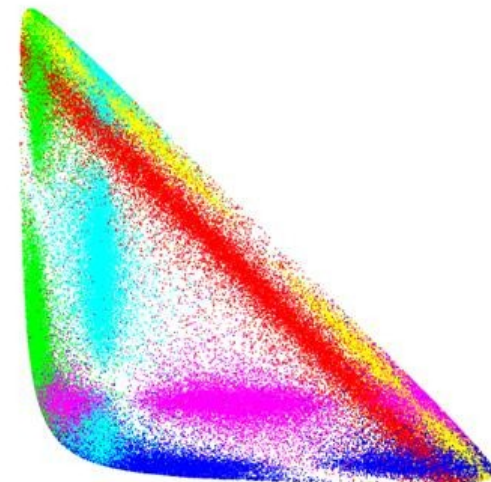
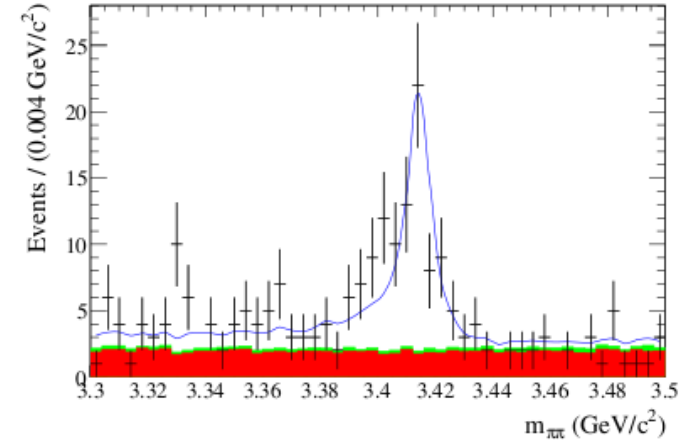
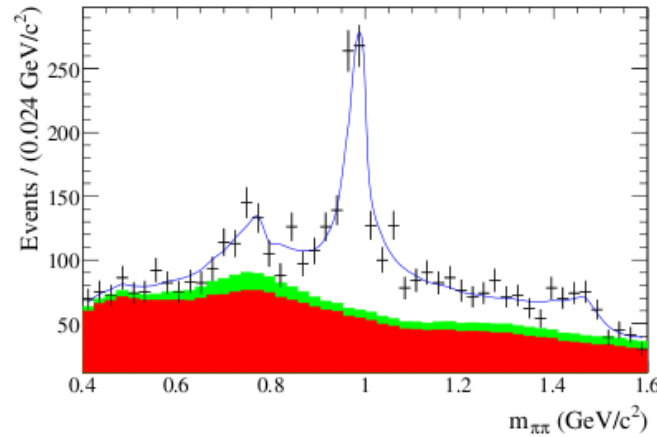
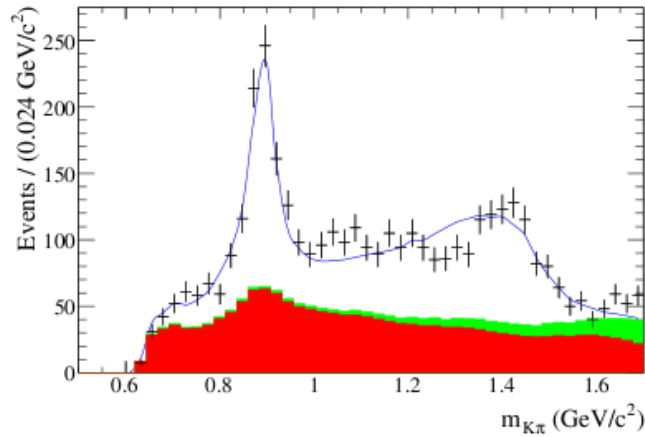


Image credit: Tom Latham

$$B^+ \rightarrow K^+ \pi^+ \pi^-$$



BaBar PRD 78 (2008) 012004
See also Belle PRL 96 (2006) 251803

Model includes:

- $K^{*0}(892)\pi^+$, $K_2^{*0}(1430)\pi^+$
- $(K\pi)_0^* \pi^+$ (LASS lineshape)
- $\rho^0(770)K^+$, $\omega(782)K^+$, $f_0(980)K^+$, $f_2(1270)K^+$, $\chi_{c0} K^+$
- $f_x(1300)K^+$, phase-space nonresonant

$B^+ \rightarrow K^+ \pi^+ \pi^-$

Events / (0.024 GeV/c²)

TABLE II: Summary of measurements of branching fractions (averaged over charge conjugate states) and CP asymmetries. Note that these results are not corrected for secondary branching fractions. The first uncertainty is statistical, the second is systematic, and the third represents the model dependence. The final column is the statistical significance of direct CP violation determined as described in the text.

Mode	Fit fraction (%)	$\mathcal{B}(B^+ \rightarrow \text{Mode})(10^{-6})$	A_{CP} (%)	DCPV sig.
$K^+ \pi^- \pi^+$ total		$54.4 \pm 1.1 \pm 4.5 \pm 0.7$	$2.8 \pm 2.0 \pm 2.0 \pm 1.2$	
$K^{*0}(892)\pi^+; K^{*0}(892) \rightarrow K^+ \pi^-$	$13.3 \pm 0.7 \pm 0.7^{+0.4}_{-0.9}$	$7.2 \pm 0.4 \pm 0.7^{+0.3}_{-0.5}$	$+3.2 \pm 5.2 \pm 1.1^{+1.2}_{-0.7}$	0.9σ
$(K\pi)_0^{*0}\pi^+; (K\pi)_0^{*0} \rightarrow K^+ \pi^-$	$45.0 \pm 1.4 \pm 1.2^{+12.9}_{-0.2}$	$24.5 \pm 0.9 \pm 2.1^{+7.0}_{-1.1}$	$+3.2 \pm 3.5 \pm 2.0^{+2.7}_{-1.9}$	1.2σ
$\rho^0(770)K^+; \rho^0(770) \rightarrow \pi^+ \pi^-$	$6.54 \pm 0.81 \pm 0.58^{+0.69}_{-0.26}$	$3.56 \pm 0.45 \pm 0.43^{+0.38}_{-0.15}$	$+44 \pm 10 \pm 4^{+5}_{-13}$	3.7σ
$f_0(980)K^+; f_0(980) \rightarrow \pi^+ \pi^-$	$18.9 \pm 0.9 \pm 1.7^{+2.8}_{-0.6}$	$10.3 \pm 0.5 \pm 1.3^{+1.5}_{-0.4}$	$-10.6 \pm 5.0 \pm 1.1^{+3.4}_{-1.0}$	1.8σ
$\chi_{c0}K^+; \chi_{c0} \rightarrow \pi^+ \pi^-$	$1.29 \pm 0.19 \pm 0.15^{+0.12}_{-0.03}$	$0.70 \pm 0.10 \pm 0.10^{+0.06}_{-0.02}$	$-14 \pm 15 \pm 3^{+1}_{-5}$	0.5σ
$K^+ \pi^- \pi^+$ nonresonant	$4.5 \pm 0.9 \pm 2.4^{+0.6}_{-1.5}$	$2.4 \pm 0.5 \pm 1.3^{+0.3}_{-0.8}$	—	—
$K_2^{*0}(1430)\pi^+; K_2^{*0}(1430) \rightarrow K^+ \pi^-$	$3.40 \pm 0.75 \pm 0.42^{+0.99}_{-0.13}$	$1.85 \pm 0.41 \pm 0.28^{+0.54}_{-0.08}$	$+5 \pm 23 \pm 4^{+18}_{-7}$	0.2σ
$\omega(782)K^+; \omega(782) \rightarrow \pi^+ \pi^-$	$0.17 \pm 0.24 \pm 0.03^{+0.05}_{-0.08}$	$0.09 \pm 0.13 \pm 0.02^{+0.03}_{-0.04}$	—	—
$f_2(1270)K^+; f_2(1270) \rightarrow \pi^+ \pi^-$	$0.91 \pm 0.27 \pm 0.11^{+0.24}_{-0.17}$	$0.50 \pm 0.15 \pm 0.07^{+0.13}_{-0.09}$	$-85 \pm 22 \pm 13^{+22}_{-2}$	3.5σ
$f_X(1300)K^+; f_X(1300) \rightarrow \pi^+ \pi^-$	$1.33 \pm 0.38 \pm 0.86^{+0.04}_{-0.14}$	$0.73 \pm 0.21 \pm 0.47^{+0.02}_{-0.08}$	$+28 \pm 26 \pm 13^{+7}_{-5}$	0.6σ

8 3.5
v/c²

- $f_X(1300)K^+$, phase-space nonresonant

BaBar PRD 78 (2008) 012004
See also Belle PRL 96 (2006) 251803

Evidence for direct CP violation
But significant model dependence

$B^+ \rightarrow K^+ \pi^+ \pi^-$

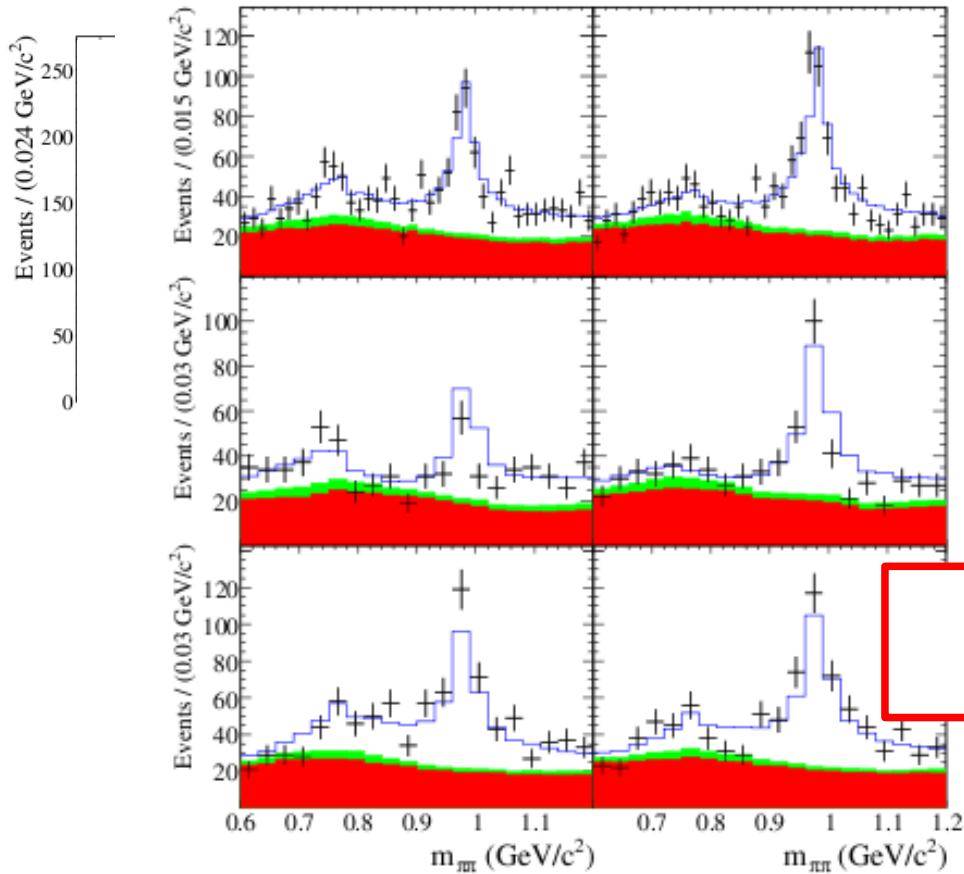


FIG. 4: Projection plots of the $\pi^+\pi^-$ invariant mass in the region of the $\rho^0(770)$ and $f_0(980)$ resonances. The left (right) plots are for B^- (B^+) candidates. The top row shows all candidates, the middle row shows those where $\cos\theta_H > 0$, and the bottom row shows those where $\cos\theta_H < 0$. The data are the black points with statistical error bars, the lower solid (red/dark) histogram is the $q\bar{q}$ component, the middle solid (green/light) histogram is the $B\bar{B}$ background contribution, while the blue open histogram shows the total fit result.

averaged over charge conjugate states) and CP asymmetries. A_{CP} and $DCPV$ are branching fractions. The first uncertainty is statistical, the second is systematic. The final column is the statistical significance of direct CP violation.

