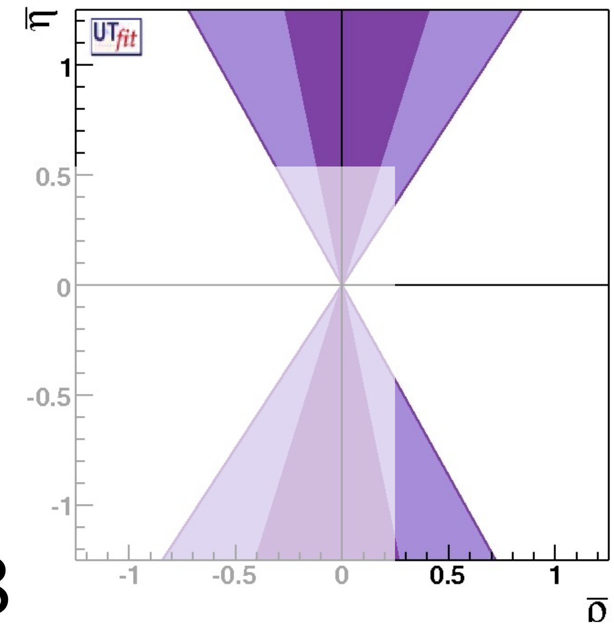
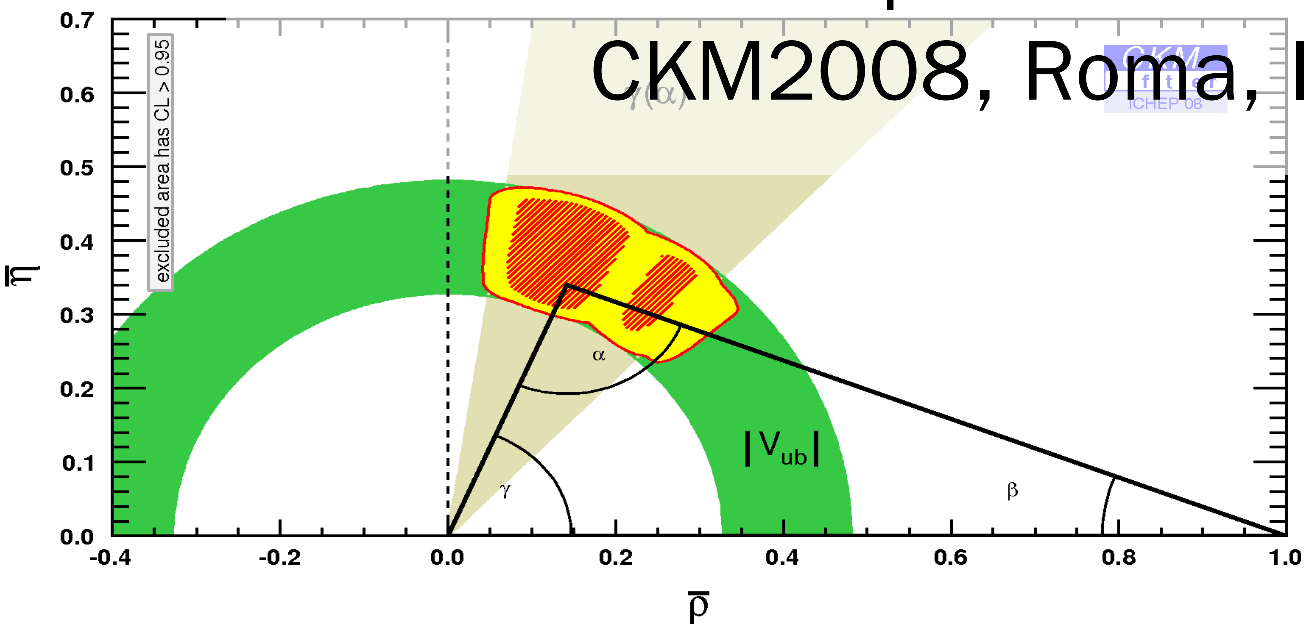
The Unitarity Triangle Angle γ
An Overview



Tim Gershon
University of Warwick

11th September 2008

CKM2008, Roma, Italy



Content

- Why measure γ?
  - How precisely do we need to measure it?
- How to measure γ?
  - The theoretically pristine $B \rightarrow DK$ approach
- Where do we stand & what remains to be done?
  - Introduce/provoke talks in the working group

Why Measure γ?

