The Angles of
The Unitarity Triangle

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Seminar in University of Lancaster
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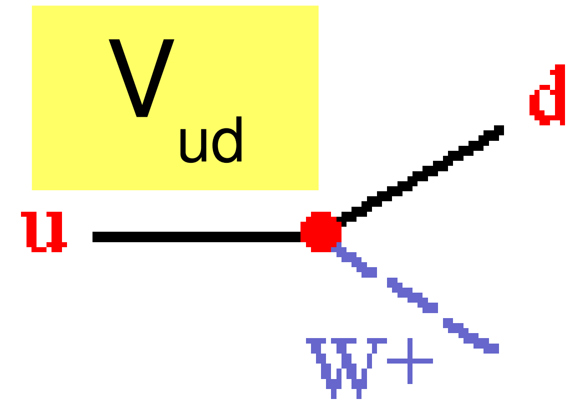
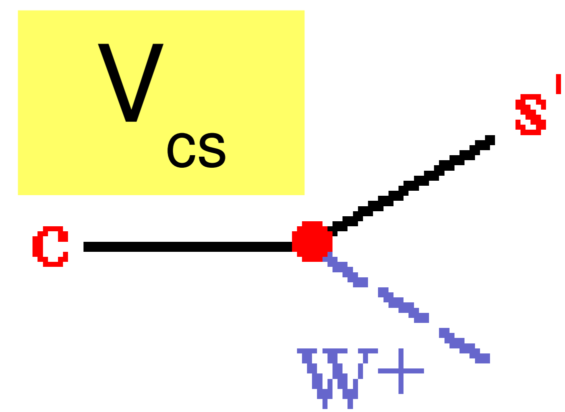
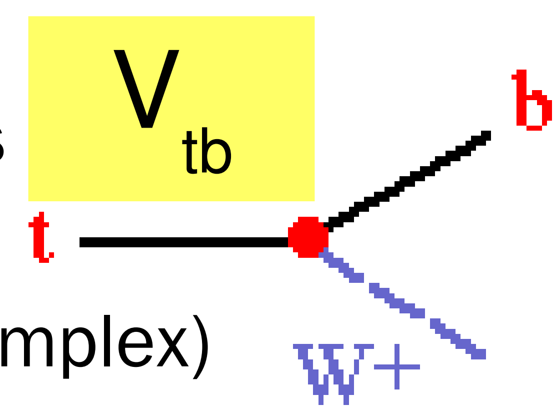
Partially based on “A Triangle That Matters”, Physics World, April 2007

CKM Matrix / KM mechanism

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$







- 3x3 matrix of complex numbers
  - ⇒ 18 parameters
- Unitary
  - ⇒ 9 parameters
- Quark fields absorb unobservable phases
  - ⇒ 4 parameters
  - ⇒ 3 mixing angles and 1 phase (V_{CKM} complex)

CP-Violation in the Renormalizable Theory of Weak Interaction

Progress of Theoretical Physics, Vol. 49 No. 2 pp. 652-657

Makoto Kobayashi and Toshihide Maskawa
Department of Physics, Kyoto University, Kyoto

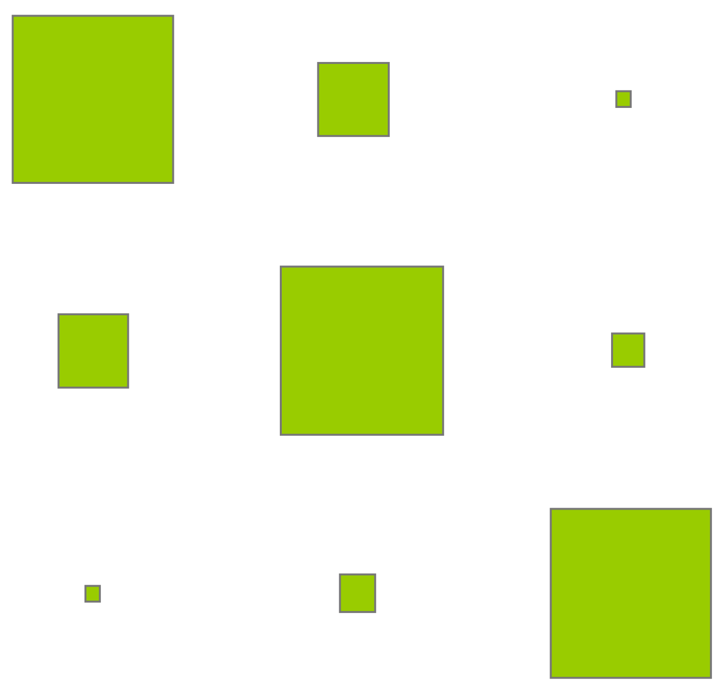
(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

Hierarchy in quark mixing

Wolfenstein parameterization – expansion in $\lambda = \sin\theta_c \sim 0.22$

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$



Unitarity

$$V^{\dagger}V = 1$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \qquad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \qquad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \qquad V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \qquad V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \qquad V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$$

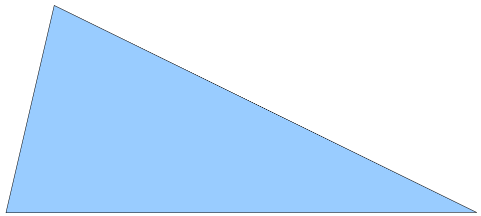
Unitarity triangles

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

$$\lambda \qquad \lambda \qquad \lambda^5$$

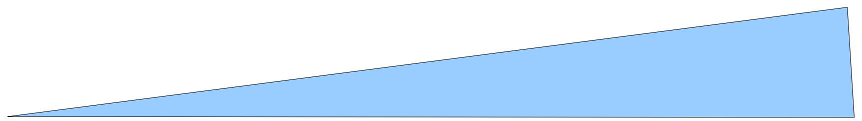
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\lambda^3 \qquad \lambda^3 \qquad \lambda^3$$



$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

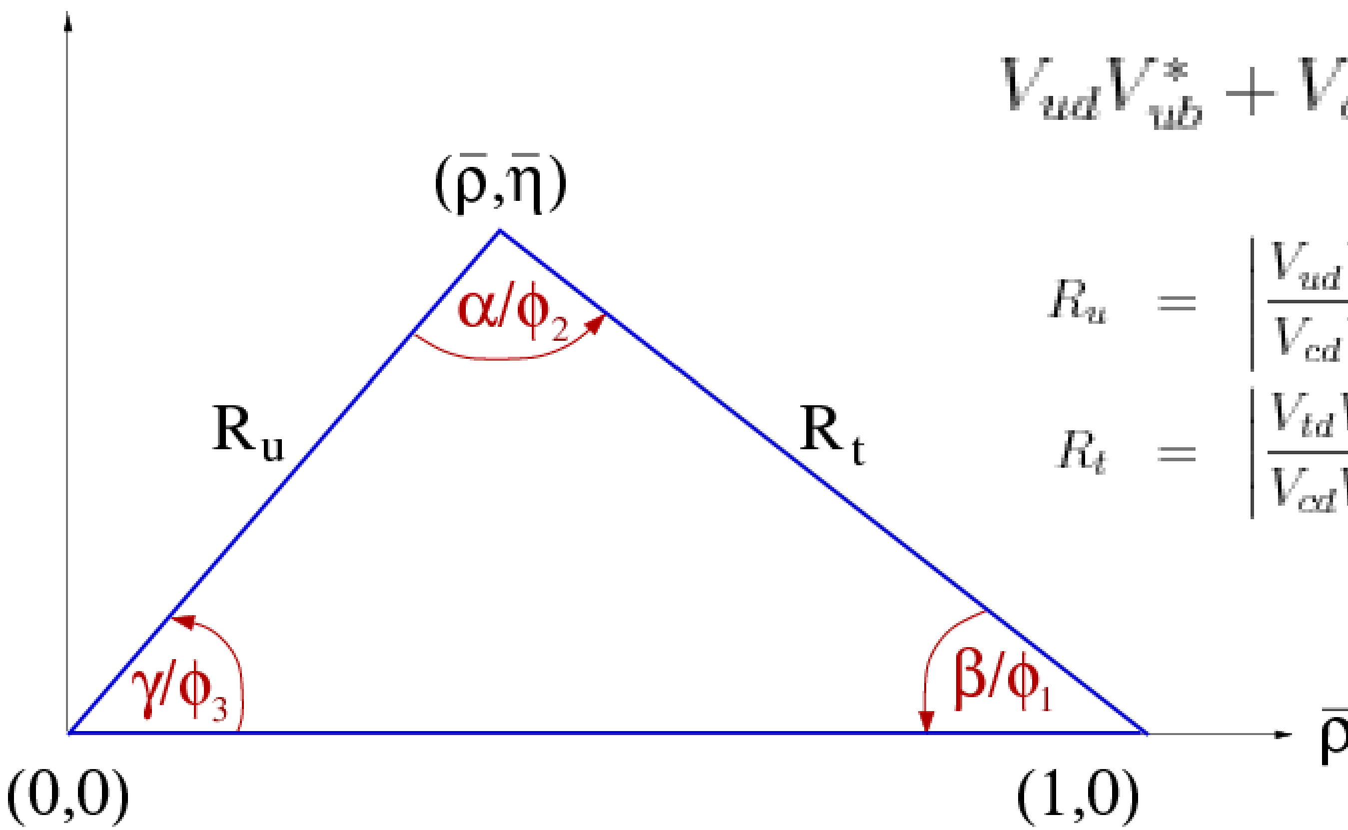
$$\lambda^4 \qquad \lambda^2 \qquad \lambda^2$$



DISCLAIMER : THESE ARE NOT TO SCALE!

The Unitarity Triangle

- Convenient method to illustrate (dis-)agreement of observables with CKM prediction



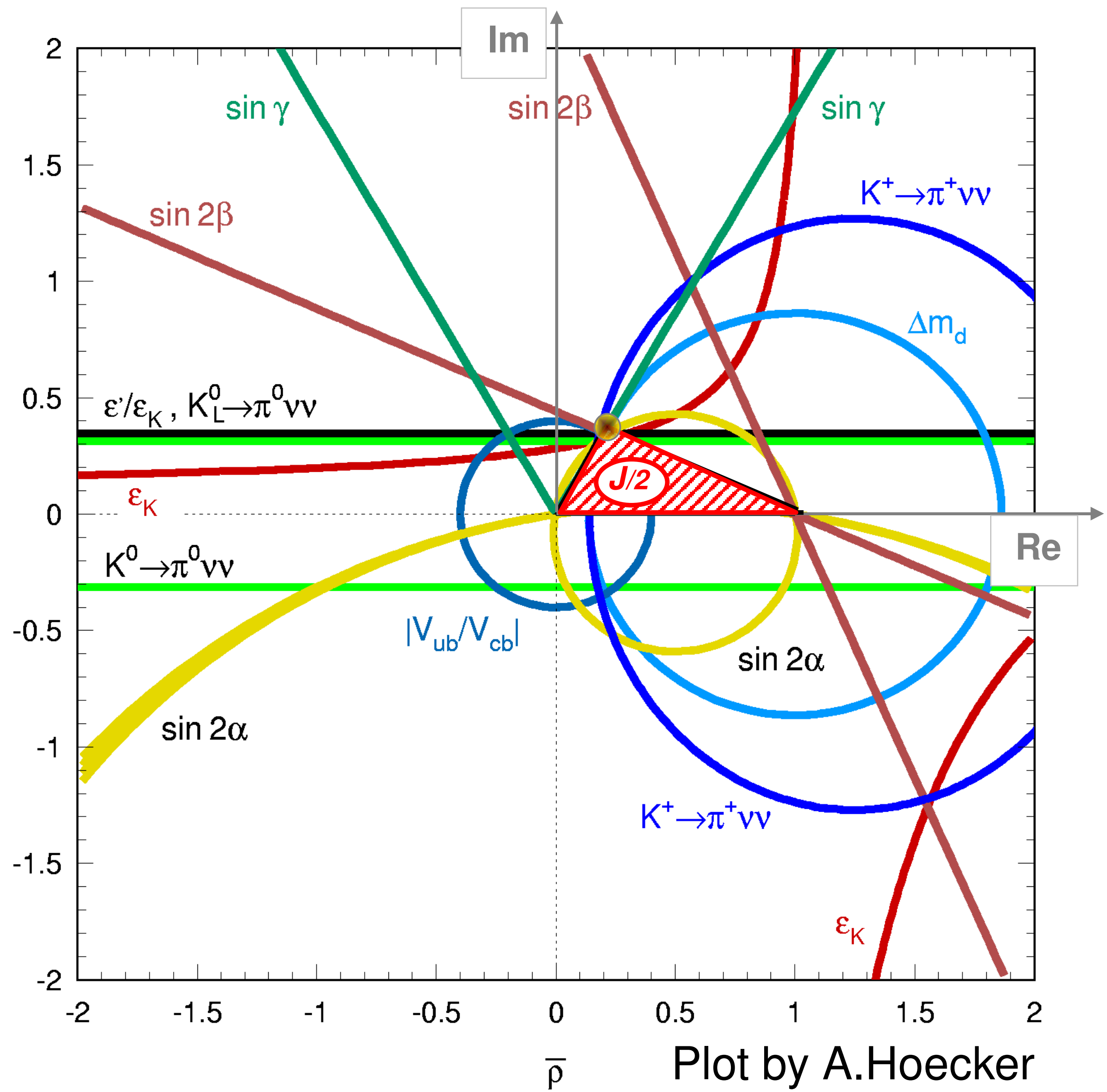
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$

$$R_u = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2},$$

$$R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}.$$

Predictive nature of KM mechanism

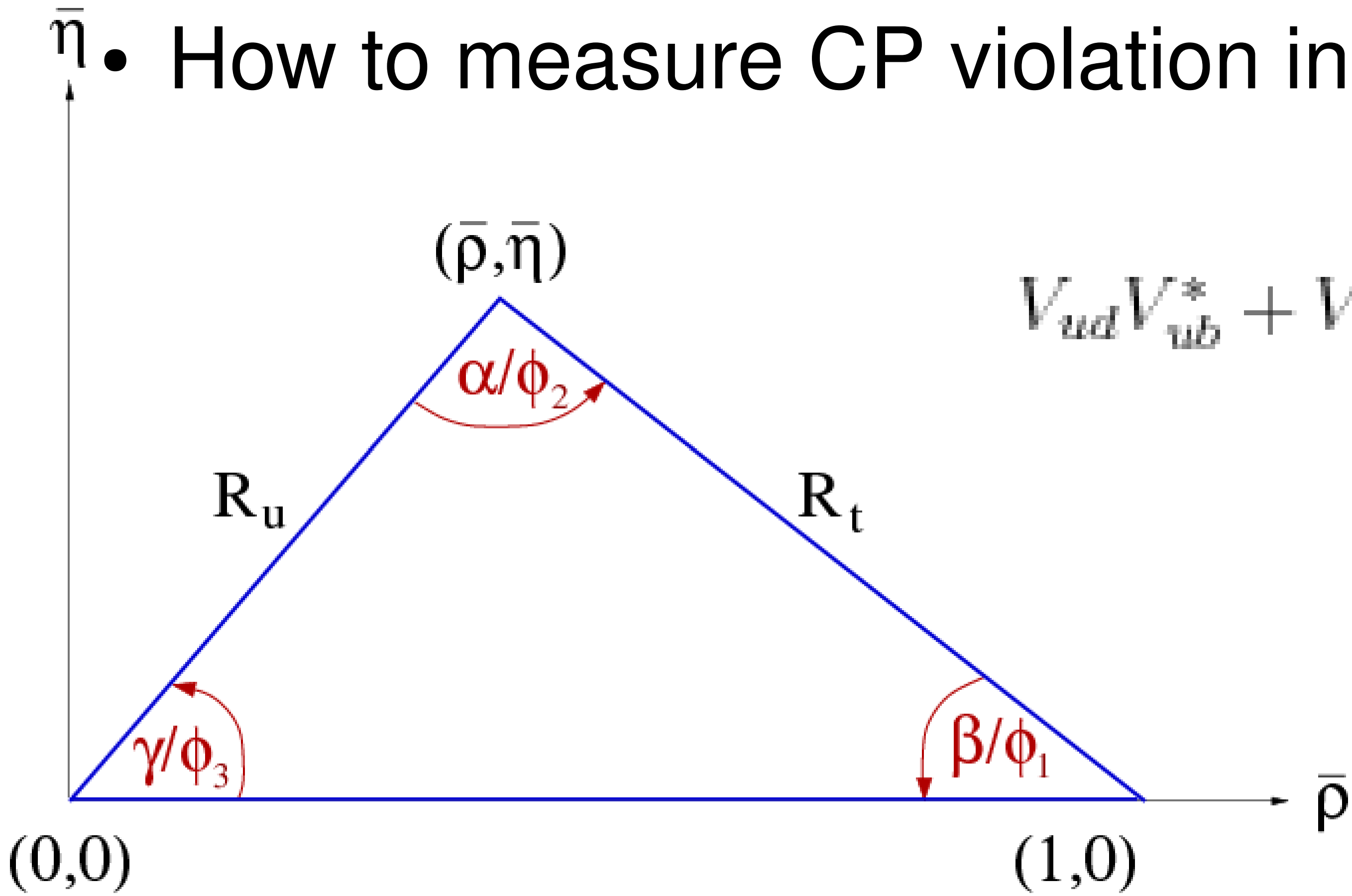
All measurements must agree



Plot by A.Hoecker

How to measure the angles?

