

# New physics and CP violation measurements at LHCb

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LHCb week, Firenze  
Common session with the Galileo Galilei Institute

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# Contents

- The CKM Matrix and CP violation
  - The need for a clean measurement of  $\gamma$
- Flavour oscillations and CP violation
  - Where are we now and where will LHCb take us?
  - $B_d$ ,  $B_s$  and charm systems
- Alternative  $\gamma$ ,  $\beta$  and  $\beta_s$  measurements

# The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A 3x3 unitary matrix
- Described by 4 real parameters – **allows CP violation**
  - PDG (Chau-Keung) parametrisation:  $\theta_{12}, \theta_{23}, \theta_{13}, \delta$
  - Wolfenstein parametrisation:  $\lambda, A, \rho, \eta$
- **Highly predictive**

# CP violation and New Physics

- CKM ansatz is **highly predictive**
- Measurements of flavour phenomena **overconstrain** the SM parameters
- **Consistency of different measurements** tests for presence of new physics
- For example, compare
  - **Clean SM reference points** (from theory or measurement)
  - **New physics sensitive measurements**
- Recall past success of CP violation in discovering new physics

# CKM Matrix – Phases

P.Harrison *et al.*,  
arXiv:0904.3077 [hep-ph]

- Can form a matrix of angles between pairs of CKM matrix elements
  - $\Phi_j =$  phase between remaining elements when row  $i$  and column  $j$  removed
  - unitarity implies sum of phases in any row or column =  $180^\circ$

$$\Phi = \begin{matrix} & \begin{matrix} d & s & b \end{matrix} \\ \begin{matrix} u \\ c \\ t \end{matrix} & \begin{pmatrix} \Phi_{ud} & \Phi_{us} & \Phi_{ub} \\ \Phi_{cd} & \Phi_{cs} & \Phi_{cb} \\ \Phi_{td} & \Phi_{ts} & \Phi_{tb} \end{pmatrix} \end{matrix} \approx \begin{matrix} & \begin{matrix} d & s & b \end{matrix} \\ \begin{matrix} u \\ c \\ t \end{matrix} & \begin{pmatrix} 1^\circ & 22^\circ & 157^\circ \\ 67^\circ & 90^\circ & 23^\circ \\ 112^\circ & 68^\circ & 0^\circ \end{pmatrix} \end{matrix}$$

$\beta \equiv \varphi_1$   
 $\alpha \equiv \varphi_2$   
 $\gamma \equiv \varphi_3$

“The Unitarity Triangle”

# CKM Matrix – Phases

P.Harrison *et al.*,  
arXiv:0904.3077 [hep-ph]

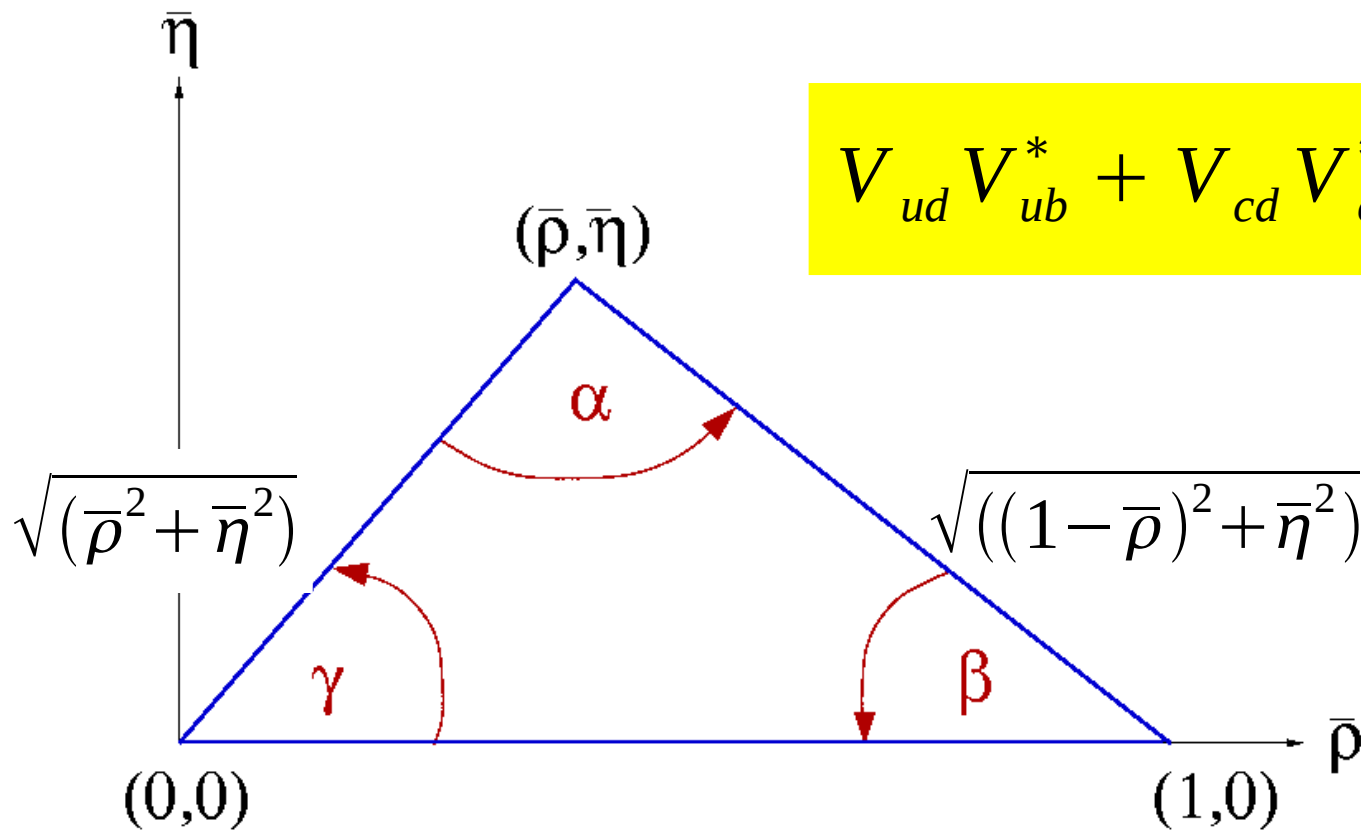
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$$\Phi = \begin{matrix} & d & s & b \\ \begin{matrix} u \\ c \\ t \end{matrix} & \begin{pmatrix} \Phi_{ud} & \Phi_{us} & \Phi_{ub} \\ \Phi_{cd} & \Phi_{cs} & \Phi_{cb} \\ \Phi_{td} & \Phi_{ts} & \Phi_{tb} \end{pmatrix} & \approx & \begin{matrix} \beta_s \approx \varphi_s/2 \\ u \\ c \\ t \end{matrix} & \begin{pmatrix} d & s & b \\ 1^\circ & 22^\circ & 157^\circ \\ 67^\circ & 90^\circ & 23^\circ \\ 112^\circ & 68^\circ & 0^\circ \end{pmatrix} & \begin{matrix} \beta \equiv \varphi_1 \\ \alpha \equiv \varphi_2 \\ \varphi_D/2 \\ \gamma \equiv \varphi_3 \end{matrix}
 \end{matrix}$$

“The Unitarity Triangle”

# The Unitarity Triangle

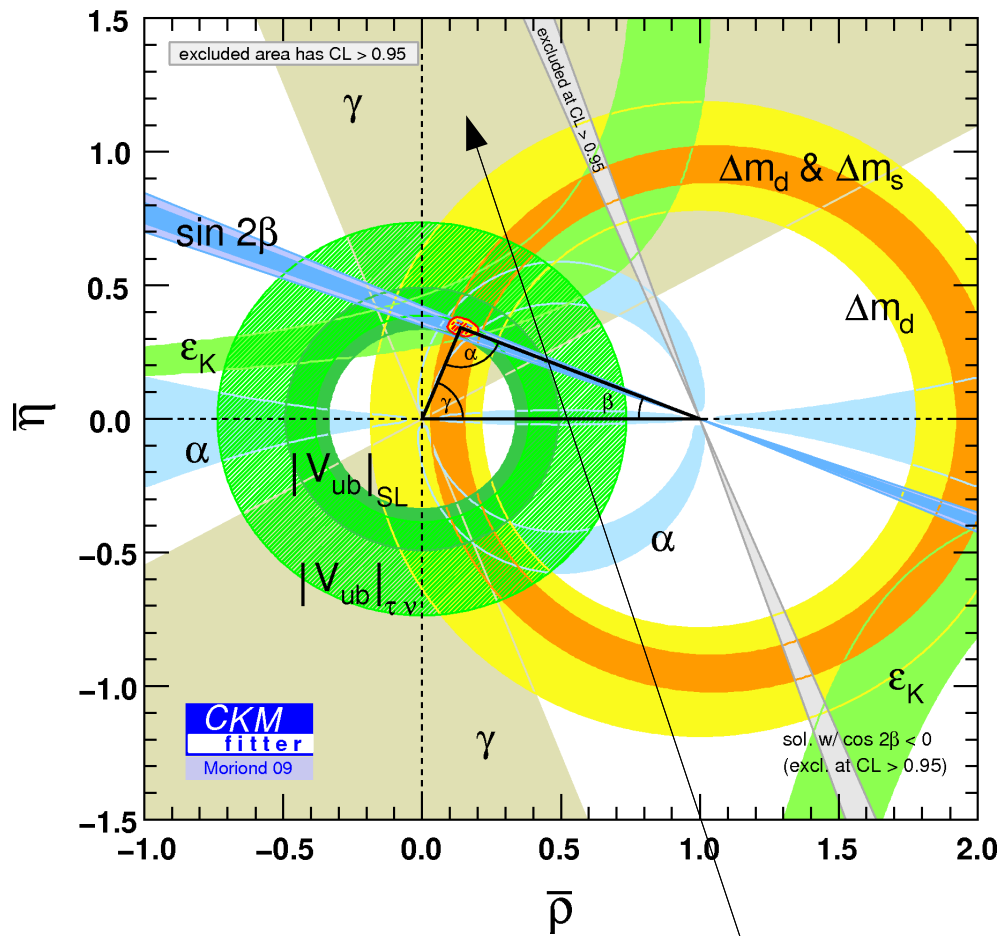
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$