- Name of the game in flavour physics is to

overconstrain the CKM matrix

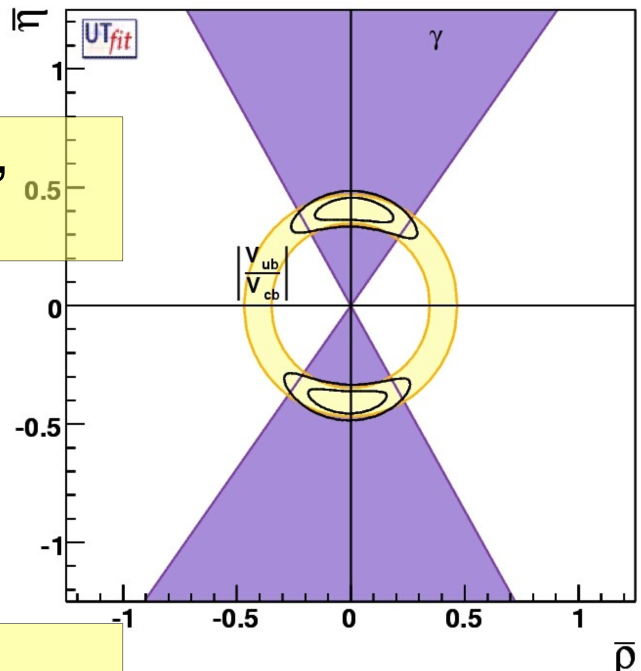
measure fundamental parameters constrain new physics effects

- Measure the 4 free parameters in various ways
  - CP conserving $\{|V_{us}|, |V_{cb}|, |V_{td}|, |V_{ub}|\}$
  - CP violating $\{\epsilon_K, \varphi_s, \beta, \gamma\}$
  - Tree level $\{\ldots, \ldots, |V_{ub}|, \gamma\}$
  - Loop processes $\{\ldots, \ldots, |V_{td}|, \beta\}$