- How to measure CP violation in the B system?

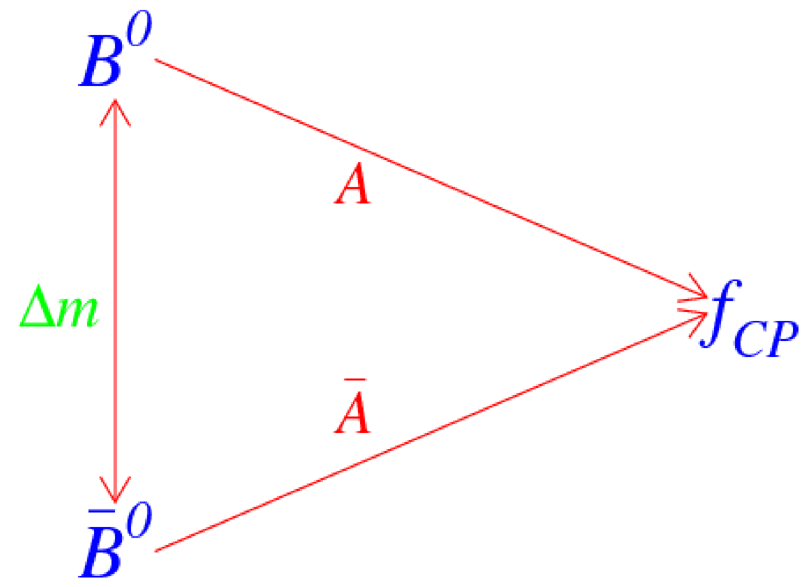


$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0,$$

$$\alpha \equiv \phi_2 = \arg\left[-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right], \quad \beta \equiv \phi_1 = \arg\left[-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right], \quad \gamma \equiv \phi_3 = \arg\left[-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right]$$

Categories of CP violation

- Consider decay of neutral particle to a CP eigenstate



$$\lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

$$\left|\frac{q}{p}\right| \neq 1$$

CP violation in mixing

$$\left|\frac{\overline{A}}{A}\right| \neq 1$$

CP violation in decay (direct CPV)

$$\Im\left(\frac{q}{p}\frac{\overline{A}}{A}\right) \neq 0$$

CP violation in interference between mixing and decay

Evolution with time

- Consider a B meson which is known to be B^0 at time t=0
- At later time t:

amplitudes

$$B^0_{(phys)}(\Delta t) =$$

$$e^{-iMt}e^{-\Gamma t/2}\cos(\Delta m \Delta t/2)\, B^0 + i\,(q/p)\, e^{-iMt}e^{-\Gamma t/2}\sin(\Delta m \Delta t/2)\, \underline{B}^0$$

CP violating mixing parameter

- Similarly

$$\underline{B}^0_{(phys)}(\Delta t) =$$

$$(p/q)\, i\, e^{-iMt}e^{-\Gamma t/2}\sin(\Delta m \Delta t/2)\, B^0 + e^{-iMt}e^{-\Gamma t/2}\cos(\Delta m \Delta t/2)\, B^0$$

Evolution with time

- Include decays to CP eigenstate

$\Gamma[B^0_{(phys)} \rightarrow f](\Delta t) \sim e^{-\Gamma t}\{1 - (C \cos(\Delta m \Delta t) - S \sin(\Delta m \Delta t))\}$

$\Gamma[\underline{B}^0_{(phys)} \rightarrow f](\Delta t) \sim e^{-\Gamma t}\{1 + (C \cos(\Delta m \Delta t) - S \sin(\Delta m \Delta t))\}$

- where
  - $C = (1 - |\lambda_{CP}|^2)/(1 + |\lambda_{CP}|^2)$
  - $S = 2\,\mathrm{Im}(\lambda_{CP})/(1 + |\lambda_{CP}|^2)$

$$\lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

- Standard Model (usual phase convention)
  - $q/p \sim e^{-2i\beta}$

The golden mode – $B^0 \to J/\psi\ K_S$

- Dominated by $b \to c\underline{c}s$ tree diagram
  - subleading $b \to sc\underline{c}$ penguin has the same weak phase
- $|\underline{A}| = |A| \Rightarrow$ no direct CP violation
- $C = 0$ & $S = -\eta_{CP} \sin(2\beta)$
- Reasonable branching fraction & experimentally clean signature

Problem

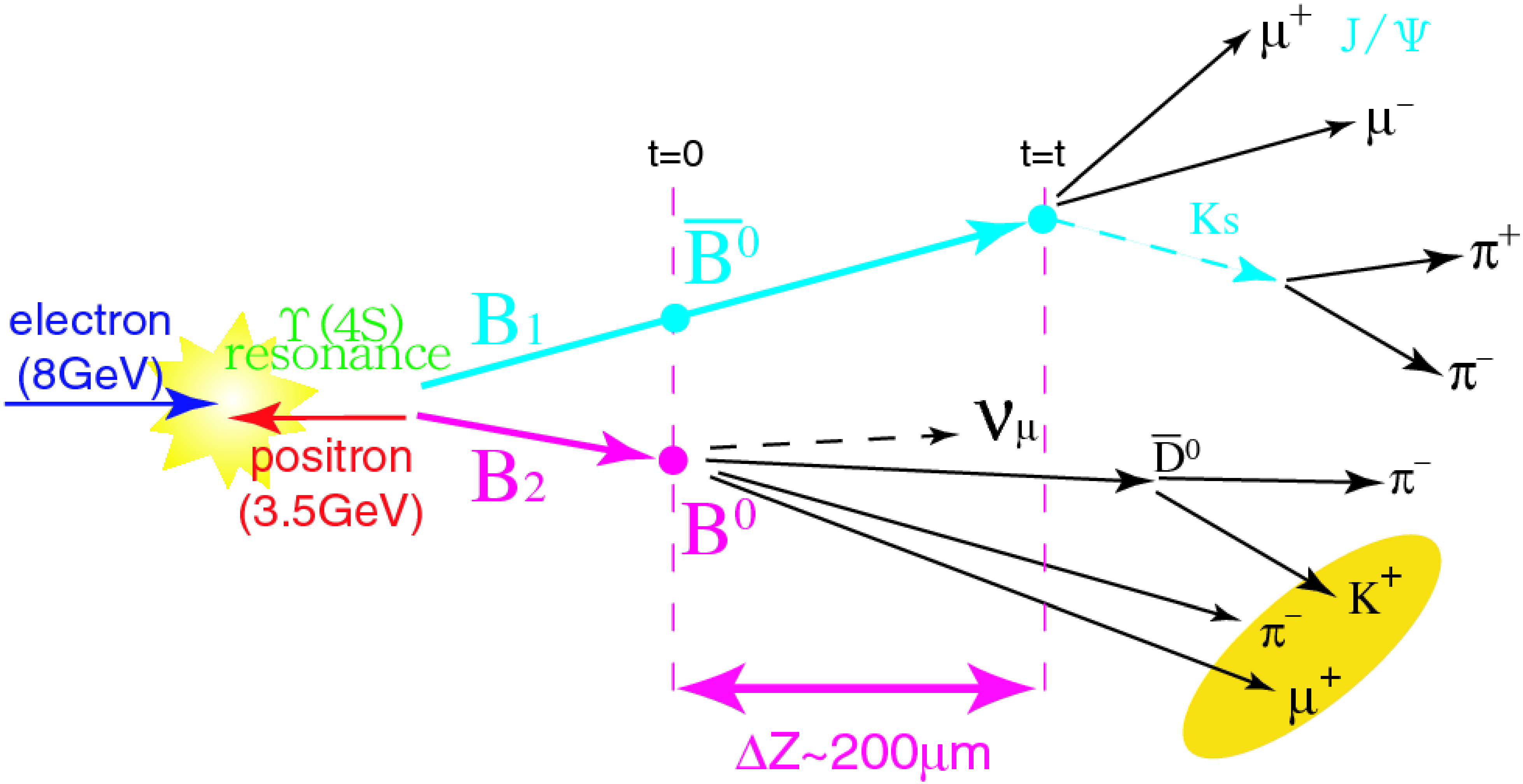
- How can we measure decay time in $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\underline{B}^0$?

- The answer: (P.Oddone)

  asymmetric-energy B factory

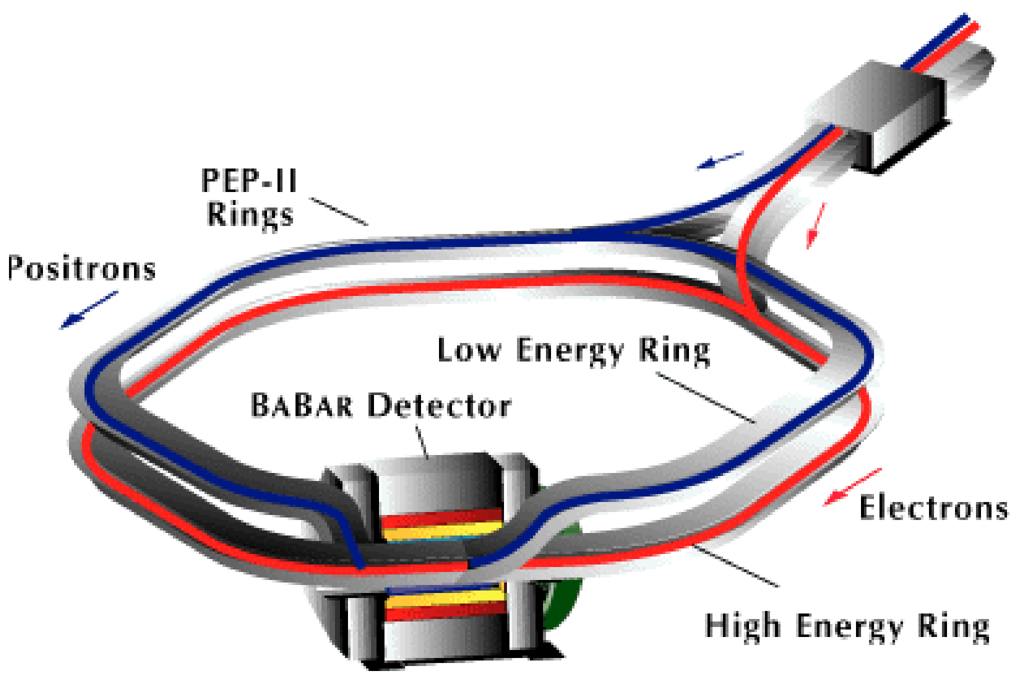
- Key points
  - $\Upsilon(4S) \rightarrow B^0\underline{B}^0$ produces coherent pairs
  - B mesons are moving in lab frame

