

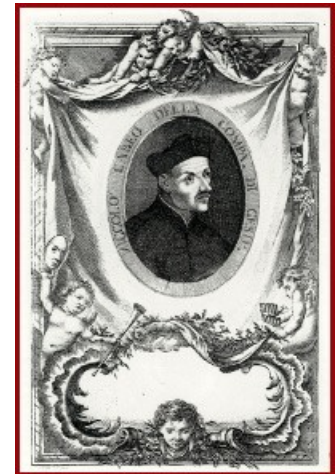
# Studying CP Violation via Amplitude Analysis (ii)

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Hadronic spectroscopy

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# Content of the lectures

- Why do we believe that multibody hadronic decays of heavy flavours may provide a good laboratory to search for new sources of CP violation?
- Which decays in particular should we look at?
- What methods can we use to study them?
- What are the difficulties we encounter when trying to do the analysis?

But first, let's look at some experiments



# BESIII Detector

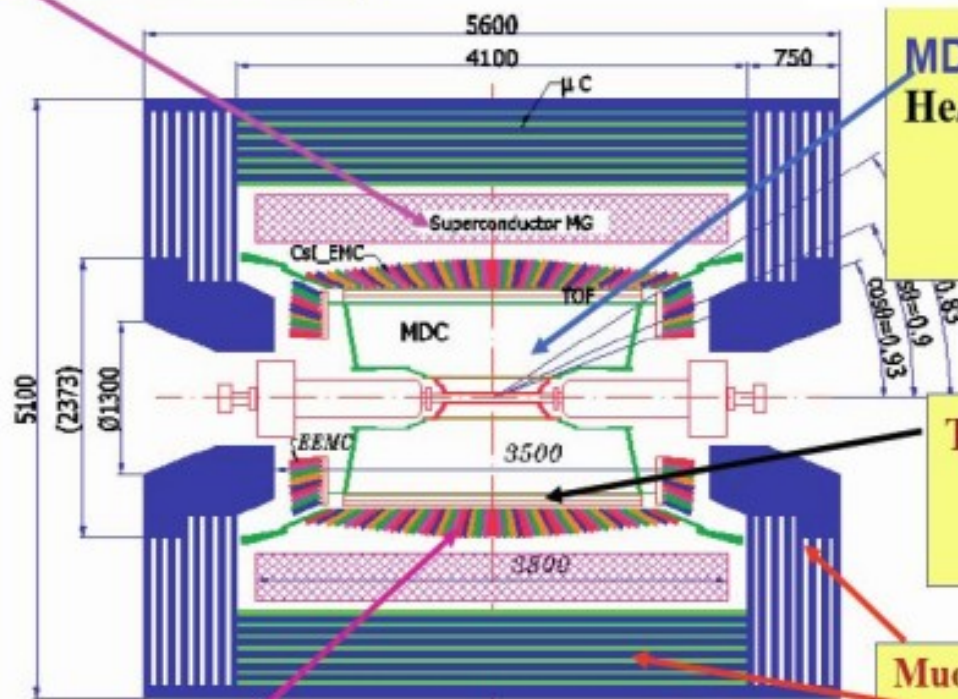
**BESIII detector: all new !**

*CsI calorimeter*

*Precision tracking*

*Time-of-flight + dE/dx PID*

**Magnet: 1 T Super conducting**



**MDC: small cell & Gas:**  
**He/C<sub>3</sub>H<sub>8</sub> (60/40), 43 layers**  
 $\sigma_{xy} = 130 \mu\text{m}$   
 $\sigma_{p/p} = 0.5\% @ 1\text{GeV}$   
 $dE/dx = 6\%$

**TOF:**  
 $\sigma_T = 100 \text{ ps}$  Barrel  
 $110 \text{ ps}$  Endcap

**Muon ID: 9 layers RPC**  
**8 layers for endcap**

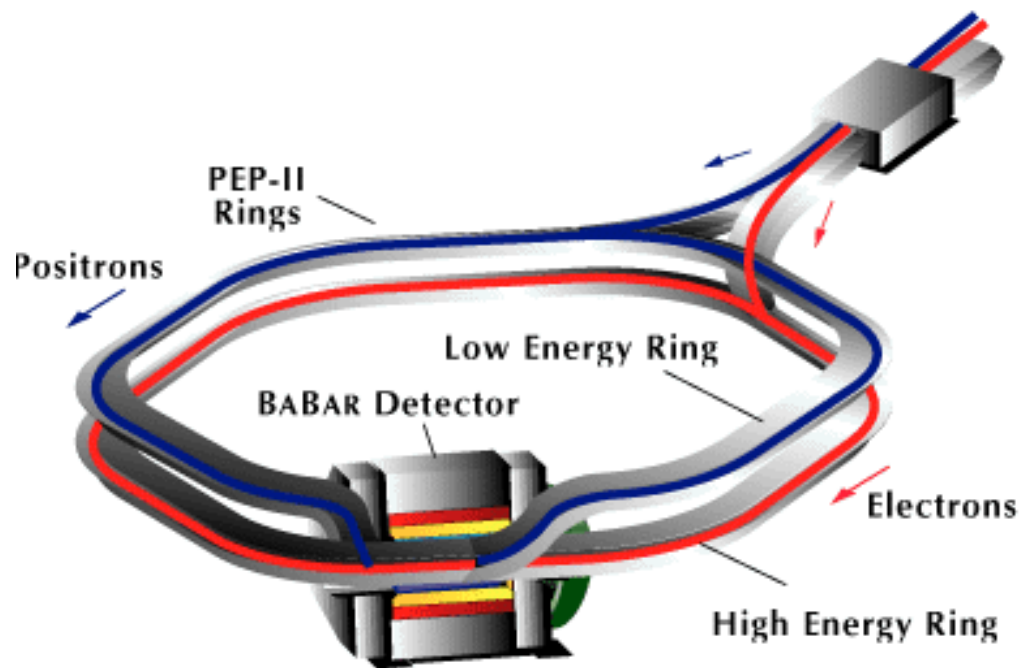
**EMC: CsI crystal, 28 cm**  
 $\Delta E/E = 2.5\% @ 1 \text{ GeV}$   
 $\sigma_z = 0.6 \text{ cm}/\sqrt{E}$