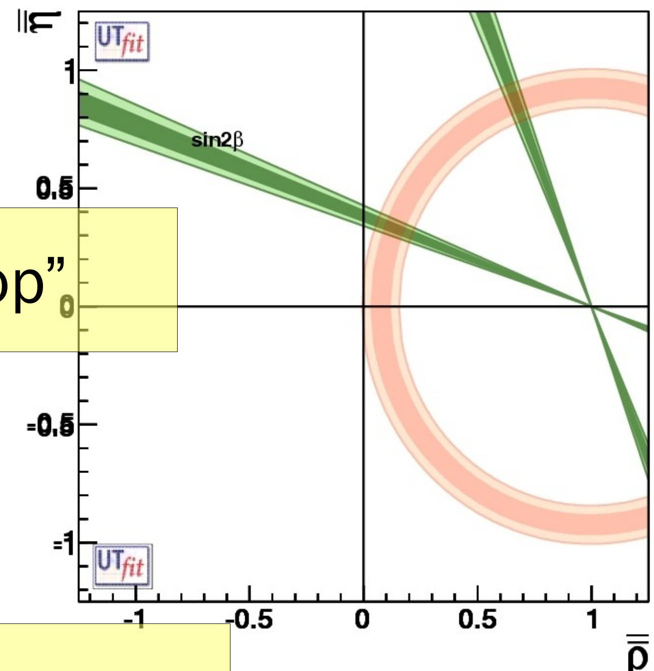
... many other possible combinations

Unitarity Triangle Comparisons

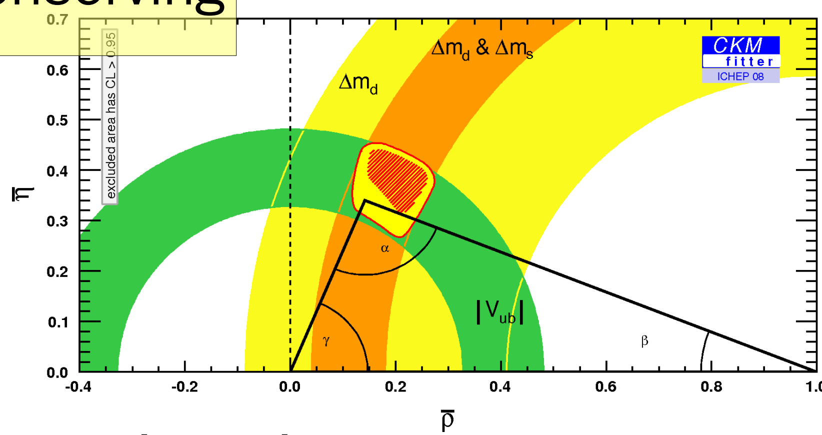
“tree”



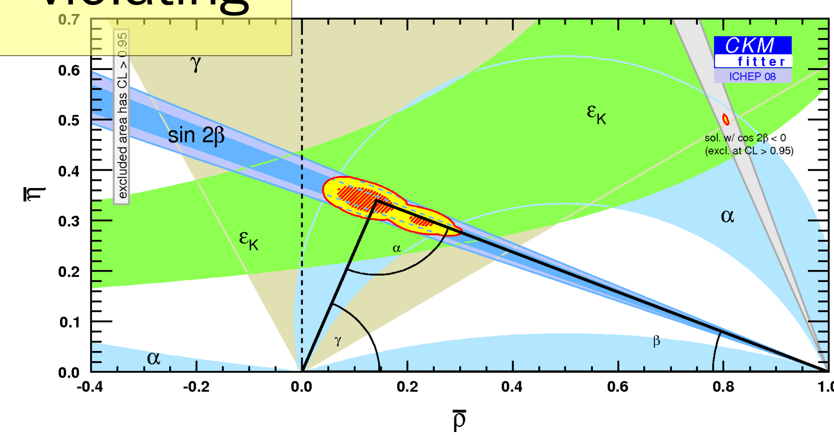
“loop”



CP conserving



CP violating



Importance of γ

- γ plays a unique role in flavour physics

the only CP violating parameter that can be measured through tree decays (*)

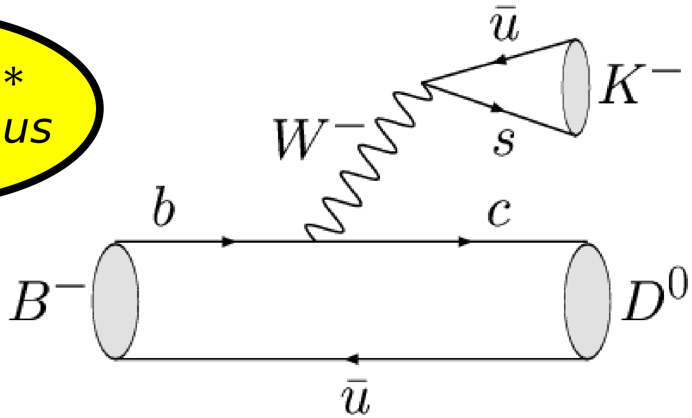
(*) more-or-less

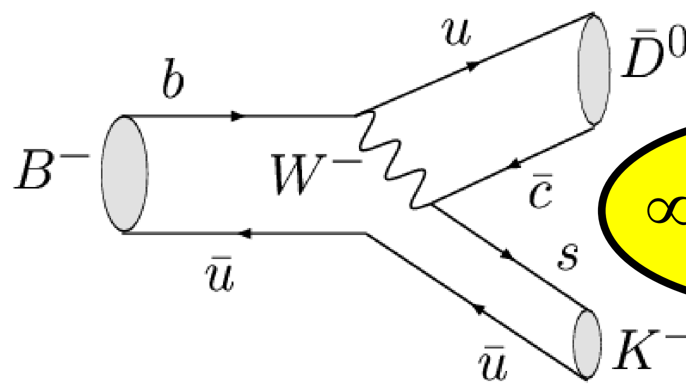
- A benchmark Standard Model reference point
  - doubly important after New Physics is observed
- How precise is precise enough?
  - 10% ⓧ At 3 sigma hardly exclude anything
  - 1% ☆ Seems the right level to test NP
  - 0.1% ⓧ Good luck if you can get the funding …