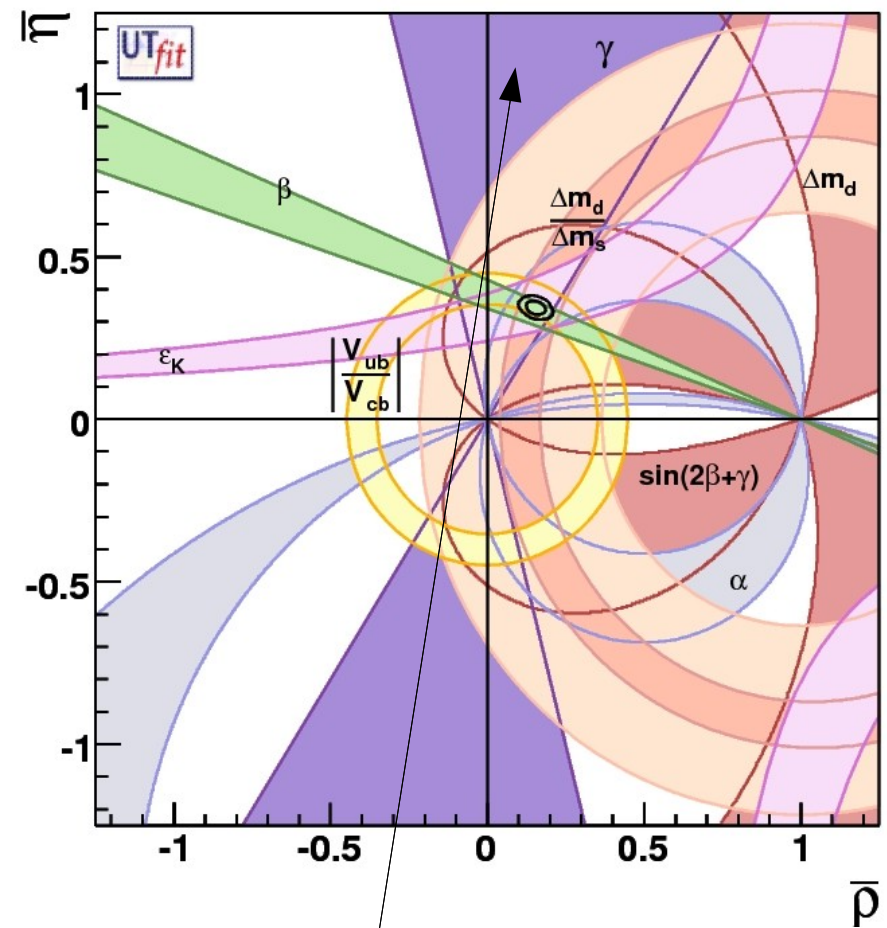
$$\alpha \equiv \phi_2 = \arg \left[ -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right], \quad \beta \equiv \phi_1 = \arg \left[ -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right], \quad \gamma \equiv \phi_3 = \arg \left[ -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]$$

# Today's Constraints on the Unitarity Triangle

<http://ckmfitter.in2p3.fr/>



<http://www.utfit.org/>



Note the relatively poor constraint on  $\gamma$



# Importance of $\gamma$

- $\gamma$  plays a unique role in flavour physics

the only CP violating parameter that can be measured through tree decays (\*)

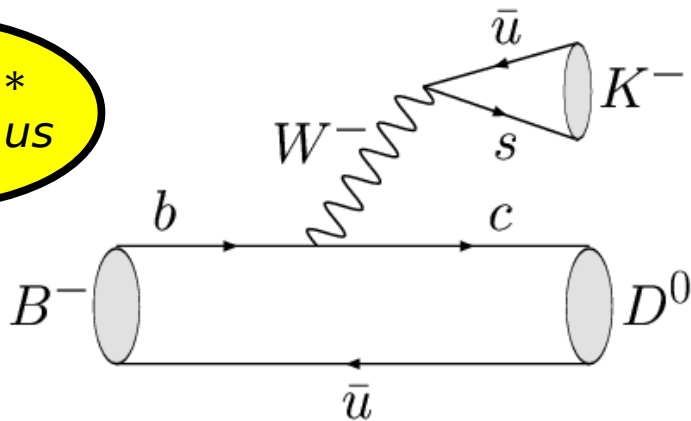
(\*) more-or-less

- A benchmark Standard Model reference point
  - doubly important after New Physics is observed
- How precise is precise enough?
  - 10% (X) At 3 sigma hardly exclude anything
  - 1% ☆ Seems the right level to test NP
  - 0.1% (X) Good luck if you can get the funding ...

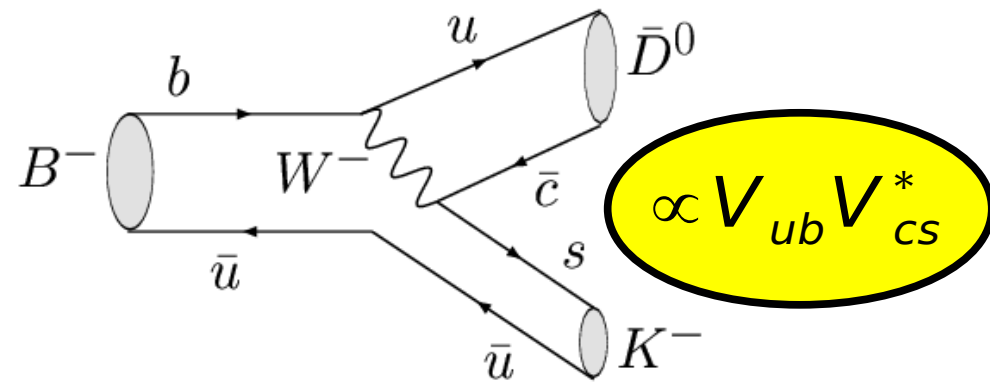
# How To Measure $\gamma$

- Focus on theoretically pristine measurement
  - Interference between

$$\propto V_{cb} V_{us}^*$$



- colour allowed
- final state contains  $D^0$



$$\propto V_{ub} V_{cs}^*$$

- colour suppressed
- final state contains  $\bar{D}^0$

- Use D decay final states accessible to both amplitudes
  - $KK$  (GLW),  $K\pi$  (ADS),  $K_s \pi\pi$  (GGSZ)
- Use also  $B_d$  decays to  $DK^{*0}$  (untagged)
- Use also  $B_s$  decays to  $D_s K$  (tagged, time-dependent)

# LHCb sensitivity to $\gamma$

$\delta_{B^0}$ ( $^\circ$ )	0	45	90	135	180
$\sigma_\gamma$ for $0.5 \text{ fb}^{-1}$ ( $^\circ$ )	8.1	10.1	9.3	9.5	7.8
$\sigma_\gamma$ for $2 \text{ fb}^{-1}$ ( $^\circ$ )	4.1	5.1	4.8	5.1	3.9

- Numbers assume nominal LHC performance
- Sensitivity to  $\delta_{\text{B}}$  inherent to  $B^0 \rightarrow DK^0$  (“quasi-two-body”) analysis
- Precision can be further improved:
  - CLEOC results on  $D \rightarrow K\pi\pi^0$  allow it to be used in ADS analysis
  - $B^0 \rightarrow DK\pi$  Dalitz plot analysis gives improved sensitivity to  $\gamma$  with reduced dependence on  $\delta_{\text{B}}$

# Flavour oscillations, CP violation and Nobel Prizes

- 1964 – Discovery of CP violation in  $K^0$  system
- 1980 – Nobel Prize to Cronin and Fitch



PRL 13 (1964) 138

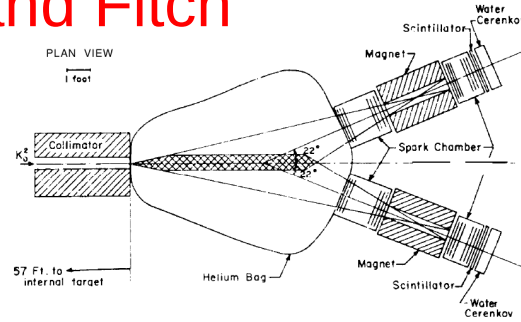


Fig. 1. Plan view of the apparatus as located at the A. G. S.