**Data Acquisition:**  
 Event rate = 4 kHz  
 Total data volume ~ 50 MB/s

# The Asymmetric B Factories

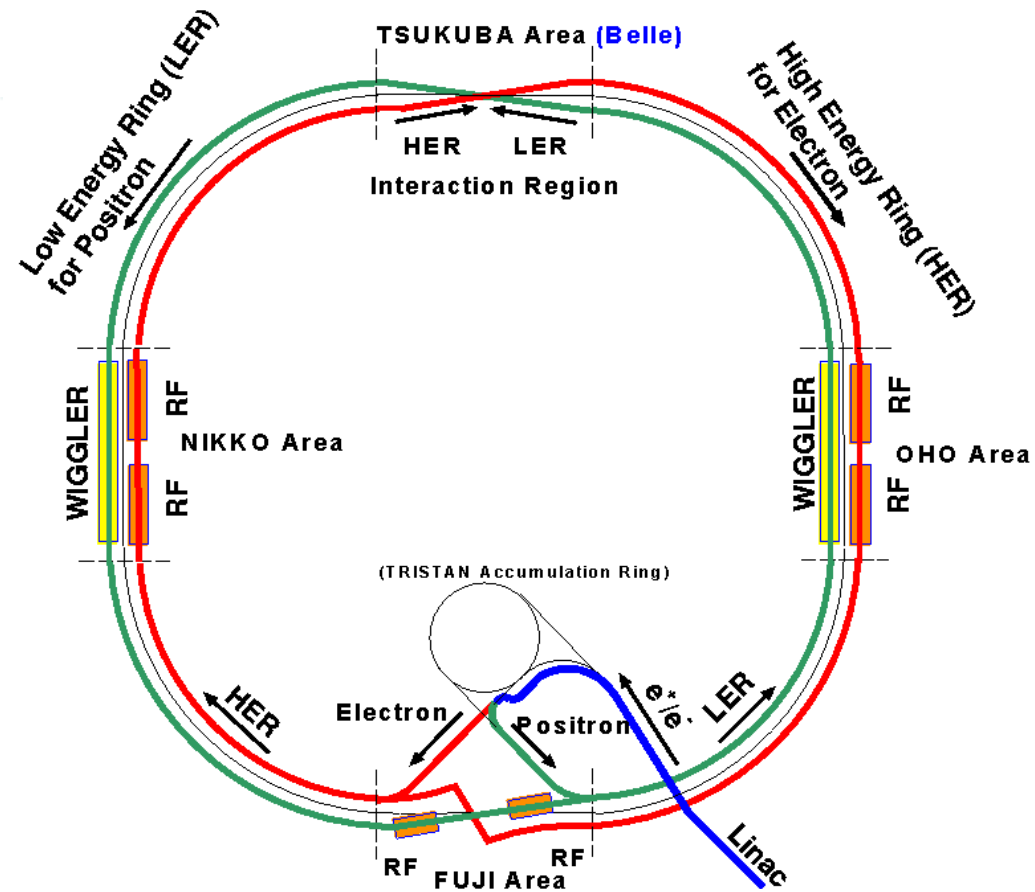
PEP-II at SLAC

9.0 GeV  $e^-$  on 3.1 GeV  $e^+$



KEKB at KEK

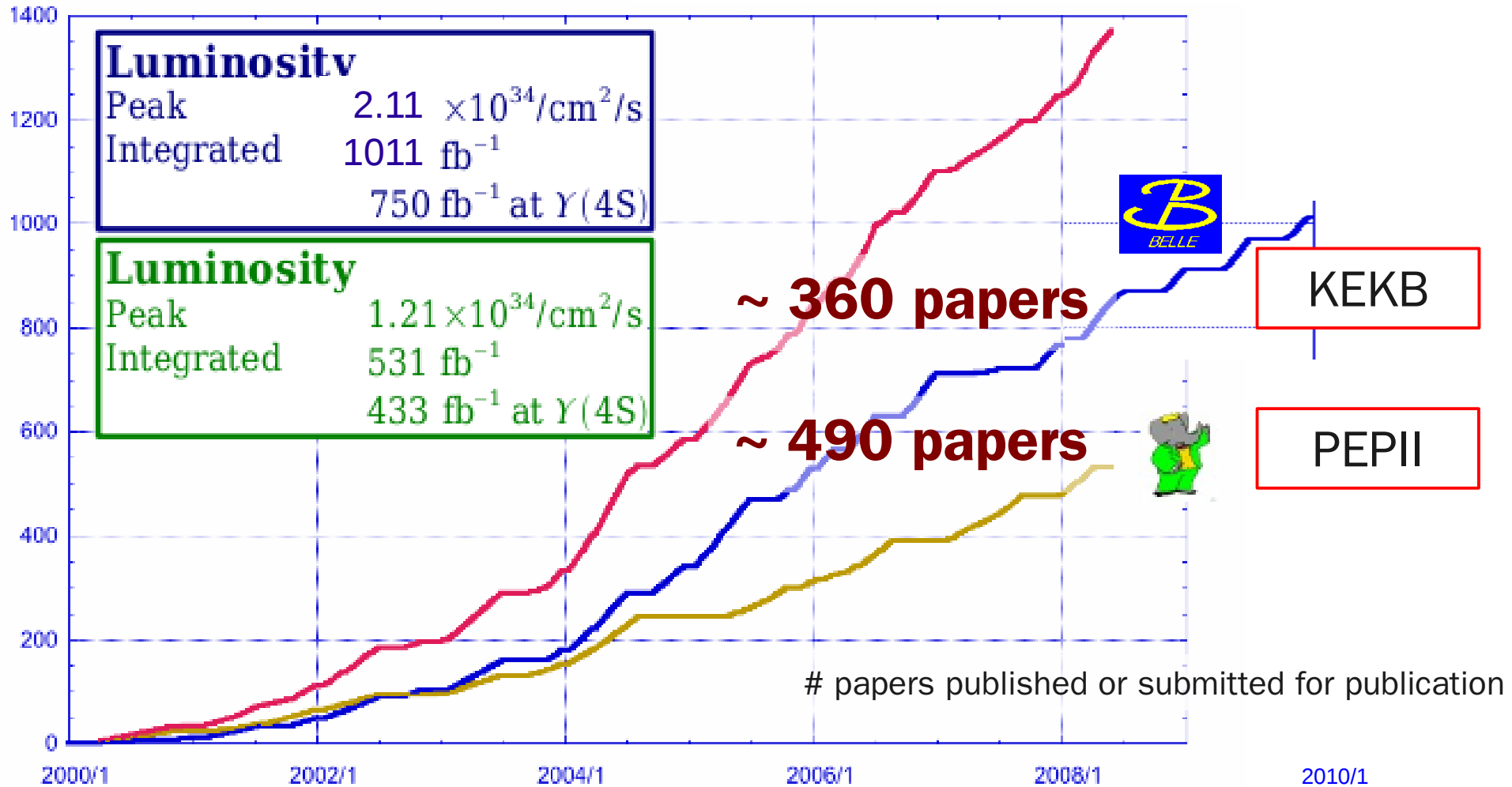
8.0 GeV  $e^-$  on 3.5 GeV  $e^+$



# B factories – World Record Luminosities

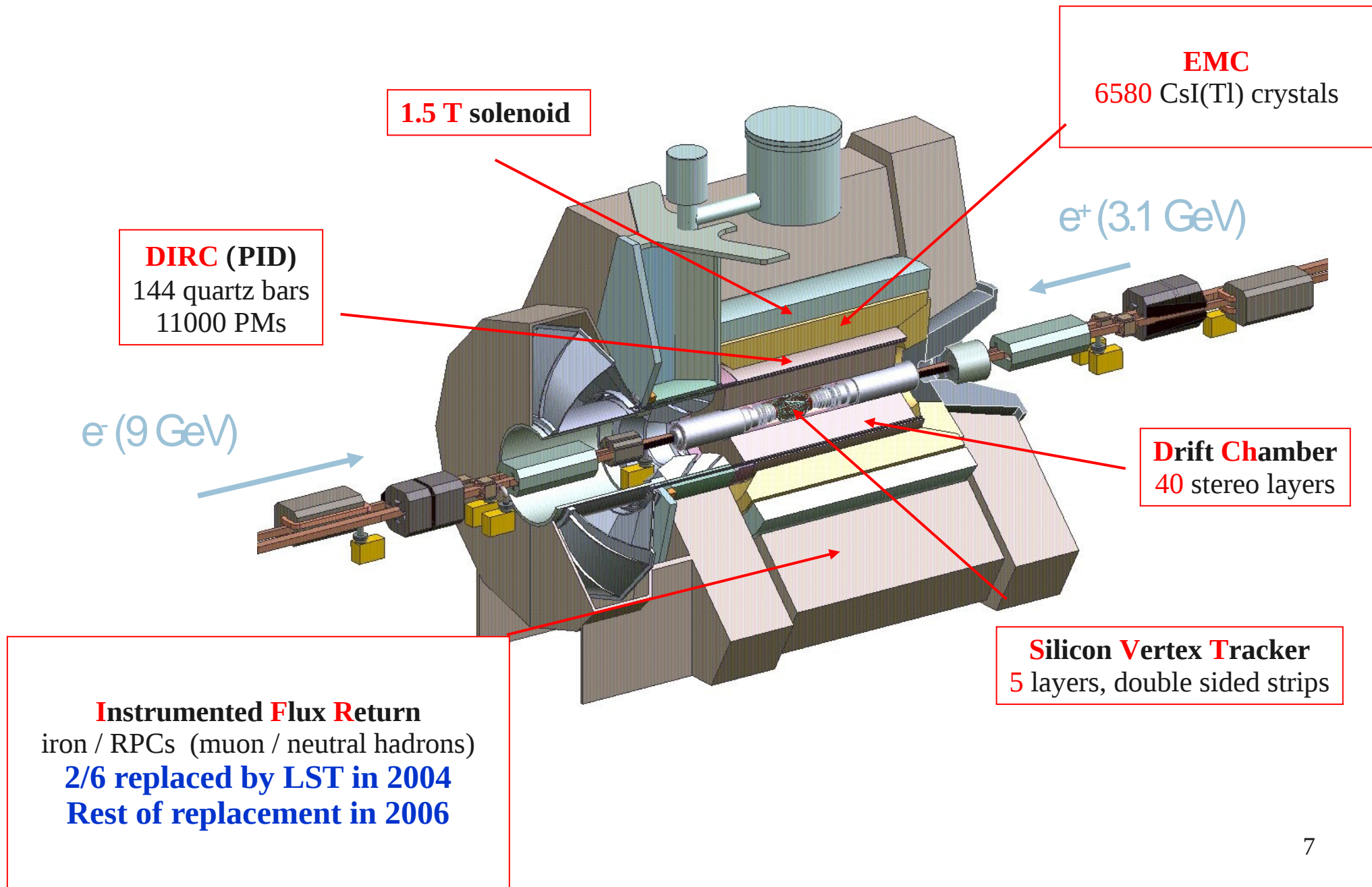
Luminosity ( $\text{fb}^{-1}$ )

Combined dataset  $> 1500 \text{ fb}^{-1}$

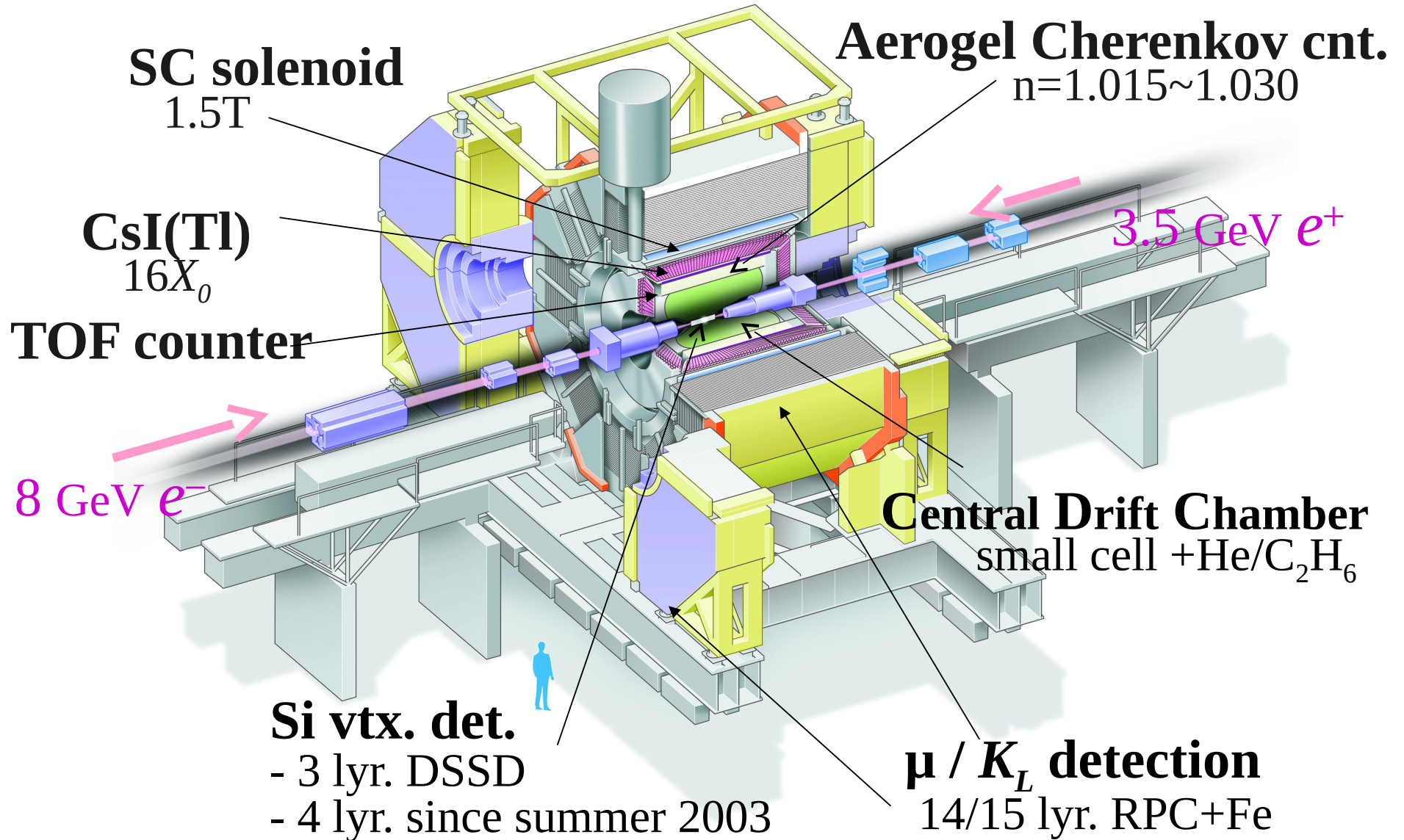




# BABAR Detector



# Belle Detector

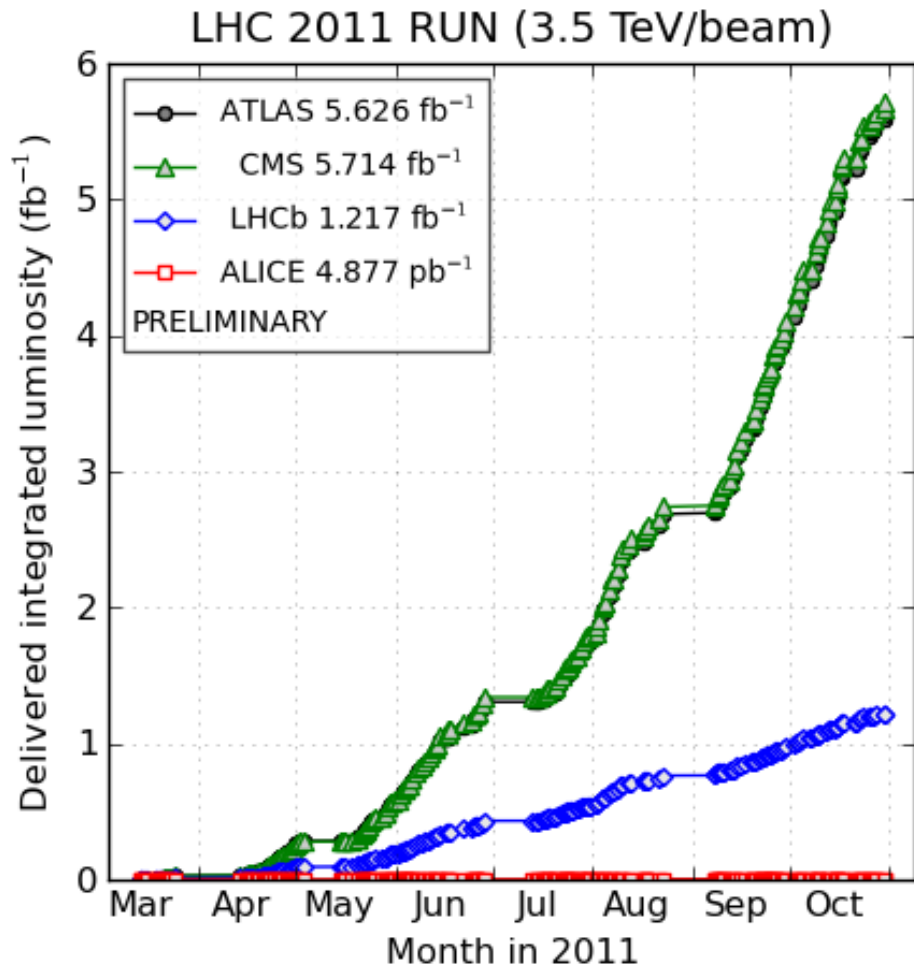




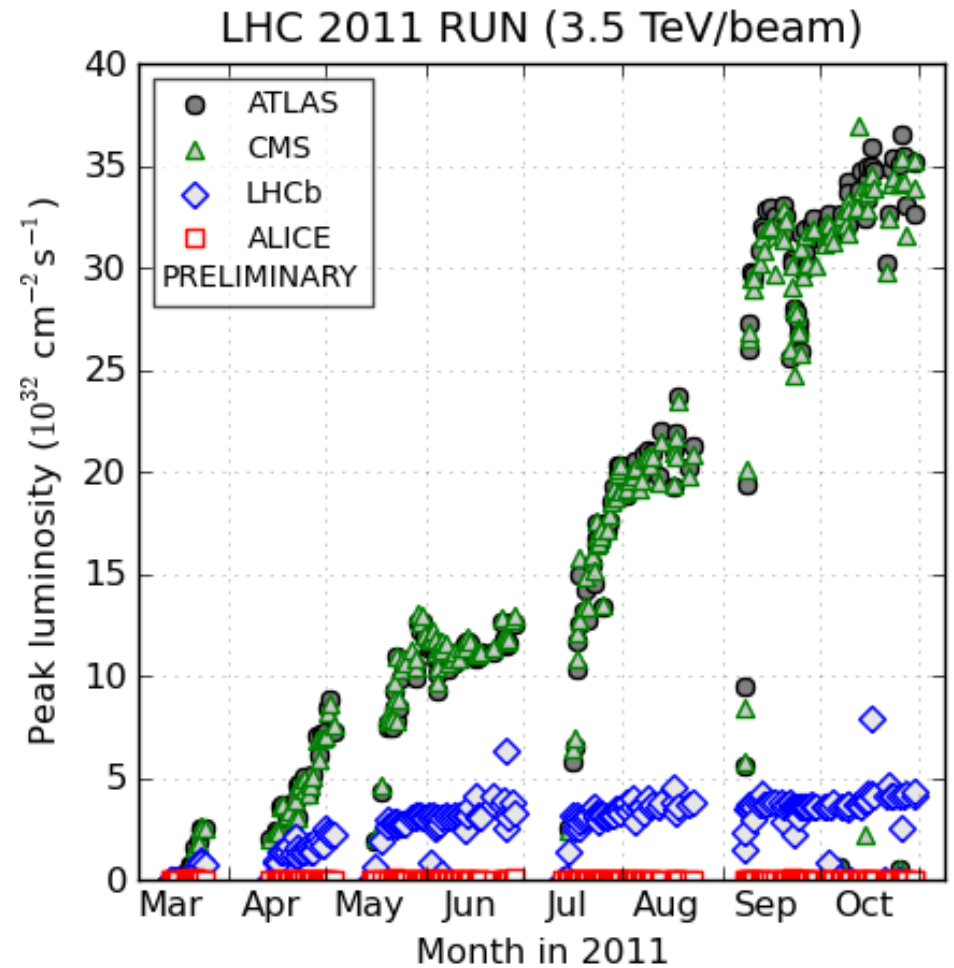
# The LHC



# LHC performance 2011



(generated 2011-12-01 19:35 including fill 2267)



(generated 2011-12-01 19:35 including fill 2267)