How To Measure γ

- Focus on theoretically pristine measurement
  - Interference between

$\propto V_{cb} V^{*}_{us}$





$\propto V_{ub} V^{*}_{cs}$

- colour allowed
- final state contains D^0

- colour suppressed
- final state contains $\bar{D}^0$

Relative magnitude of suppressed amplitude is r_B

Relative weak phase is –γ, relative strong phase is δ_B

One Method, Many Modes

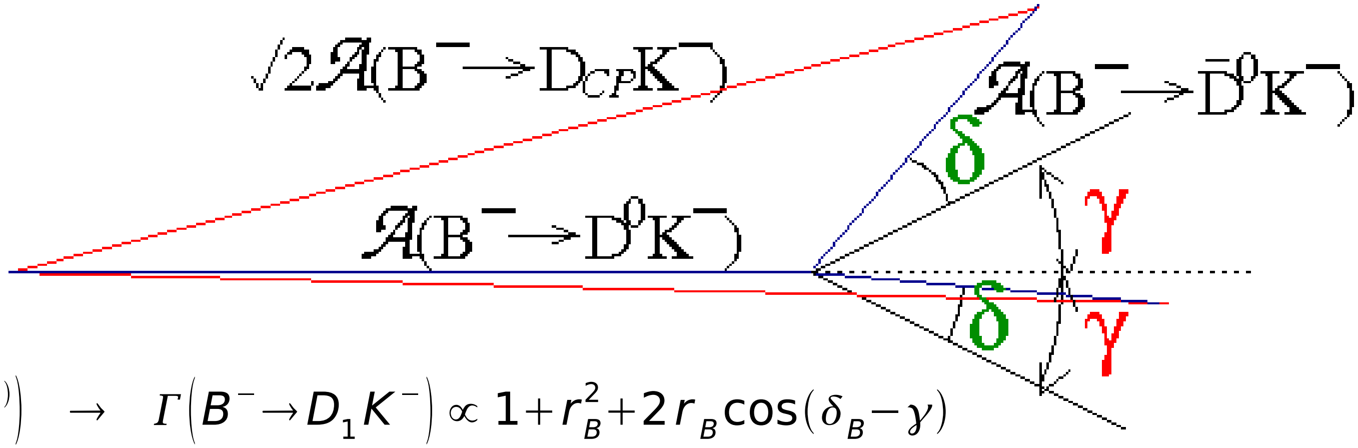
- $B \to DK$ with any D decay mode that is accessible to both D^0 and $\overline{D}^0$ is sensitive to γ
  - M.Gronau & D.Wyler, PLB 253, 483 (1991)
  - M.Gronau & D.London, PLB 265, 172 (1991)
  - D.Atwood, I.Dunietz and A.Soni, PRL 78, 3257 (1997); PRD 63, 036005 (2001)
- Different D decay modes in use
  - CP eigenstates (eg. K^+K^-, $K_S\pi^0$) "GLW"
  - Doubly-suppressed decays (eg. $K\pi$) "ADS"
  - Singly-suppressed decays (eg. KK^*) "GLS"
  - Three-body decays (eg. $K_S\pi^+\pi^-$) "GGSZ / Dalitz"
  - Other possibilities exist …

PRD 67, 071301 (2003)