Asymmetric B factory principle

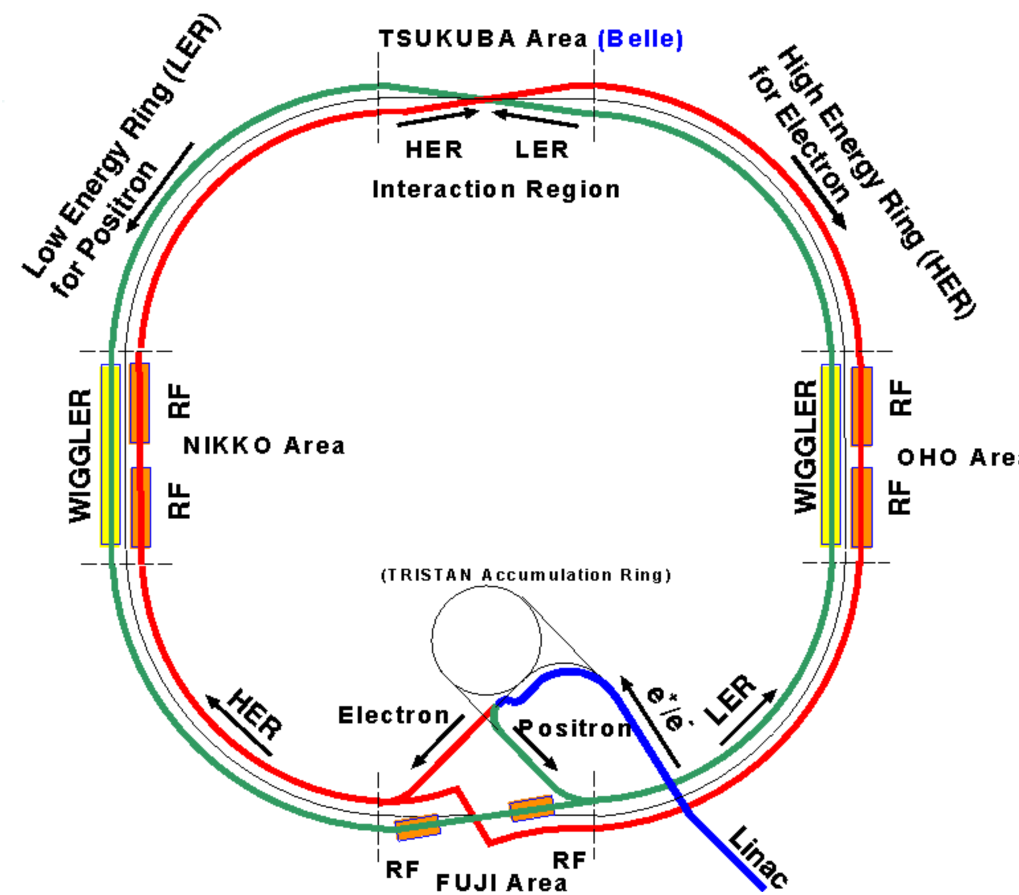


Asymmetric B Factories

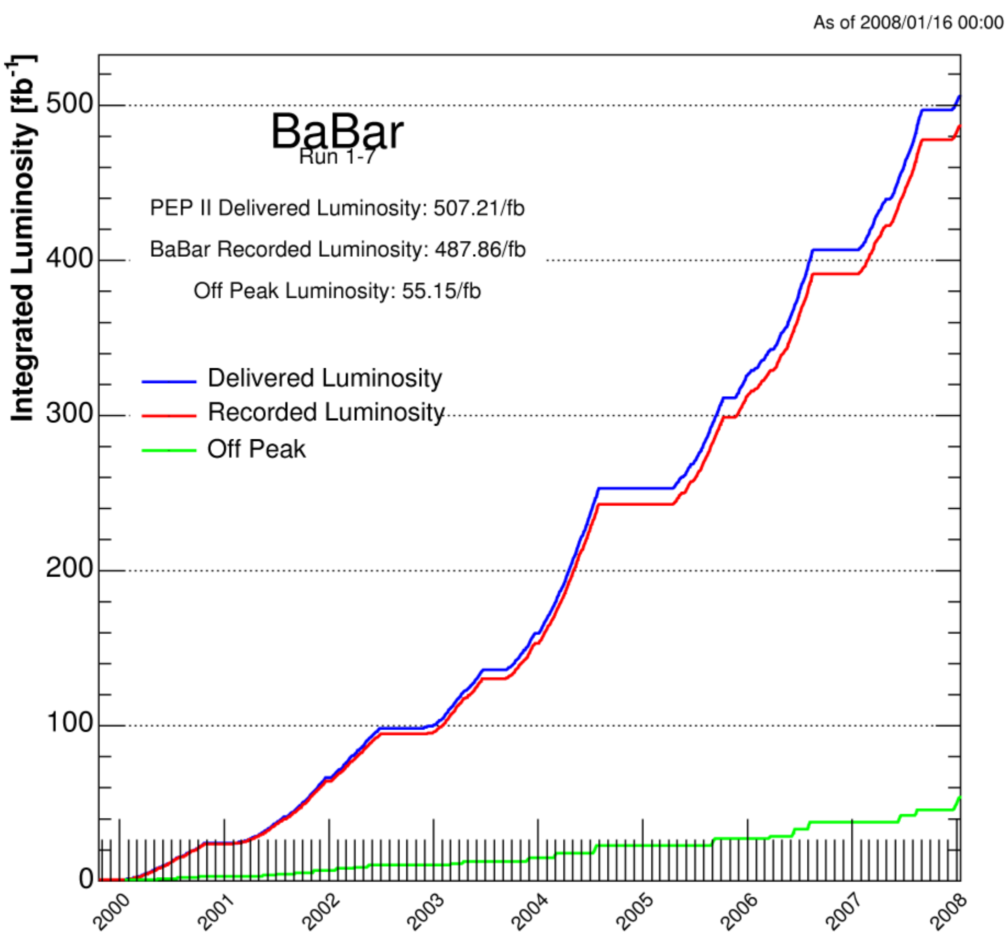
PEPII at SLAC
9.0 GeV e^- on 3.1 GeV e^+

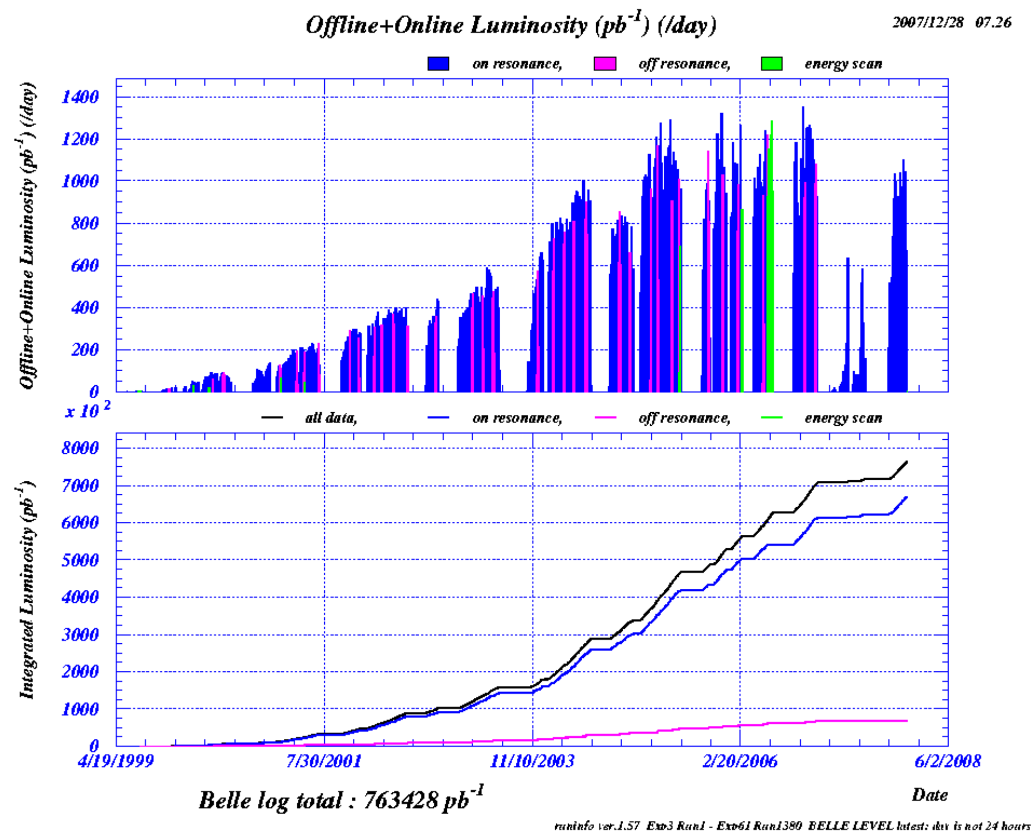


KEKB at KEK
8.0 GeV e^- on 3.5 GeV e^+



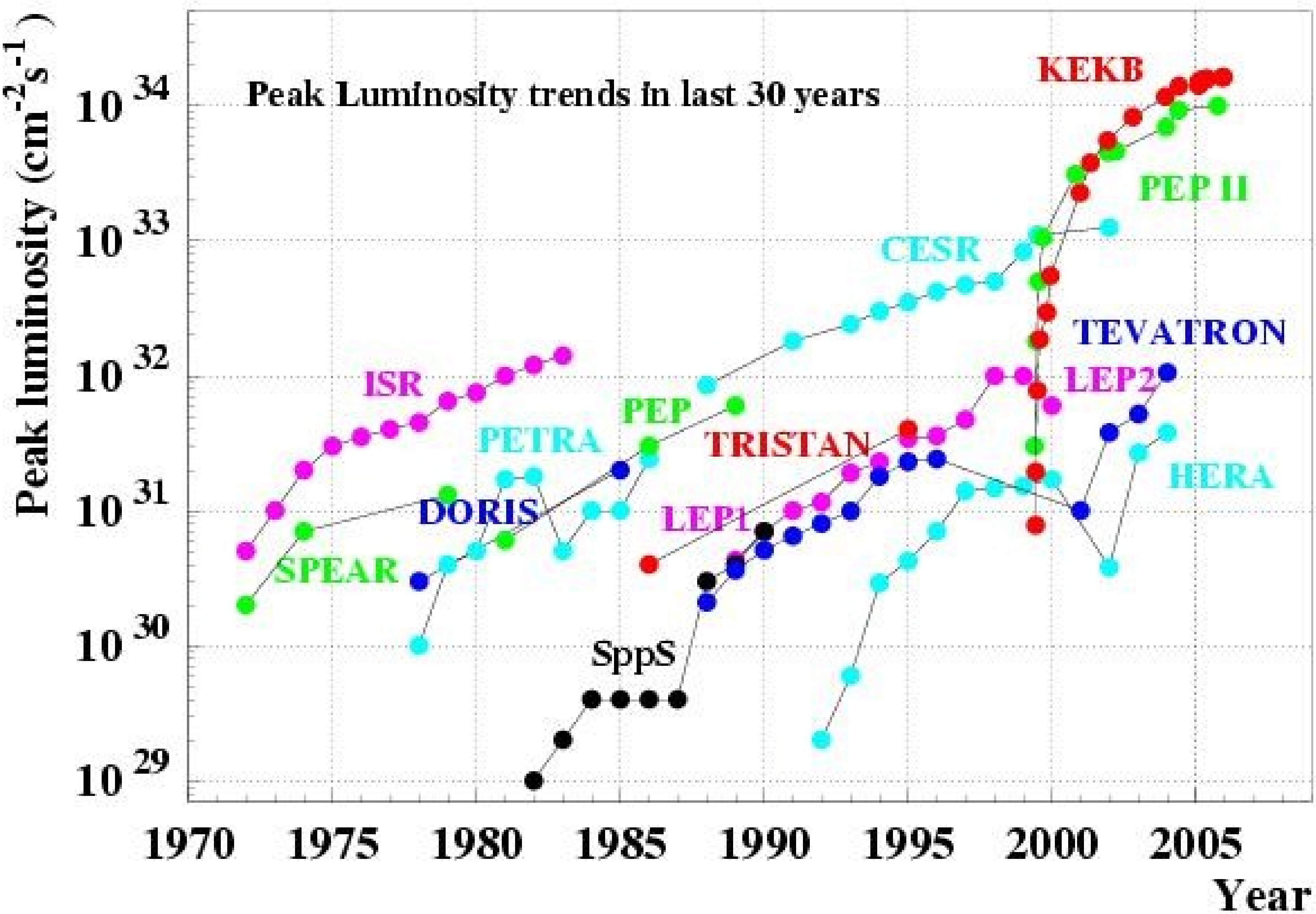
B Factories – World Record Luminosities





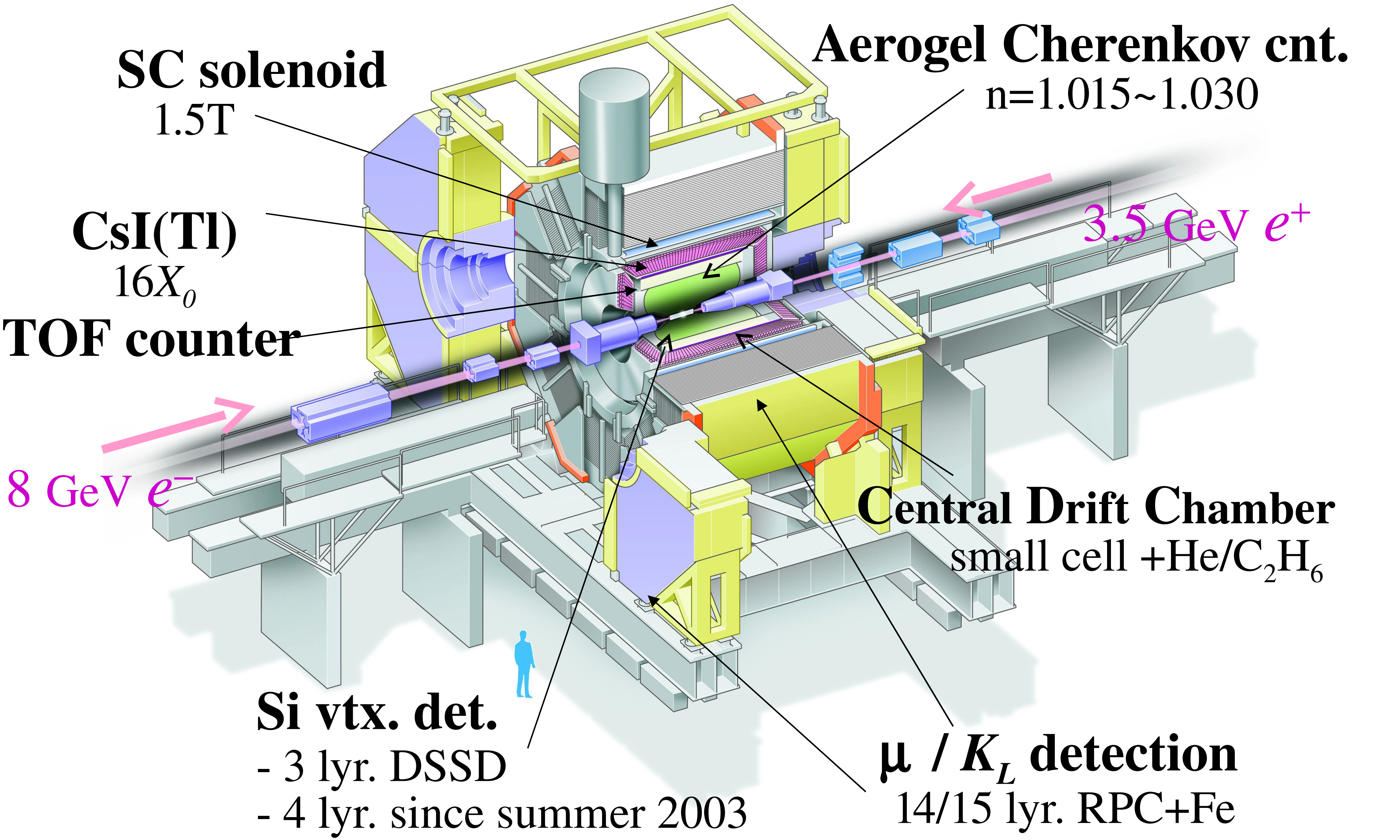
~ 430/fb on Υ(4S)

~ 660/fb on Υ(4S)

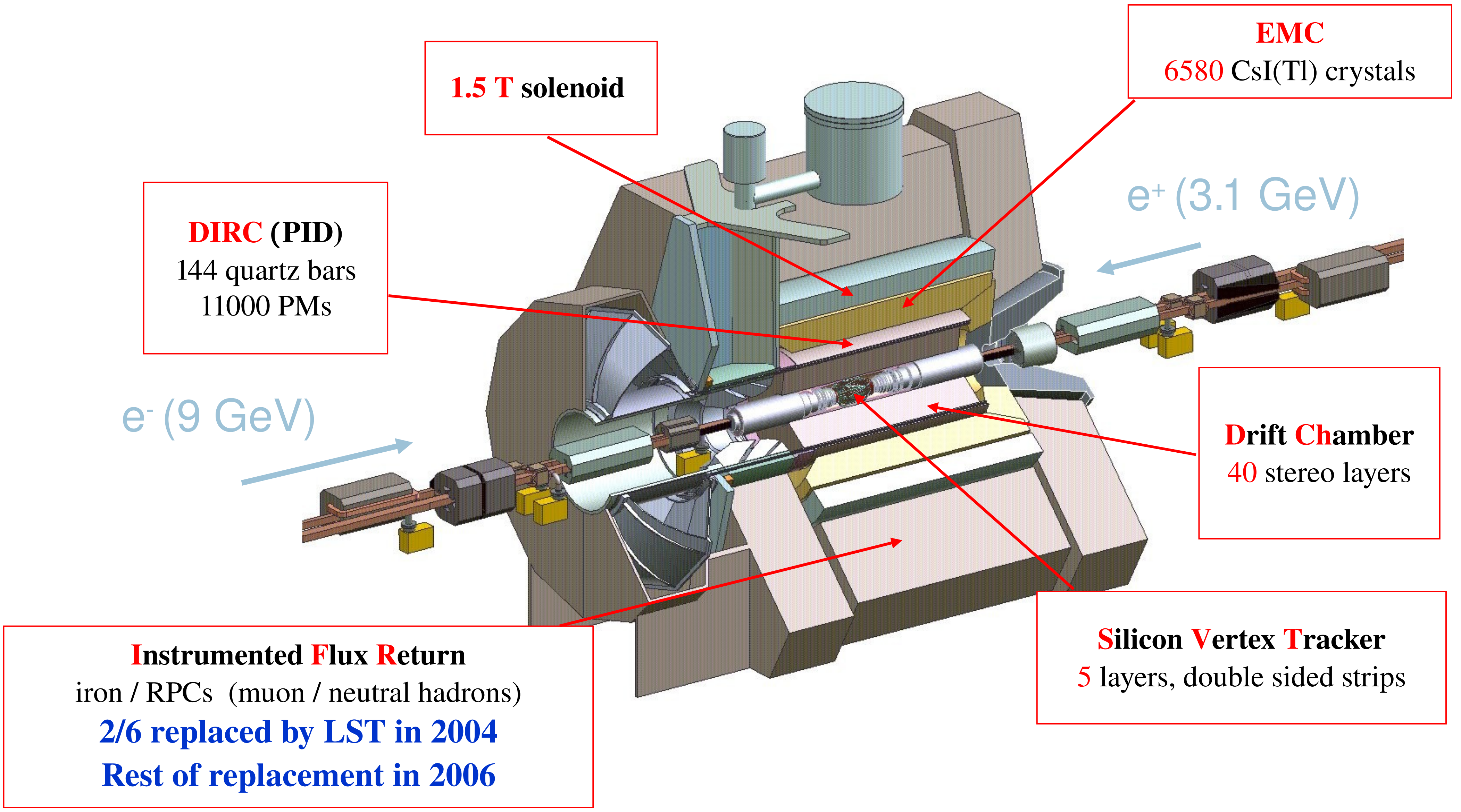


Peak Luminosity trends in last 30 years
Peak luminosity (cm-2s-1)
10 34
10 33
10 32
10 31
10 30
10 29
1970
1975
1980
1985
1990
1995
2000
2005
Year
KEKB
PEP II
CESR
TEVATRON
ISR
LEP2
PEP
PETRA
TRISTAN
HERA
DORIS
LEP1
SPEAR
SppS