LHCb design luminosity:  $2 \cdot 10^{32} / \text{cm}^2 / \text{s}$

# What does $\int \mathcal{L} dt = 1/\text{fb}$ mean?

- Measured cross-section, in LHCb acceptance

$$\sigma(pp \rightarrow b\bar{b}X) = (75.3 \pm 5.4 \pm 13.0) \mu\text{b}$$

PLB 694 (2010) 209

- So, number of  $b\bar{b}$  pairs produced

$$10^{15} \times 75.3 \times 10^{-6} \sim 10^{11}$$

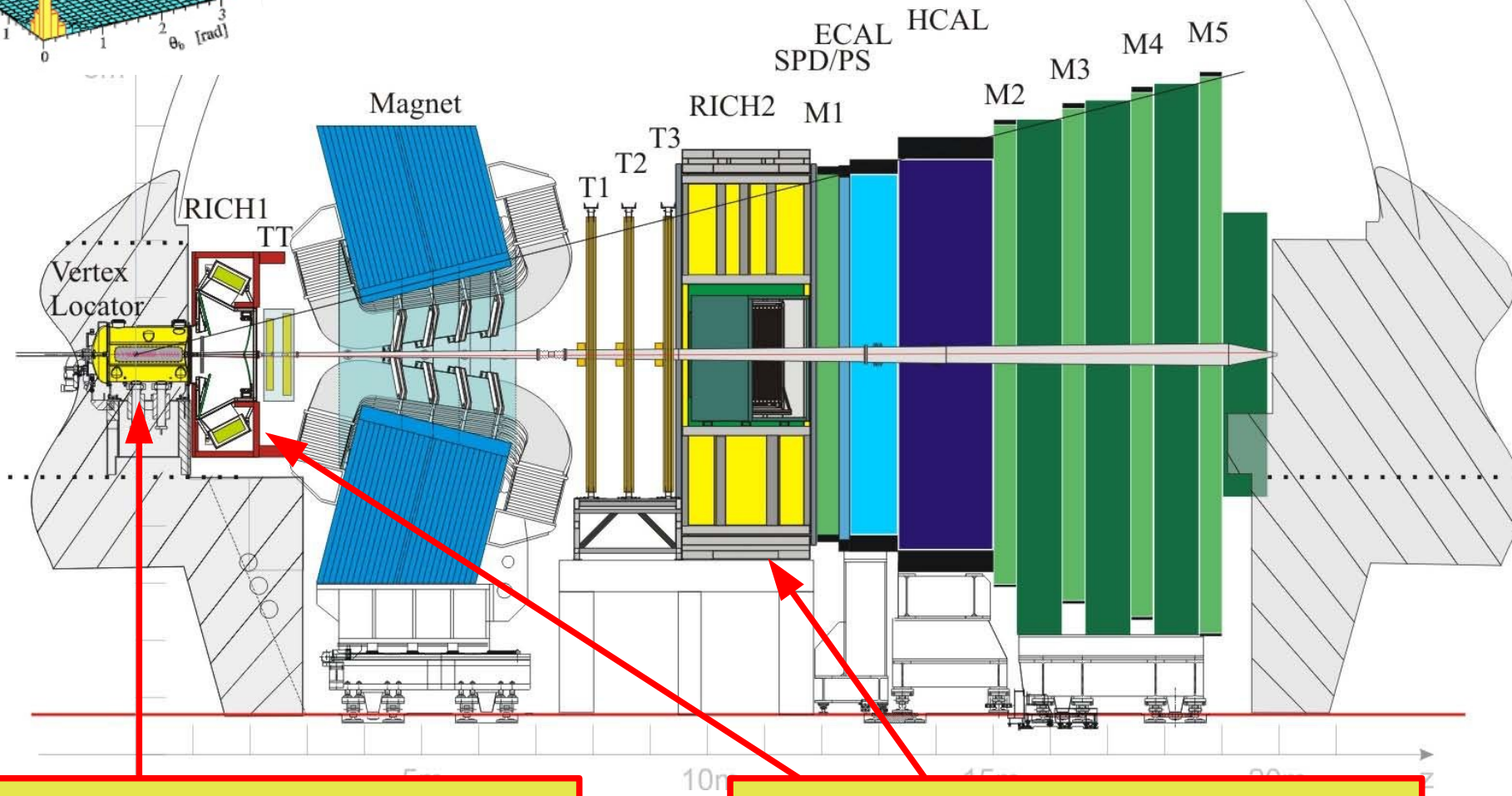
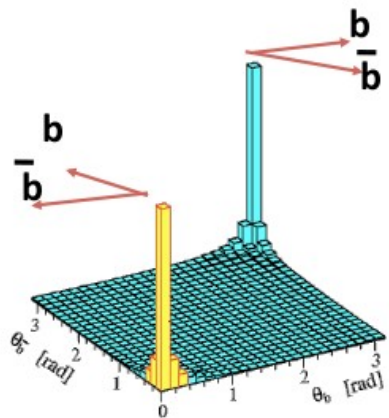
- Compare to combined data sample of  $e^+e^-$  “B factories”  
BaBar and Belle of  $\sim 10^9$   $B\bar{B}$  pairs

for any channel where the (trigger, reconstruction, stripping, offline) efficiency is not too small, LHCb has world's largest data sample

- p.s.: for charm,  $\sigma(pp \rightarrow c\bar{c}X) = (6.10 \pm 0.93) \text{mb}$

LHCb-CONF-2010-013

# The LHCb detector



Precision primary and secondary vertex measurements

Excellent  $K/\pi$  separation capability



# Lepton vs. hadron colliders

- All these examples can be put into one of two categories
  - $e^+e^-$  colliders (KLOE, CLEOc, BES, BaBar, Belle, etc.)
    - produce meson-antimeson pair in coherent state
  - hadron colliders (NA48, CDF, D0, LHCb, etc.)
    - produce hadrons from various mechanisms, such as gluon splitting
- **What are relative advantages and disadvantages of the two approaches?**
  - (More specific: in which do you expect the background to be lower?)

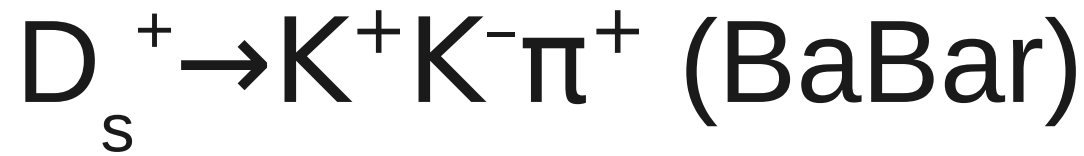


What methods can we use to study multibody hadronic decays of heavy flavours (and search for CP violation)?

# Methods

- **Two-sample comparison tests**
  - To ask: is there CP violation? Yes/No
  - (If yes, can extend to ask: where on the Dalitz plot does it occur?)
- **Quantitative determinations of CP phases**
  - Model independent approaches
  - **Amplitude analyses**
    - suffer from hard-to-quantify model dependence
    - improve by using better models ...
    - ... using data to provide insights into hadronic effects
    - **example: partial wave analysis**

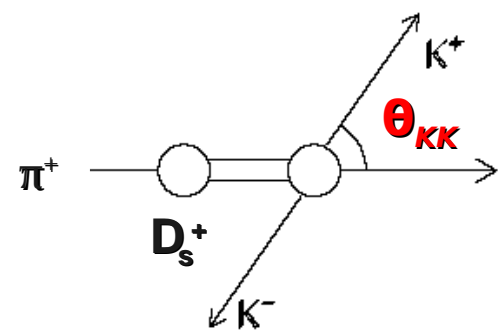
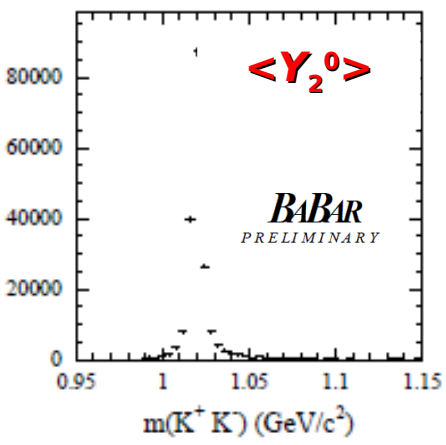
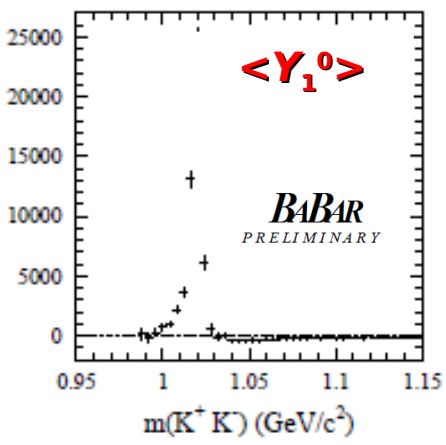
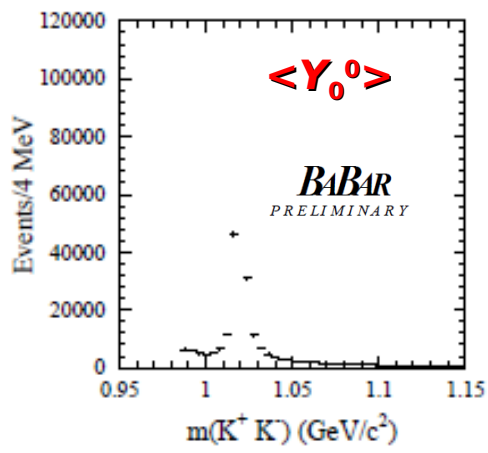
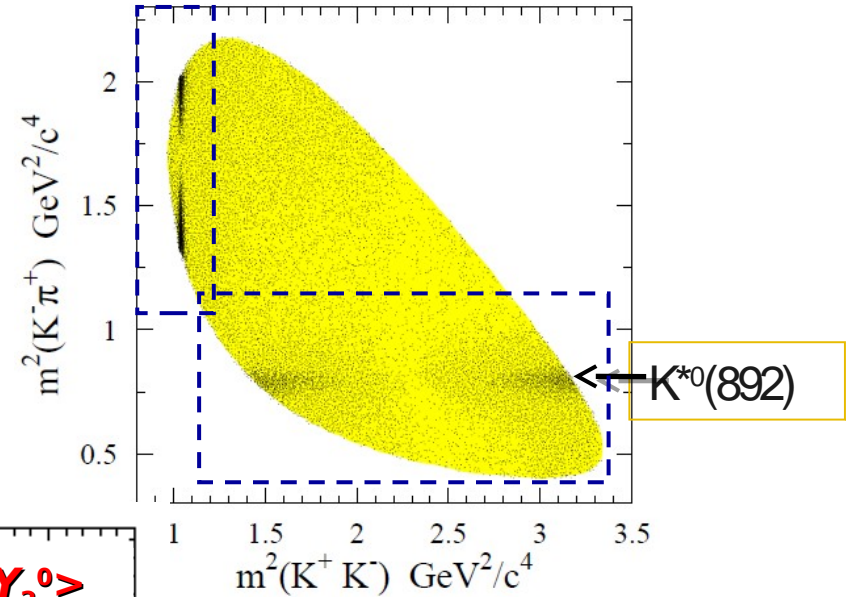
# Example partial wave analysis:



Plot  $m(K^+K^-)$ , weighting events by factors  $Y_L^0(\cos \theta_{KK})/\epsilon$  to obtain “moments  $\langle Y_L^0(m) \rangle$ ”

$$\begin{aligned} \sqrt{4\pi} \langle Y_0^0 \rangle &= |S|^2 + |P|^2 \\ \sqrt{4\pi} \langle Y_2^0 \rangle &= \frac{2}{\sqrt{5}} |P|^2, \\ \sqrt{4\pi} \langle Y_1^0 \rangle &= 2|S||P| \cos \phi_{SP} \end{aligned}$$

$\phi(1020)$  ~101K Events  
96% purity

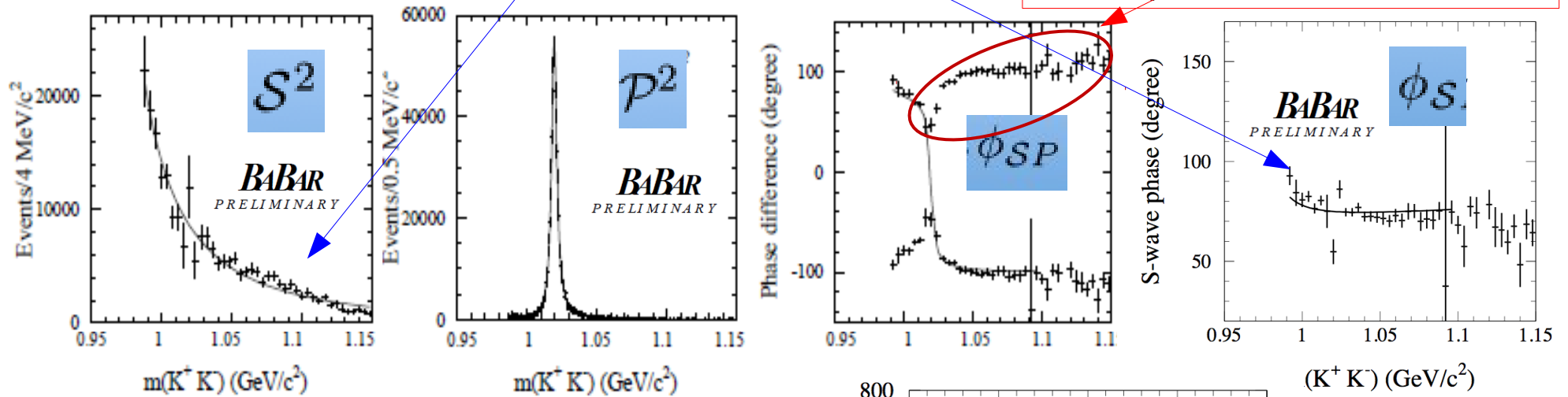


# Example partial wave analysis:

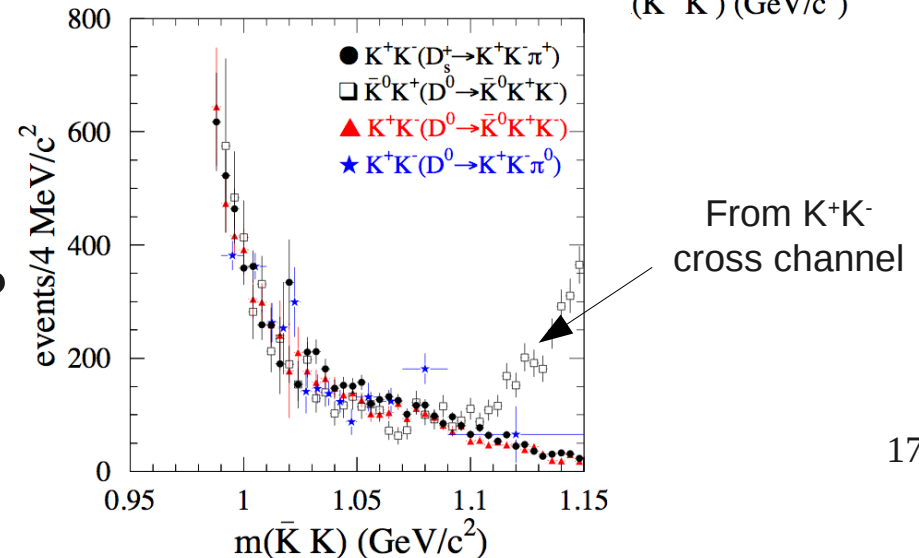
$$D_s^+ \rightarrow K^+ K^- \pi^+ \text{ (BaBar)}$$