$\mathcal{B}(B^+ \rightarrow \text{Mode})(10^{-6})$	A_{CP} (%)	DCPV sig.
$54.4 \pm 1.1 \pm 4.5 \pm 0.7$	$2.8 \pm 2.0 \pm 2.0 \pm 1.2$	
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$24.5 \pm 0.9 \pm 2.1^{+7.0}_{-1.1}$	$+3.2 \pm 3.5 \pm 2.0^{+2.7}_{-1.9}$	1.2σ
$15.6 \pm 0.45 \pm 0.43^{+0.38}_{-0.15}$	$+44 \pm 10 \pm 4^{+5}_{-13}$	3.7σ
$10.3 \pm 0.5 \pm 1.3^{+1.5}_{-0.4}$	$-10.6 \pm 5.0 \pm 1.1^{+3.4}_{-1.0}$	1.8σ

Evidence for direct CP violation
But significant model dependence

$1.85 \pm 0.41 \pm 0.28^{+0.54}_{-0.08}$	$+5 \pm 23 \pm 4^{+10}_{-7}$	0.2σ
$1.09 \pm 0.13 \pm 0.02^{+0.03}_{-0.04}$	—	—
$1.50 \pm 0.15 \pm 0.07^{+0.13}_{-0.09}$	$-85 \pm 22 \pm 13^{+22}_{-2}$	3.5σ
$1.73 \pm 0.21 \pm 0.47^{+0.02}_{-0.08}$	$+28 \pm 26 \pm 13^{+7}_{-5}$	0.6σ

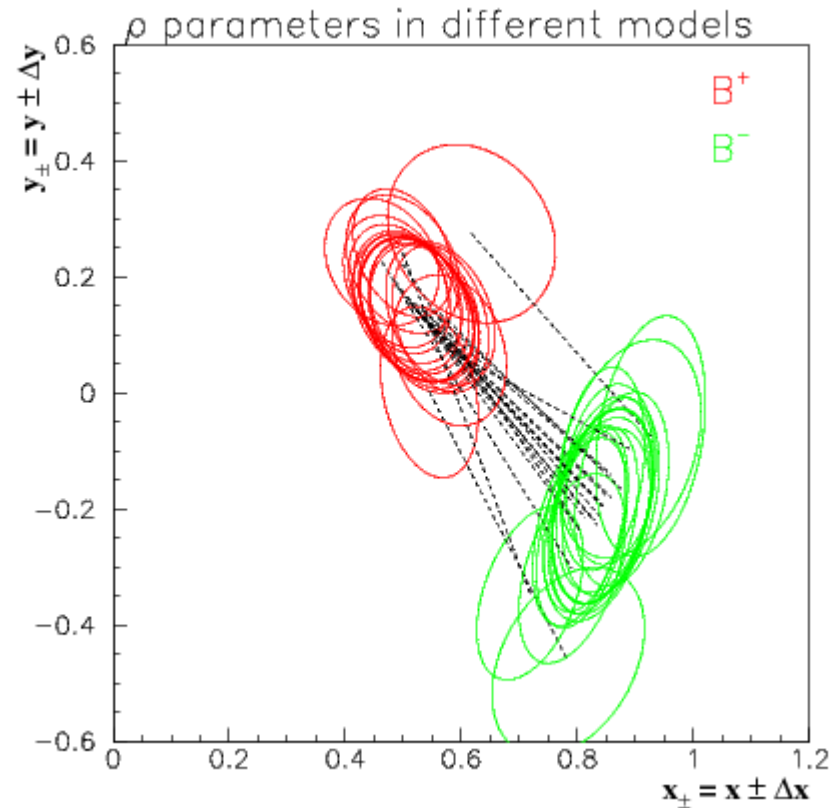
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BaBar PRD 78 (2008) 012004
See also Belle PRL 96 (2006) 251803

$B^+ \rightarrow K^+ \pi^+ \pi^-$ – model dependence

Complex coefficients parametrised as $x + iy$

$\rightarrow (x \pm \Delta x) + i(y \pm \Delta y)$ with CP violation



Ellipses correspond to fitted parameters obtained with different Dalitz plot models

Significance of CP violation corresponds to the lack of overlap of the ellipses

Sources of model dependence

- Lineshapes
 - coupled channels, threshold effects, etc.
- Isobar formalism
 - “sum of Breit-Wigners” model violates unitarity
 - problem most severe for broad, overlapping resonances
 - even talking about “mass” and “width” for such states is not strictly correct (process dependent) – can only be defined by pole position
- Nonresonant contributions
 - such terms are small for D decays, but are found to be large for some B decays (not well understood why)
 - interference with other (S-wave) terms can lead to unphysical phase variations

Are methods used for D decay Dalitz plots also valid for B decays?

Dalitz plots also valid for B decays?

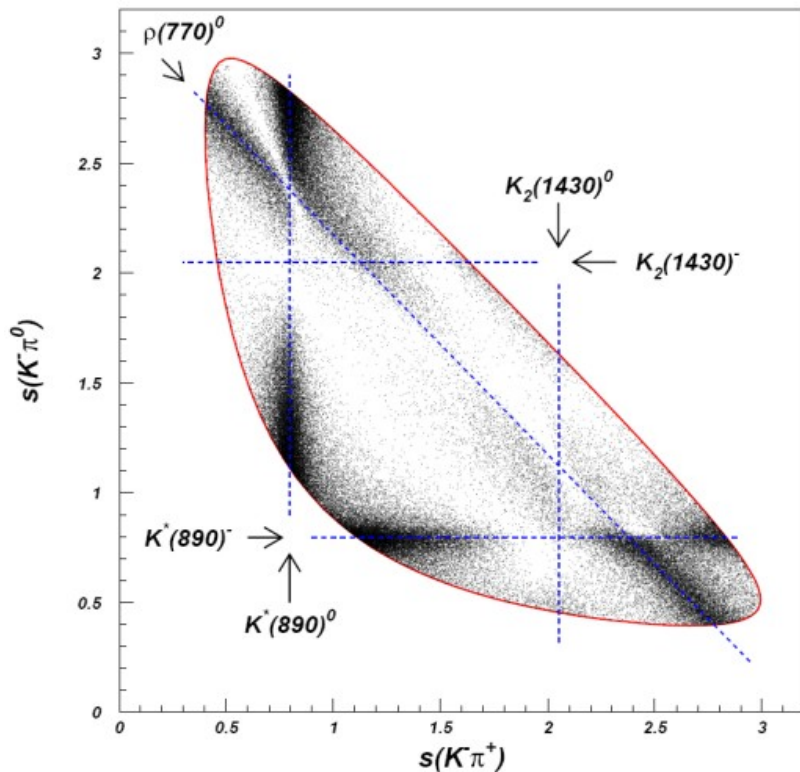
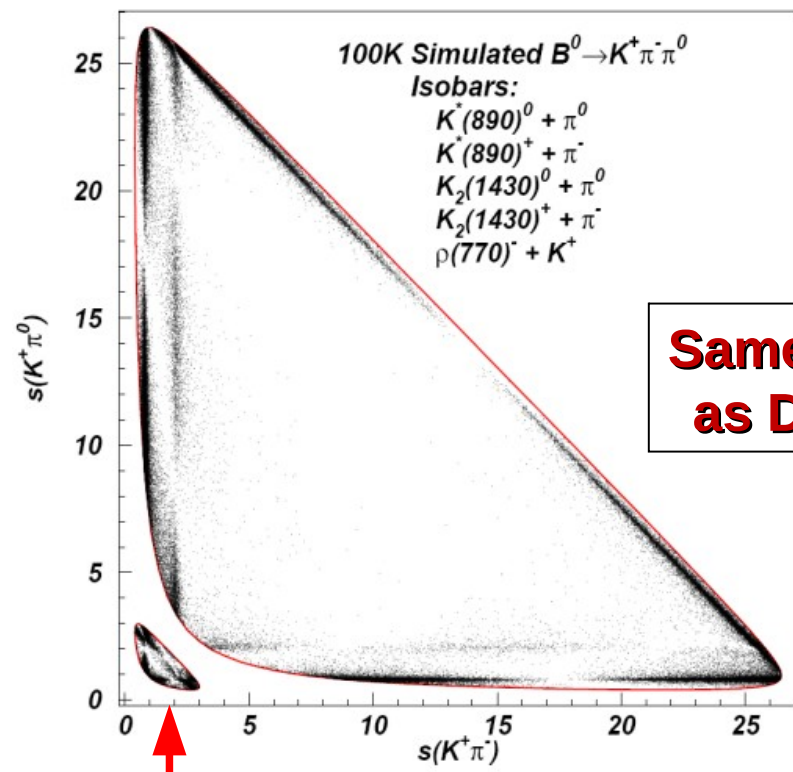