Belle Detector



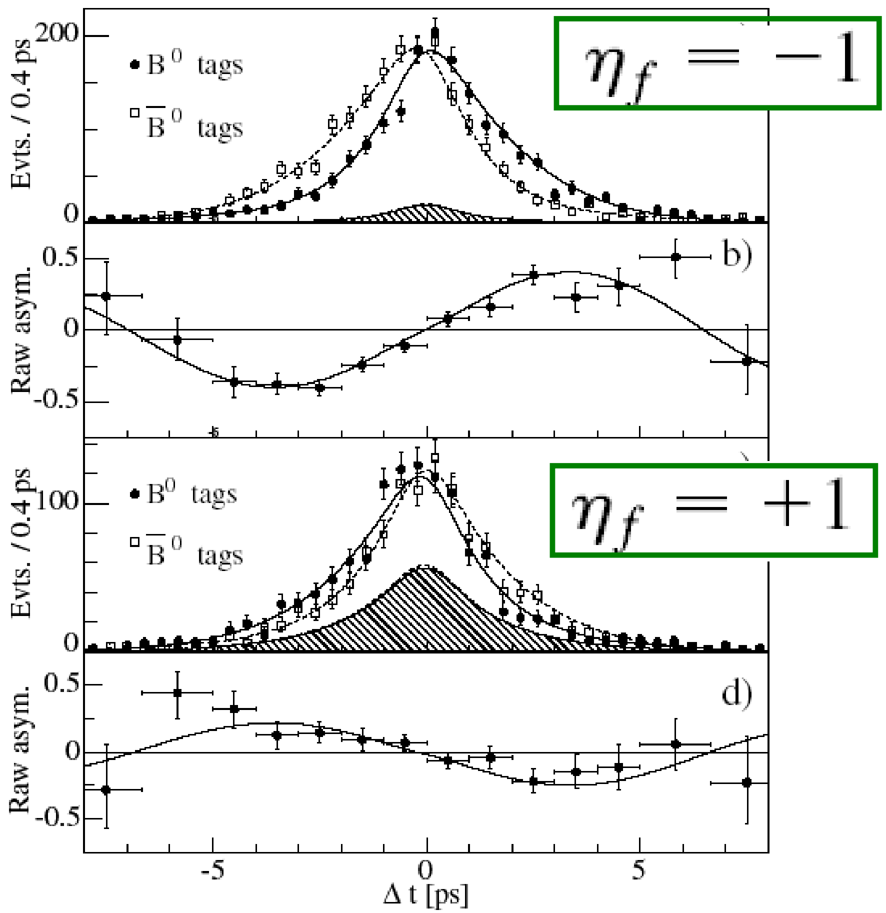
BaBar Detector



Results for the golden mode

$B^0 \to J/\psi K^0$

BABAR



PRL **94,** 161803 (2005)

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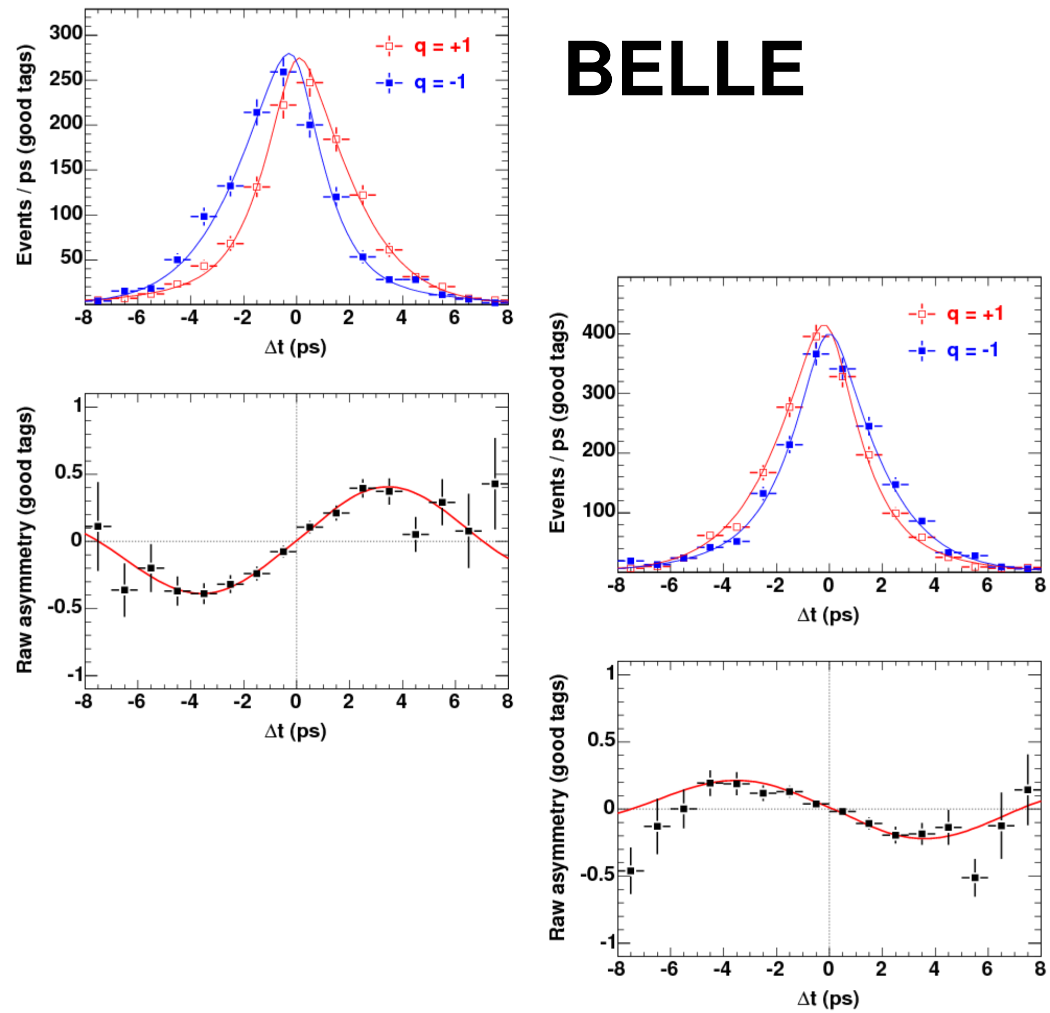
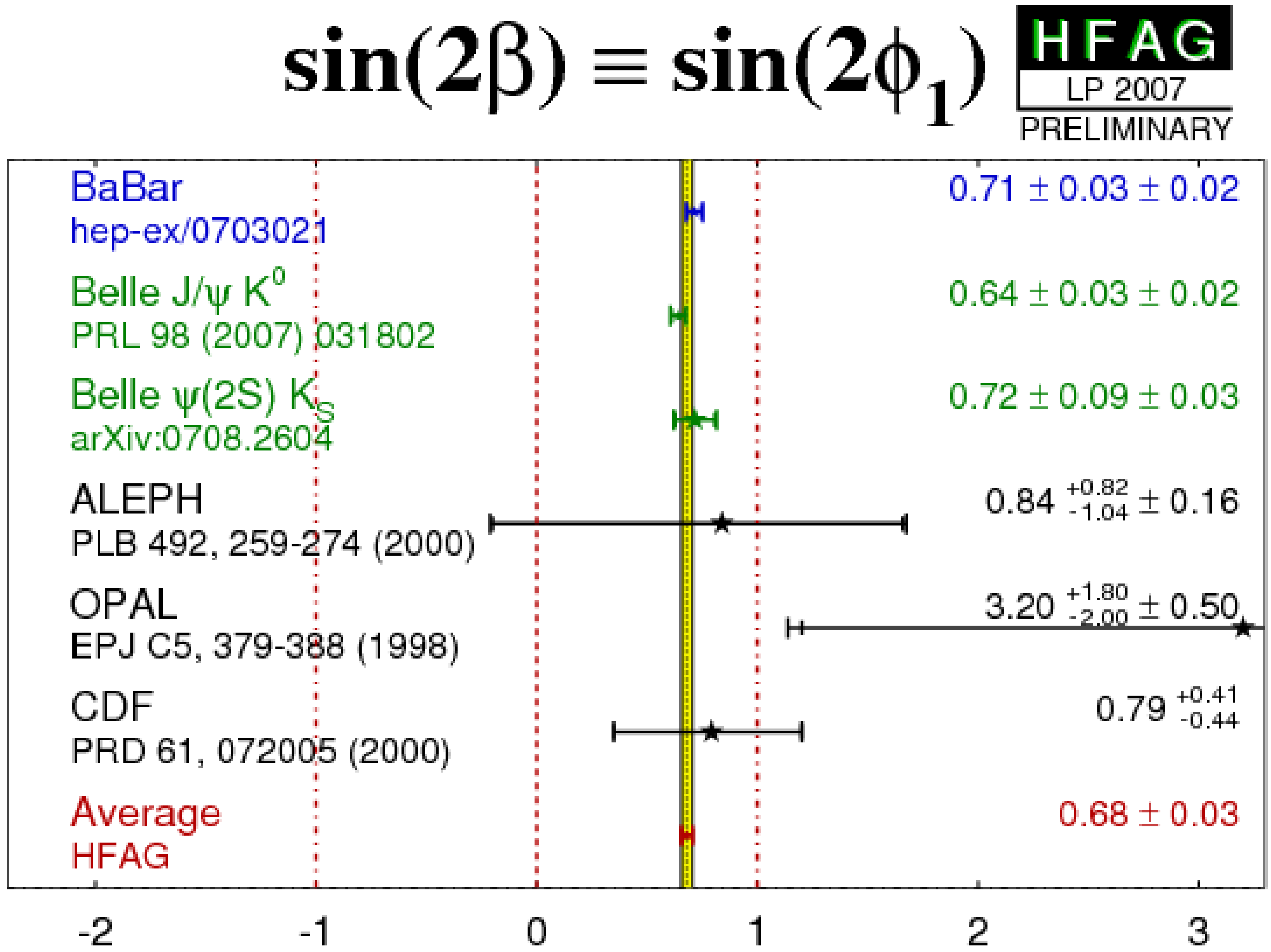
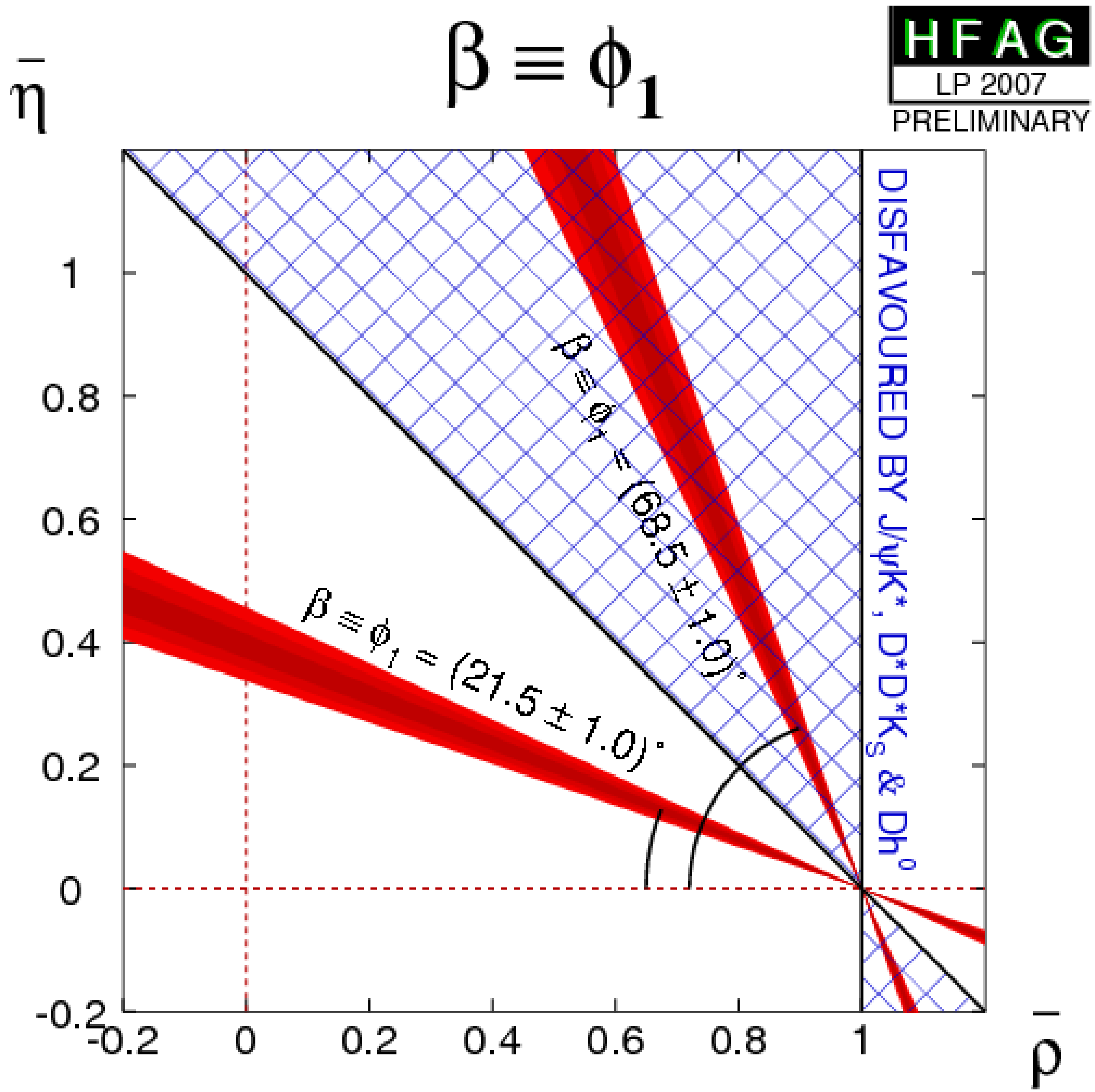


PRL 98, 031802 (2007)

Compilation of results



Constraint from β measurement



Measurement of α

- Time-dependent CP violation in modes dominated by $b \rightarrow u\underline{u}d$ tree diagrams probes α (or π–(β+γ))
  - $C = 0$ & $S = +\eta_{CP} \sin(2\alpha)$
- $b \rightarrow du\underline{u}$ penguin transitions contribute to same final states ⇒ "penguin pollution"
  - $C \neq 0 \Leftrightarrow$ direct CP violation can occur
  - $S \neq +\eta_{CP} \sin(2\alpha)$
- Two approaches (optimal approach combines both)
  - try to use modes with small penguin contribution
  - correct for penguin effect (isospin analysis)