(Approximately) model-independent information on the KK S-wave magnitude and phase

Ambiguity in  $\phi_{SP}$  resolved by knowledge of  $\phi(1020)$  phase variation

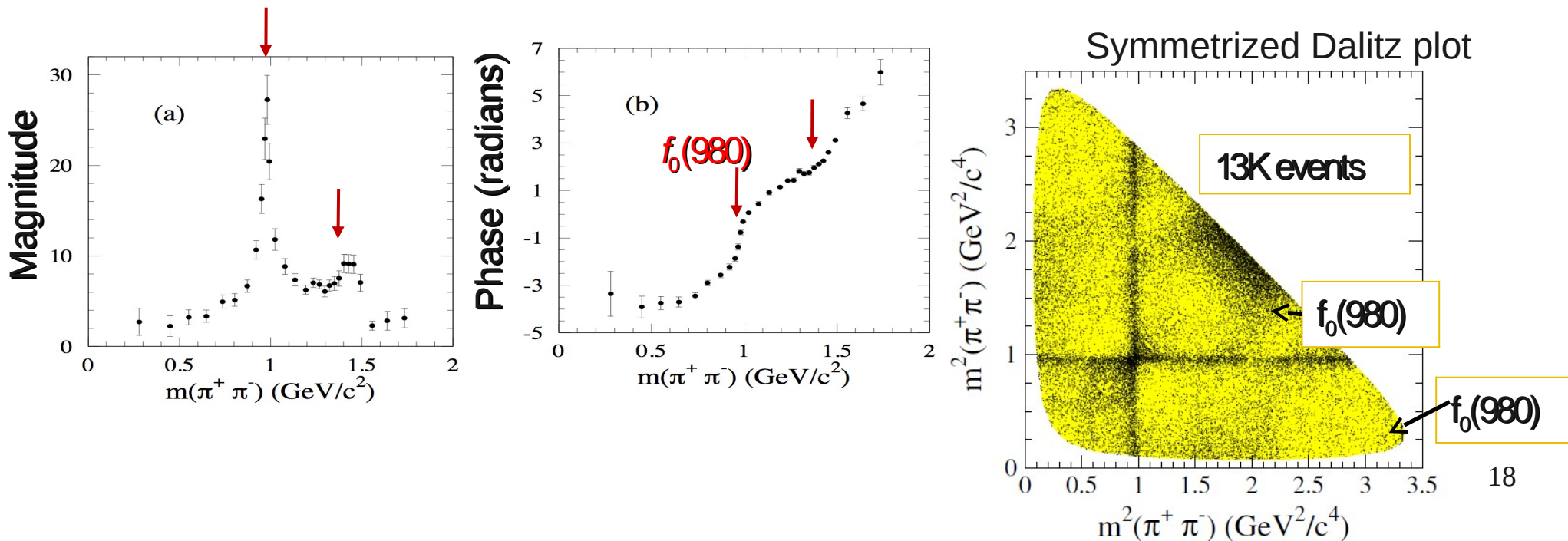


KK S-wave seems process independent?



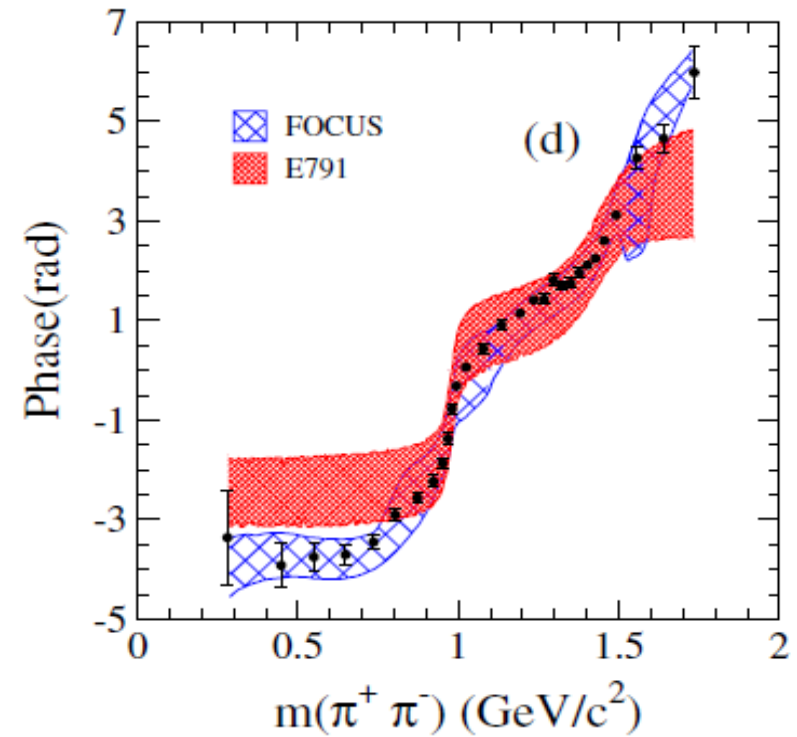
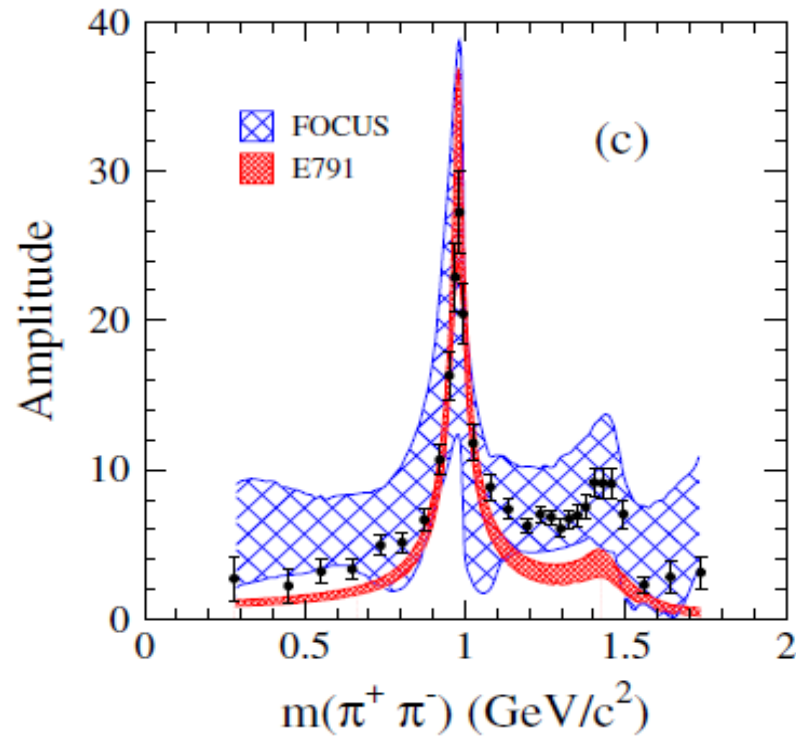
# Quasi-model-independent partial wave analysis

- Pioneered by E791 (B.Meadows) in  $D^+ \rightarrow K^- \pi^+ \pi^+$
- Describe S-wave by complex spline (many free parameters)
- Example:  $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$  from BaBar



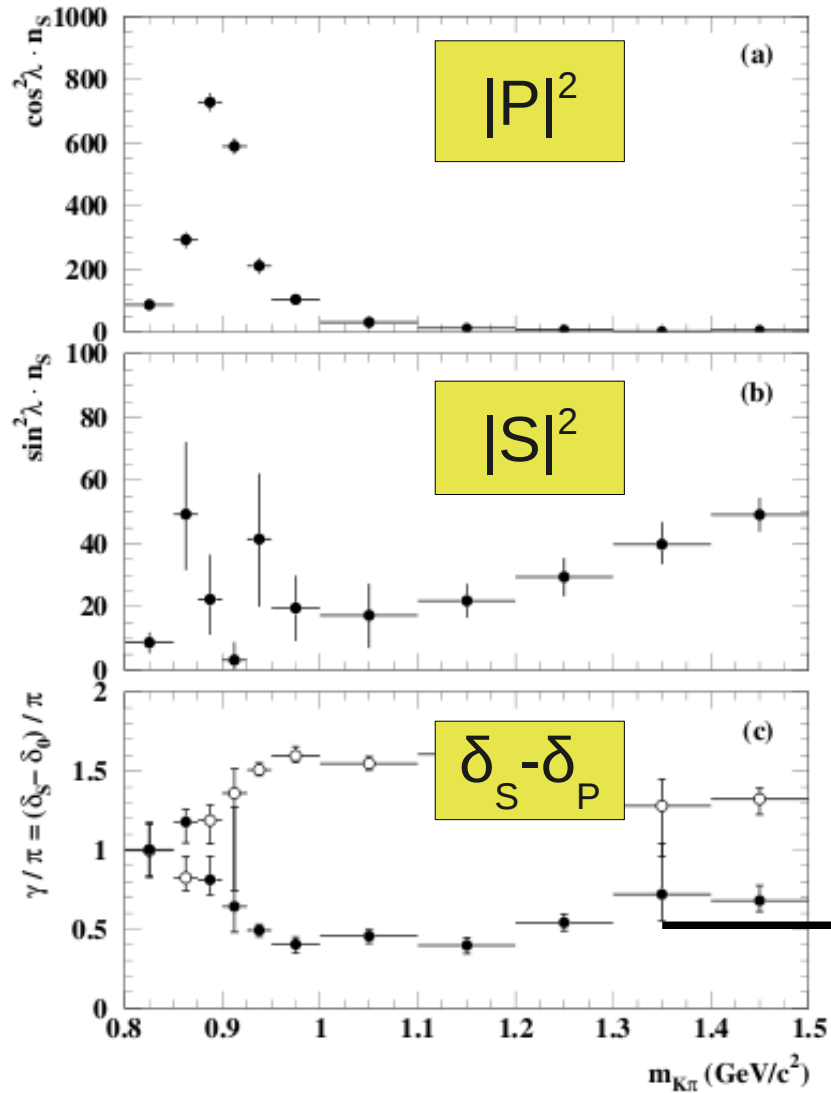


# $\pi\pi$ S-wave comparison

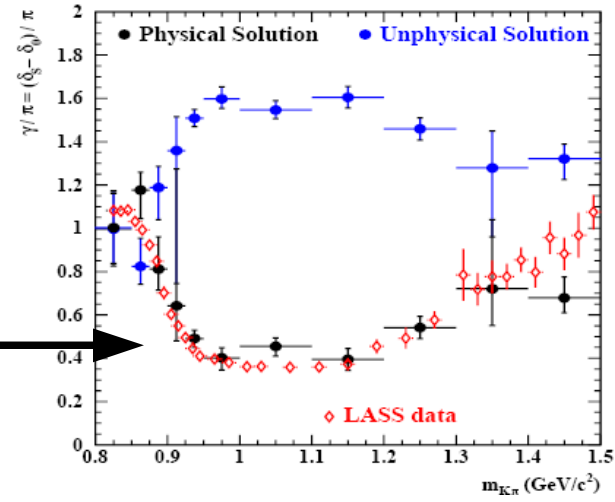
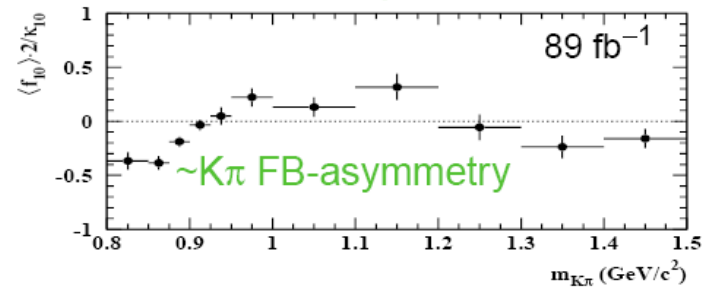