PRD 68, 054018 (2003) & Belle

How It Works

- Consider D→CP eigenstates as an example



$$A\left(B^- \to D_1 K^-\right) \propto \frac{1}{\sqrt{2}}\left(1 + r_B e^{i(\delta_B - \gamma)}\right) \quad \to \quad \Gamma\left(B^- \to D_1 K^-\right) \propto 1 + r_B^2 + 2 r_B \cos(\delta_B - \gamma)$$

$$A\left(B^- \to D_2 K^-\right) \propto \frac{1}{\sqrt{2}}\left(1 - r_B e^{i(\delta_B - \gamma)}\right) \quad \to \quad \Gamma\left(B^- \to D_2 K^-\right) \propto 1 + r_B^2 - 2 r_B \cos(\delta_B - \gamma)$$

$$A\left(B^+ \to D_1 K^+\right) \propto \frac{1}{\sqrt{2}}\left(1 + r_B e^{i(\delta_B + \gamma)}\right) \quad \to \quad \Gamma\left(B^+ \to D_1 K^+\right) \propto 1 + r_B^2 + 2 r_B \cos(\delta_B + \gamma)$$

$$A\left(B^+ \to D_2 K^+\right) \propto \frac{1}{\sqrt{2}}\left(1 - r_B e^{i(\delta_B + \gamma)}\right) \quad \to \quad \Gamma\left(B^+ \to D_2 K^+\right) \propto 1 + r_B^2 - 2 r_B \cos(\delta_B + \gamma)$$

In practice, measure asymmetries and ratios where possible to reduce systematics

Theoretically Pristine

- Try to draw a diagram for $B^- \to DK^-$ with a different weak phase to those on previous slide
  - if you succeed, estimate effect on γ extraction
- Largest effects due to
  - charm mixing
  - charm CP violation

  } negligible PRD 72 031501 (2005)
  - B mixing ($\Delta\Gamma$) for neutral B decays

  $\Rightarrow$ can be controlled PRD 69 113003 (2004), PLB 649 61 (2007)
- BUT, must obtain hadronic parameters from data
  - Includes (r_B, δ_B) as well as D decay model

Even more modes ...

- As well as different D decays, can use
  - Different B decays (DK, D^*K, DK^*)
    - different hadronic factors (r_B, δ_B) for each
    - benefit from $D^* \rightarrow D\pi^0$ & $D^* \rightarrow D\gamma$
    - some care required due to K^* width

PRD 70, 091503 (2004)

PLB 557, 198 (2003)

  - Neutral B decays
    - different hadronic factors (r_B, δ_B) – larger r_B
- Useful rule-of-thumb: NIMSBHO principle (A.Soffer)
  - Not Inherently More Sensitive But Helps Overall
- Best sensitivity by combining all measurements

Where Do We Stand?

Total data sizes to date: BABAR 465 M BB, Belle ~800 M BB, CDF ~4/fb

only charged B results shown	$B \to DK$	$B \to D^*K$: $D^* \to D\pi^0$	$B \to D^*K$: $D^* \to D\gamma$	$B \to DK^*$
GLW	BABAR (382M $B\bar{B}$) Belle (275M $B\bar{B}$) CDF (1.1 fb^{-1})	BABAR (383M $B\bar{B}$) Belle (275M $B\bar{B}$)		BABAR (379M $B\bar{B}$)
ADS ($K\pi$)	BABAR (232M $B\bar{B}$) Belle (657M $B\bar{B}$)	BABAR (232M $B\bar{B}$)	BABAR (232M $B\bar{B}$)	BABAR (379M $B\bar{B}$)
ADS ($K\pi\pi^0$)	BABAR (226M $B\bar{B}$)			
$D \to K_S\pi^+\pi^-$	BABAR (383M $B\bar{B}$) Belle (657M $B\bar{B}$)	BABAR (383M $B\bar{B}$) Belle (657M $B\bar{B}$)	BABAR (383M $B\bar{B}$)	BABAR (383M $B\bar{B}$) Belle (386M $B\bar{B}$)
$D \to K_SK^+K^-$	BABAR (383M $B\bar{B}$)	BABAR (383M $B\bar{B}$)	BABAR (383M $B\bar{B}$)	
$D \to \pi^+\pi^-\pi^0$	BaBar (324M $B\bar{B}$)			