$B^0 \rightarrow \pi^+\pi^-$ -- Experimental Situation

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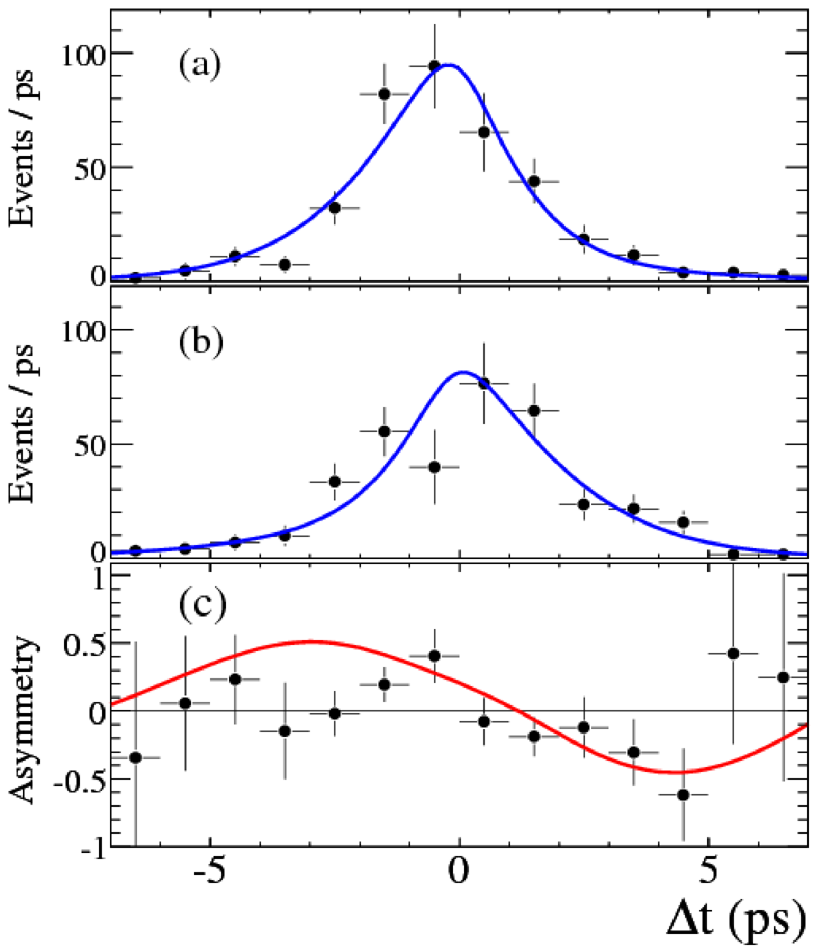

PRL 99 (2007) 021603

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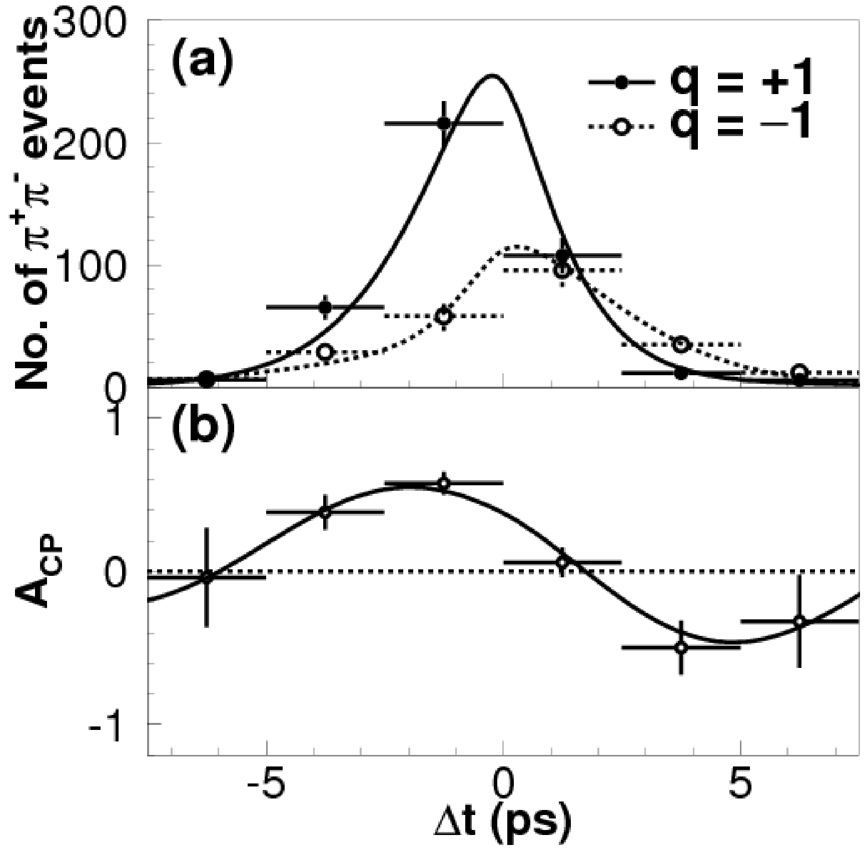
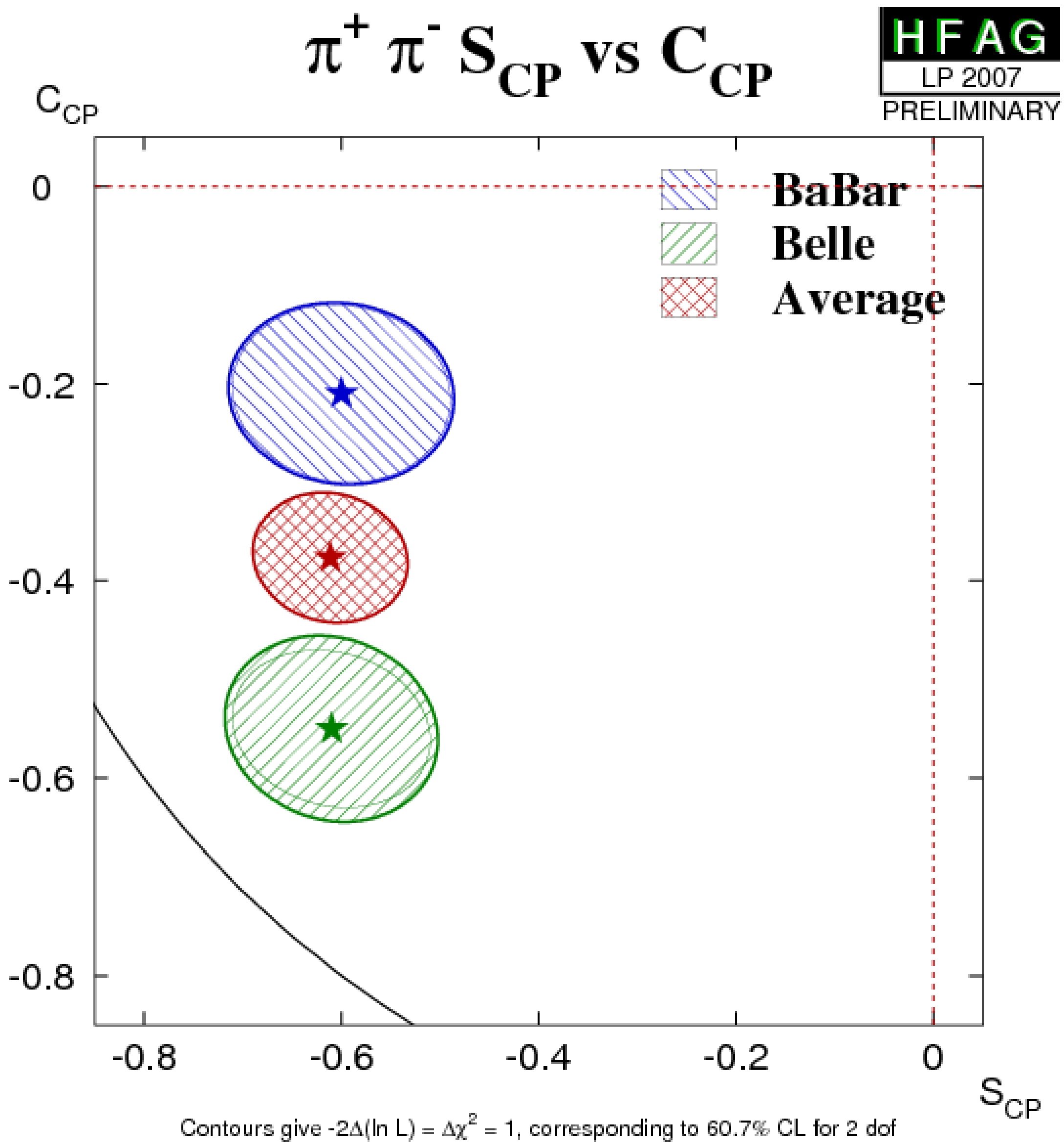
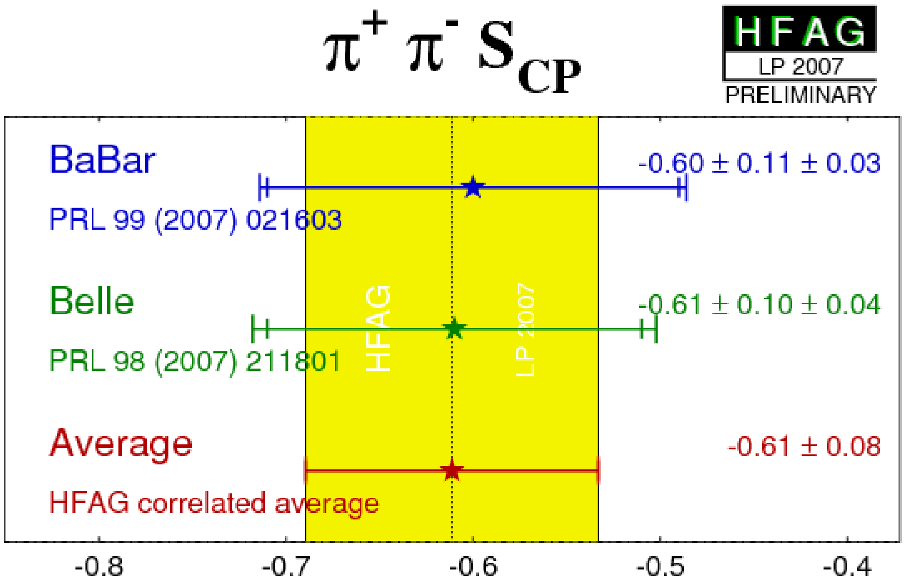
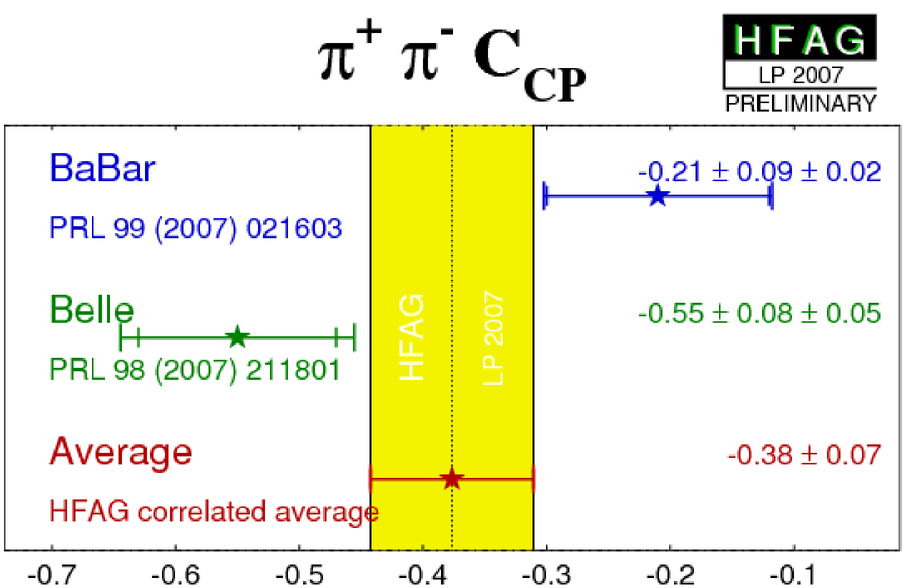