Data points from BaBar

# $B \rightarrow J/\psi K\pi$



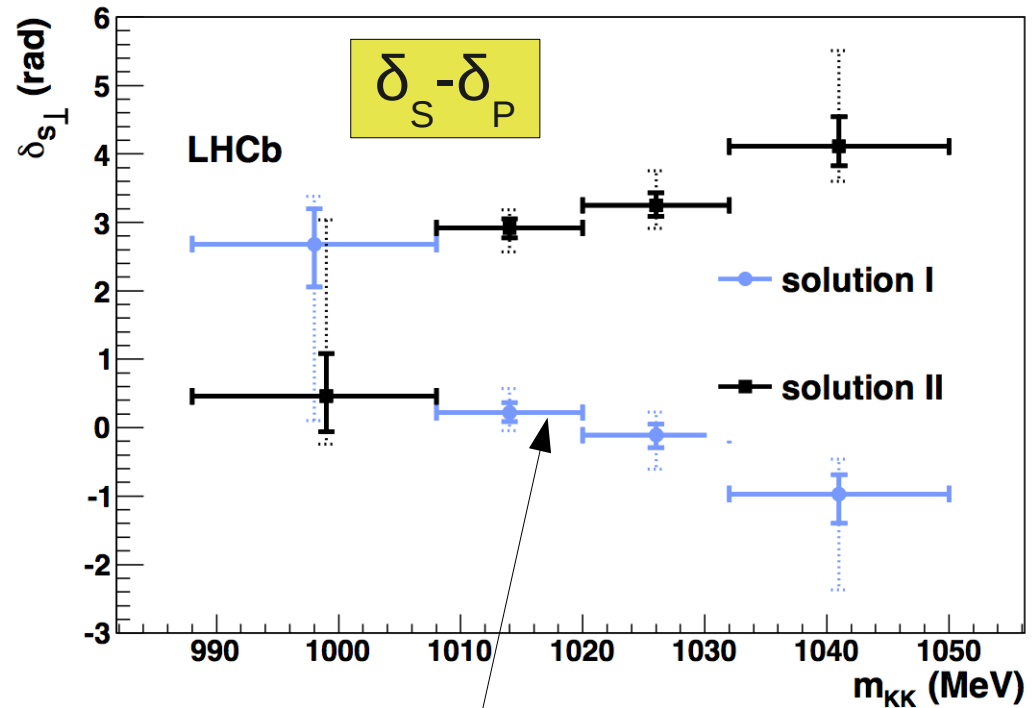
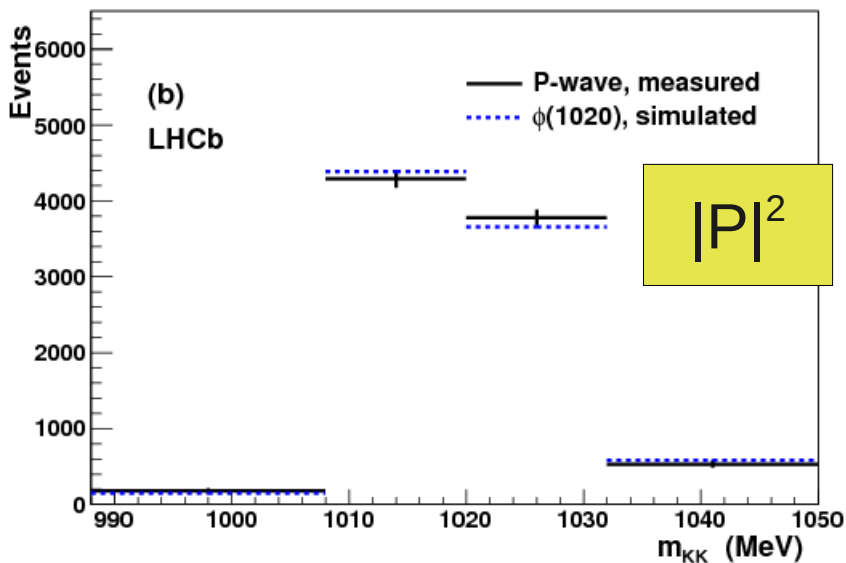
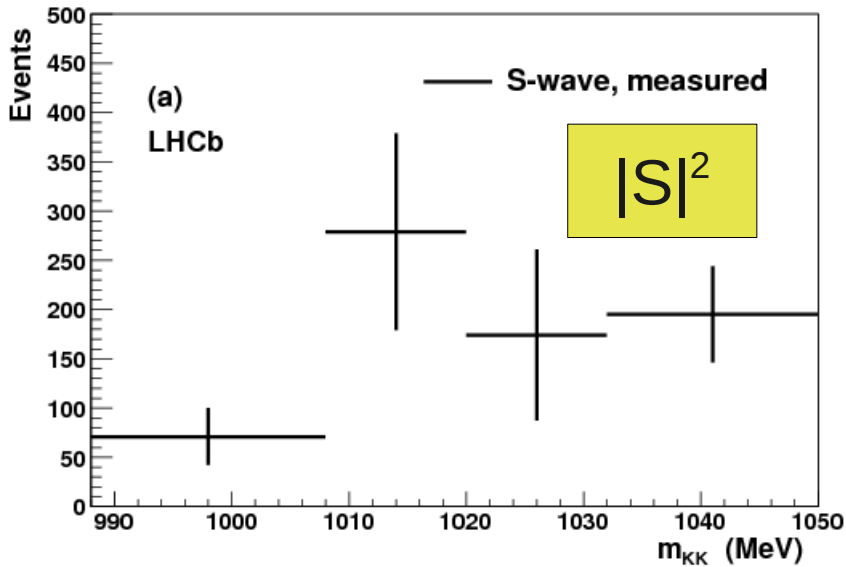
Similar idea (complicated by spin of J/ψ)  
 BaBar PRD 71 (2005) 032005  
 See also Belle PRL 95 (2005) 091601



Essential input to unambiguous measurement of  $\cos(2\beta)$  using  $B \rightarrow J/\psi K_S \pi^0$

# $B_s \rightarrow J/\psi KK$

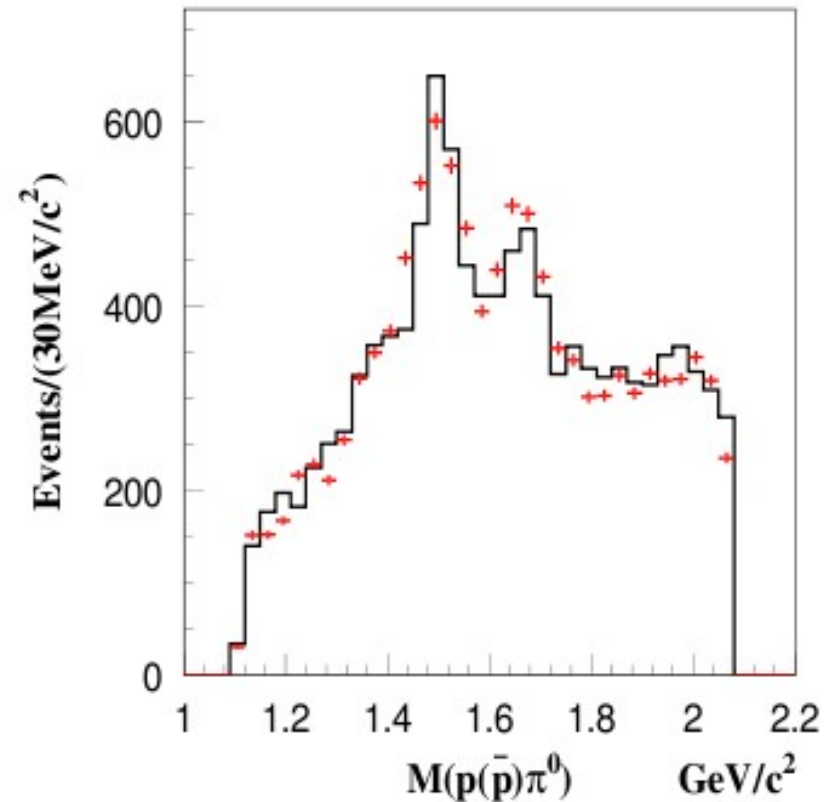
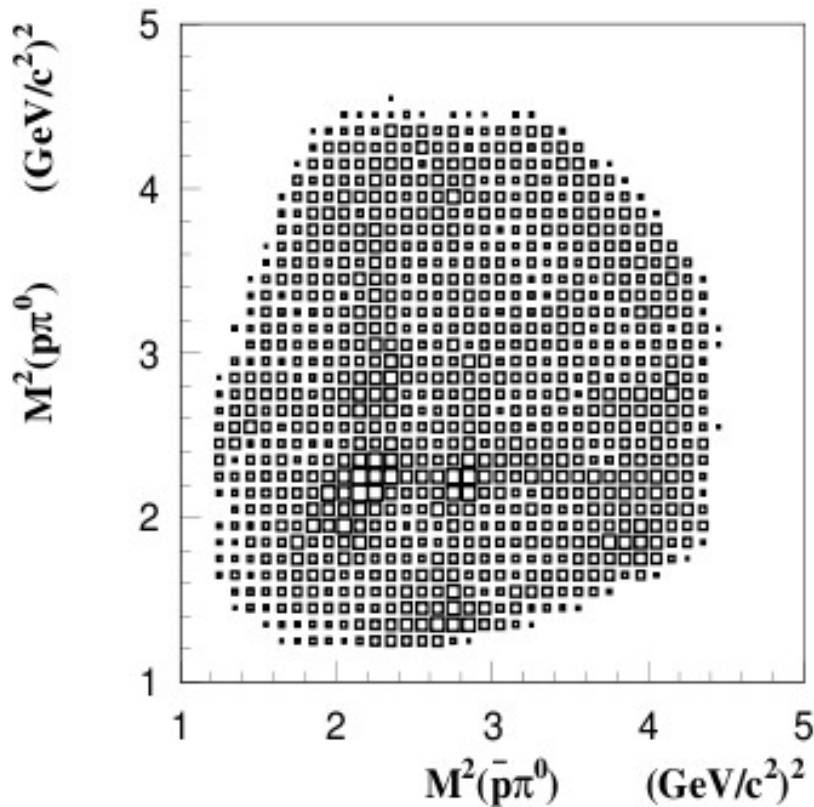
Similar idea (complicated by spin of  $J/\psi$ )  
LHCb arXiv:1202.4717



Physical solution corresponds to  $\Delta\Gamma_s > 0$   
and value of  $\phi_s$  consistent with SM

# “Partial wave analysis of $J/\psi \rightarrow p\bar{p}\pi^0$ at BESII

PRD 80 (2009) 052004



An important and interesting amplitude analysis ... but not a partial wave analysis in the (quasi- model-independent) sense that I have been using <sup>22</sup>

What are the difficulties we encounter when trying to do the analysis?



# Difficulties, difficulties ...

- Backgrounds
- Efficiency
- Misreconstruction & resolution
- Speed
- Parametrisations and conventions
- Goodness of fit
- Model dependence

# Backgrounds

- Do you expect the background to be lower in lepton or hadron colliders?

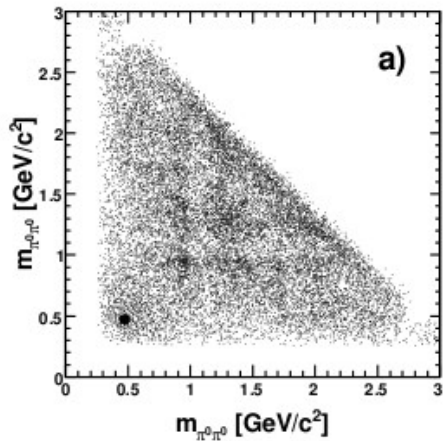
# Backgrounds

- Do you expect the background to be lower in lepton or hadron colliders?
  - It depends (of course ...)
  - Overall multiplicity much lower in  $e^+e^-$  collisions
    - very low backgrounds if you reconstruct everything in the event
    - but if signal is, e.g., B meson from  $Y(4S)$  decay, still have background from “the rest of the event”
  - Particles produced in hadron collisions have high momenta
    - can efficiently reduce background using variables related to flight distance and transverse momenta
    - extreme example: charged kaon beams

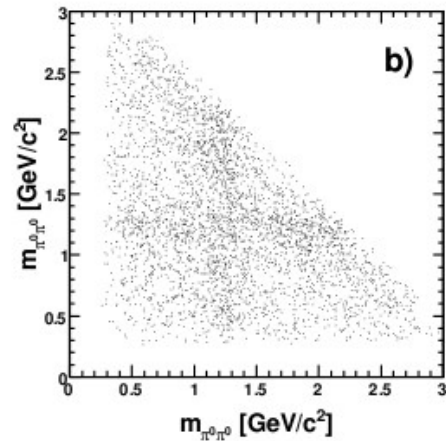
# $\psi(2S) \rightarrow \gamma\chi_{cJ} \rightarrow \gamma(4\pi^0)$ at BESIII

PRD 83 (2011) 012006

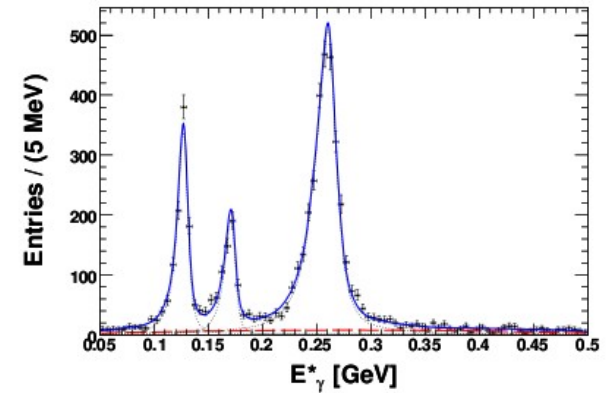
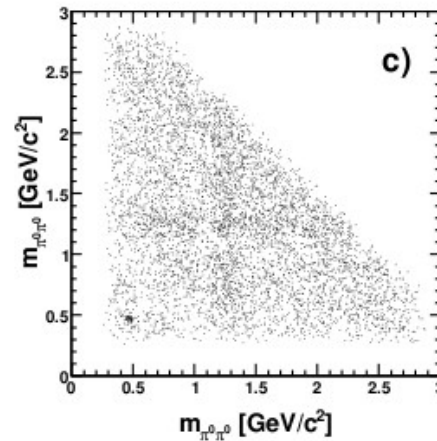
$\chi_{c0}$