NEW today!
see V.Tisserand's talk

- Huge & impressive effort
- Still much to be done

How to Determine D Decay Models

- To extract γ need to understand D decays
  - GLW modes
    - need constraints on direct CP violation in D decay
  - ADS modes
    - r_D – can be measured from flavour-tagged D mesons
    - δ_D – need CP tagged D mesons
    - multibody modes have additional coherence parameter
  - Dalitz modes
    - model dependent – need model, including phase variation
    - model independent – need c_i and s_i parameters

$\psi(3770) \rightarrow DD$ (CLEOc & BESIII)

NEW tomorrow – see J.Libby's talk

NEW tomorrow – see J.Rademacker's talk

Model-Independent Dalitz Measurements

- Revert to a "counting" analysis by binning the DP
  - Model not needed … BUT
  - Best to use model to define bins

PRD 68, 054018 (2003)
EPJC47, 347 (2006)
& arXiv:0801.0840

- Measure cos and sin of average $D^0 – \bar{D}^0$ strong phase difference in each bin
  - c_i from $K_S\pi^+\pi^-$ *vs.* CP tags
  - s_i (and c_i) from $K_S\pi^+\pi^-$ *vs.* $K_S\pi^+\pi^-$

$\psi(3770) \to D\bar{D}$
(CLEOc & BESIII)

- How to implement this?
  - Define common model & binning; expts count events per bin
  - OR Expts make data available in common format

Alternatives to $B \to DK$

- $B \to D^{(*)}\pi$ measures $\sin(2\beta+\gamma)$ (interference between decays with and without mixing) PLB 427, 179 (1998)
  - r_B very small (~0.02)
    - small modulation on a large signal $\Rightarrow$ systematics!
    - cannot extract r_B (only r_B^2) $\Rightarrow$ need input, eg. SU(3)
- $B \to D^*\rho$ similar but can get r_B from interference between helicity amplitudes PRL 80, 3706 (1998)
  - but now you need to deal with slow π/π^0 related systematics
- $B_s \to D_s K$ similar (measures $\sin(\varphi_s+\gamma)$) but now r_B is larger
  - a promising channel for LHCb NPB 671, 459 (2003)

Definitions of parameters

- GLW:

$$A_{CP\pm} = \frac{\Gamma\left(B^-\to D_\pm K^-\right)-\Gamma\left(B^+\to D_\pm K^+\right)}{\Gamma\left(B^-\to D_\pm K^-\right)+\Gamma\left(B^+\to D_\pm K^+\right)} = \frac{\pm 2 r_B \sin(\delta_B)\sin(\gamma)}{1+r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)}$$

$$R_{CP\pm} = \frac{\Gamma\left(B^-\to D_\pm K^-\right)+\Gamma\left(B^+\to D_\pm K^+\right)}{\Gamma\left(B^-\to D_{fav} K^-\right)+\Gamma\left(B^+\to D_{fav} K^+\right)} = 1+r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)$$

- ADS:

$$A_{ADS} = \frac{\Gamma\left(B^-\to D_{ADS} K^-\right)-\Gamma\left(B^+\to D_{ADS} K^+\right)}{\Gamma\left(B^-\to D_{ADS} K^-\right)+\Gamma\left(B^+\to D_{ADS} K^+\right)} = \frac{2 r_B r_D \sin(\delta_B+\delta_D)\sin(\gamma)}{r_B^2+r_D^2+2 r_B r_D \cos(\delta_B+\delta_D)\cos(\gamma)}$$

$$R_{ADS} = \frac{\Gamma\left(B^-\to D_{ADS} K^-\right)+\Gamma\left(B^+\to D_{ADS} K^+\right)}{\Gamma\left(B^-\to D_{fav} K^-\right)+\Gamma\left(B^+\to D_{fav} K^+\right)} = r_B^2+r_D^2+2 r_B r_D \cos(\delta_B+\delta_D)\cos(\gamma)$$

- Dalitz:

$$x_+ = r_B \cos(\delta_B+\gamma) \qquad x_- = r_B \cos(\delta_B-\gamma)$$
$$y_+ = r_B \sin(\delta_B+\gamma) \qquad y_- = r_B \sin(\delta_B-\gamma)$$