PRL 98 (2007) 211801

$B^0 \rightarrow \pi^+\pi^-$ – Experimental Situation



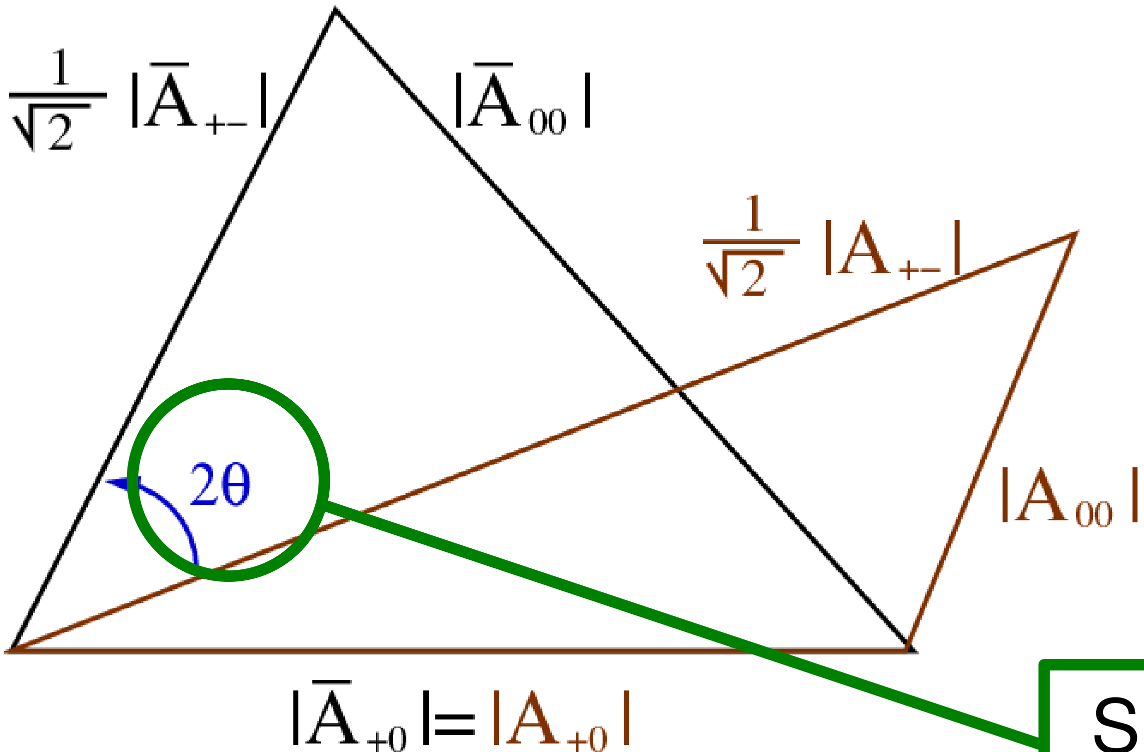




Isospin Analysis

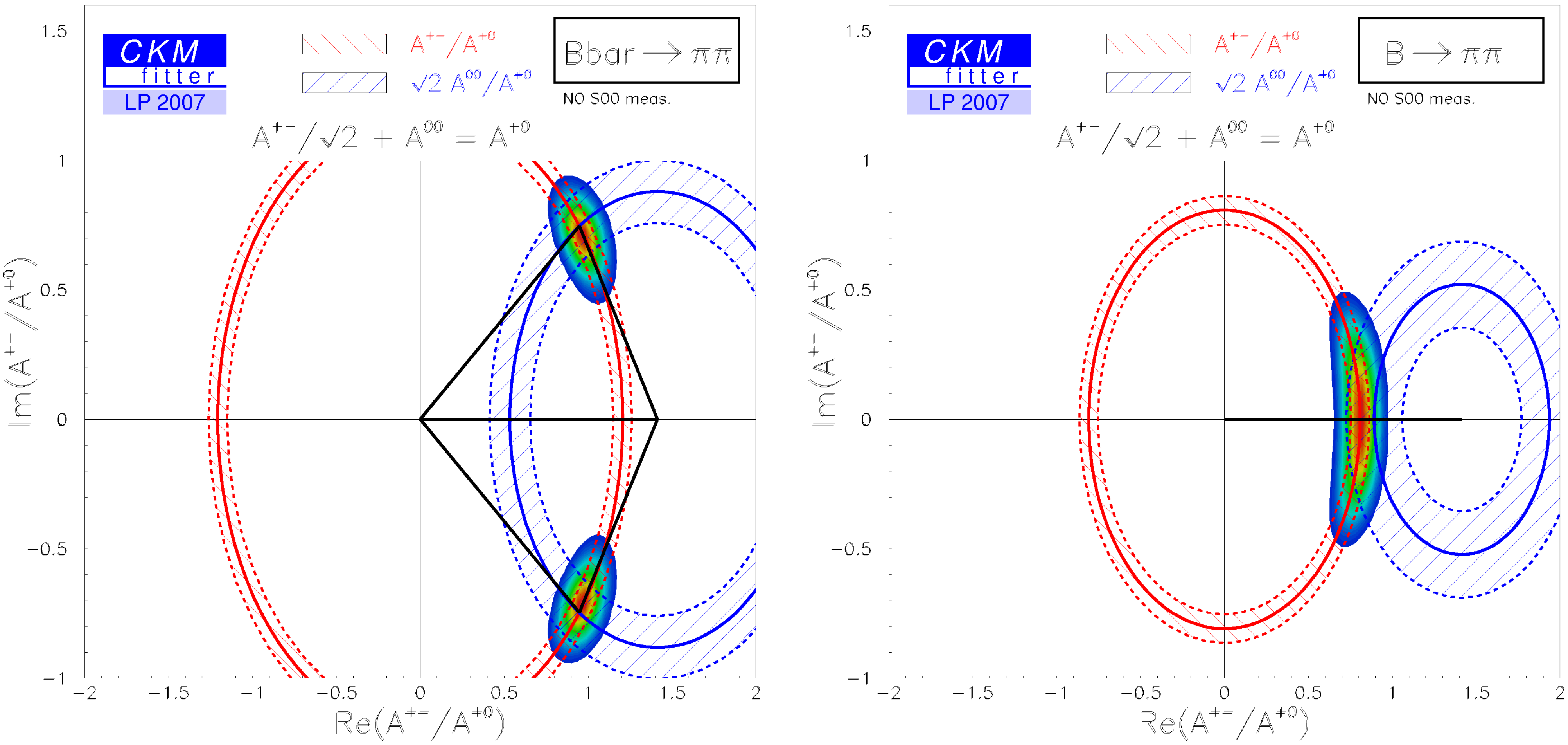
$$A_{+-} = \langle \pi^+\pi^-|H|B^0\rangle = -A_{1/2} + \frac{1}{\sqrt{2}}A_{3/2} - \frac{1}{\sqrt{2}}A_{5/2},$$
$$A_{00} = \langle \pi^0\pi^0|H|B^0\rangle = \frac{1}{\sqrt{2}}A_{1/2} + A_{3/2} - A_{5/2},$$
$$A_{+0} = \langle \pi^+\pi^0|H|B^+\rangle = \frac{3}{2}A_{3/2} + A_{5/2},$$

Negligible in SM weak Hamiltonian



$S_{\pi\pi} = \sin(2\alpha+2\theta)/\sqrt{(1-C_{\pi\pi}^2)}$

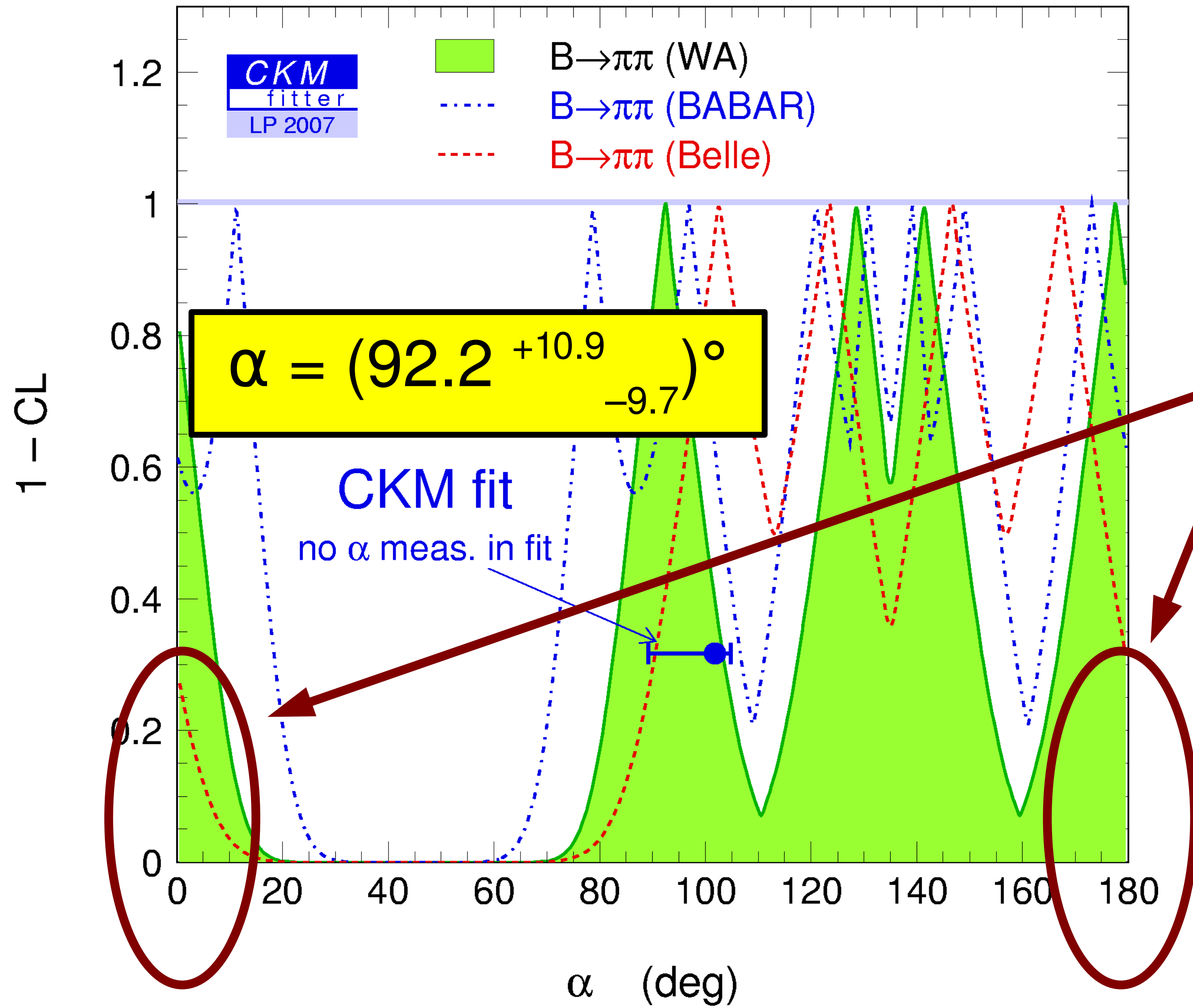
Isospin Triangles – B→ππ



- Large penguins
- Large direct CP violation

- Large $\mathcal{B}(B\to\pi^0\pi^0)$
- Large correction θ

Measurement of α – B→ππ



THESE SOLUTIONS RULED OUT BY OBSERVATION OF DIRECT CP VIOLATION IN $B^0 \to \pi^+\pi^-$

$B^0 \rightarrow \rho^+\rho^-$ -- Experimental Situation

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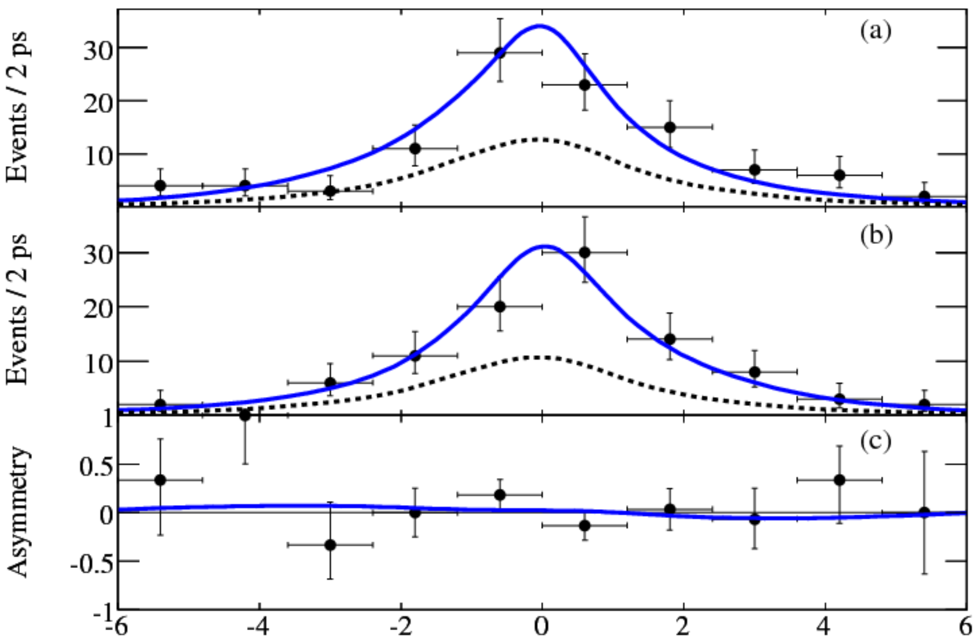

PRD 76 (2007) 052007

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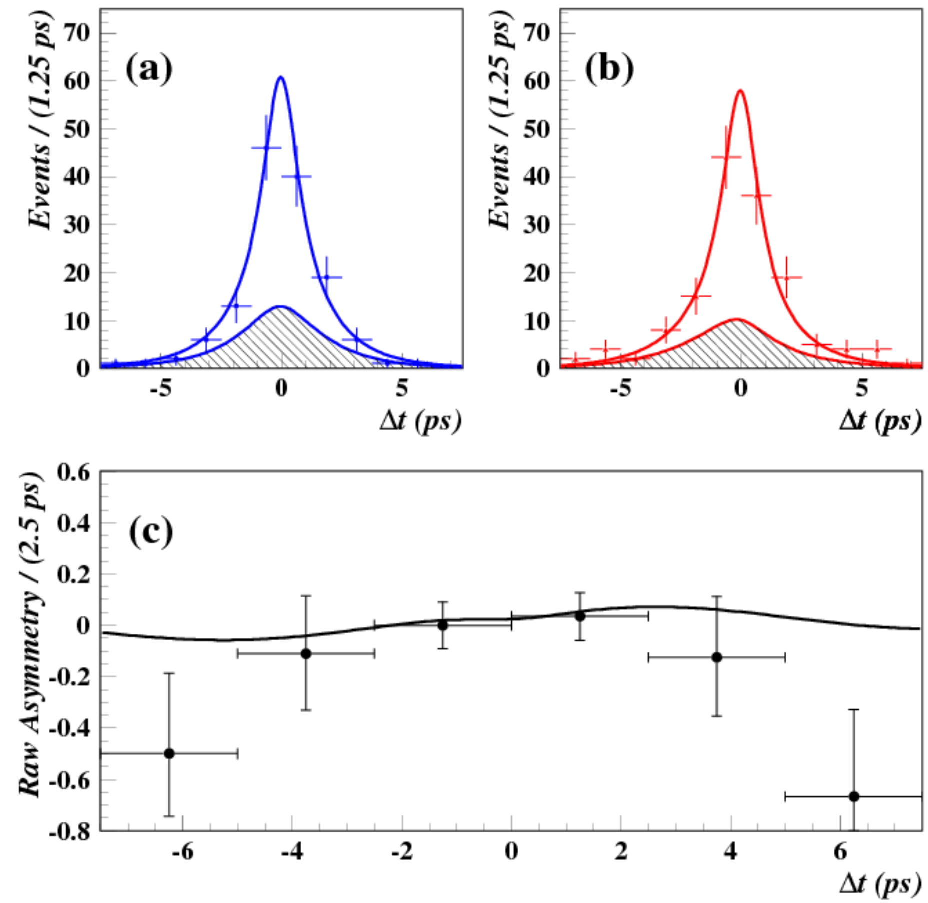
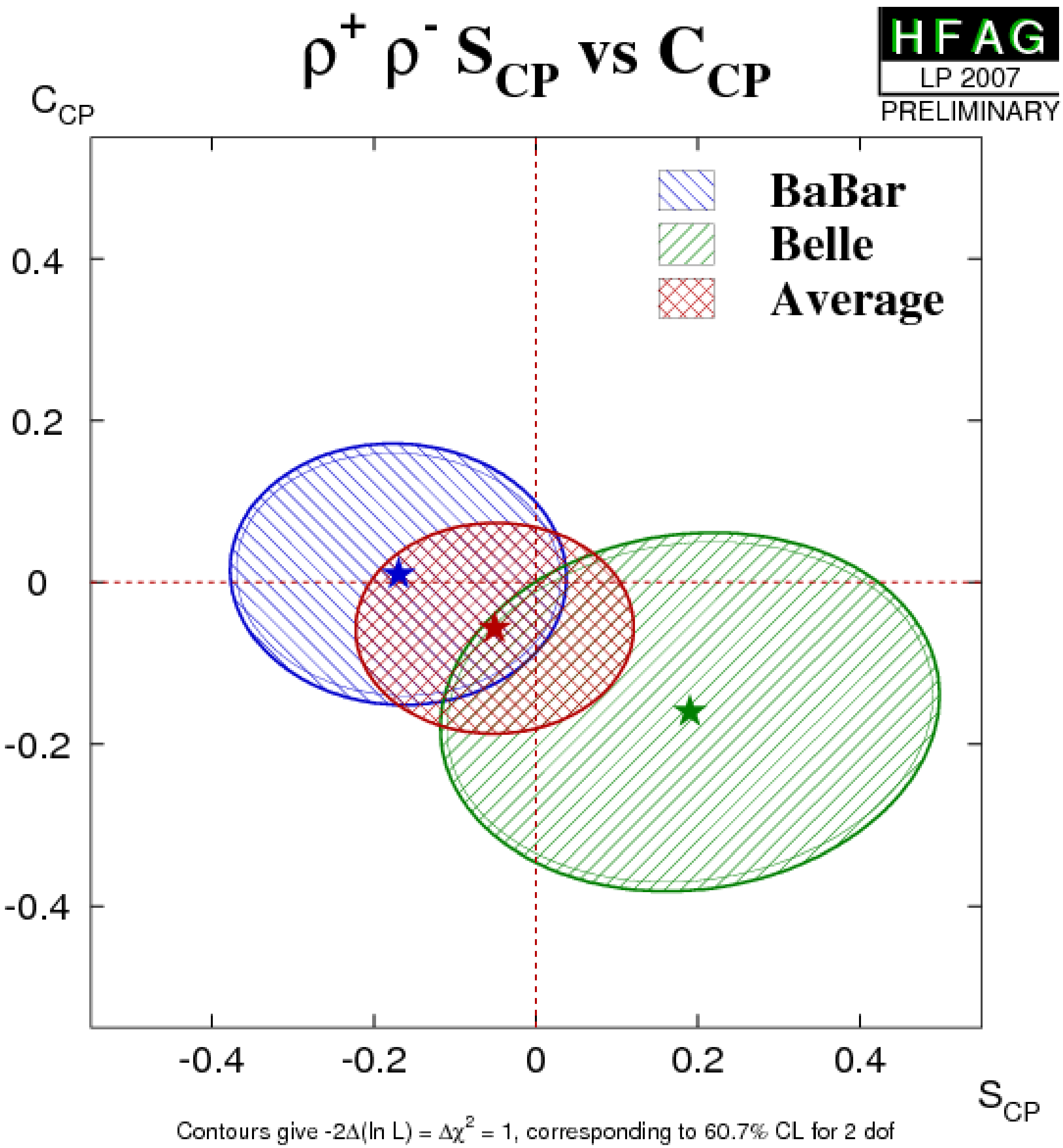
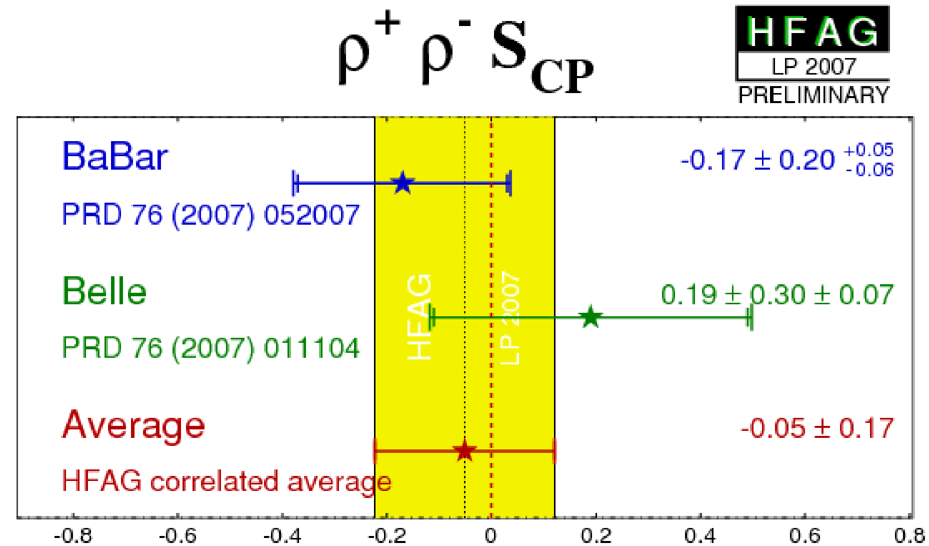
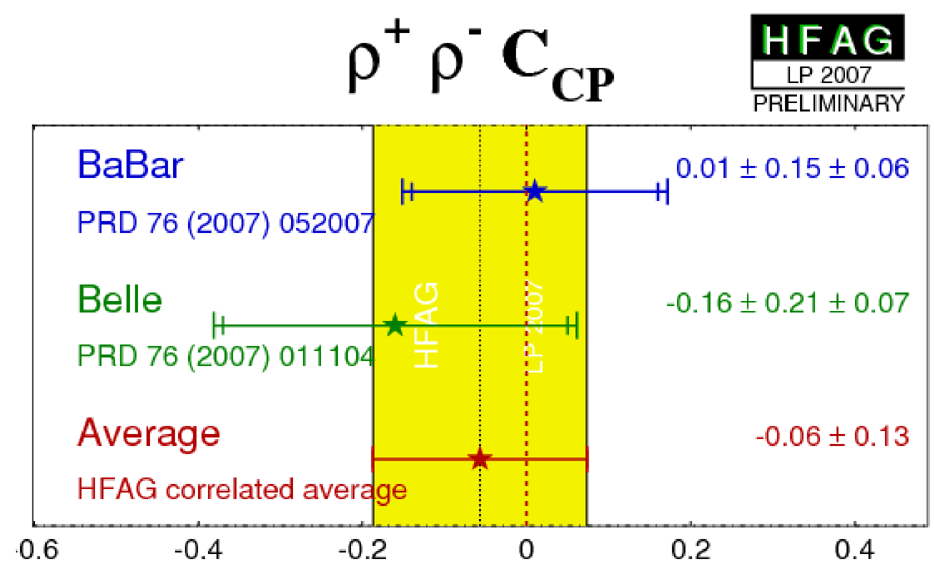