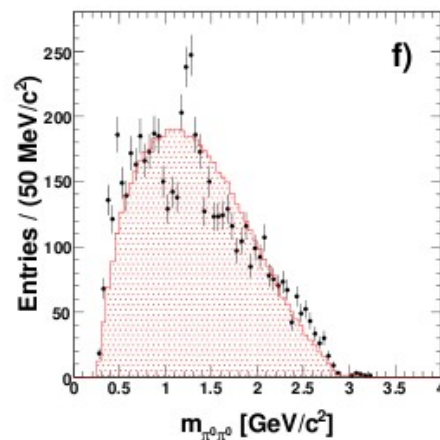
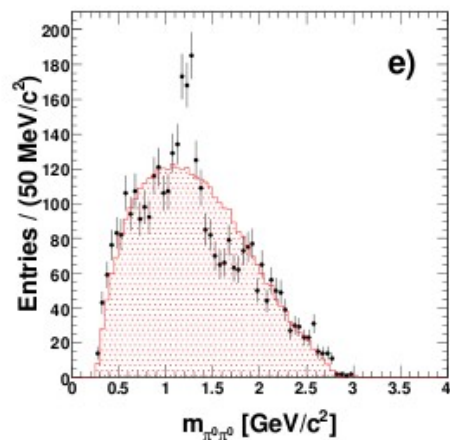
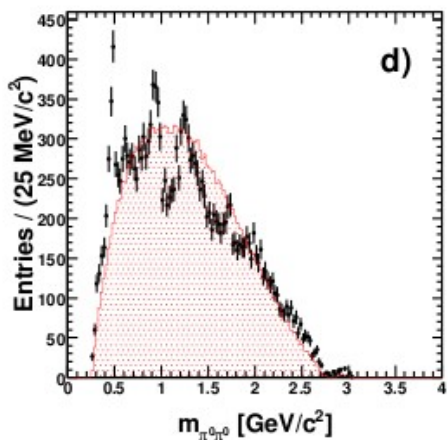
$\chi_{c1}$



$\chi_{c2}$



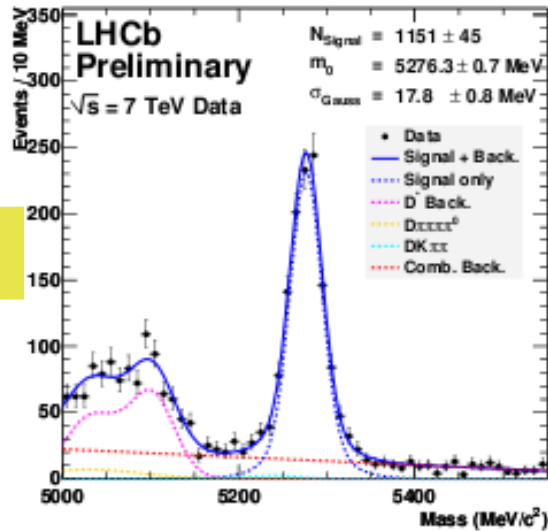
N.B. Not Dalitz plots!



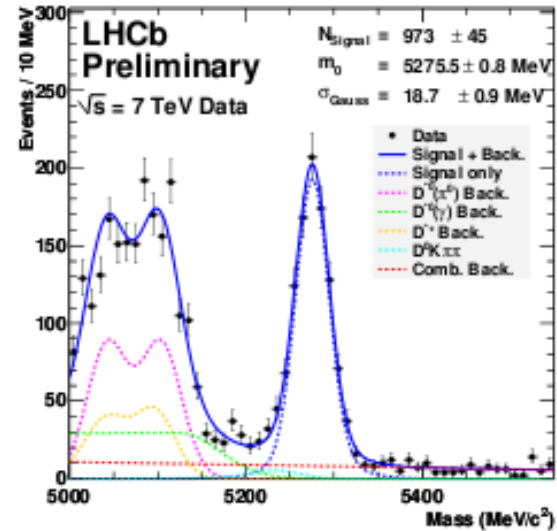
# $X_b \rightarrow X_c 3\pi$ at LHCb

PRD 84 (2011) 092001

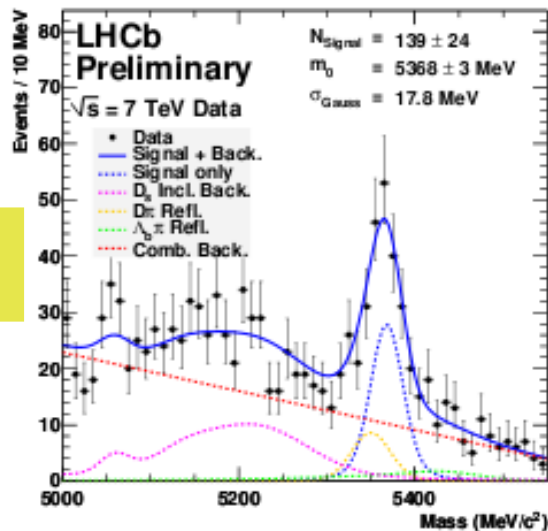
$D^+(3\pi)^-$



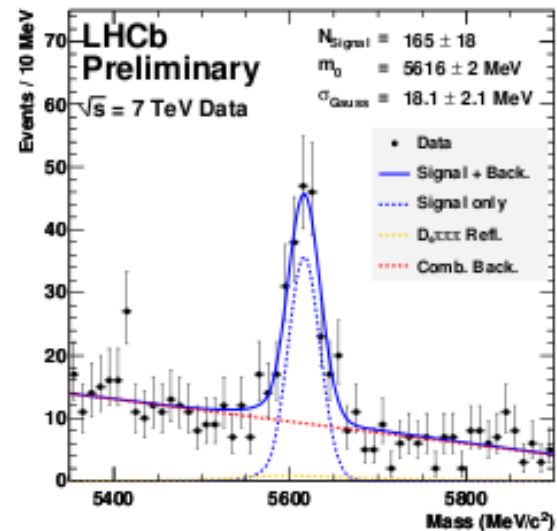
$D^0(3\pi)^-$



$D_s^+(3\pi)^-$



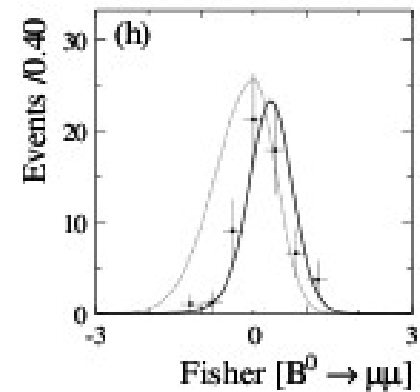
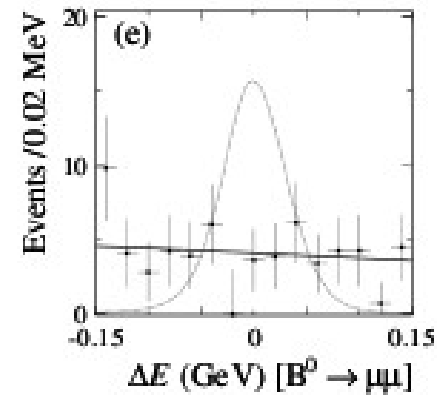
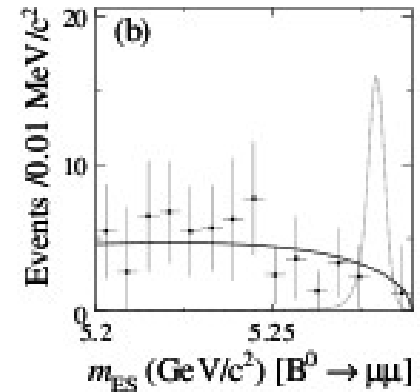
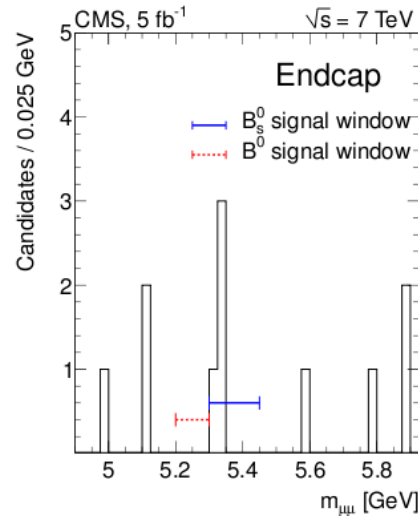
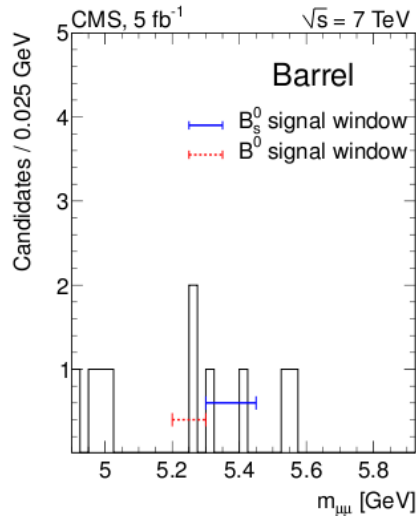
$\Lambda_c^+(3\pi)^-$



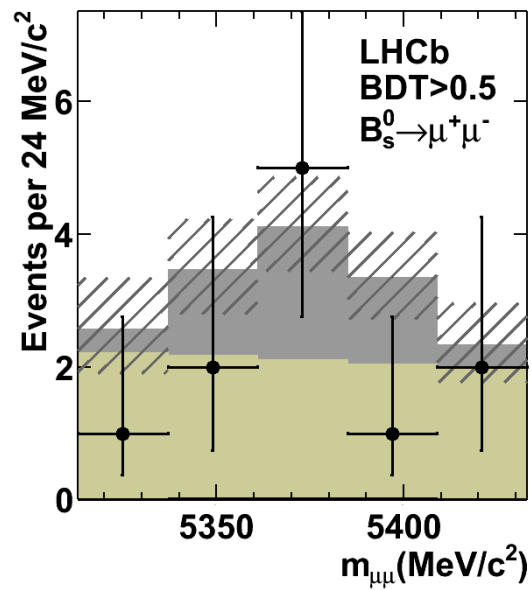
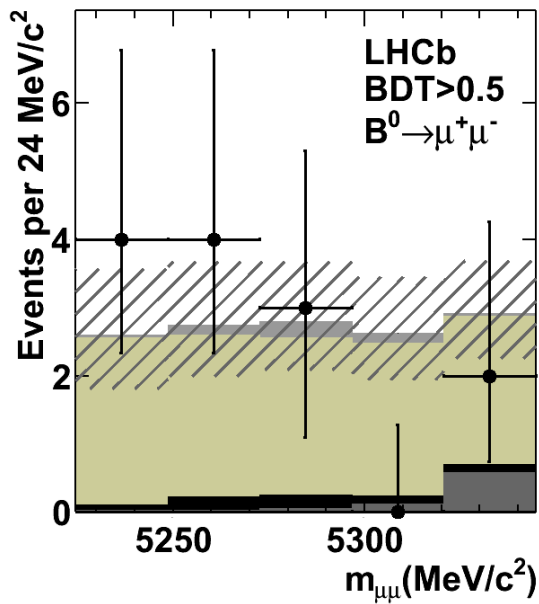
# $B \rightarrow \mu^+ \mu^-$ comparison

CMS JHEP 04 (2012) 033

BaBar PRD 77 (2008) 032007



LHCb arXiv:1203.4493





# Maximum likelihood fit

$$L = \prod_{i=1}^N P_i$$

likelihood can also be “extended” to include Poisson probability to observe N events

$$-2 \ln L = -2 \sum_{i=1}^N \ln(P_i)$$

need to obtain background distributions and to know background fraction (or event-by-event background probability)

$$P_i = P_{i, sig} + P_{i, bkg}$$

$$P_{i, sig} = P_{i, phys} * R_{det}$$

convolution with detector response: includes efficiency and resolution

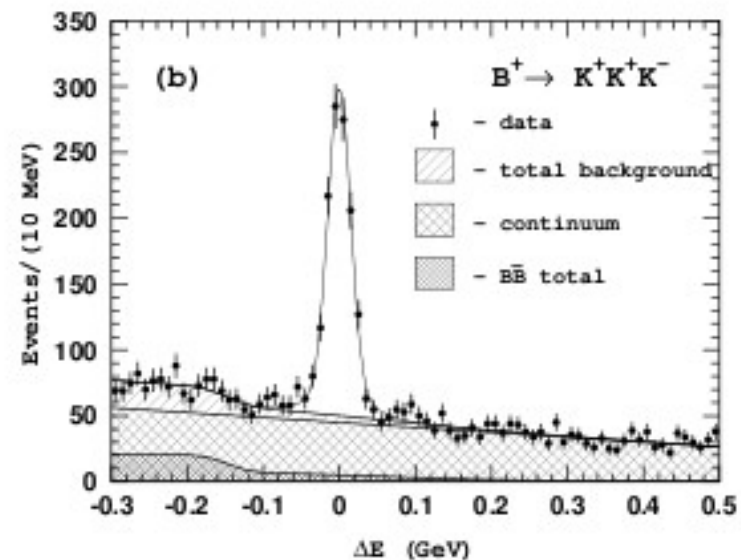
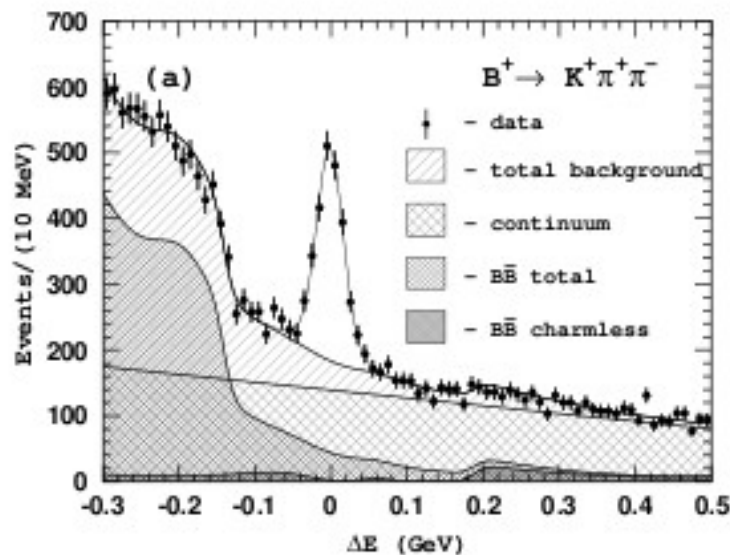
$P_{i, phys}$  contains the physics ...  
but must be coded in a way that allows reliable determination of the model parameters

In the case of a binned fit to data, sum over events is replaced by sum over bins