Image credit: Brian Meadows

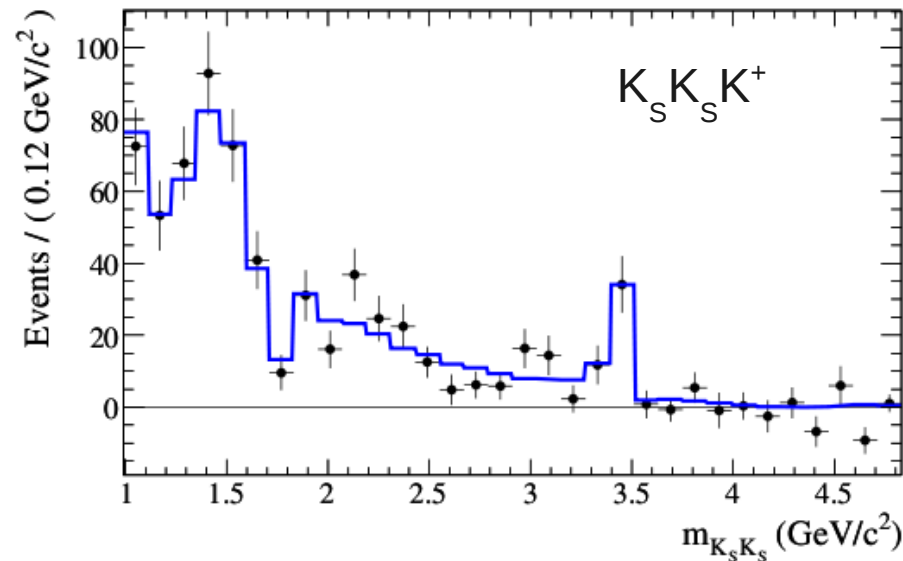
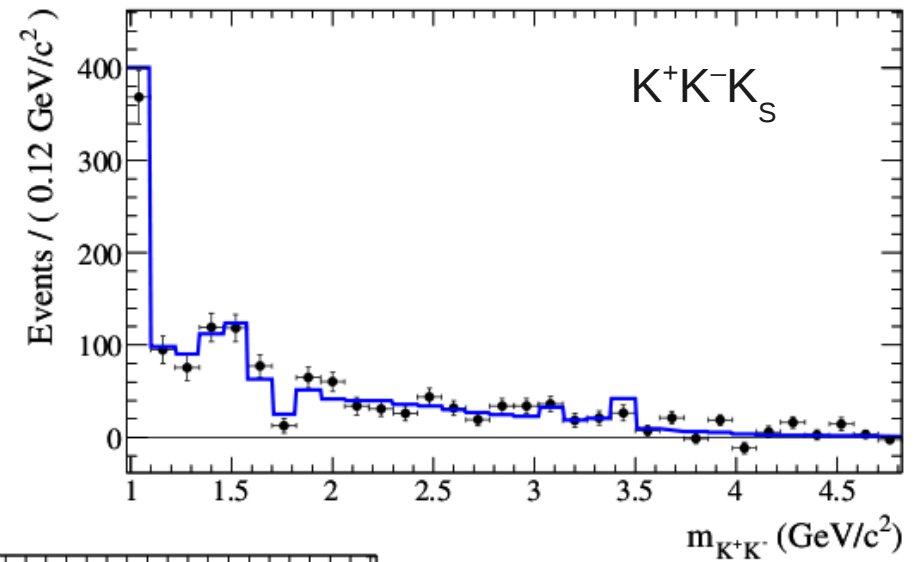
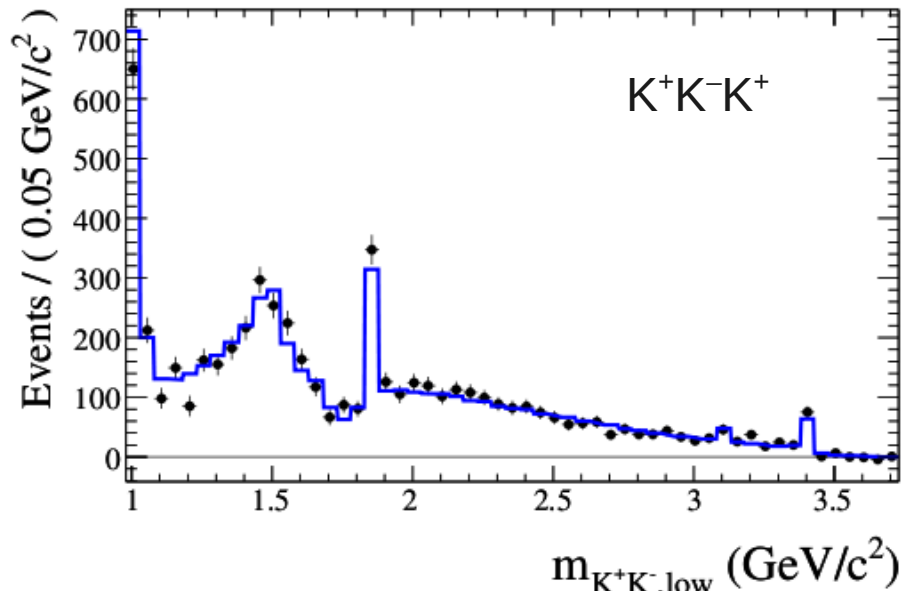


Same model as D decay

D Dalitz plot on same scale

Nonresonant contribution to $B \rightarrow KKK$

BaBar arXiv:1201.5897
See also Belle PRD 71 (2005) 092003
PRD 82 (2010) 073011



Which decays in particular should we look at?

Extracting weak phases from Dalitz plots

- Many methods exist in the literature
 - some have been used to date, others not yet
 - most results are statistically limited
 - still plenty of room for good new ideas
- Examples (there are many more!)
 - Snyder-Quinn method to measure α from $B \rightarrow \pi^+\pi^-\pi^0$
 - GGSZ/BP method to measure γ from $B^\pm \rightarrow DK^\pm$ with $D \rightarrow K_S\pi^+\pi^-$
 - Measurement of charm oscillation parameters using $D \rightarrow K_S\pi^+\pi^-$
 - Various methods to measure γ from three-body charmless B decays ($B_{\{u,d,s\}} \rightarrow \pi\pi\pi, K\pi\pi, KK\pi, KKK$)
 - Penguin-free measurements of β & β_s from $D\pi^+\pi^-$ & DK^+K^- , respectively
- I will mention just a couple of these examples ...

Searching for CP violation in charm Dalitz plots

- Standard Model effects are small
 - negligible in Cabibbo-favoured decays
 - not more than $O(10^{-3})$ in singly-Cabibbo-suppressed decays (see, e.g., PRD 75 (2007) 036008)
 - at least, this was the thinking until ~6 months ago ...
 - can be enhanced in various NP models
- Good channel for model-independent analysis
 - new LHCb analysis based on 'Miranda' approach
 - search for CP violation in $D^{\pm} \rightarrow K^+K^-\pi^{\pm}$
 - exploit $D_s^{\pm} \rightarrow K^+K^-\pi^{\pm}$ as control sample
 - care taken over binning to optimise sensitivity

(digression)

Evidence for CP violation in $D \rightarrow h^+h^-$ decays

LHCb PRL 108 (2012) 111602

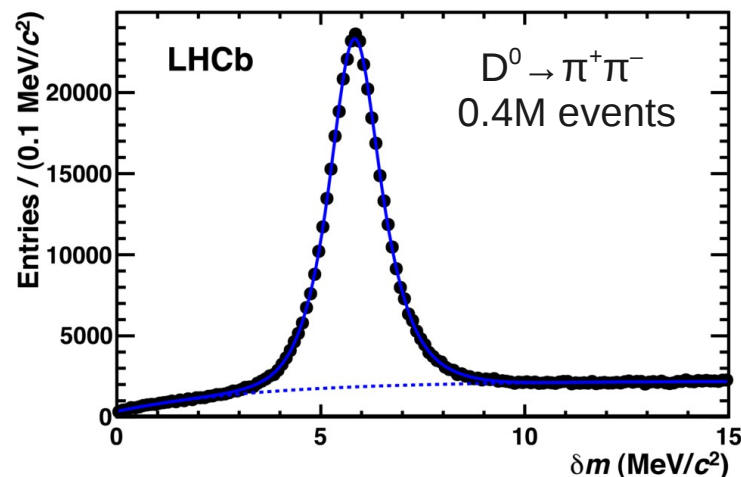
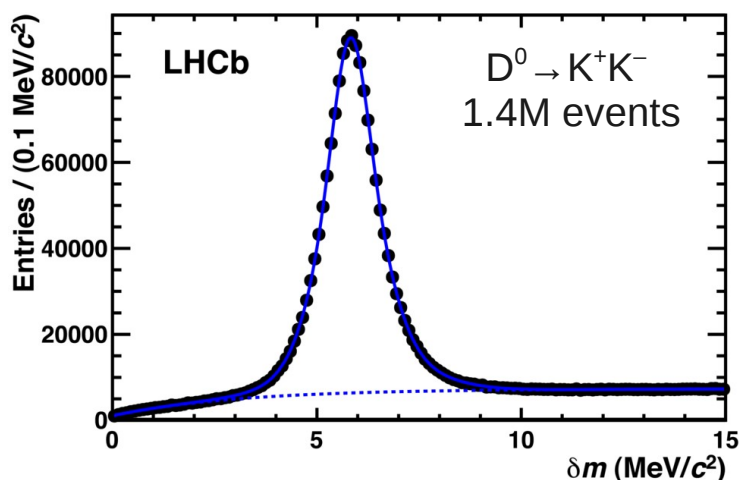
Measurement of CP asymmetry at pp collider requires knowledge of production and detection asymmetries; e.g. for $D^0 \rightarrow f$, where D meson flavour is tagged by $D^{*+} \rightarrow D^0\pi^+$ decay

$$A_{\text{raw}}(f) = A_{CP}(f) + A_D(f) + A_D(\pi_s^+) + A_P(D^{*+}).$$

final state detection asymmetry
vanishes for CP eigenstate

Cancel asymmetries by taking difference of raw asymmetries in two different final states
(Since A_D and A_P depend on kinematics, must bin or reweight to ensure cancellation)

$$\Delta A_{CP} = A_{\text{raw}}(K^-K^+) - A_{\text{raw}}(\pi^-\pi^+).$$



(digression)

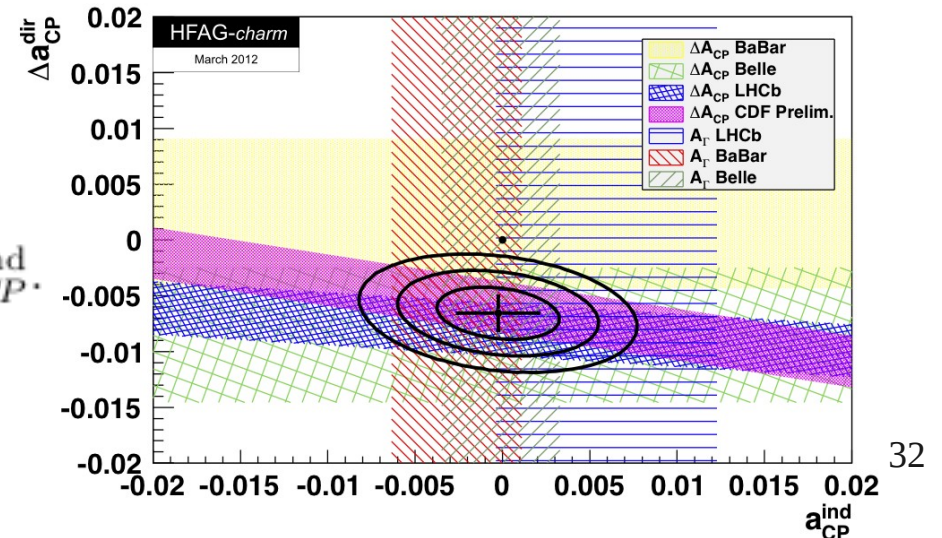
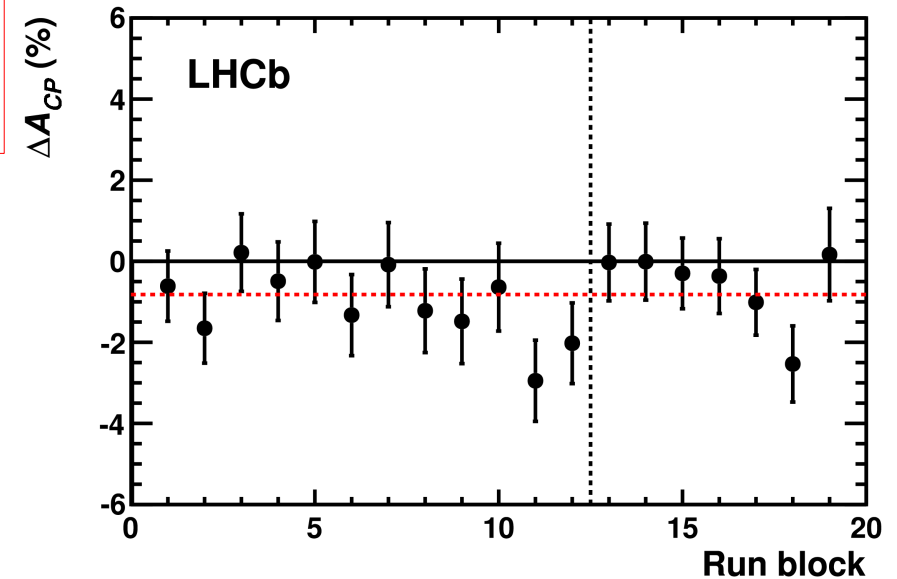
Evidence for CP violation in $D \rightarrow h^+h^-$ decays