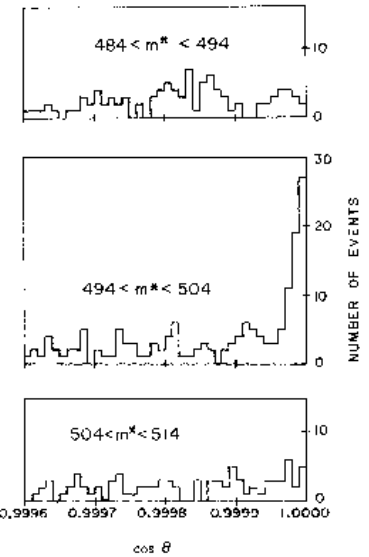
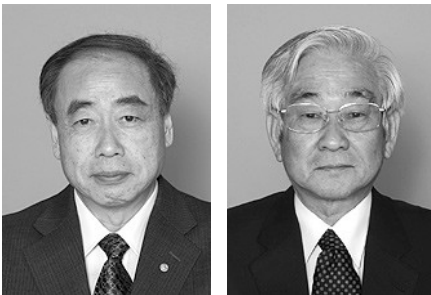
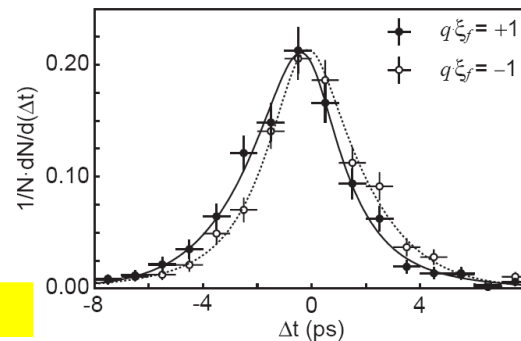


FIG. 3. Angular distribution in three mass ranges for events with  $\cos\theta > 0.9995$ .

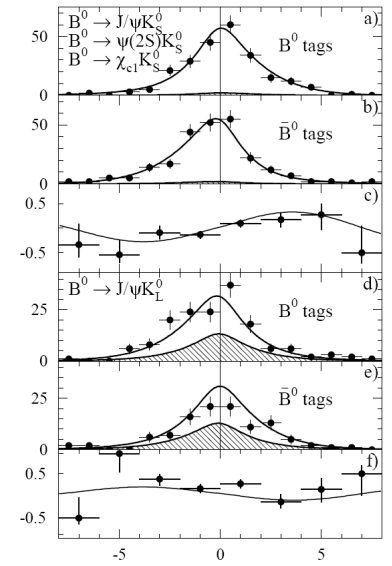
- 2001 – Discovery of CP violation in  $B_d$  system
- 2008 – Nobel Prize to Kobayashi and Maskawa



Prog.Theor.Phys. 49 (1973) 652



Belle PRL 87 (2001) 091802

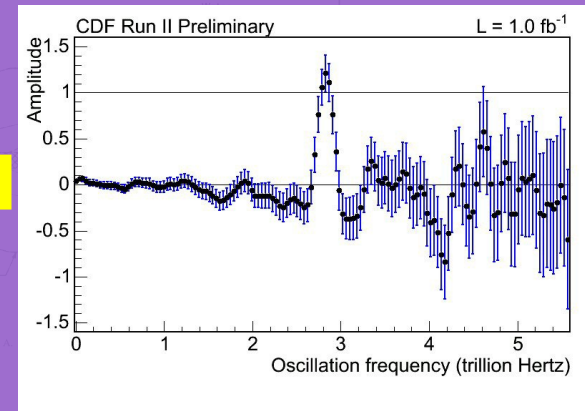


BABAR PRL 87 (2001) 091801

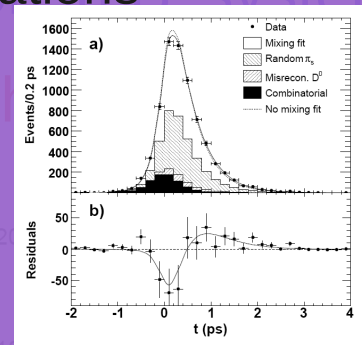
# Flavour oscillations, CP violation and Nobel Prizes

- 1964 • Discovery of oscillations → discovery of CP violation → Nobel Prize
  - can take 30 years
- 1980 – Nobel Prize to Cronin and Fitch
  - 2006 Discovery of  $B_s$  oscillations
    - no discovery of CP violation yet
- 2001 • 2007 Discovery of charm oscillations
- 2008 • no CP violation yet

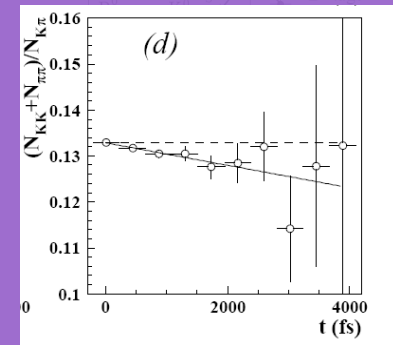
484 < m\* < 494



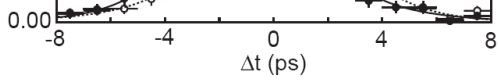
CDF PRL 97 (2006) 242003



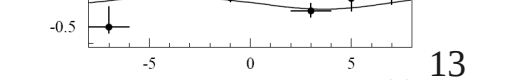
BABAR PRL 98 (2007) 211802



Belle PRL 98 (2007) 211803



Belle PRL 87 (2001) 091802



BABAR PRL 87 (2001) 091801

# Time-dependent CP Violation Formalism

- Generic (but shown for  $B_s$ ) decays to CP eigenstates

$$\Gamma(B_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} + \mathcal{A}_{\text{CP}}^{\text{dir}} \cos(\Delta m t) + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} + \mathcal{A}_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right]$$

$$\Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} (1 + a) e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} - \mathcal{A}_{\text{CP}}^{\text{dir}} \cos(\Delta m t) + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} - \mathcal{A}_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right].$$

# Time-dependent CP Violation Formalism

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$$\Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} (1 - a) e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} - \mathcal{A}_{CP}^{dir} \cos(\Delta m t) + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} - \mathcal{A}_{CP}^{mix} \sin(\Delta m t) \right].$$

CP violating asymmetries

CP conserving parameter

$$(\mathcal{A}_{CP}^{dir})^2 + (\mathcal{A}_{\Delta\Gamma})^2 + (\mathcal{A}_{CP}^{mix})^2 = 1$$

# Time-dependent CP Violation Formalism

- Generic (but shown for  $B_s$ ) decays to CP eigenstates

$$\Gamma(B_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} \quad \text{[red oval]} + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} \quad \text{[red oval]} \right]$$

$$\Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} (1 + a) e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} \quad \text{[red oval]} + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} \quad \text{[red oval]} \right].$$

- Untagged analyses still sensitive to some interesting physics



# Time-dependent CP Violation Formalism

- Generic (but shown for  $B_s$ ) decays to CP eigenstates

$$\Gamma(B_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} + 0 + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} + \mathcal{A}_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right]$$

$$\Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} (1 + 0) e^{-\Gamma t} \times \left[ \cosh \frac{\Delta\Gamma t}{2} - 0 + \mathcal{A}_{\Delta\Gamma} \sinh \frac{\Delta\Gamma t}{2} - \mathcal{A}_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right].$$

- In some channels, expect no direct CP violation
- and/or no CP violation in mixing

# Time-dependent CP Violation Formalism

- Generic (but shown for  $B_s$ ) decays to CP eigenstates

$$\Gamma(B_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} e^{-\Gamma t} \times \left[ \mathbf{1} + \mathcal{A}_{\text{CP}}^{\text{dir}} \cos(\Delta m t) + \mathbf{0} + \mathcal{A}_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right]$$

$$\Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} (1 + a) e^{-\Gamma t} \times \left[ \mathbf{1} - \mathcal{A}_{\text{CP}}^{\text{dir}} \cos(\Delta m t) + \mathbf{0} - \mathcal{A}_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right].$$

- In some channels, expect no direct CP violation
- $B_d$  case:  $\Delta\Gamma$  negligible

# Time-dependent CP Violation Formalism

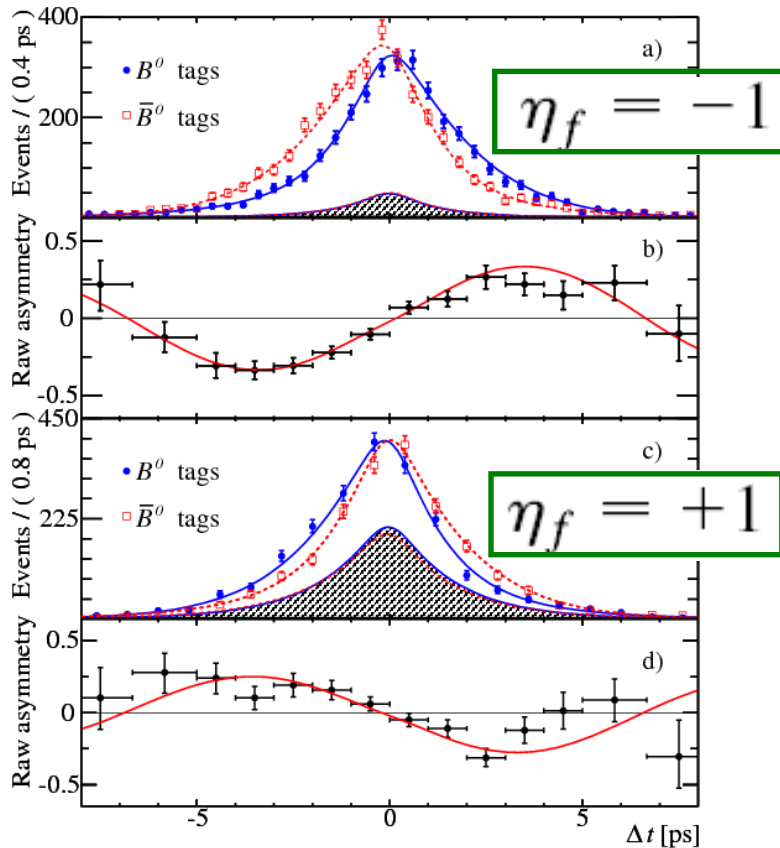
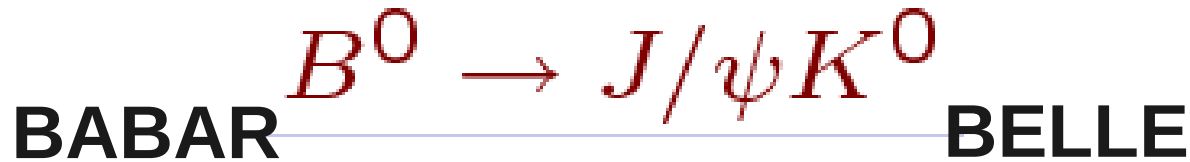
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$$\Gamma(B_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} e^{-\Gamma t} \times \left[ \mathbf{1} + \mathcal{A}_{\text{CP}}^{\text{dir}} \mathbf{1} + \mathcal{A}_{\Delta\Gamma} y\Gamma t + \mathcal{A}_{\text{CP}}^{\text{mix}} x\Gamma t \right]$$