- Notes
  - Dalitz parameters (x_+,y_+), (x_-, y_-) from B^+, B^- independently
  - GLW-Dalitz relation: $x_\pm = \left(R_{CP+}(1\mp A_{CP+})-R_{CP-}(1\mp A_{CP-})\right)/4$
  - Additional coherence factors needed for DK^* or $D\to K\pi\pi^0$

Putting It All Together

- Evidence for CP violation in both GLW & Dalitz

Mode	Experiment	A_{CP+}	A_{CP-}	R_{CP+}	R_{CP-}	Reference
$D_{CP}K^-$	BaBar N(BB)=382M	0.27 ± 0.09 ± 0.04	−0.09 ± 0.09 ± 0.02	1.06 ± 0.10 ± 0.05	1.03 ± 0.10 ± 0.05	PRD 77 (2008) 111102
	Belle N(BB)=275M	0.06 ± 0.14 ± 0.05	−0.12 ± 0.14 ± 0.05	1.13 ± 0.16 ± 0.08	1.17 ± 0.14 ± 0.14	PRD 73, 051106 (2006)
	CDF	0.39 ± 0.17 ± 0.04	-	1.30 ± 0.24 ± 0.12	-	ICHEP 2008 preliminary
	Average	0.24 ± 0.07 χ^2/ndf = 2.3/2 (CL=0.32 ⇒ 1.0σ)	−0.10 ± 0.08 χ^2 = 0.03 (CL=0.86 ⇒ 0.2σ)	1.10 ± 0.09 χ^2/ndf = 0.7/2 (CL=0.70 ⇒ 0.4σ)	1.06 ± 0.10 χ^2 = 0.4 (CL=0.54 ⇒ 0.6σ)	HFAG

GLW results can be translated into $x_+ = -0.082 \pm 0.045$, $x_- = 0.103 \pm 0.045$, $r_B^2 = 0.08 \pm 0.07$

Mode	Experiment	x+	y+	x-	y-	Correlation	Reference
DK^- $D\to K_S\pi^+\pi^-$ & $D\to K_SK^+K^-$	BaBar N(BB)=383M	−0.067 ± 0.043 ± 0.014 ± 0.011	−0.015 ± 0.055 ± 0.006 ± 0.008	0.090 ± 0.043 ± 0.015 ± 0.011	0.053 ± 0.056 ± 0.007 ± 0.015	(stat) (syst) (model)	PRD 78 (2008) 034023
	Belle N(BB)=657M	−0.107 ± 0.043 ± 0.011 ± 0.055	−0.067 ± 0.059 ± 0.018 ± 0.063	0.105 ± 0.047 ± 0.011 ± 0.064	0.177 ± 0.060 ± 0.018 ± 0.054	(stat) (model)	arXiv:0803.3375
	Average No model error	−0.087 ± 0.032	−0.037 ± 0.041	0.104 ± 0.033	0.111 ± 0.042	(stat+syst)	HFAG correlated average χ^2 = 3.1/4 dof (CL=0.54 ⇒ 0.6σ)

Combining GLW & Dalitz gives $x_+ = -0.085 \pm 0.026$, $x_- = 0.103 \pm 0.027$

$x_- - x_+ = 0.189 \pm 0.037$ **5.1σ from zero**

Making Constraints on γ

- Previous discussion suggests we should get a fairly precise world average for γ

(at least, neglecting model uncertainties)

- However, extracting γ is non-trivial
  - simple trigonometry fails (beware non-Gaussian errors)
- Complicated statistical treatment is necessary
  - From Dalitz modes,

    PRD 78 (2008) 034023

    - BaBar obtain $\gamma = (76 \pm 22 \pm 5 \pm 5)°$ (from DK^-, D^*K^- & DK^{*-})

    arXiv:0803.3375

    - Belle obtain $\varphi_3 = (76\ ^{+12}_{-13} \pm 4 \pm 9)°$ (from DK^- & D^*K^-)

Concluding Questions

- Why has neither experiment published a combined constraint on γ from its B→DK measurements?
- What auxiliary measurements should be made?
  - eg. DK^* hadronic parameters in $DK_S\pi$ DP analysis?
- How can we solve the problem of model dependence in Dalitz plot analyses?
  - Will we reach necessary agreement to enable model independent analysis?
- How much data is needed before statistical issues in γ extraction become irrelevant?