LHCb PRL 108 (2012) 111602

Result, based on 0.62/fb of 2011 data
 $\Delta A_{CP} = [-0.82 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})]\%$

ΔA_{CP} related mainly to direct CP violation
(contribution from indirect CPV suppressed by
difference in mean decay time)

$$\begin{aligned}\Delta A_{CP} &\equiv A_{CP}(K^-K^+) - A_{CP}(\pi^-\pi^+) \\ &= [a_{CP}^{\text{dir}}(K^-K^+) - a_{CP}^{\text{dir}}(\pi^-\pi^+)] + \frac{\Delta\langle t \rangle}{\tau} a_{CP}^{\text{ind}}.\end{aligned}$$



(digression)

Evidence for CP violation in $D \rightarrow h^+h^-$ decays

- Naive SM expectation is for decays to be tree-dominated
- Penguin contributions are possible for singly-Cabibbo-suppressed decays but CKM suppression is severe
- So CP violation effects should be $O(10^{-4})$... or should they?
 - Implications of the LHCb Evidence for Charm CP Violation arXiv:1111.4987
 - Direct CP violation in two-body hadronic charmed meson decays arXiv:1201.0785
 - CP asymmetries in singly-Cabibbo-suppressed D decays to two pseudoscalar mesons arXiv:1201.2351
 - Direct CP violation in charm and flavor mixing beyond the SM arXiv:1201.6204
 - New Physics Models of Direct CP Violation in Charm Decays arXiv:1202.2866
 - Repercussions of Flavour Symmetry Breaking on CP Violation in D-Meson Decays arXiv:1202.3795
 - On the Universality of CP Violation in $\Delta F = 1$ Processes arXiv:1202.5038
 - The Standard Model confronts CP violation in $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ arXiv:1203.3131
 - A consistent picture for large penguins in $D \rightarrow \pi^+\pi^-, K^+K^-$ arXiv:1203.6659

... and many others! Further experimental input needed
to clarify whether CPV is SM or NP

→ further motivation for Dalitz plot analyses of CPV in charm decays

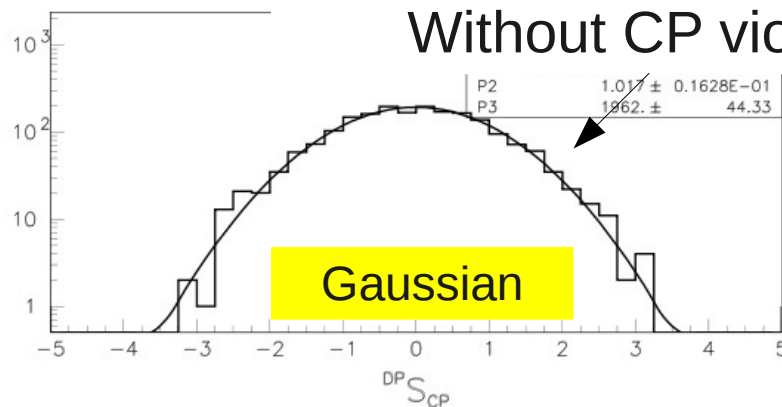
“Miranda” procedure a.k.a. Dalitz plot anisotropy

$$D_{p} S_{CP} \equiv \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}}$$

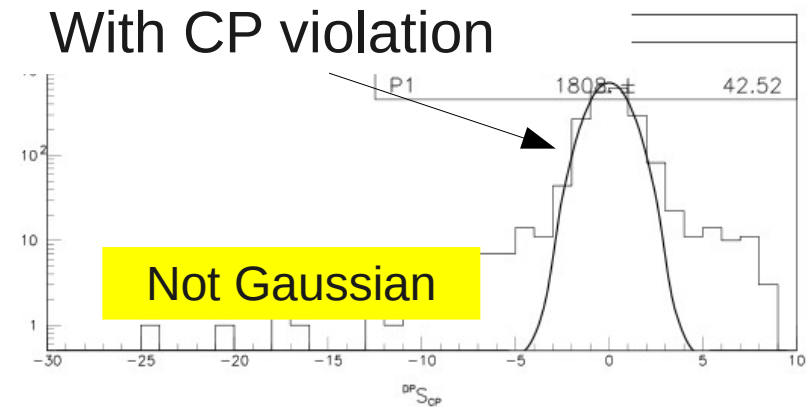
PRD 80 (2009) 096006
see also arXiv:1205.3036

Toy model (using $B^+ \rightarrow K^+ \pi^+ \pi^-$)

Without CP violation



With CP violation



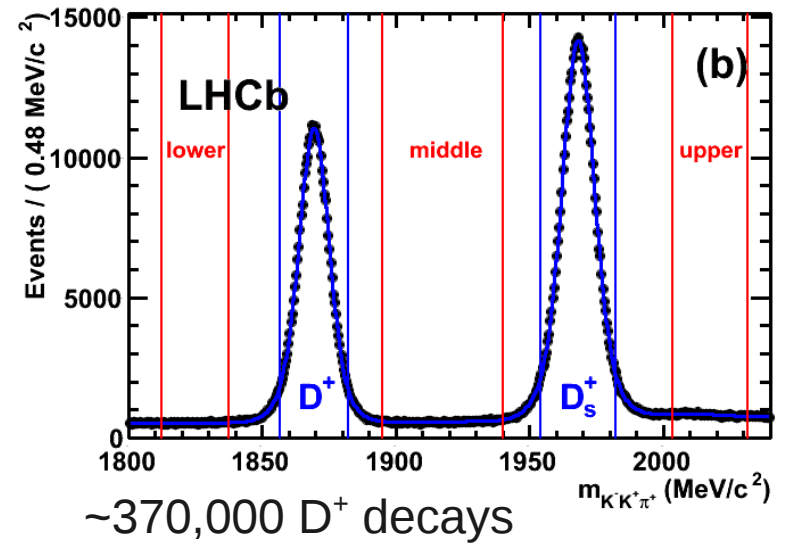
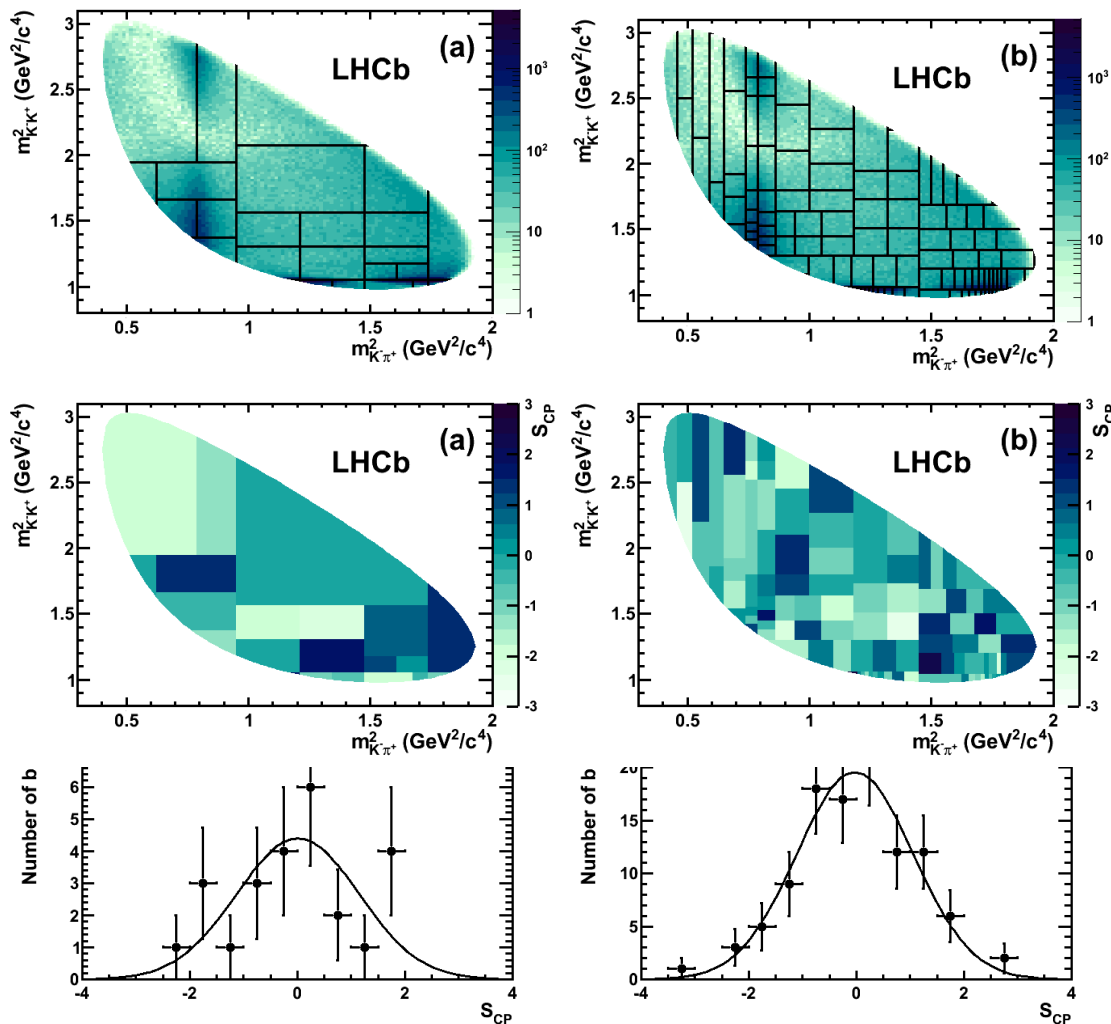
- Good model-independent way to identify CP violation
 - could be sufficient to identify non-SM physics in, e.g., charm decays
- Constant (DP independent) systematic asymmetries can be accounted for
- Can isolate region of the Dalitz plot where CP violation effects occur

But does not provide quantitative measure of weak phase

Search for CP violation in $D^+ \rightarrow K^- K^+ \pi^+$ decays

PRD 84 (2011) 112008

Two of the different binning schemes used



No evidence for CP violation found

Prospects for improved analyses

- These new results put stringent limits on CP violation effects in $D^\pm \rightarrow K^+K^-\pi^\pm$ decays
 - (albeit in a manner that is slightly hard to quantify)
- Improved statistical sensitivity is guaranteed since LHCb has much more data on tape
- Further improvements possible using an alternative, unbinned method (PRD 84 (2011) 054015)

Unbinned, model-independent CP violation search (arXiv:1105.5338)