# Background fractions and distributions

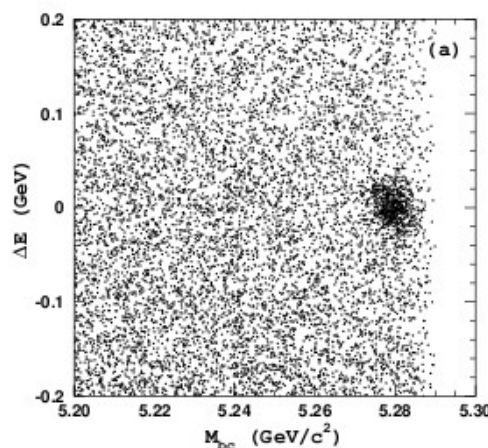
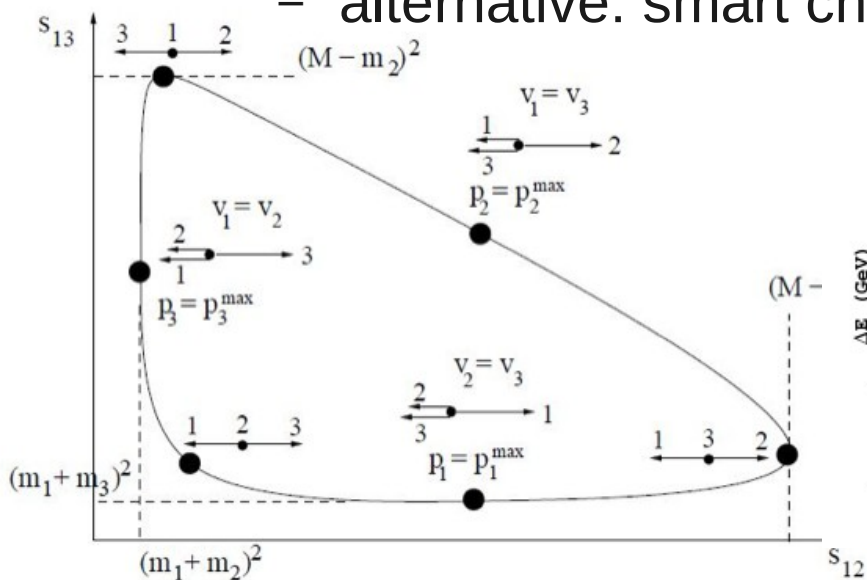
- It is usually possible to determine the background fraction by fitting some kinematic variable (e.g. invariant mass)
  - Can be done prior to, or simultaneously with, the fit to the Dalitz plot
- The background distribution can then be studied from sidebands of this variable
  - Care needed: background composition may be different in the signal and sideband regions

Belle PRD71 (2005) 092003

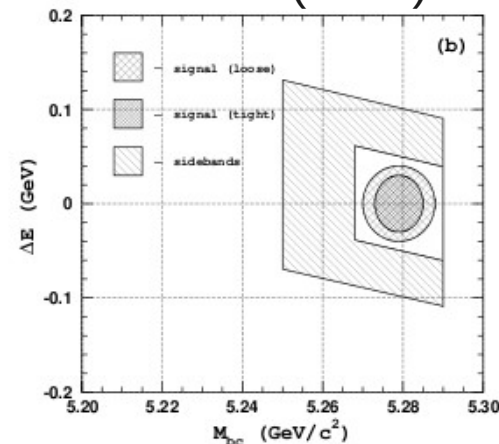


# Background distribution issues

- Boundary of Dalitz plot depends on 3-body invariant mass
  - To have a unique DP, and to improve resolution for substructure, apply 3-body mass constraint
  - This procedure distorts the background shape
    - noticeable if narrow resonances are present in the sideband
    - can be alleviated by averaging upper and lower sidebands (not always possible)
    - alternative: smart choice of sidebands (not always possible)



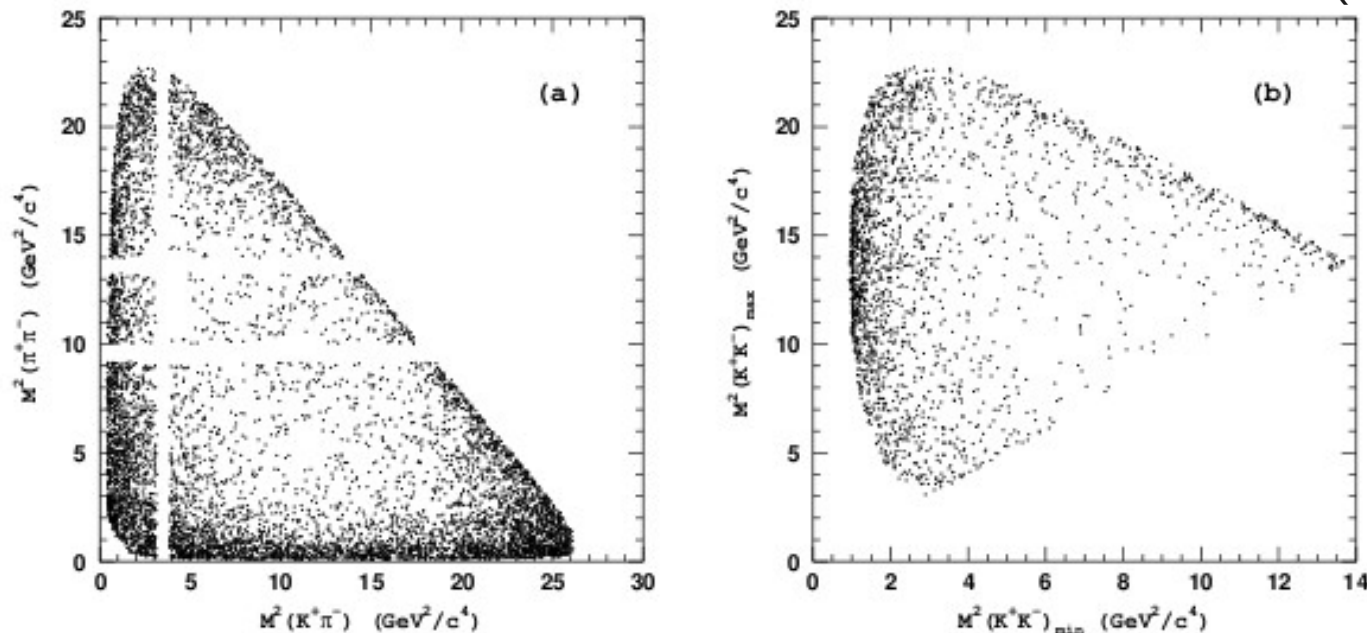
Belle PRD71 (2005) 092003



# Background distribution issues

- In a binned fit, the background can be subtracted
- In an unbinned fit, the background PDF must be described, either
  - parametrically (usually some smooth function plus incoherent sum of narrow states)

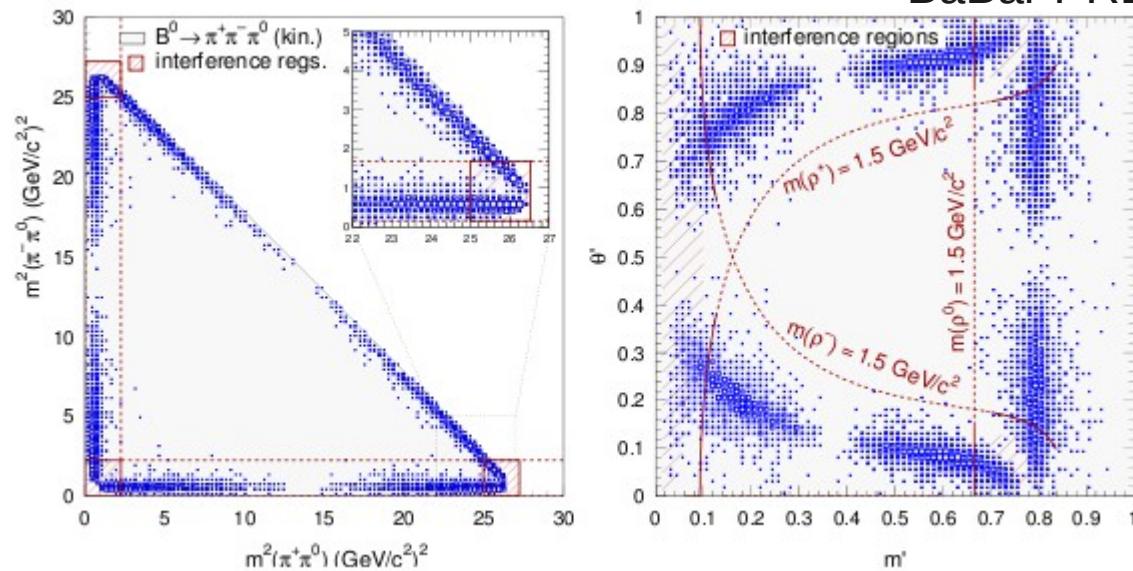
Belle PRD71 (2005) 092003



# Background distribution issues

- In a binned fit, the background can be subtracted
- In an unbinned fit, the background PDF must be described, either
  - **nonparametrically (usually as a histogram)**
    - since background tends to cluster near DP boundaries, advantageous to use “square Dalitz plot”

BaBar PRD 76 (2007) 012004



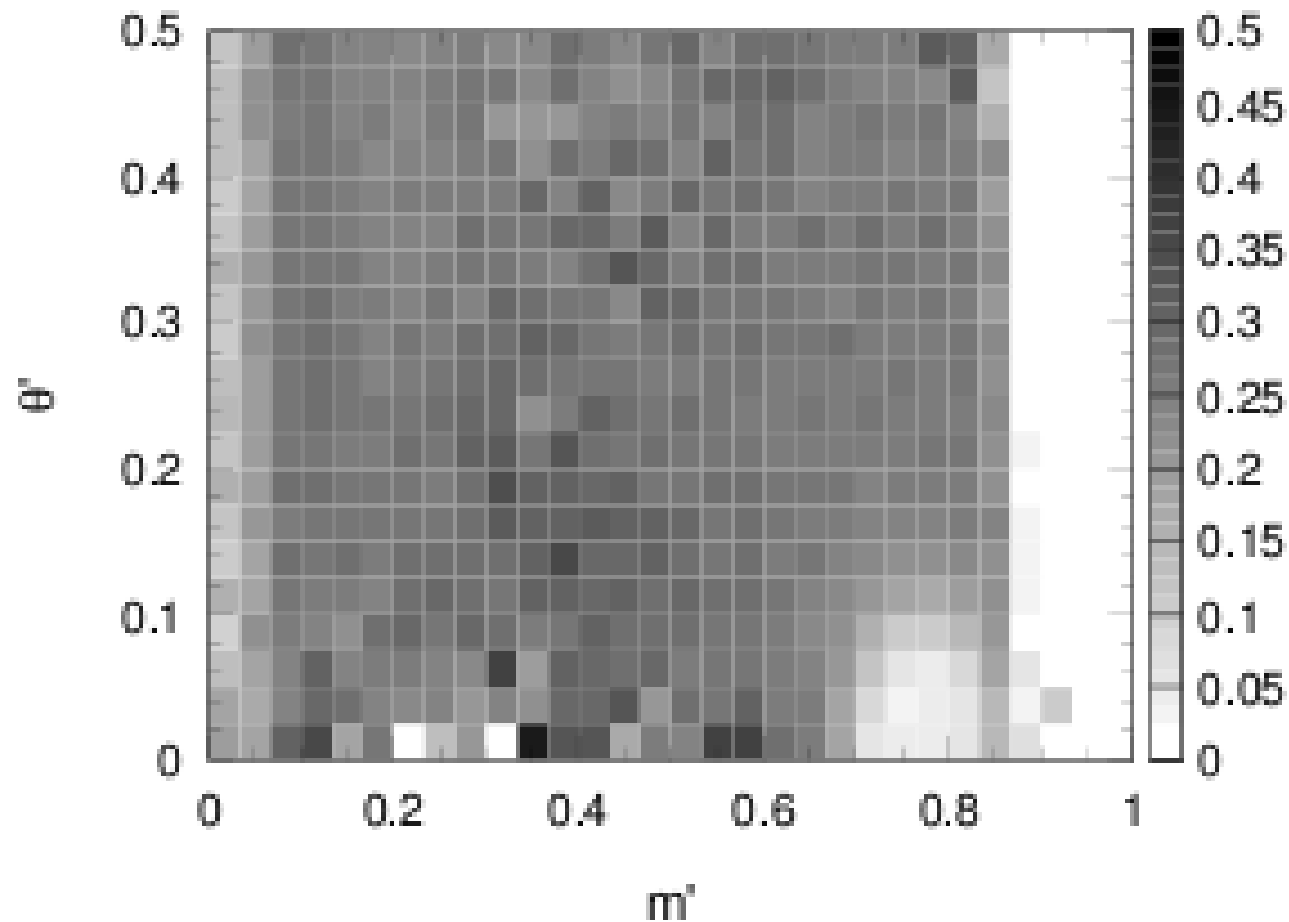
# Detector response – efficiency

- Key point:
  - If the efficiency is uniform across the phase-space, we can ignore it in the maximum likelihood fit
- Efficiency non-uniformity must be accounted for
  - Choose selection variables to minimise effect
  - Determine residual variation from Monte Carlo simulation (validated/corrected using data where possible)
  - Can either
    - explicitly correct for efficiency (event-by-event)
      - usually implemented as a histogram (using square DP or otherwise)
    - determine overall effect from MC simulation with same model parameters
      - only viable approach for high-dimensional problems