PRD 76 (2007) 011104

$B^0 \rightarrow \rho^+\rho^-$ -- Experimental Situation







Extra information – $S(\rho^0\rho^0)$

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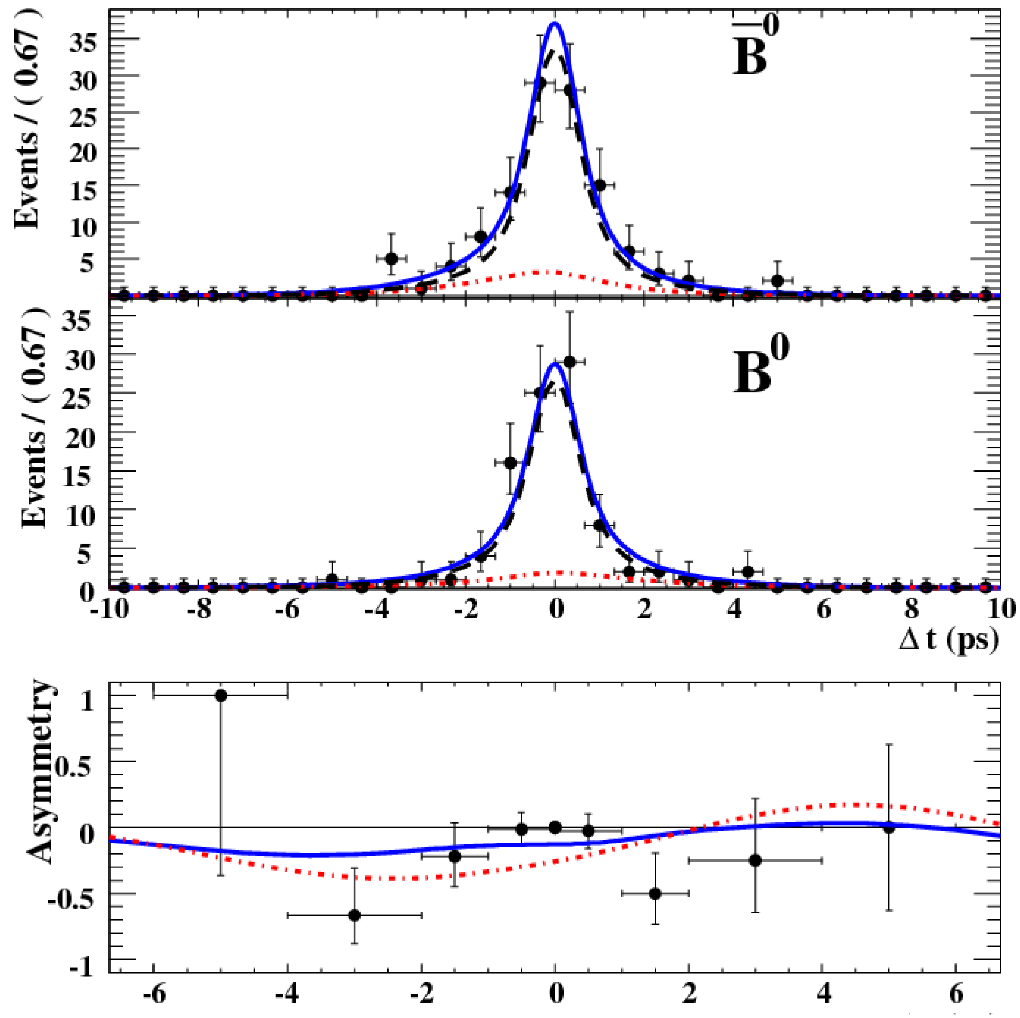
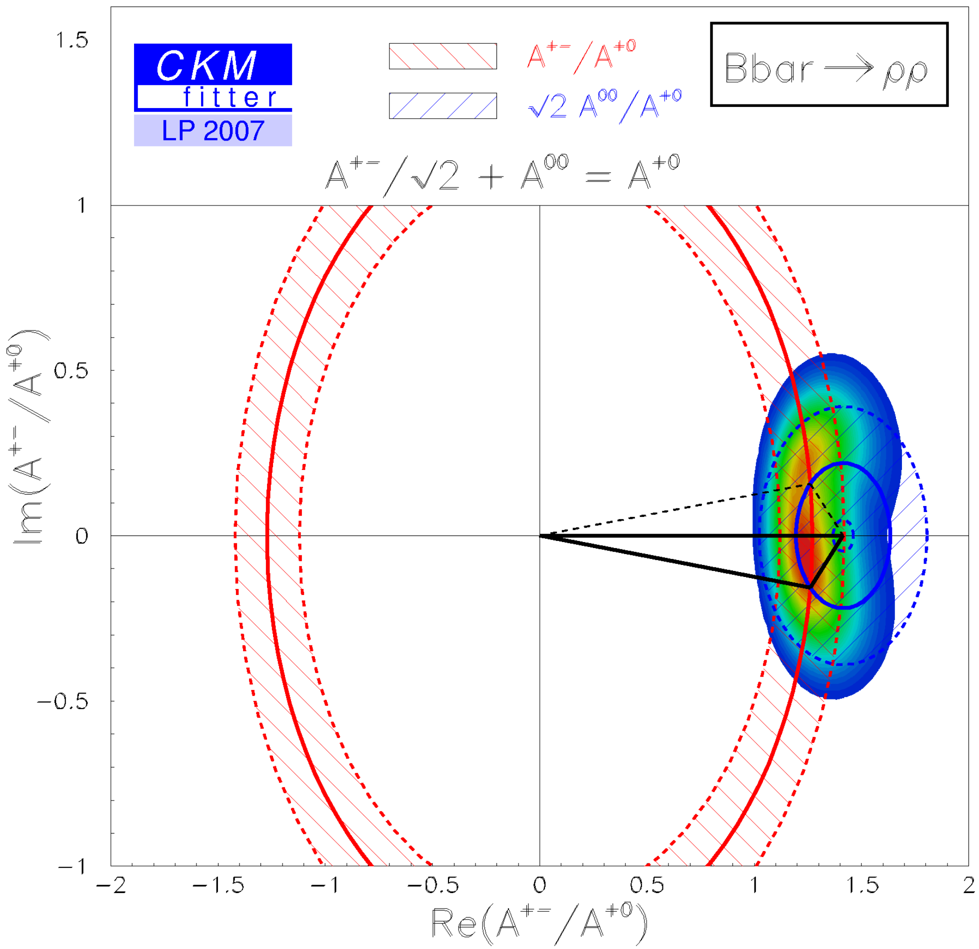
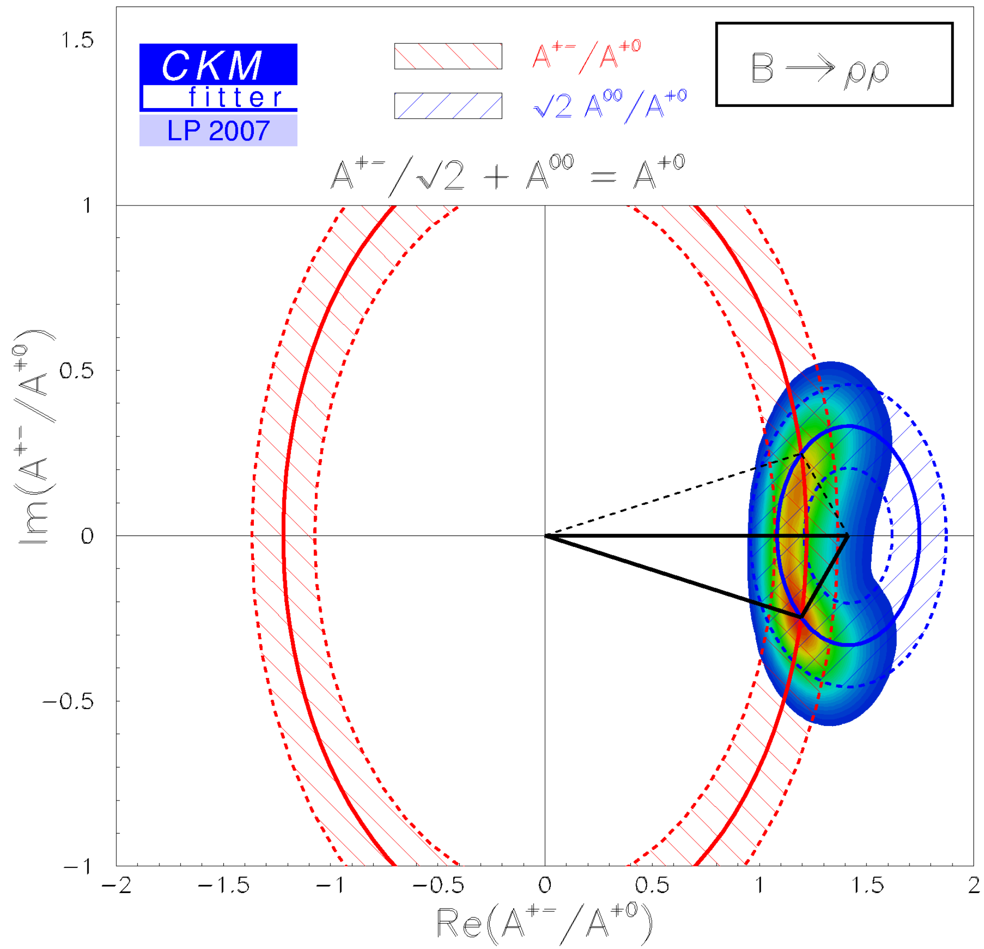