The following test statistic correlates the difference between the $X \rightarrow abc$ and c.c. p.d.f.s, denoted by $f(\vec{x})$ and $\bar{f}(\vec{x})$, respectively, at different points in the multivariate space [13] [14]:

$$\begin{aligned}
 T &= \frac{1}{2} \int \int (f(\vec{x}) - \bar{f}(\vec{x})) (f(\vec{x}') - \bar{f}(\vec{x}')) \\
 &\quad \times \psi(|\vec{x} - \vec{x}'|) d\vec{x} d\vec{x}' \\
 &= \frac{1}{2} \int \int [f(\vec{x})f(\vec{x}') + \bar{f}(\vec{x})\bar{f}(\vec{x}') - 2f(\vec{x})\bar{f}(\vec{x}')] \\
 &\quad \times \psi(|\vec{x} - \vec{x}'|) d\vec{x} d\vec{x}', \quad (3)
 \end{aligned}$$

where $\psi(|\vec{x} - \vec{x}'|)$ is a weighting function. T can be estimated without the need for any knowledge about the forms of f and \bar{f} using $X \rightarrow abc$ and c.c. data as

$$\begin{aligned}
 T &\approx \frac{1}{n(n-1)} \sum_{i,j>i}^n \psi(\Delta\vec{x}_{ij}) \\
 &\quad + \frac{1}{\bar{n}(\bar{n}-1)} \sum_{i,j>i}^{\bar{n}} \psi(\Delta\vec{x}_{ij}) - \frac{1}{n\bar{n}} \sum_{i,j}^{n,\bar{n}} \psi(\Delta\vec{x}_{ij}), \quad (4)
 \end{aligned}$$

where $\Delta\vec{x}_{ij} = |\vec{x}_i - \vec{x}_j|$ and n (\bar{n}) is the number of $X \rightarrow abc$ (c.c.) events. *N.b.*, in the order in which they appear in Eq. [4] the sums are over pairs of $X \rightarrow abc$ events, pairs of c.c. events and pairs consisting of an $X \rightarrow abc$ event and a c.c. event, respectively.

“energy test”

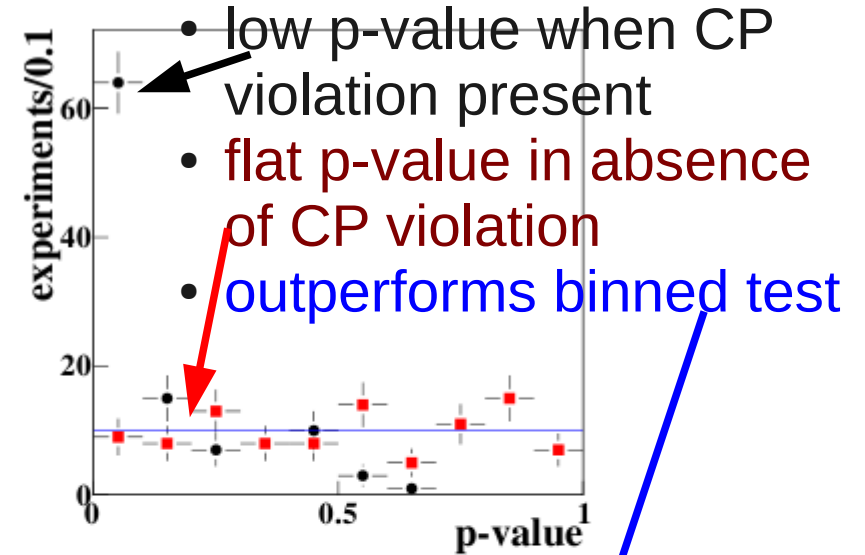


FIG. 4: p -value distributions obtained using the energy test on the CP -conserving (red squares) and CP -violating (black circles) ensembles of data sets. The (solid blue) line shows the expected distribution for the CP -conserving case.

test	$1\sigma(\%)$	$2\sigma(\%)$	$3\sigma(\%)$
χ^2	38 ± 5	3 ± 2	0 ± 1
energy	87 ± 3	52 ± 5	13 ± 3

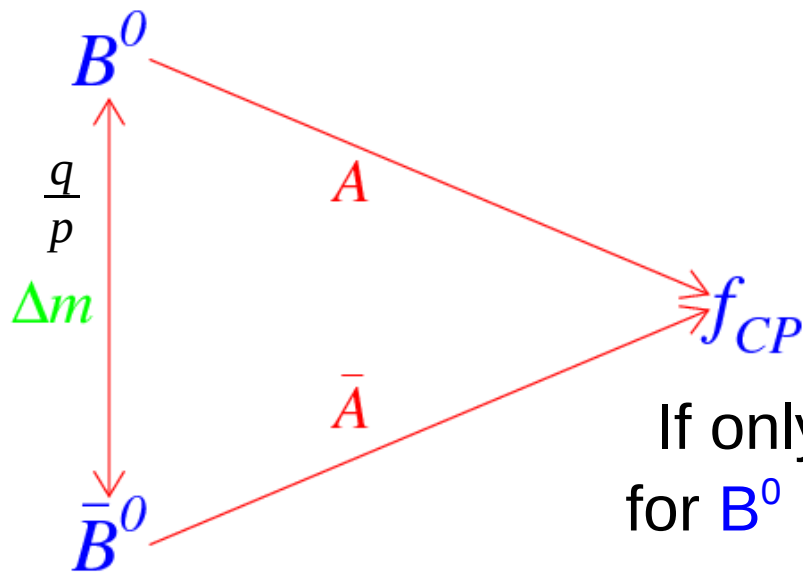
TABLE II: Observed deviation levels for the CP -violating ensemble of data sets for the χ^2 and energy tests. 37

Time-Dependent CP Violation in the $B^0-\bar{B}^0$ System

- For a B meson known to be 1) B^0 or 2) \bar{B}^0 at time $t=0$, then at later time t :

$$\Gamma(B_{phys}^0 \rightarrow f_{CP}(t)) \propto e^{-\Gamma t} (1 - (S \sin(\Delta m t) - C \cos(\Delta m t)))$$

$$\Gamma(\bar{B}_{phys}^0 \rightarrow f_{CP}(t)) \propto e^{-\Gamma t} (1 + (S \sin(\Delta m t) - C \cos(\Delta m t)))$$



$$S = \frac{2 \Im(\lambda_{CP})}{1 + |\lambda_{CP}^2|} \quad C = \frac{1 - |\lambda_{CP}^2|}{1 + |\lambda_{CP}^2|} \quad \lambda_{CP} = \frac{q}{p} \frac{\bar{A}}{A}$$

If only tree diagrams contribute
for $B^0 \rightarrow J/\psi K_S$, $S = \sin(2\beta)$, $C=0$

for $B^0 \rightarrow \pi^+\pi^-$, $S = \sin(2\alpha)$, $C=0$

... but if penguin “pollution” occurs need methods to correct for it

Snyder-Quinn method for α

PHYSICAL REVIEW D

VOLUME 48, NUMBER 5

1 SEPTEMBER 1993

Measuring CP asymmetry in $B \rightarrow \rho\pi$ decays without ambiguities

Arthur E. Snyder and Helen R. Quinn

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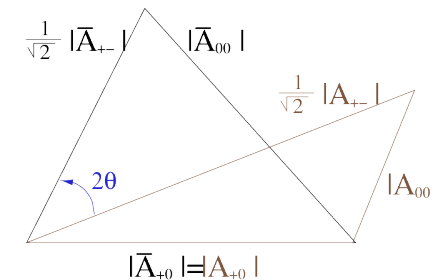
(Received 24 February 1993)

PRD 48 (1993) 2139

- Methods to measure α exploit time-dependent CP violation in B_d decays via $b \rightarrow u$ transitions (eg. $B_d \rightarrow \pi^+\pi^-$)

PRL 65 (1990) 3381

- Penguin “pollution” can be subtracted using Gronau-London isospin triangles built from $A(\pi^+\pi^-)$, $A(\pi^+\pi^0)$, $A(\pi^0\pi^0)$
- Expect discrete ambiguities in the solution for α



- Ambiguities can be resolved if you measure both real and imaginary parts of $\lambda = (q/p)(\bar{A}/A)$
 - ie. measure $\cos(2\alpha)$ as well as $\sin(2\alpha)$

Toy model for $B \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot

Contributions only from $\rho^+ \pi^-$, $\rho^- \pi^+$ and $\rho^0 \pi^0$

PRD 48 (1993) 2139

TABLE I. The time and kinematic dependence of contributions to the distribution of events.

Time dependence	Kinematic form	Amplitude measured	α dependence (all $P_i=0$)
1	$f^+ f^{+*}$	$S_3 S_3^* + \bar{S}_4 \bar{S}_4^*$	1
$\cos(\Delta Mt)$	$f^+ f^{+*}$	$S_3 S_3^* - \bar{S}_4 \bar{S}_4^*$	1
$\sin(\Delta Mt)$	$f^+ f^{+*}$	$\text{Im}(q \bar{S}_4 S_3^*)$	$\sin(2\alpha)$
1	$f^- f^{-*}$	$S_4 S_4^* + \bar{S}_3 \bar{S}_3^*$	1
$\cos(\Delta Mt)$	$f^- f^{-*}$	$S_4 S_4^* - \bar{S}_3 \bar{S}_3^*$	1
$\sin(\Delta Mt)$	$f^- f^{-*}$	$\text{Im}(q \bar{S}_3 S_4^*)$	$\sin(2\alpha)$
1	$f^0 f^{0*}$	$(S_5 S_5^* + \bar{S}_5 \bar{S}_5^*)/4$	1
$\cos(\Delta Mt)$	$f^0 f^{0*}$	$(S_5 S_5^* - \bar{S}_5 \bar{S}_5^*)/4$	1
$\sin(\Delta Mt)$	$f^0 f^{0*}$	$\text{Im}(q \bar{S}_5 S_5^*)/4$	$\sin(2\alpha)$
1	$\text{Re}(f^+ f^{-*})$	$\text{Re}(S_3 S_4^* + \bar{S}_4 \bar{S}_3^*)$	1
$\cos(\Delta Mt)$	$\text{Re}(f^+ f^{-*})$	$\text{Re}(S_3 S_4^* - \bar{S}_4 \bar{S}_3^*)$	1
$\sin(\Delta Mt)$	$\text{Re}(f^+ f^{-*})$	$\text{Im}(q \bar{S}_4 S_4^* - q^* S_3 \bar{S}_3^*)$	$\sin(2\alpha)$
1	$\text{Im}(f^+ f^{-*})$	$\text{Im}(S_3 S_4^* + \bar{S}_4 \bar{S}_3^*)$	1
$\cos(\Delta Mt)$	$\text{Im}(f^+ f^{-*})$	$\text{Im}(S_3 S_4^* - \bar{S}_4 \bar{S}_3^*)$	1
$\sin(\Delta Mt)$	$\text{Im}(f^+ f^{-*})$	$\text{Re}(q \bar{S}_4 S_4^* - q^* S_3 \bar{S}_3^*)$	$\cos(2\alpha)$
1	$\text{Re}(f^+ f^{0*})$	$\text{Re}(S_3 S_5^* + \bar{S}_4 \bar{S}_5^*)/2$	1
$\cos(\Delta Mt)$	$\text{Re}(f^+ f^{0*})$	$\text{Re}(S_3 S_5^* - \bar{S}_4 \bar{S}_5^*)/2$	1
$\sin(\Delta Mt)$	$\text{Re}(f^+ f^{0*})$	$\text{Im}(q \bar{S}_4 S_5^* + q^* S_3 \bar{S}_5^*)/2$	$\sin(2\alpha)$
1	$\text{Im}(f^+ f^{0*})$	$\text{Im}(S_3 S_5^* + \bar{S}_4 \bar{S}_5^*)/2$	1
$\cos(\Delta Mt)$	$\text{Im}(f^+ f^{0*})$	$\text{Im}(S_3 S_5^* - \bar{S}_4 \bar{S}_5^*)/2$	1
$\sin(\Delta Mt)$	$\text{Im}(f^+ f^{0*})$	$\text{Re}(q \bar{S}_4 S_5^* - q^* S_3 \bar{S}_5^*)/2$	$\cos(2\alpha)$
1	$\text{Re}(f^- f^{0*})$	$\text{Re}(S_4 S_5^* + \bar{S}_3 \bar{S}_5^*)/2$	1
$\cos(\Delta Mt)$	$\text{Re}(f^- f^{0*})$	$\text{Re}(S_4 S_5^* - \bar{S}_3 \bar{S}_5^*)/2$	1
$\sin(\Delta Mt)$	$\text{Re}(f^- f^{0*})$	$\text{Im}(q \bar{S}_3 S_5^* - q^* S_4 \bar{S}_5^*)$	$\sin(2\alpha)$
1	$\text{Im}(f^- f^{0*})$	$\text{Im}(S_4 S_5^* + \bar{S}_3 \bar{S}_5^*)/2$	1
$\cos(\Delta Mt)$	$\text{Im}(f^- f^{0*})$	$\text{Im}(S_4 S_5^* - \bar{S}_3 \bar{S}_5^*)/2$	1
$\sin(\Delta Mt)$	$\text{Im}(f^- f^{0*})$	$\text{Re}(q \bar{S}_3 S_5^* - q^* S_4 \bar{S}_5^*)/2$	$\cos(2\alpha)$