$B \rightarrow DK^{*}$

- Following PLB 557 198 (2003)
- Suppressed & favoured amplitudes vary across $B \rightarrow DK\pi$ phase space in both magnitude and phase

$$A_{CP\pm} = \frac{\pm 2 r_B \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2 r_B \cos(\delta_B) \cos(\gamma)} \qquad R_{CP\pm} = 1 + r_B^2 \pm 2 r_B \cos(\delta_B) \cos(\gamma)$$

$$A_{CP\pm} = \frac{\pm 2 \kappa r_s \sin(\delta_s) \sin(\gamma)}{1 + r_s^2 \pm 2 \kappa r_s \cos(\delta_s) \cos(\gamma)} \qquad R_{CP\pm} = 1 + r_s^2 \pm 2 \kappa r_s \cos(\delta_s) \cos(\gamma)$$

$$r_s = \sqrt{\frac{\int_{K^*} |\overline{A}|^2 dPS}{\int_{K^*} |A|^2 dPS}} \qquad \kappa e^{i\delta_s} = \frac{\int_{K^*} |\overline{A}||A| e^{i(arg(\overline{A}) - arg(A))} dPS}{\sqrt{\int_{K^*} |\overline{A}|^2 dPS \int_{K^*} |A|^2 dPS}}$$

  - r_s, κ, δ_s depend on K^* selection
  - In $DK_S\pi^-$ do not expect (DK_S) or $(D\pi^-)$ resonances

$D \to K\pi\pi^0$, $D \to K\pi\pi\pi$

- Following PRD 68 033003 (2003)
- Suppressed & favoured amplitudes vary across phase space in both magnitude and phase
- Usual expressions get modified

$$A_{ADS} = \frac{2 r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)} \qquad R_{ADS} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)$$

$$A_{ADS} = \frac{2 r_B R_F r_D^F \sin(\delta_B + \delta_D^F)\sin(\gamma)}{r_B^2 + (r_D^F)^2 + 2 r_B R_F r_D \cos(\delta_B + \delta_D^F)\cos(\gamma)} \qquad R_{ADS} = r_B^2 + (r_D^F)^2 + 2 r_B R_F r_D \cos(\delta_B + \delta_D^F)\cos(\gamma)$$

$$r_D^F = \sqrt{\frac{\int_{PS} |\overline{A}|^2 dPS}{\int_{PS} |A|^2 dPS}} \qquad R_F e^{-i\delta_D^F} = \frac{\int_{PS} |\overline{A}||A| e^{-i(arg(\overline{A}) - arg(A))} dPS}{\sqrt{\int_{PS} |\overline{A}|^2 dPS \int_{PS} |A|^2 dPS}}$$

Coherence factor
measure from $\psi(3770) \to DD$

B→DK, D→πππ^0

- In typical Dalitz analysis, parameters (x_+, y_+), (x_-, y_-) determined independently from B^+, B^- samples
- However, imagine extreme example: D→$(XYZ)_{CP}$
  - DP distributions contain **no** sensitivity to γ
  - Rates & asymmetries **are** sensitivity to γ (as GLW)
- Parameter of "CP-specificity"

$$x_0 = -\int_{DP} \Re\left(A\overline{A}^*\right) dDP$$

  - x_0 = 0.850 for D→πππ^0
  - BaBar fit for $\rho_\pm = \left|z_\pm - x_0\right| \quad \theta_\pm = \tan^{-1}\left(\frac{\Im\left(z_\pm\right)}{\Re\left(z_\pm\right) - x_0}\right)$

PRL 99 (2007) 251801

$$z_\pm = x_\pm + iy_\pm$$