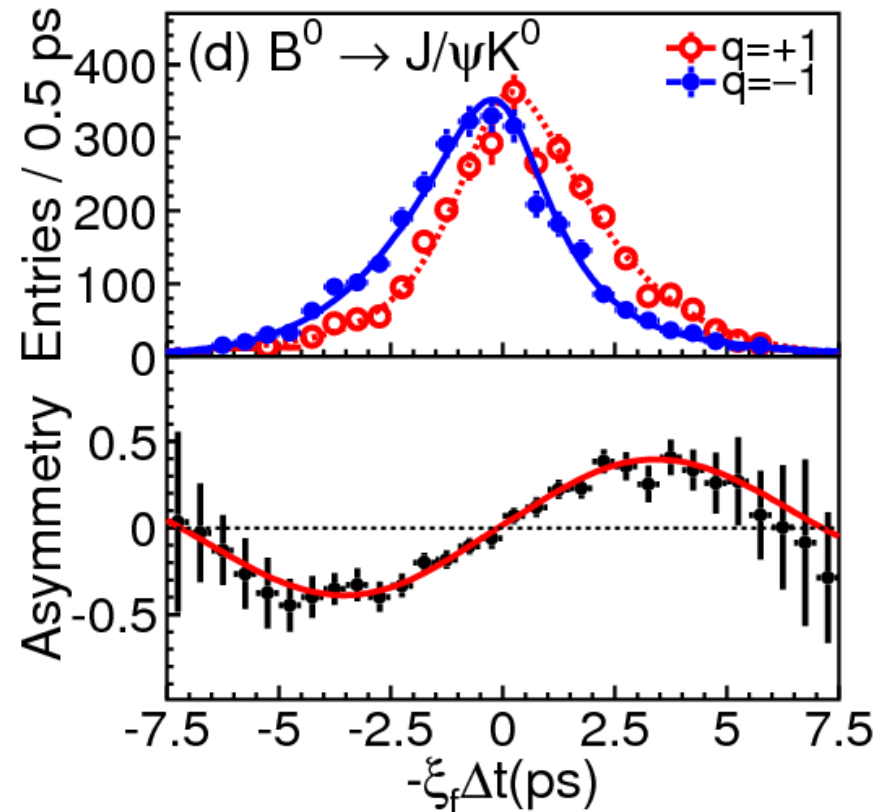
$$\Gamma(\bar{B}_s(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1 + |\lambda_f|^2}{2} (1 + a) e^{-\Gamma t} \times \left[ \mathbf{1} - \mathcal{A}_{\text{CP}}^{\text{dir}} \mathbf{1} + \mathcal{A}_{\Delta\Gamma} y\Gamma t - \mathcal{A}_{\text{CP}}^{\text{mix}} x\Gamma t \right].$$

- In some channels, expect no direct CP violation
- $B_d$  case:  $\Delta\Gamma$  negligible
- $D^0$  case: both  $x = \Delta m/\Gamma$  and  $y = \Delta\Gamma/2\Gamma$  small

# (Almost) latest measurements of $\beta$

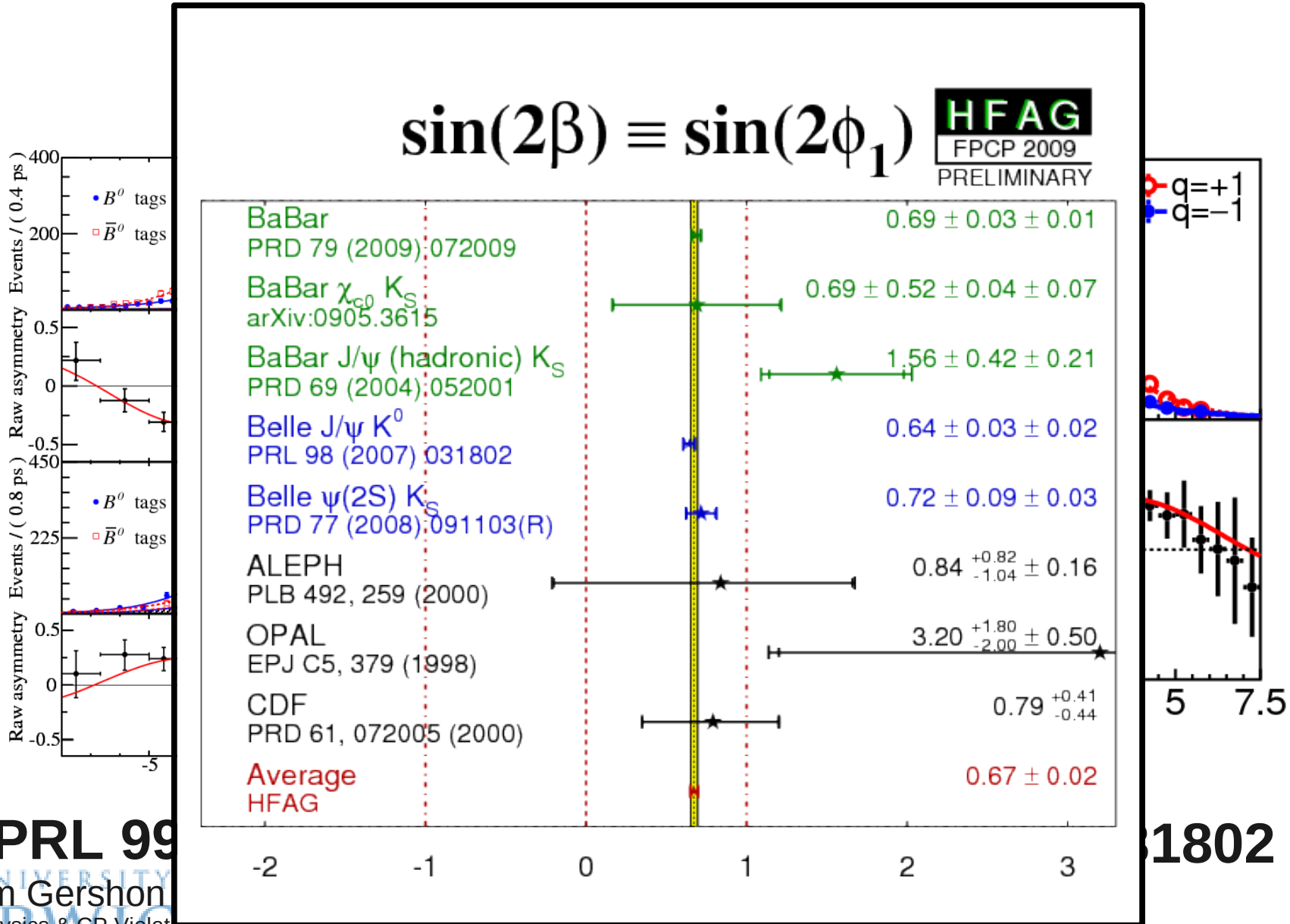


**PRL 99, 171803 (2007)**



**PRL 98 (2007) 031802**

# (Almost) latest measurements of $\beta$



**PRL 99**

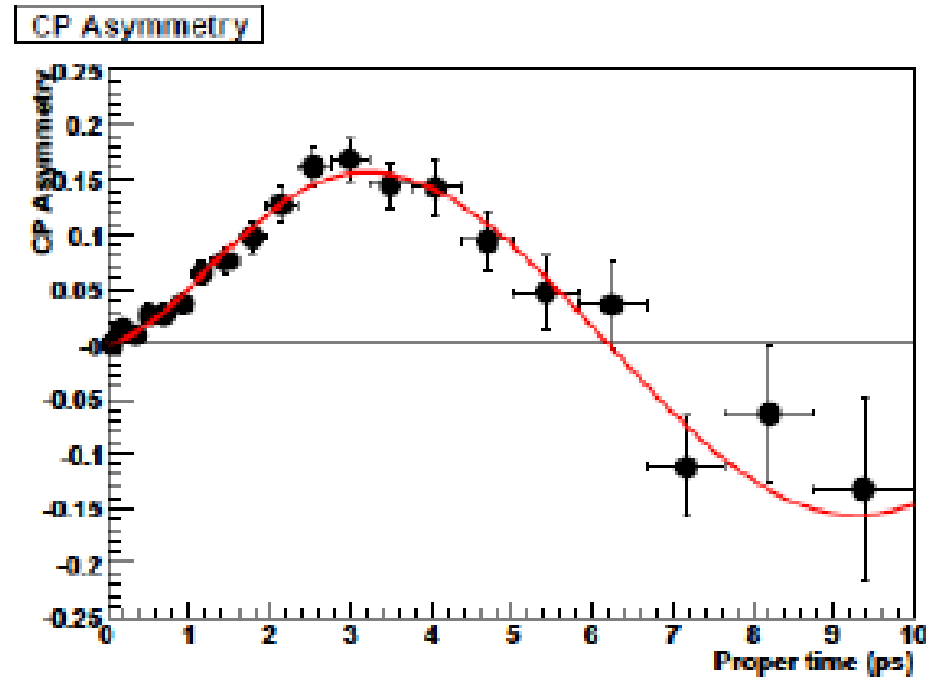
Tim Gershon

New Physics & CP Violation

**1802**

# LHCb prospects for $\beta$

- “Yesterday's sensation is tomorrow's calibration”