Note: physical observables depend on either $\sin(2\alpha)$ or $\cos(2\alpha)$ – never “directly” on α

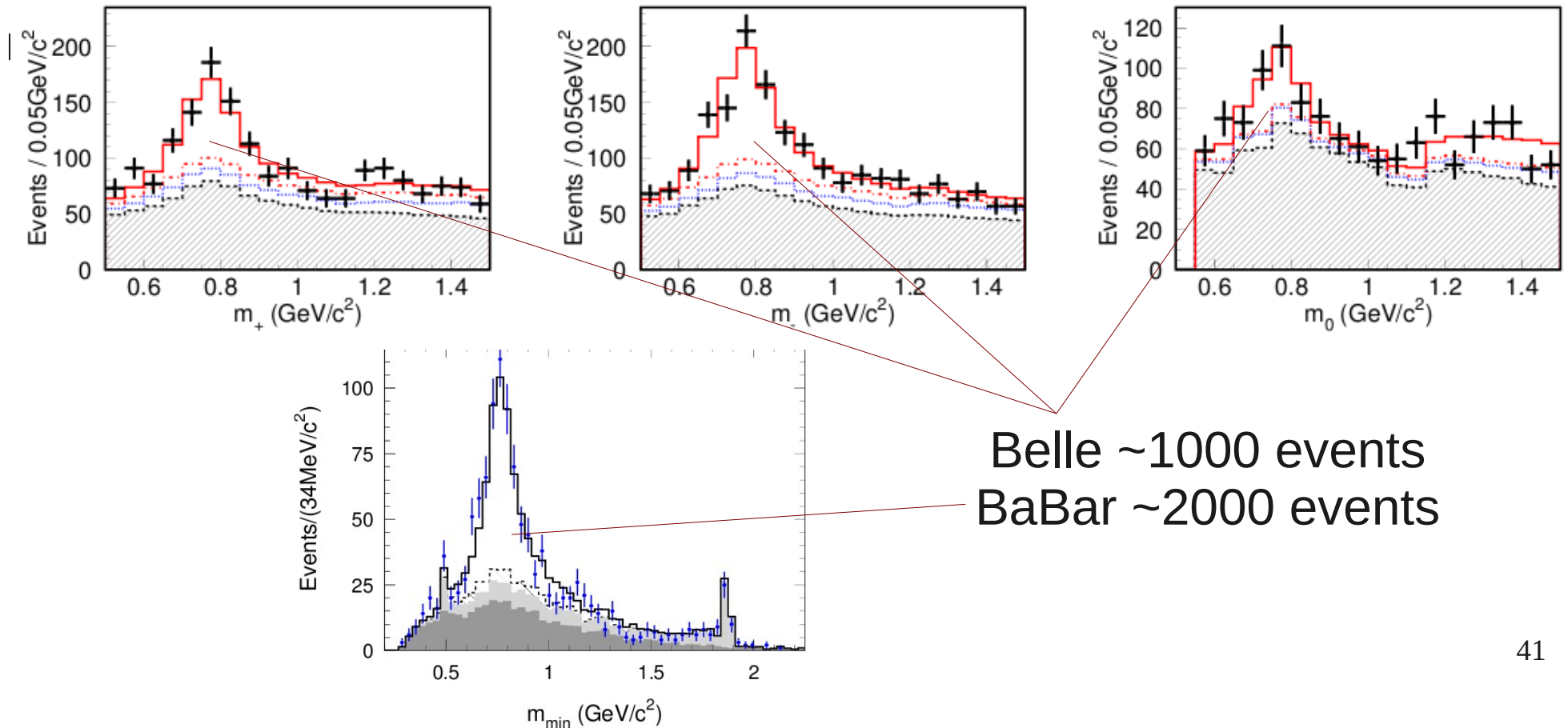
f terms contain hadronic physics (lineshape, spin)

$S_3 = A(\rho^+ \pi^-)$, $S_4 = A(\rho^- \pi^+)$, $S_5 = A(\rho^0 \pi^0)$,

$B \rightarrow \pi^+ \pi^- \pi^0$ – B factory results

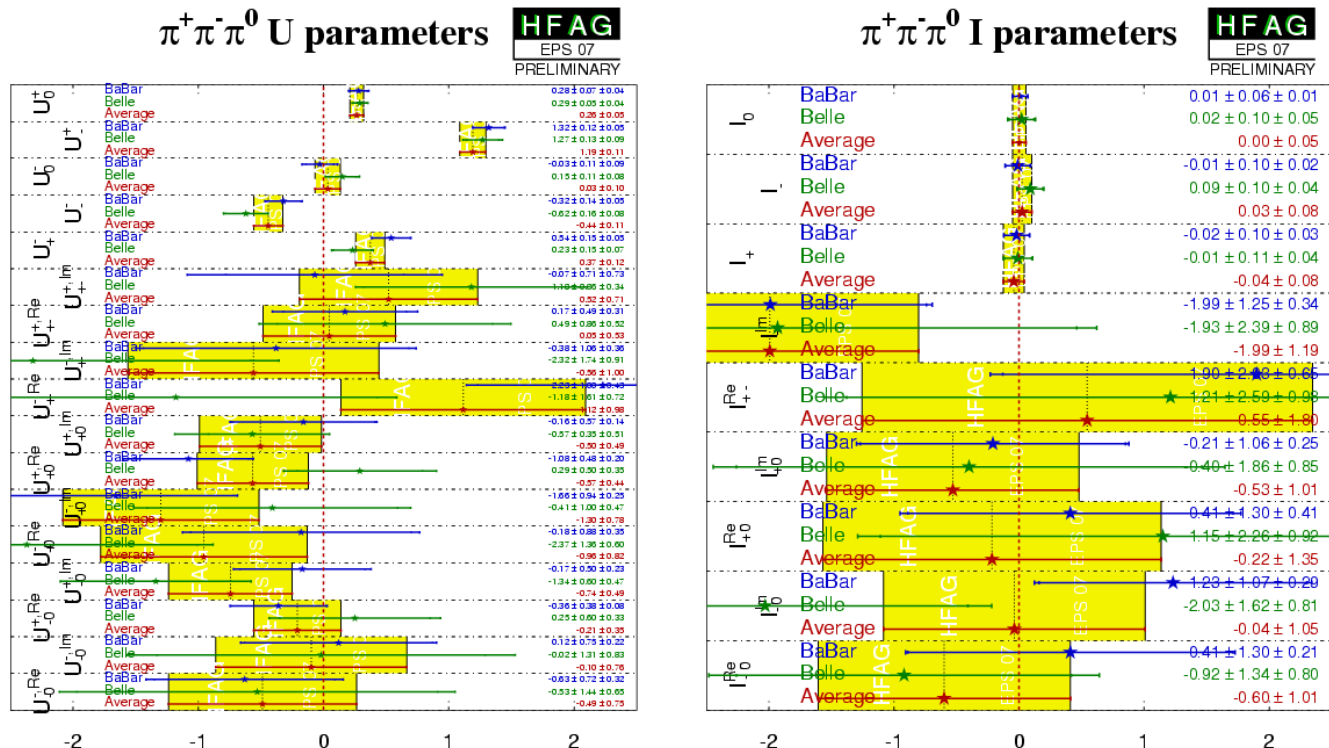
- Results from

- Belle, 449 M BB pairs: PRL 98 (2007) 221602, PRD 77 (2008) 072001
- BaBar, 375 M BB pairs: PRD 76 (2007) 012004



$B \rightarrow \pi^+ \pi^- \pi^0$ – B factory results

- Results from
 - Belle, 449 M BB pairs: PRL 98 (2007) 221602, PRD 77 (2008) 072001
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$B \rightarrow \pi^+ \pi^- \pi^0$ – B factory results

- Results from

- Belle, 449 M BB pairs: PRL 98 (2007) 221602, [PRD 77 \(2008\) 072001](#)
- BaBar, 375 M BB pairs: PRD 76 (2007) 012004

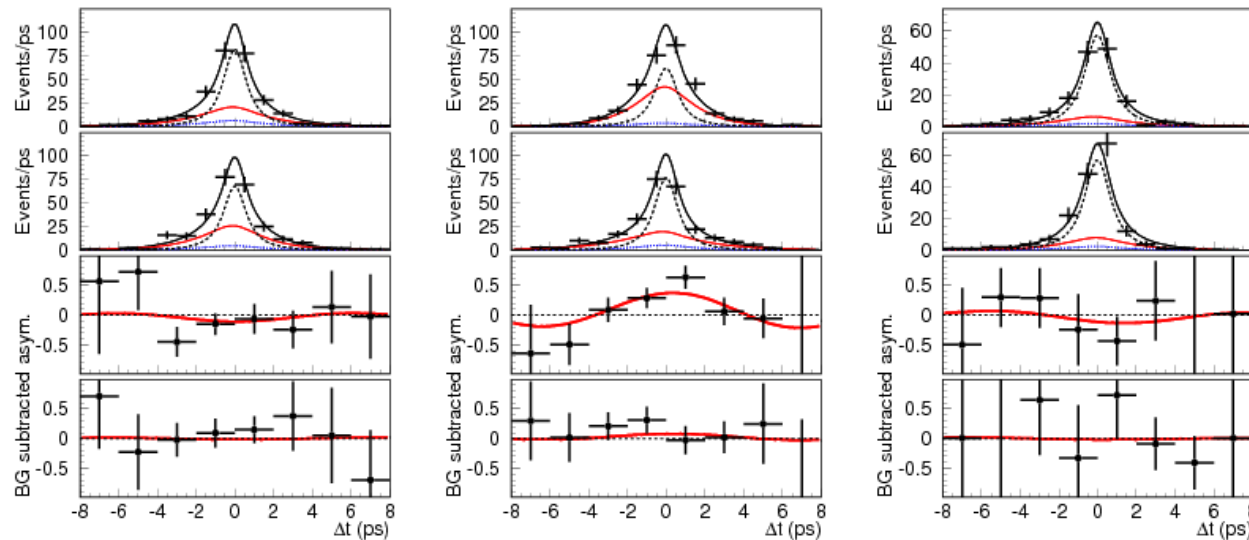


FIG. 10: Proper time distributions of good tag ($r > 0.5$) regions for $f_{\text{tag}} = B^0$ (upper) and $f_{\text{tag}} = \bar{B}^0$ (middle upper), in $\rho^+ \pi^-$ (left), $\rho^- \pi^+$ (middle), $\rho^0 \pi^0$ (right) enhanced regions, where solid (red), dotted, and dashed curves correspond to signal, continuum, and $B\bar{B}$ PDFs. The middle lower and lower plots show the background-subtracted asymmetries in the good tag ($r > 0.5$) and poor tag ($r < 0.5$) regions, respectively. The significant asymmetry in the $\rho^- \pi^+$ enhanced region (middle) corresponds to a non-zero value of U_{π^-} .