# Example of efficiency variation

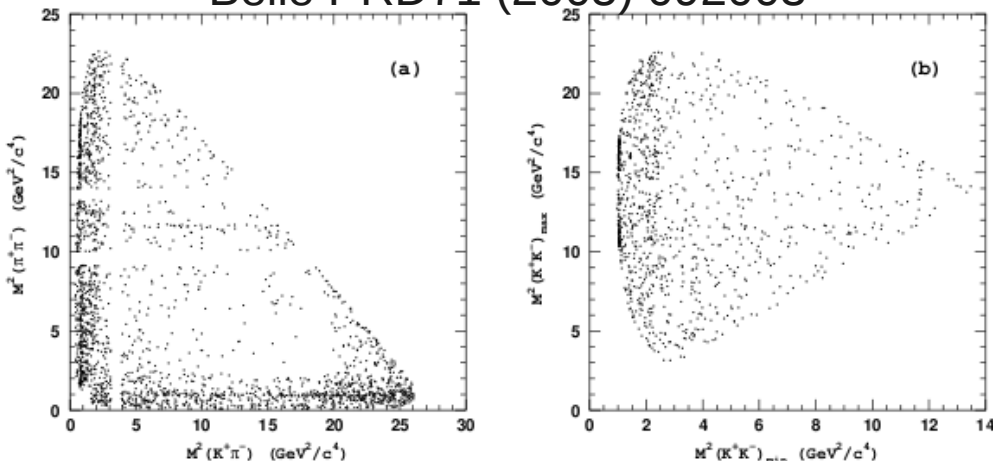
BaBar  $B \rightarrow \pi^+ \pi^- \pi^0$  PRD 76 (2007) 012004



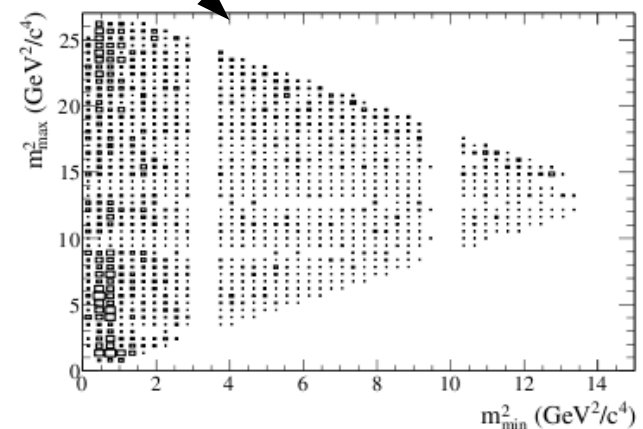
# Visualisation of the Dalitz plot

- Obviously important to present the data to the world
- How to present it?
  - 2D scatter plot of events in the signal region
    - unbinned, hence most information
    - but contains background and not corrected for efficiency
  - Binned 2D (or 1D) projections
    - can correct for background and efficiency
      - sPlots is a useful tool
    - but tend to wash out some of the fine structure

Belle PRD71 (2005) 092003



BaBar PRD 79 (2009) 072006

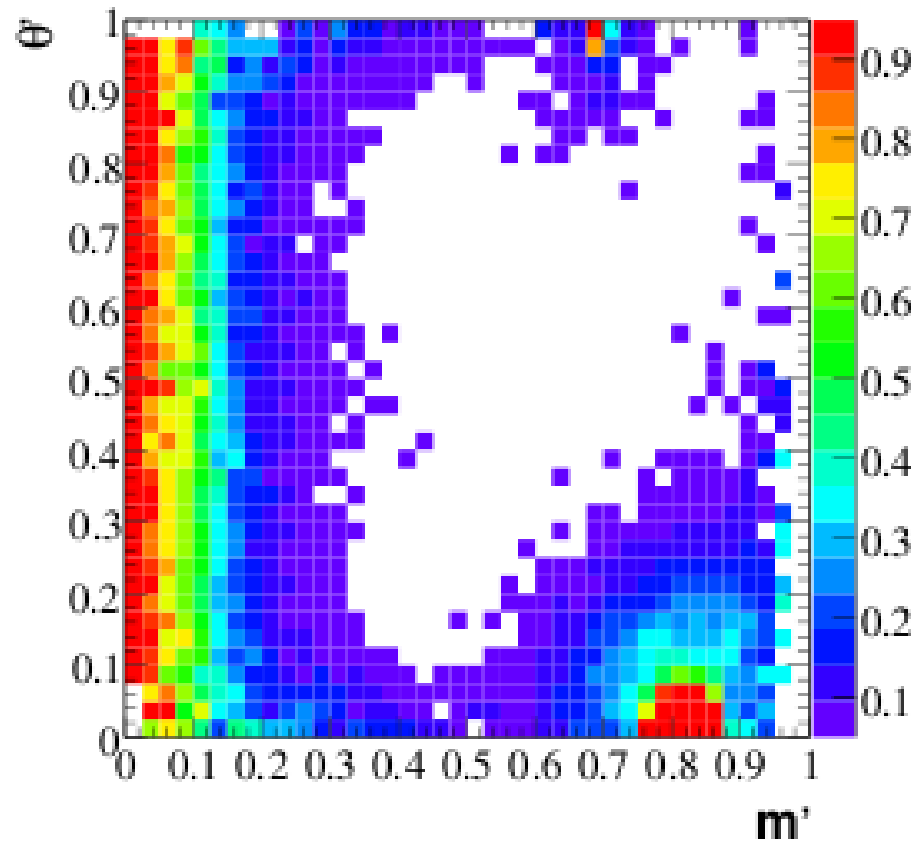


# Resolution and misreconstruction

- Key point:
  - If resolution is  $\ll$  width of narrowest structure on the Dalitz plot, we can ignore it
- Applying 3-body mass constraint helps, but
  - Some Dalitz plots contain narrow structures ( $\omega$ ,  $\varphi$ ,  $D^*$ )
  - Misreconstruction effects (“self-cross-feed”) can lead to significant non-Gaussian tails
    - complicated smearing of events across the Dalitz plot
    - hard to model
    - relies on Monte Carlo simulation – hard to validate with data
    - significant for states with multiple soft particles at B factories

# Example SCF fraction

BaBar  $B \rightarrow K^+\pi^-\pi^0$  PRD 78 (2008) 052005



# Parametrisations

- Fit parameters are complex coefficients of the contributing amplitudes
  - allowing for CP violation, 4 parameters for each
    - usually necessary to fix (at least) two reference parameters
  - many possible parametrisations
    - $r \exp(i\delta) \rightarrow (r \pm \Delta r) \exp(i(\delta \pm \Delta\delta))$
    - $r \exp(i\delta) \rightarrow r \exp(i\delta) (1 \pm \Delta\rho \exp(i\Delta\phi))$
    - $x+iy \rightarrow (x \pm \Delta x) + i(y \pm \Delta y)$
  - there is no general best choice of “well-behaved parameters”
    - unbiased, Gaussian distributed, uncertainties independent of other parameters
    - (correlations allowed in Gaussian limit – important to report full covariance matrix)
  - some partial solutions available, but often not applicable
    - e.g. Snyder-Quinn parametrisation for  $B \rightarrow \pi^+ \pi^- \pi^0$ 
      - #parameters explodes for >3 resonances

# Conventions

- There are many different ways to write the lineshapes, spin factors, etc.
  - choice of normalisation is important
- Even if all code is bug-free, it is very hard to present unambiguously all information necessary to allow the Dalitz plot model to be reproduced
- Important to present results in convention-independent form (as well as other ways)
  - e.g. fit fractions and interference fit fractions



# Example fit fraction matrix

TABLE I: Fit fractions [matrix](#) of the best fit. The diagonal elements  $F_{kk}$  correspond to component fit fractions shown in the paper in Table I. The off-diagonal elements give the fit fractions of the interference terms defined as  $F_{kl} = 2\Re \int \mathcal{M}_k \mathcal{M}_l^* ds_{23} ds_{13} / \int |\mathcal{M}|^2 ds_{23} ds_{13}$ .

$F_{kl} \times 100\%$	$\phi$	$f_0(980)$	$X_0(1550)$	$f_0(1710)$	$\chi_{c0}$	NR
$\phi$	<b>11.8 ± 0.9 ± 0.8</b>	-0.94 ± 0.18 ± 0.11	-1.71 ± 0.36 ± 0.24	0.01 ± 0.10 ± 0.03	0.11 ± 0.02 ± 0.05	3.54 ± 0.38 ± 0.40
$f_0(980)$		<b>19 ± 7 ± 4</b>	53 ± 12 ± 7	-4.5 ± 2.9 ± 1.2	-0.9 ± 0.2 ± 0.5	-85 ± 21 ± 14
$X_0(1550)$			<b>121 ± 19 ± 6</b>	-30 ± 11 ± 4	-1.1 ± 0.3 ± 0.5	-140 ± 26 ± 7
$f_0(1710)$				<b>4.8 ± 2.7 ± 0.8</b>	-0.10 ± 0.07 ± 0.07	4 ± 6 ± 3
$\chi_{c0}$					<b>3.1 ± 0.6 ± 0.2</b>	3.9 ± 0.4 ± 1.9
NR						<b>141 ± 16 ± 9</b>

# Goodness of fit

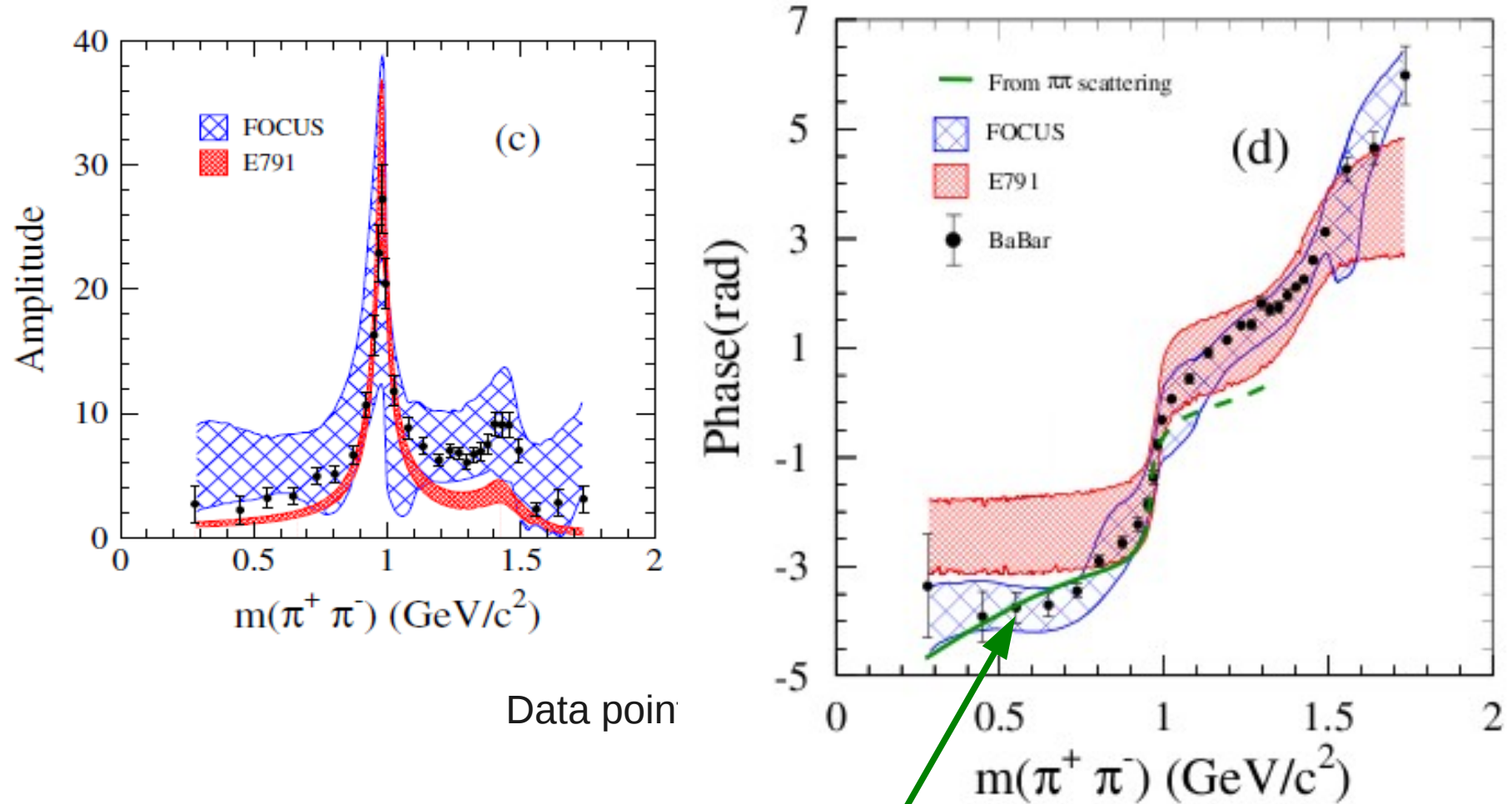
- How do I know that my fit is good enough?
- You don't (sorry) ... but some guidelines can tell you if there are serious problems
  - Is your fit model physical?
    - sometimes there may be little choice but to accept this
  - Do you get an acceptable  $\chi^2/n.d.f.$  for various projections (1D and 2D)?
    - if no, is the disagreement localised in the Dalitz plot?
    - with high statistics it is extremely difficult to get an acceptable p-value; check if the disagreement is compatible with experimental systematics
    - some unbinned goodness-of-fit tests are now becoming available
  - Do you get an excessive sum of fit fractions?
    - values >100% are allowed due to interference, but very large values are usually indicative of unphysical interference patterns (possibly because the model is not physical)
  - Do you think you have done the best that you possibly can?
    - eventually it is better to publish with an imperfect model than to suppress the data

# Summary

- It must be clear by now that Dalitz plot analyses are extremely challenging
  - both experimentally and theoretically
- So let's recall that the motivation justifies the effort
  - hadronic effects: improved understanding of QCD, including possible exotic states
  - CP violation effects: potential sensitivity to discover new sources of matter-antimatter asymmetry
- We have an obligation to exploit the existing and coming data to the maximum of our abilities
  - and if that is not enough, we will have to improve our abilities!

**THE END**

# $\pi\pi$ S-wave comparison



Prediction from theory:  
Kaminski et al. PRD77:054015,2008