- Sensitivity  $\sigma(\sin(2\beta)) \approx 0.06$  with 200/pb of data
- Validate **selection**, **vertexing**, **resolution**, **tagging**

# Charm mixing and CP violation

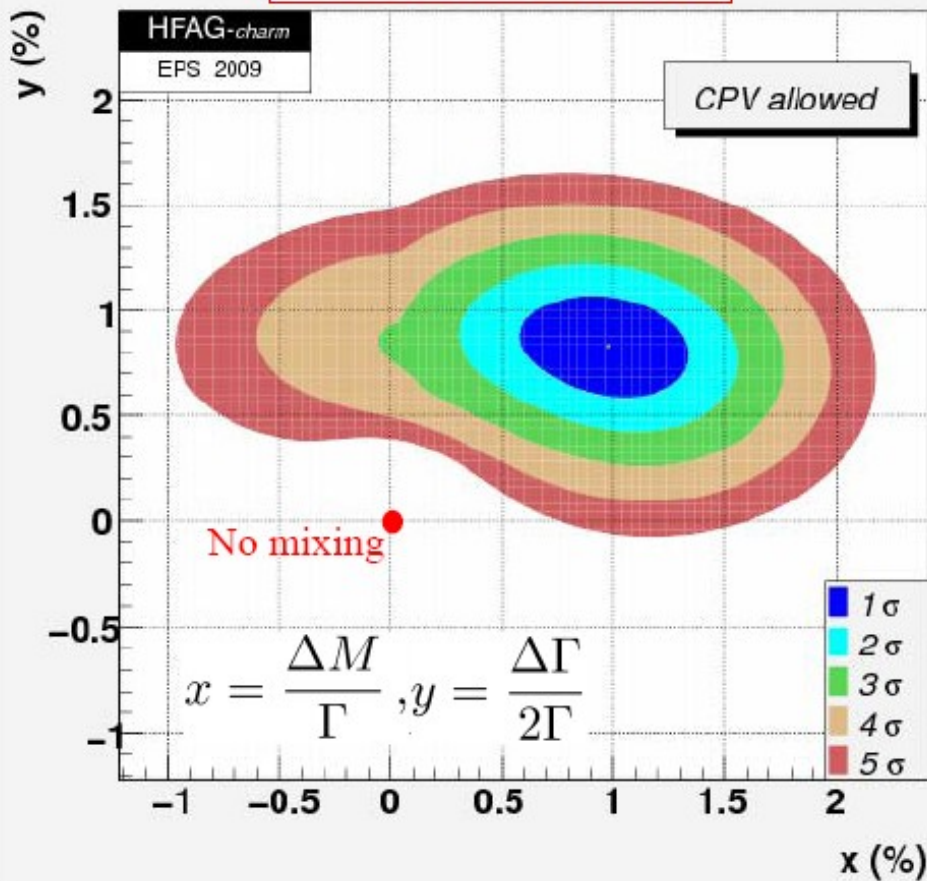
Including results from BABAR, Belle, CDF, CLEO(c), FOCUS

Latest new results Belle arXiv:0905.4185 [hep-ex]

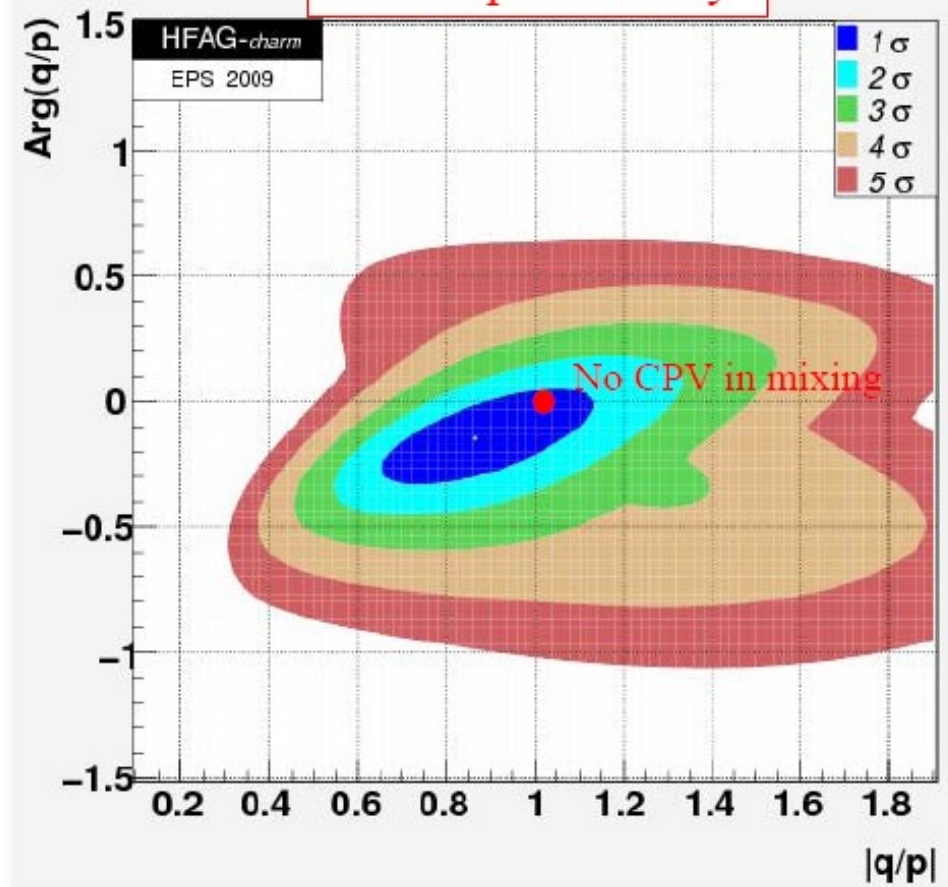
BABAR arXiv:0908.0761 [hep-ex]

A.Bevan at EPS'09

HFAG preliminary



HFAG preliminary

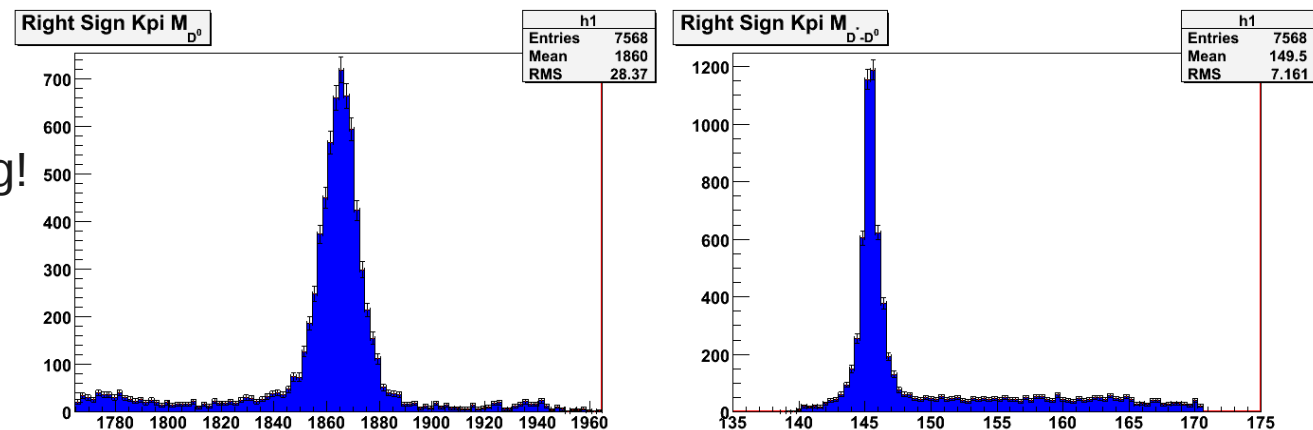


- Mixing established (though still no single measurement  $> 5\sigma$ )
- No indication of CP violation

# LHCb Prospects for Charm

- Most sensitive channels
  - $D \rightarrow K^+K^-, \pi^+K^-, K_s^0 \pi^+ \pi^-$
  - All well-suited to LHCb
- Tagging provided by  $D^{*\pm} \rightarrow D\pi^\pm$
- Huge cross-section for prompt charm production
  - plus large rates of secondary charm
- With 100/pb accumulate  $\sim 2.5M$  tagged  $D \rightarrow K^+K^-$

Expected  $D \rightarrow \pi^+K^-$  yields  
from 15 minutes of running!