$\rho^+ \pi^-$

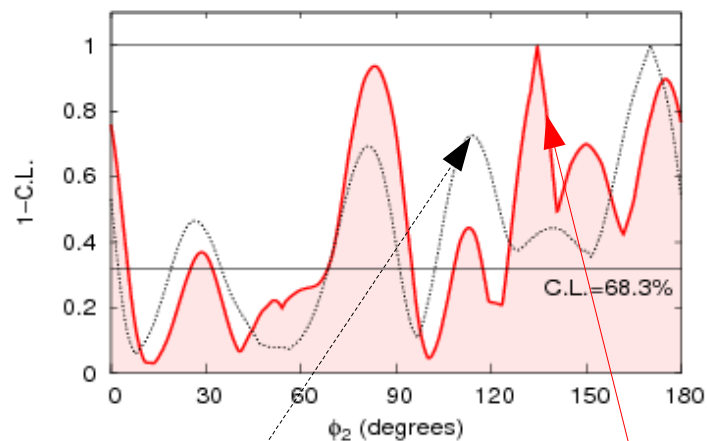
$\rho^- \pi^+$

$\rho^0 \pi^0$

$B \rightarrow \pi^+ \pi^- \pi^0$ – B factory results

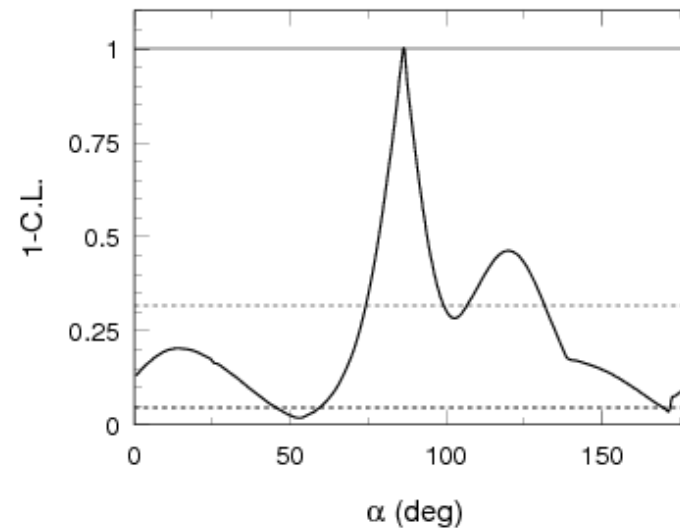
- Results from

- Belle, 449 M BB pairs: PRL 98 (2007) 221602, PRD 77 (2008) 072001
- BaBar, 375 M BB pairs: PRD 76 (2007) 012004



Contour from $B \rightarrow \pi^+ \pi^- \pi^0$ only

Including also information on
 $B^+ \rightarrow \rho^+ \pi^0$ and $B^+ \rightarrow \rho^0 \pi^+$



Importance of γ from $B \rightarrow DK$

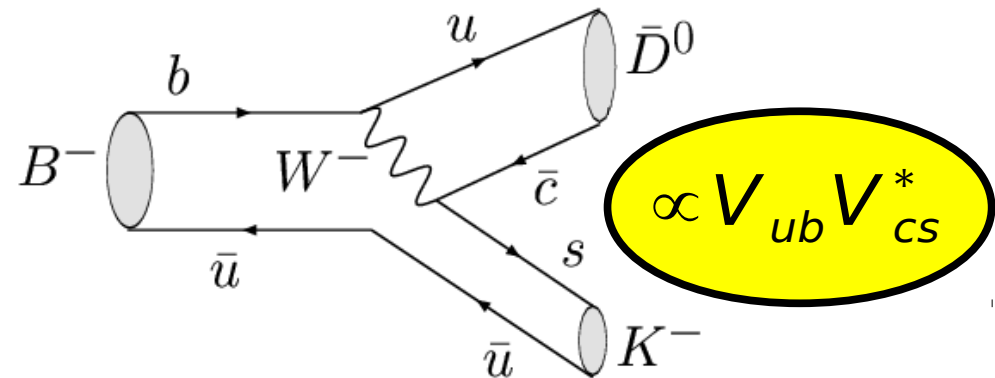
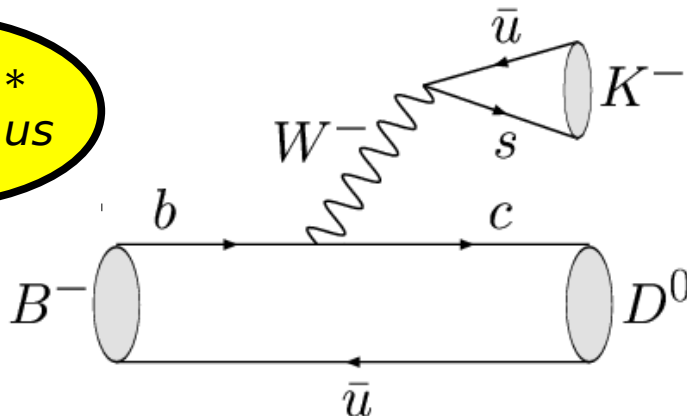
- γ plays a unique role in flavour physics

the only CP violating parameter that can be measured through tree decays ^(*)

^(*) more-or-less

- A benchmark Standard Model reference point
 - doubly important after New Physics is observed

$$\propto V_{cb} V_{us}^*$$



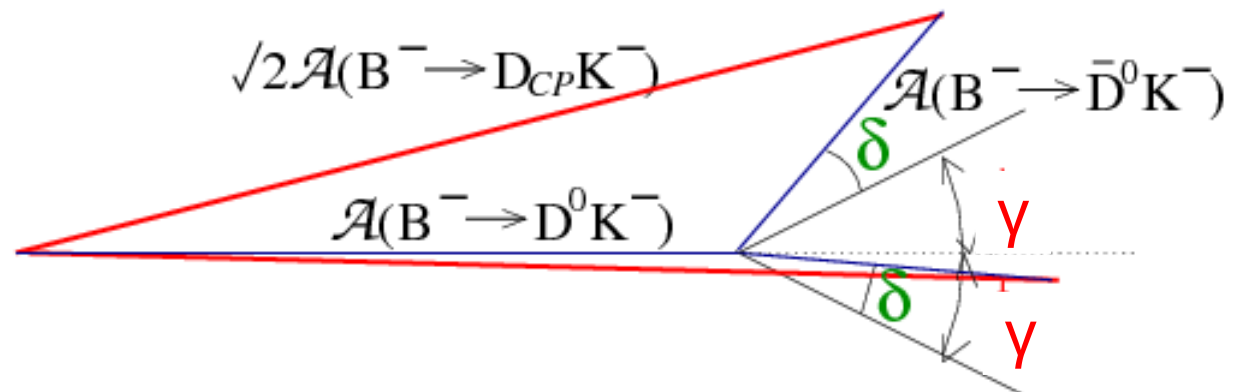
$$\propto V_{ub} V_{cs}^*$$

Variants use different B or D decays

require a final state common to both D^0 and \bar{D}^0

Why is $B \rightarrow DK$ so nice?

- For theorists:
 - theoretically clean: no penguins; factorisation works
 - all parameters can be determined from data
- For experimentalists:
 - many different observables (different final states)
 - all parameters can be determined from data
 - γ & δ_B (weak & strong phase differences), r_B (ratio of amplitudes)



B → DK methods

- Different D decay final states
 - CP eigenstates, e.g. K^+K^- (GLW)
 - doubly-Cabibbo-suppressed decays, e.g. $K^+\pi^-$ (ADS)
 - singly-Cabibbo-suppressed decays, e.g., $K^{*+}K^-$ (GLS)
 - self-conjugate multibody decays, e.g., $K_S \pi^+ \pi^-$ (GGSZ)
- Different B decays
 - $B^- \rightarrow DK^-, D^*K^-, DK^{*-}$
 - $B^0 \rightarrow DK^{*0}$ (or $B \rightarrow DK\pi$ Dalitz plot analysis)
 - $B^0 \rightarrow DK_S, B_S^0 \rightarrow D\phi$ (with or without time-dependence)
 - $B_S^0 \rightarrow D_S K, B^0 \rightarrow D^{(*)}\pi$ (time-dependent)
 - Search for direct CP violation caused by $\gamma \neq 0$
 - All parameters from data – no theory input needed



Interference between $b \rightarrow u$ and $b \rightarrow c$ amplitudes when D is reconstructed in final state common to D^0 and \bar{D}^0 provides sensitivity to γ

$$|M_\pm(m_+^2, m_-^2)|^2 = |f_D(m_+^2, m_-^2) + re^{i\delta_B \pm i\phi_3} f_D(m_-^2, m_+^2)|^2$$

$$= \left| \begin{array}{c} \text{[Plot of } f_D(m_+^2, m_-^2)] \\ + re^{i\delta_B \pm i\phi_3} \text{ [Plot of } f_D(m_-^2, m_+^2)] \end{array} \right|^2$$

Model ($f_D(m_+^2, m_-^2)$) taken from measurements of $|f_D|^2$ using flavour tagging D^* decays – model dependence

BaBar obtain
 $\gamma = (68^{+15}_{-14} \pm 4 \pm 3)^\circ$
 (from DK^- , D^*K^- & DK^{*+})