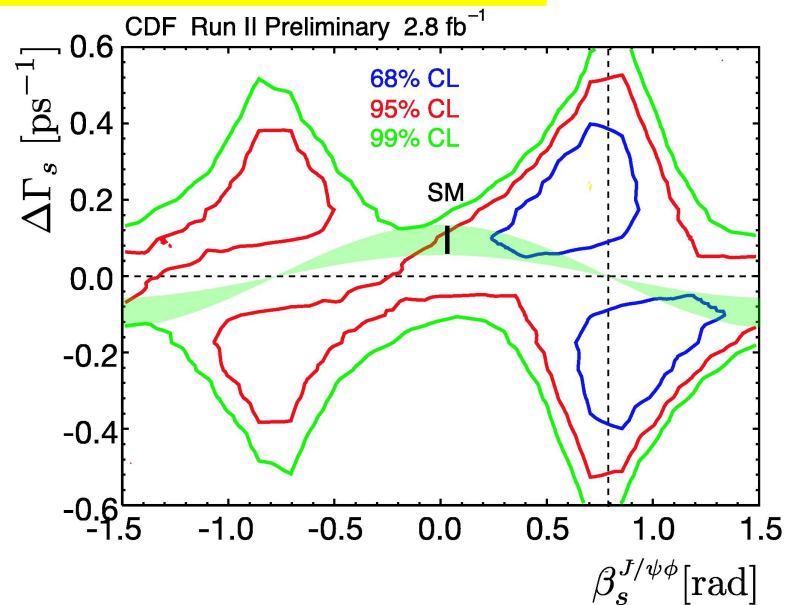




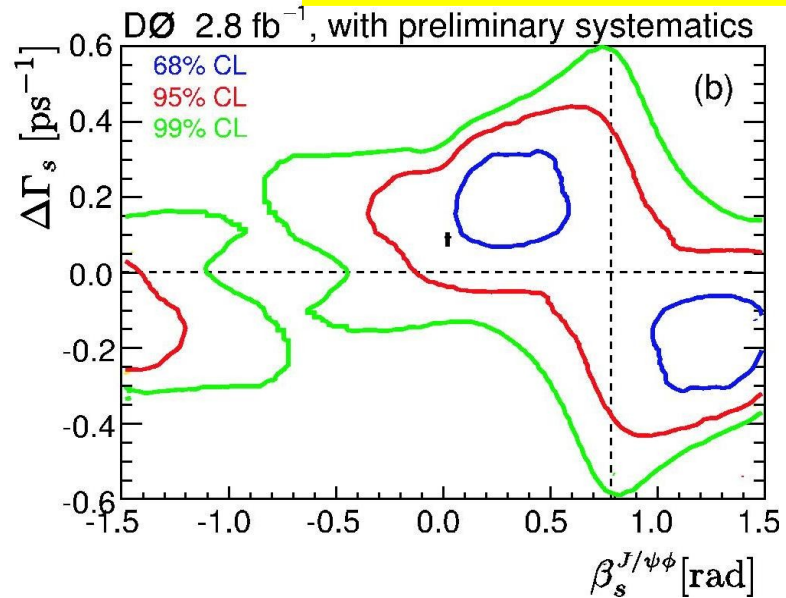
# $B_s$ oscillations and CP violation

- Tevatron measurements using tagged  $B_s \rightarrow J/\psi\phi$
- Angular analyses of vector-vector final state
- Results depend on  $\Delta\Gamma$

CDF note 9787

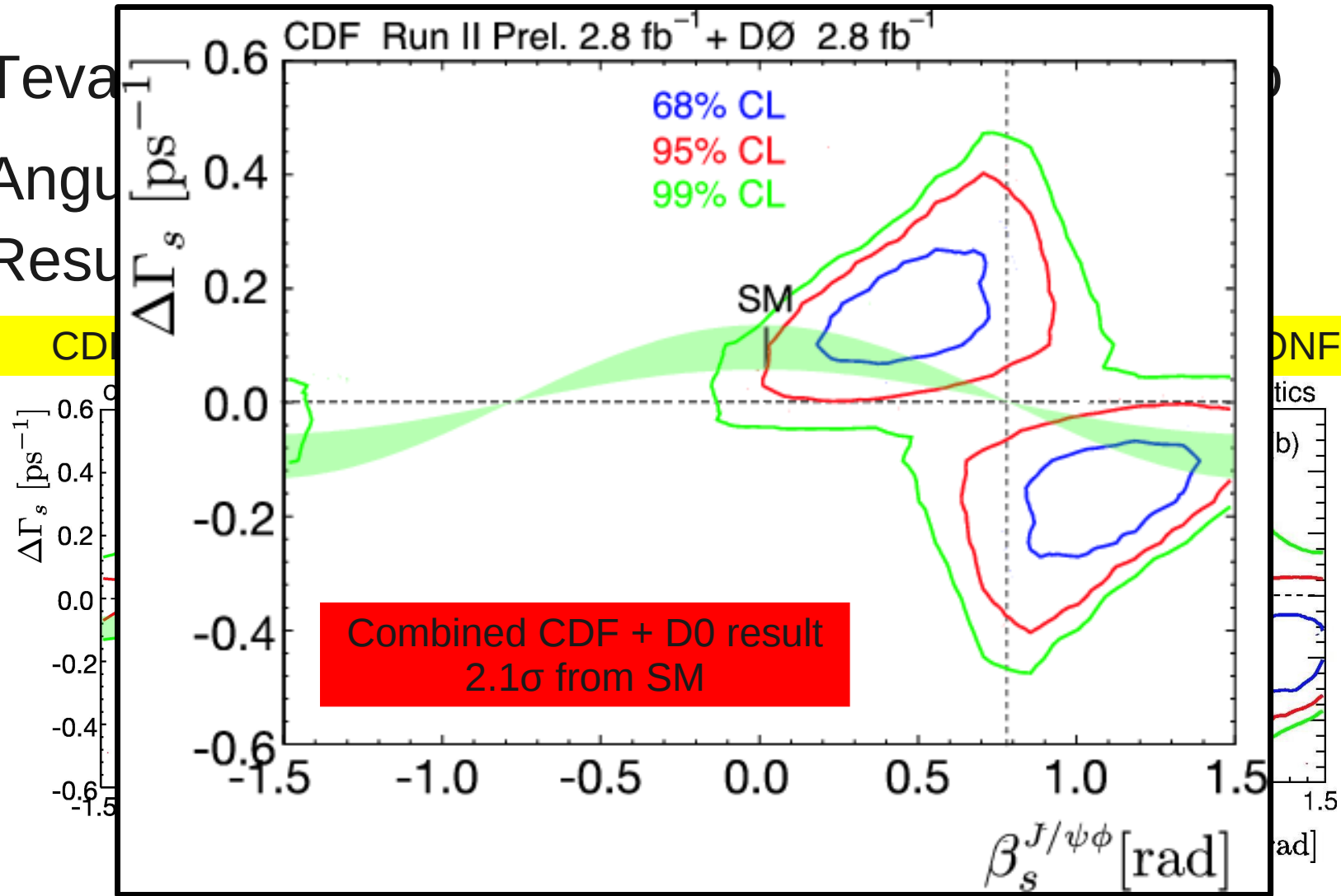


D0 5928-CONF



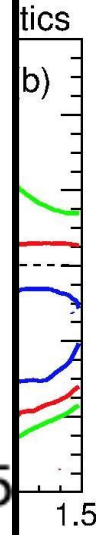
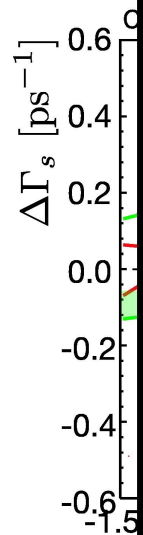
# $B_s$ oscillations and CP violation

- Teva
- Angu
- Resu

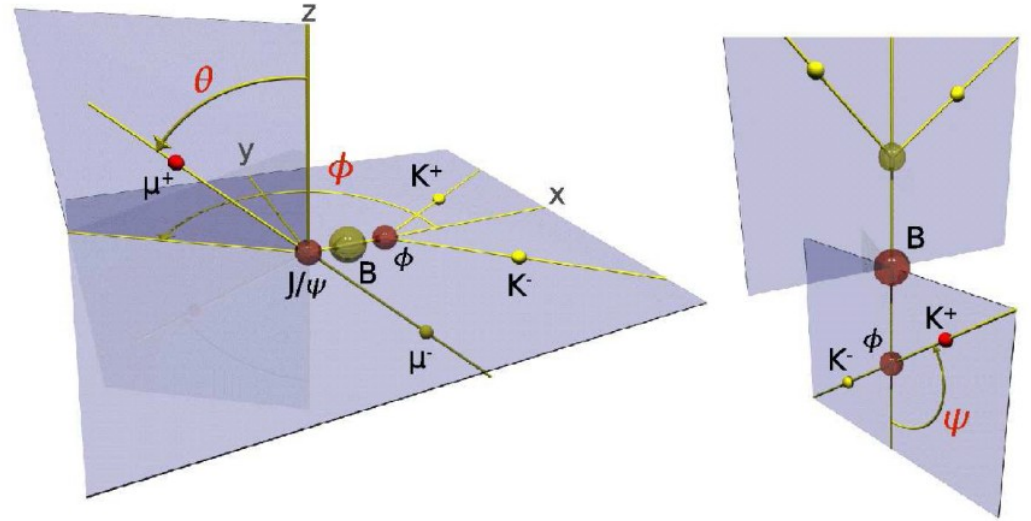


CD

DNF



# Complications of $B_s \rightarrow J/\psi\phi$

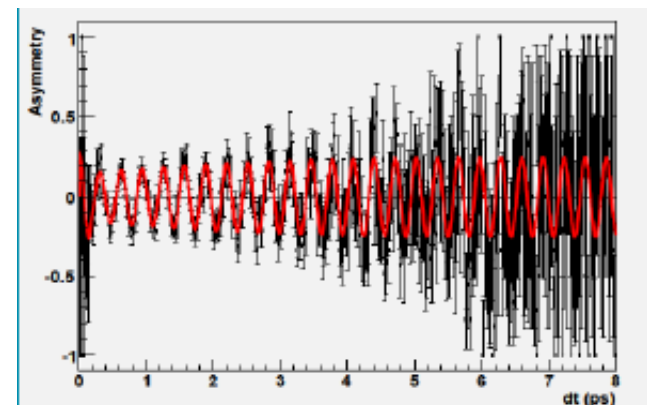
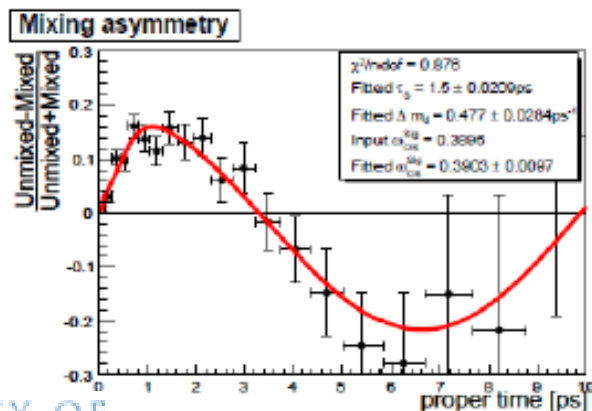
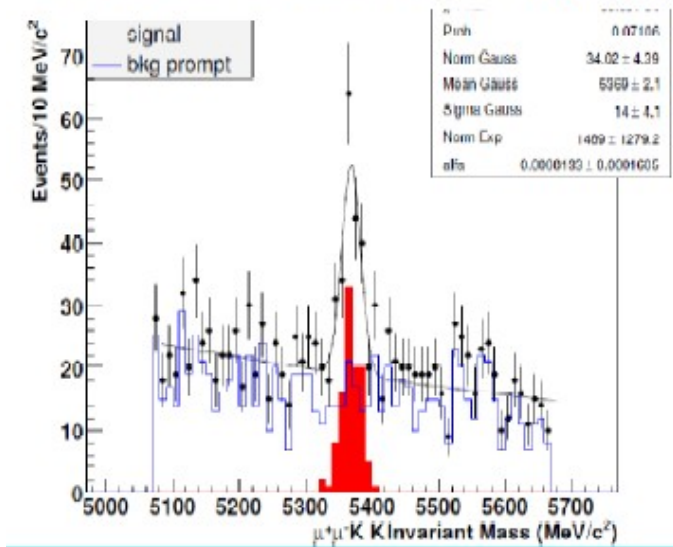


- VV final state
  - three helicity amplitudes – mixture of CP-even and CP-odd
  - can be disentangled using angular & time-dependent distributions → **additional sensitivity**
- $\Phi$  width not negligible
  - contribution from KK S-wave can be handled in the analysis
  - $B_s \rightarrow J/\psi f_0$ ,  $f_0 \rightarrow \pi\pi$  can also be studied (CP-eigenstate)

# Aspects of $B_s \rightarrow J/\psi\phi$ analysis

- Selection
  - $O(10^5)$  signal events/2/fb
  - High efficiency for dimuon trigger
  - Largest background from prompt  $J/\psi$
- Use of control samples
  - Calibrate and validate resolution, acceptance, tagging

Full MC: inclusive  $J/\psi(\mu\mu)$



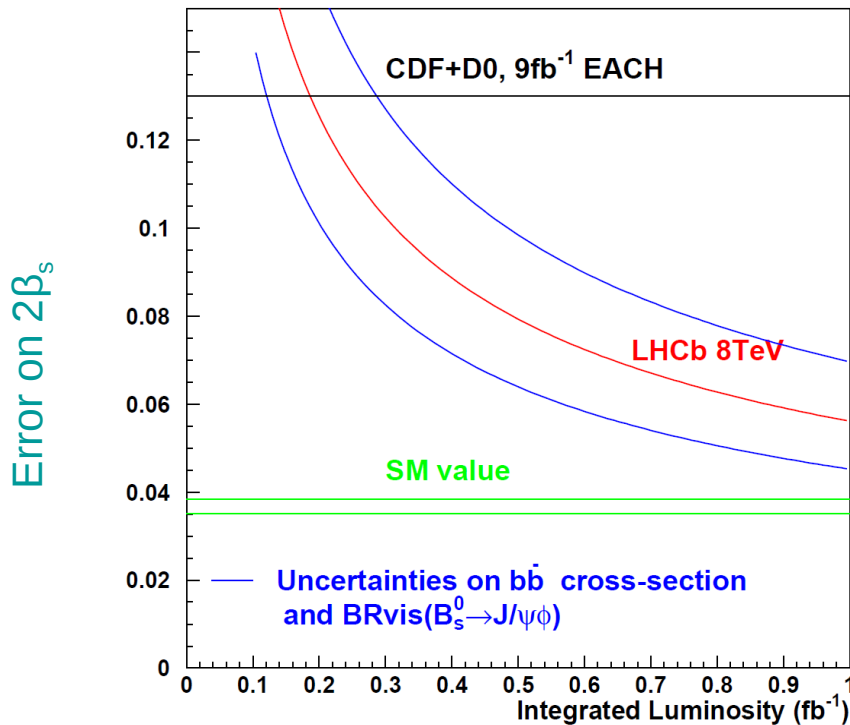
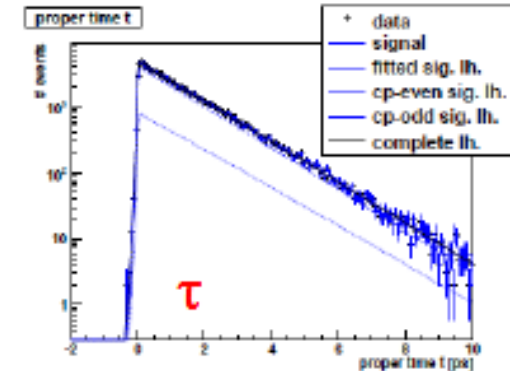
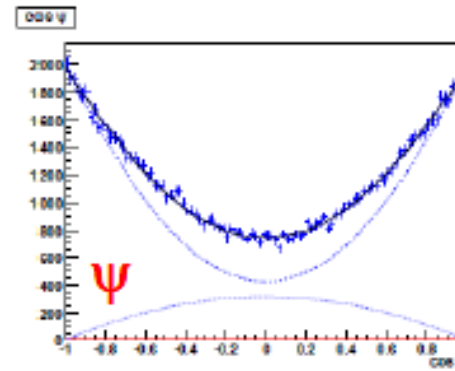
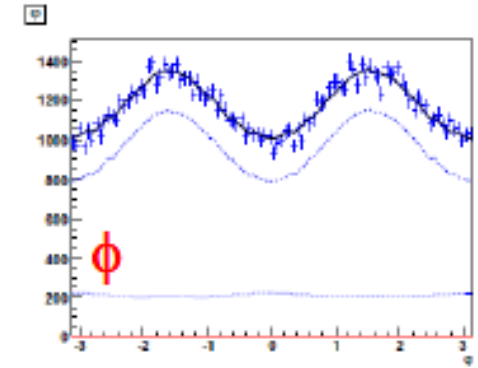
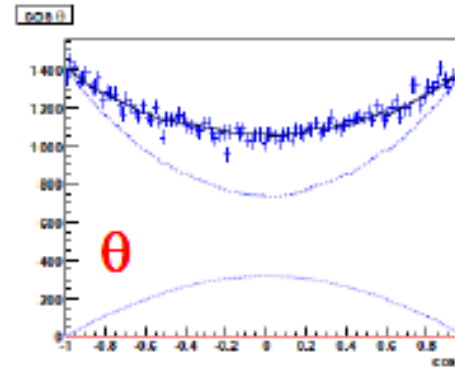
Note  $\Delta m_s$  oscillations

$$B_d \rightarrow J/\psi K^0$$

$$B_s \rightarrow D_s \pi$$

# Sensitivity to $\beta_s$

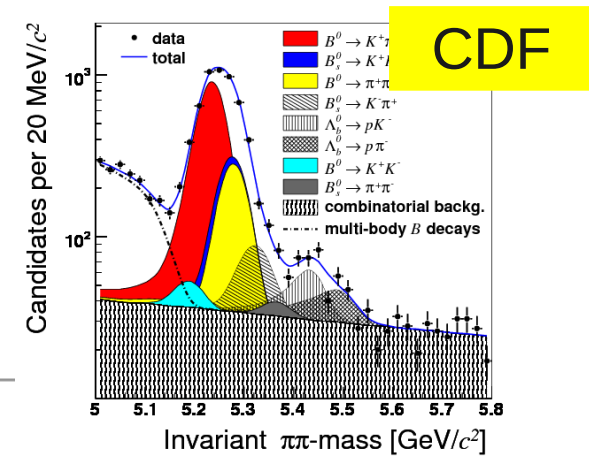
Expect  $\sigma(2\beta_s) \approx 0.03$  from 2/fb