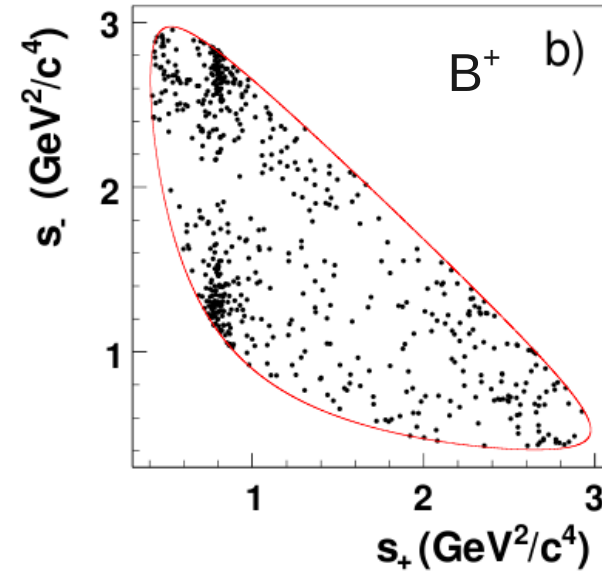
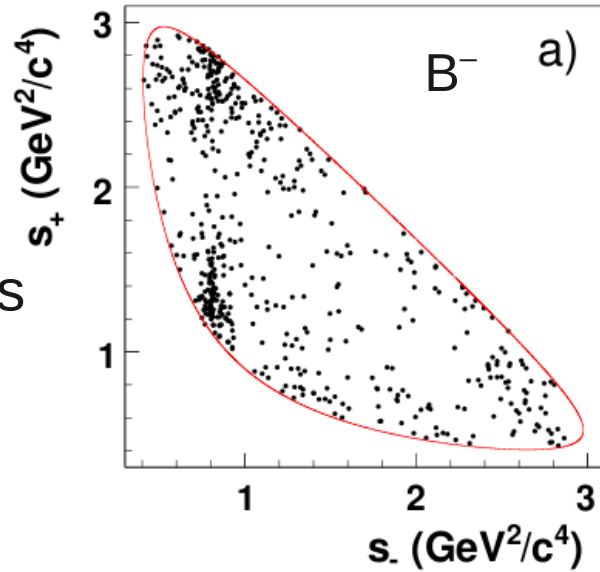
PRL 105 (2010) 121801

Belle obtain
 $\phi_3 = (78^{+11}_{-12} \pm 4 \pm 9)^\circ$
 (from DK^- & D^*K^-)

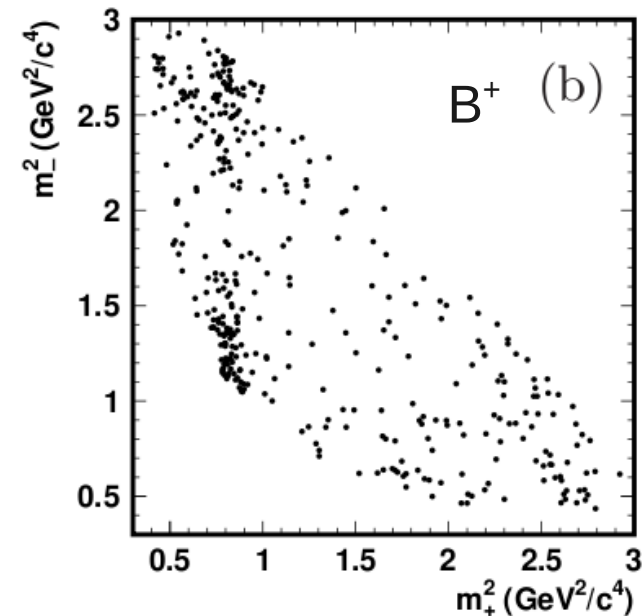
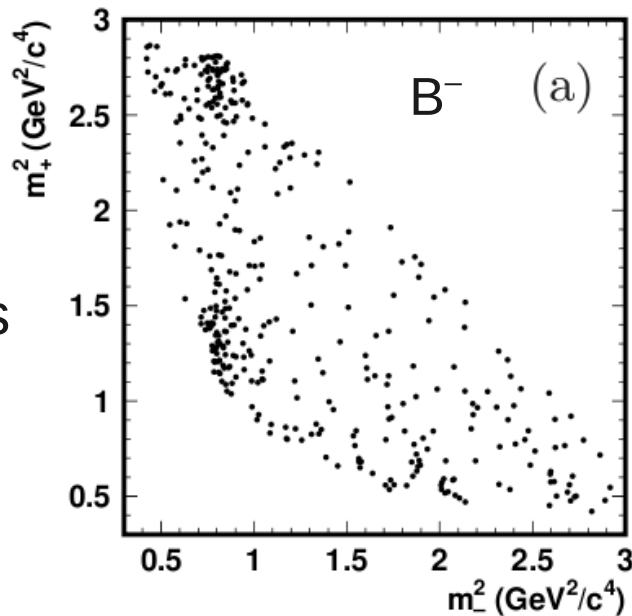
PRD 81 (2010) 112002

$$B^{\pm} \rightarrow DK^{\pm} \text{ with } D \rightarrow K_S \pi^+ \pi^-$$

BaBar
~900 events



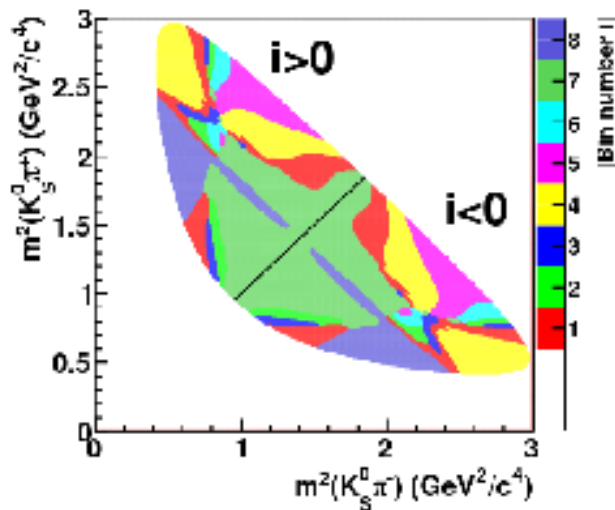
Belle
~525 events





Solution to model uncertainty – bin the Dalitz plot and use $\psi(3770) \rightarrow D\bar{D}$ events (CLEOc, BES) to measure per-bin phases

PRD 68, 054018 (2003), EPJ C 47, 347 (2006); EPJ C 55, 51 (2008)
(unusual bin shapes to attempt to optimise sensitivity)



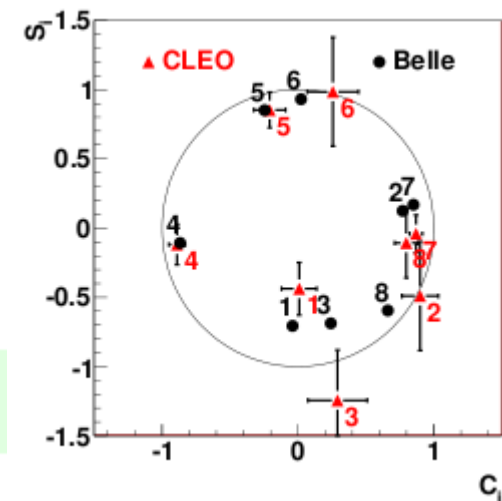
$$M_i^\pm = h\{K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_\pm c_i + y_\pm s_i)\}$$

$$x_\pm = r_B \cos(\delta_B \pm \phi_3) \quad y_\pm = r_B \sin(\delta_B \pm \phi_3)$$

c_i, s_i measured by CLEO
PRD 82, 112006 (2010)

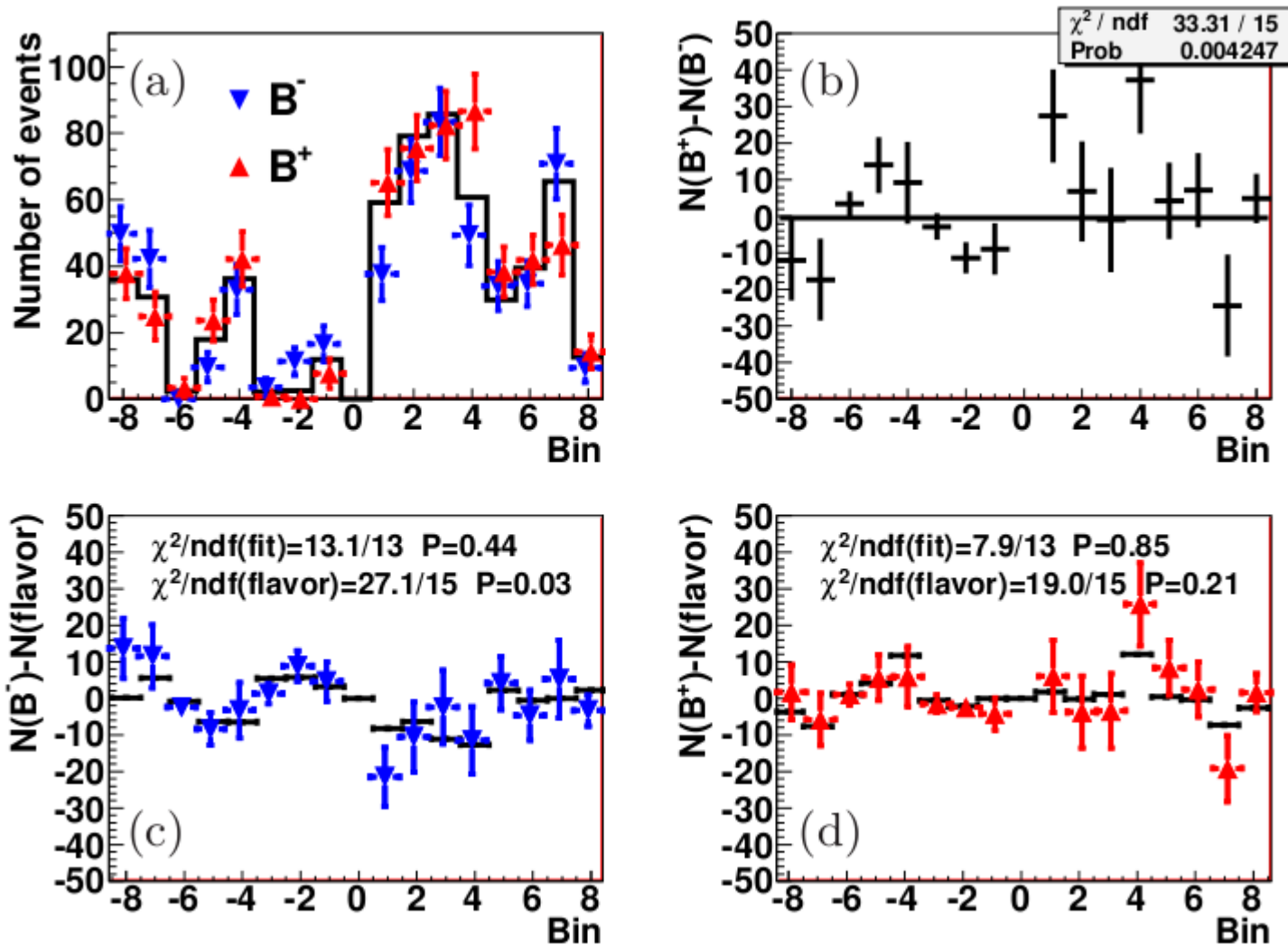
First model independent measurement
of γ in this mode by Belle

Belle obtain
 $\phi_3 = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$





Belle arXiv:1204.6561



Summary

- Dalitz plot analyses provide promising methods to measure weak phases and CP violation
- Many attractive features ...
- ... but significant complications due to model dependence
- Need progress on several fronts
 - Understand better $(\pi\pi)$, $(K\pi)$, (KK) , $(D\pi)$, (DK) systems
 - “Nonresonant” contributions and 3-body unitarity
 - Methods to combat model-dependence