If true  $\beta_s$  value is at current Tevatron central value, can measure it with 200/pb (even with reduced LHC energy)

# Alternative approaches to $\gamma$ , $\beta$ and $\beta_s$

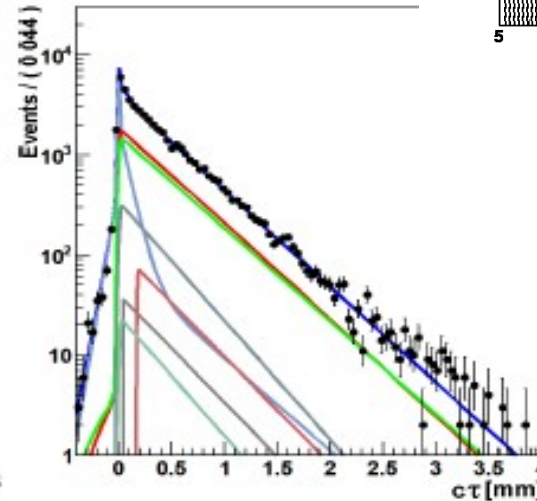
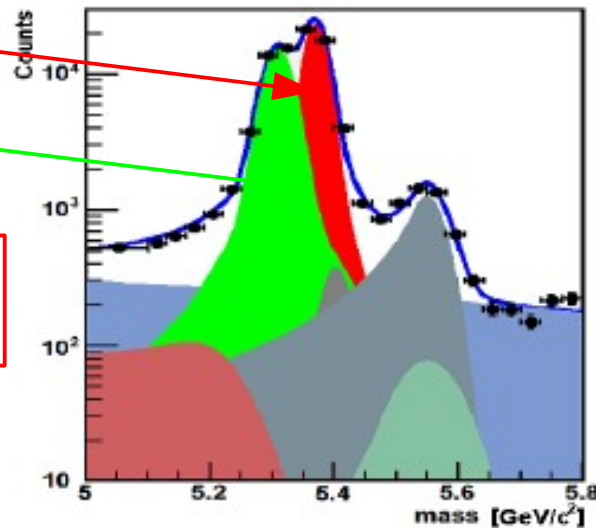
- All methods discussed so far use  $b \rightarrow c$  decay transitions
- Search for new physics** by testing if  $b \rightarrow s$  loop dominated decays give consistent results
- $\gamma$ :  $B_s \rightarrow hh'$  decays
  - eg.  $B_s \rightarrow K^+K^-$  cf.  $B_d \rightarrow \pi^+\pi^-$  via U-spin



$B_s \rightarrow K^+K^-$

$B_d \rightarrow K^+\pi^-$

kaon identification applied



# Alternative approaches to $\gamma$ , $\beta$ and $\beta_s$

- All methods discussed so far use  $b \rightarrow c$  decay transitions
- Search for new physics** by testing if  $b \rightarrow s$  loop dominated decays give consistent results

## $\beta$ and $\beta_s$

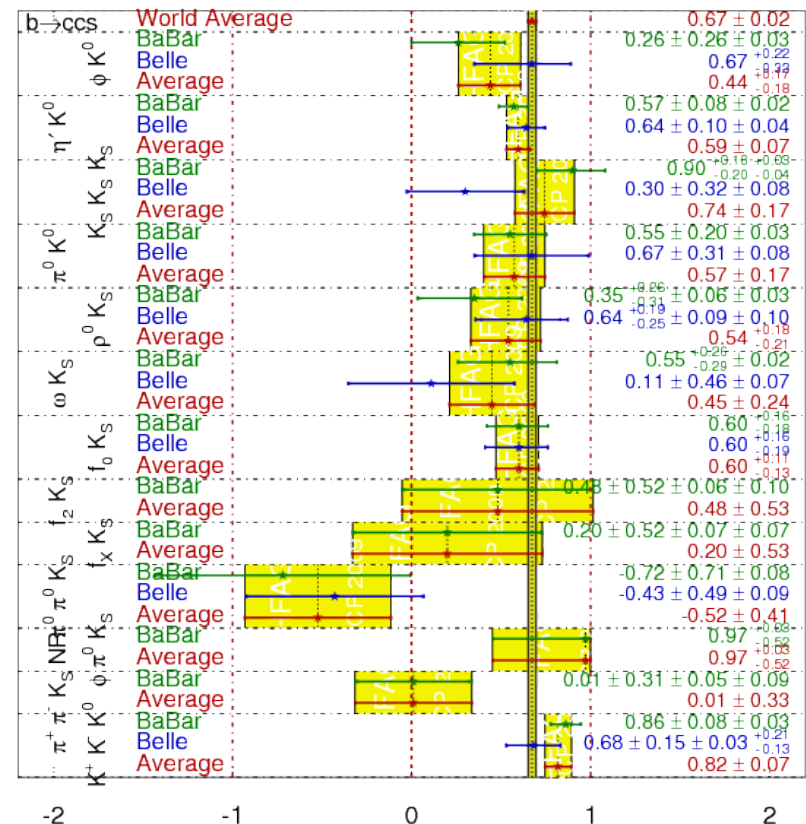
- Promising sensitivity for

$$- B_d \rightarrow \phi K_S$$

$$- B_s \rightarrow \phi\phi$$

$$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}}) \quad \text{HFAG}$$

FPCP 2009  
PRELIMINARY



# Conclusion

LHCb well placed to further our understanding of  
CP violation ...

... and, perhaps, to find new physics!



# $B_s \rightarrow J/\psi\phi$ formalism

Differential decay rate:

$$\frac{d^4\Gamma(B_s^0 \rightarrow J/\psi\phi)}{dt d\cos\theta d\varphi d\cos\psi} \equiv \frac{d^4\Gamma}{dt d\Omega} \propto \sum_{k=1}^6 h_k(t) f_k(\Omega)$$

$B_s$

$\bar{B}_s$

$k$	$h_k(t)$	$h_k(t)$	$f_k(\theta, \psi, \varphi)$
1	$ A_0(t) ^2$	$ \bar{A}_0(t) ^2$	$2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi)$
2	$ A_{\parallel}(t) ^2$	$ \bar{A}_{\parallel}(t) ^2$	$\sin^2 \psi (1 - \sin^2 \theta \sin^2 \varphi)$
3	$ A_{\perp}(t) ^2$	$ \bar{A}_{\perp}(t) ^2$	$\sin^2 \psi \sin^2 \theta$
4	$\Im\{A_{\parallel}^*(t)A_{\perp}(t)\}$	$\Im\{\bar{A}_{\parallel}^*(t)\bar{A}_{\perp}(t)\}$	$-\sin^2 \psi \sin 2\theta \sin \varphi$
5	$\Re\{A_0^*(t)A_{\parallel}(t)\}$	$\Re\{\bar{A}_0^*(t)\bar{A}_{\parallel}(t)\}$	$\frac{1}{\sqrt{2}} \sin 2\psi \sin^2 \theta \sin 2\varphi$
6	$\Im\{A_0^*(t)A_{\perp}(t)\}$	$\Im\{\bar{A}_0^*(t)\bar{A}_{\perp}(t)\}$	$\frac{1}{\sqrt{2}} \sin 2\psi \sin 2\theta \cos \varphi$

$A_0(0) \rightarrow$  CP even  
 $A_{\parallel}(0) \rightarrow$  CP even  
 $A_{\perp}(0) \rightarrow$  CP odd

$\pm$  signs differ for  $B_s$  and  $\bar{B}_s$

$$|\bar{A}_0(t)|^2 = |\bar{A}_0(0)|^2 e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - \cos\Phi \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - \sin\Phi \sin(\Delta m_s t) \right],$$

$$|\bar{A}_{\parallel}(t)|^2 = |\bar{A}_{\parallel}(0)|^2 e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - \cos\Phi \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - \sin\Phi \sin(\Delta m_s t) \right],$$

$$|\bar{A}_{\perp}(t)|^2 = |\bar{A}_{\perp}(0)|^2 e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + \cos\Phi \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + \sin\Phi \sin(\Delta m_s t) \right],$$

$$\Im\{\bar{A}_{\parallel}^*(t)\bar{A}_{\perp}(t)\} = |\bar{A}_{\parallel}(0)||\bar{A}_{\perp}(0)| e^{-\Gamma_s t} \left[ -\cos(\delta_{\perp} - \delta_{\parallel}) \sin\Phi \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - \sin(\delta_{\perp} - \delta_{\parallel}) \cos(\Delta m_s t) + \cos(\delta_{\perp} - \delta_{\parallel}) \cos\Phi \sin(\Delta m_s t) \right],$$

$$\Re\{\bar{A}_0^*(t)\bar{A}_{\parallel}(t)\} = |\bar{A}_0(0)||\bar{A}_{\parallel}(0)| e^{-\Gamma_s t} \cos\delta_{\parallel} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - \cos\Phi \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - \sin\Phi \sin(\Delta m_s t) \right] \text{ and}$$

$$\Im\{\bar{A}_0^*(t)\bar{A}_{\perp}(t)\} = |\bar{A}_0(0)||\bar{A}_{\perp}(0)| e^{-\Gamma_s t} \left[ -\cos\delta_{\perp} \sin\Phi \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - \sin\delta_{\perp} \cos(\Delta m_s t) + \cos\delta_{\perp} \cos\Phi \sin(\Delta m_s t) \right].$$