arXiv:0708.1630

Isospin Triangles – B→ρρ

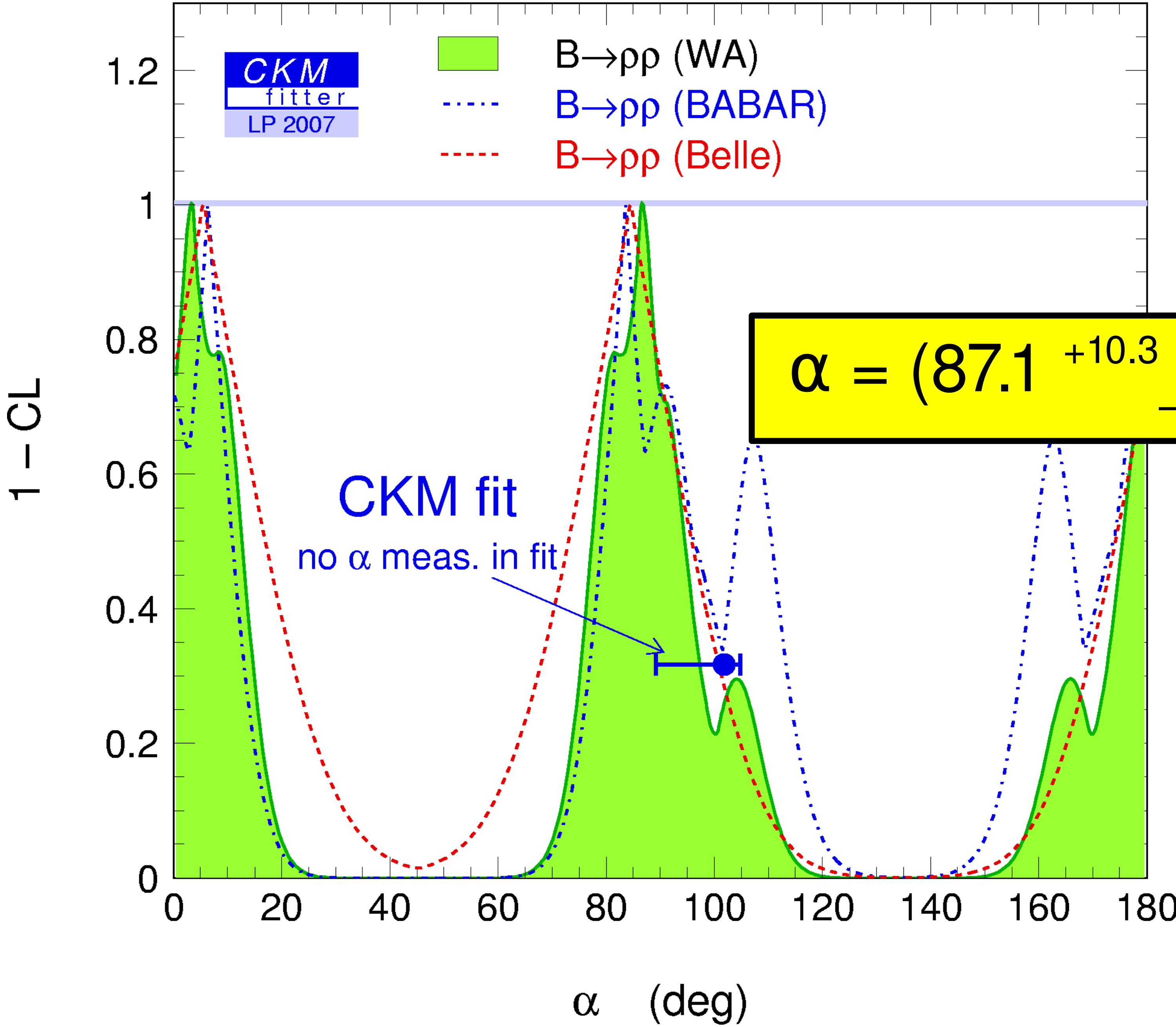




- Small penguins
- Small direct CP violation

- Small $B(B\to\rho^0\rho^0)$
- Small correction θ

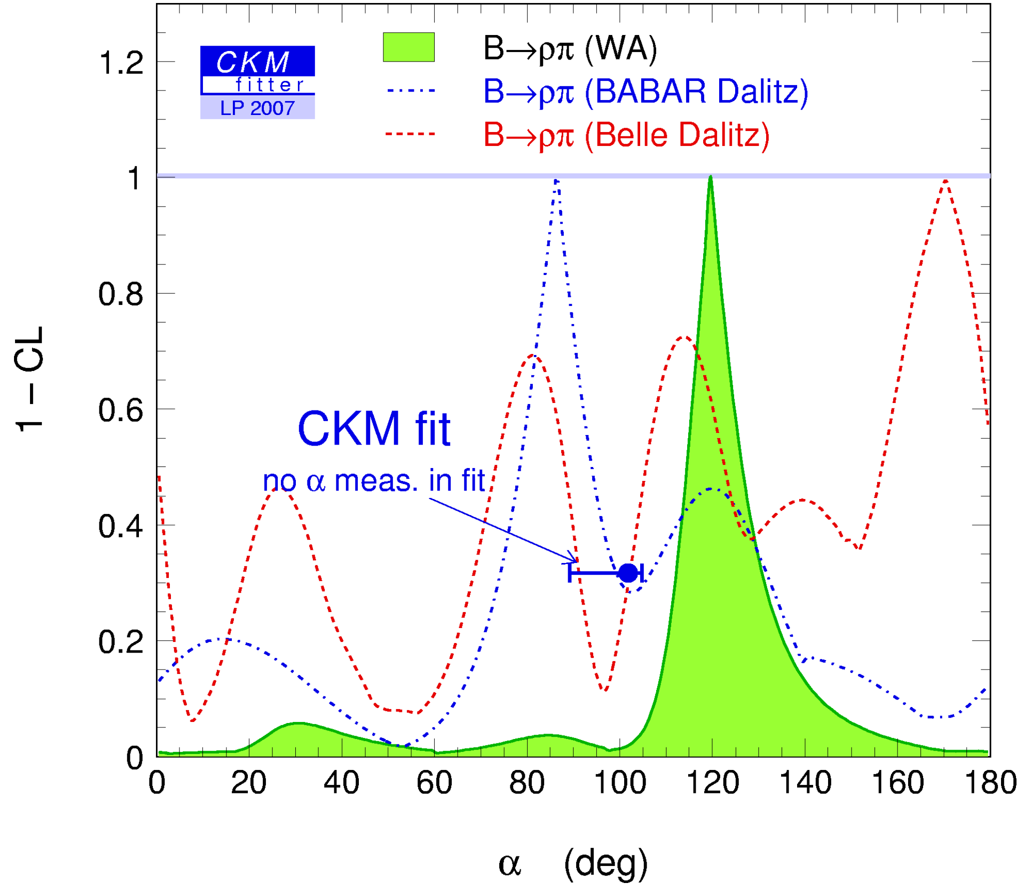
Measurement of α – B→ρρ



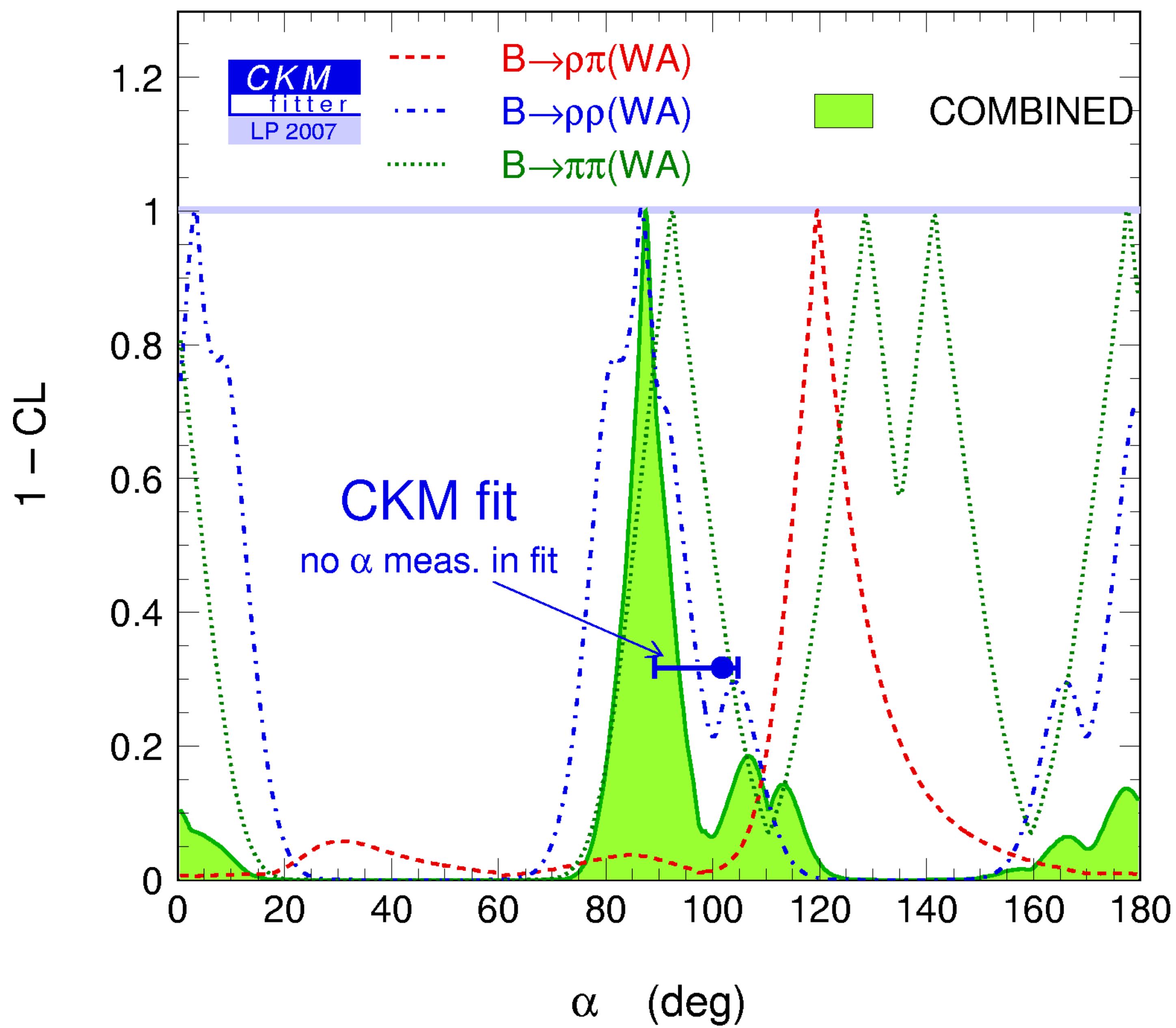
$\alpha = (87.1\,^{+10.3}_{-11.5})^\circ$

Measurement of α

- Particularly elegant approach uses $B \to (\rho\pi)^0 \to \pi^+\pi^-\pi^0$
  - Time-dependent Dalitz plot analysis
    - Highly detailed and complex analysis
  - Helps with
    - theory uncertainties
    - ambiguities



Measurement of α



Measurement of γ

- Charmless B decays, eg. $B^0 \rightarrow K^+\pi^-$
  - contributions from
    - P : $b \rightarrow s\underline{u}u$ penguin
    - T : $b \rightarrow us\underline{u}$ tree
  - relative weak (CP violating) phase is γ
  - relative strong (CP conserving) phase δ

$$A_{CP} = 2|P||T|\sin(\gamma)\sin(\delta)/\{|P|^2+|T|^2+2|P||T|\cos(\gamma)\cos(\delta)\}$$

- Hadronic uncertainties:
  - even if we observe $A_{CP} \neq 0$, cannot easily extract γ
  - other processes also contribute

Direct CP violation in the B system

$$B^0 \to K^+\pi^-$$

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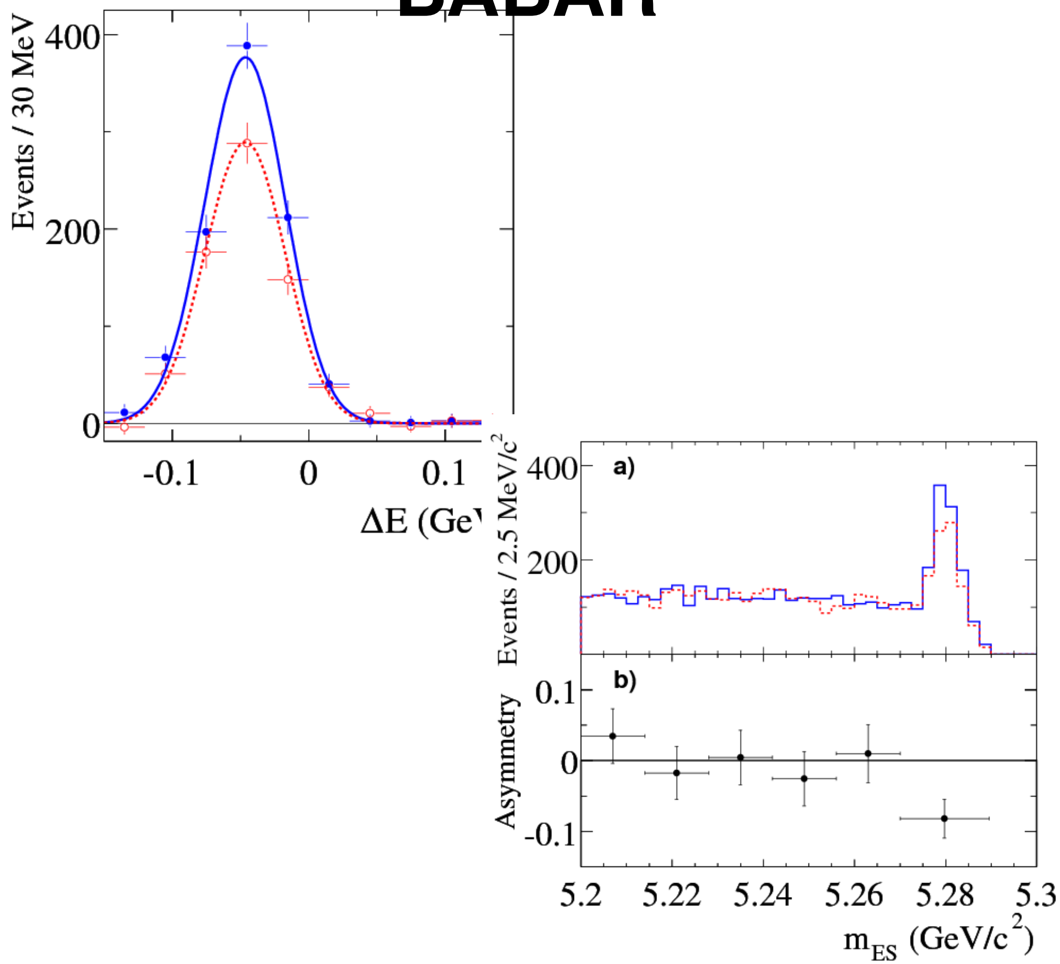

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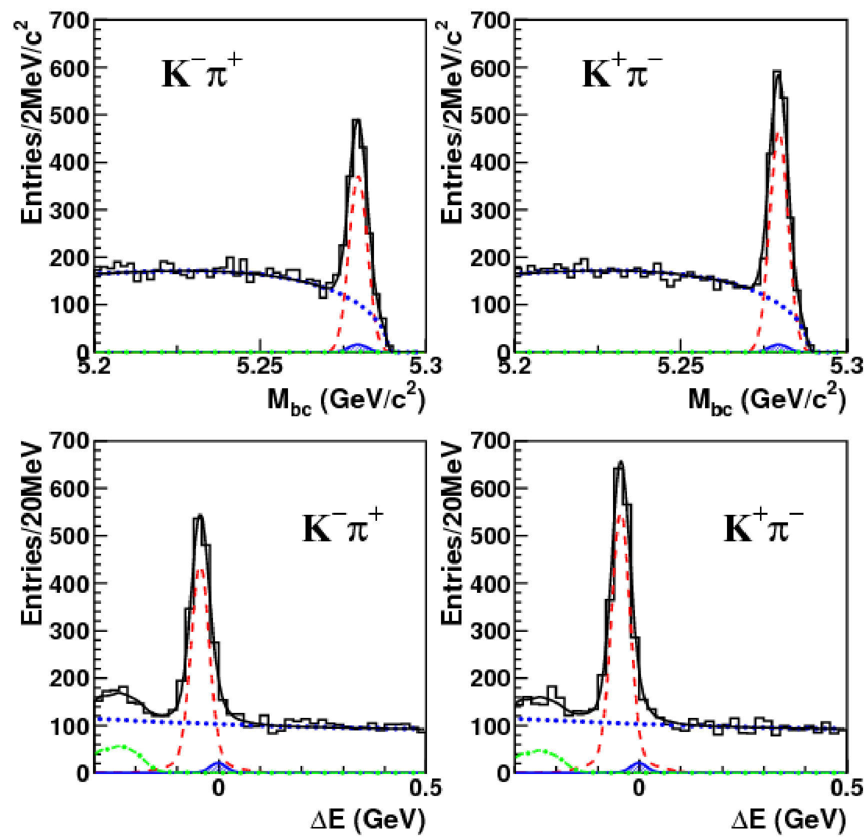


PRL 93 (2004) 131801

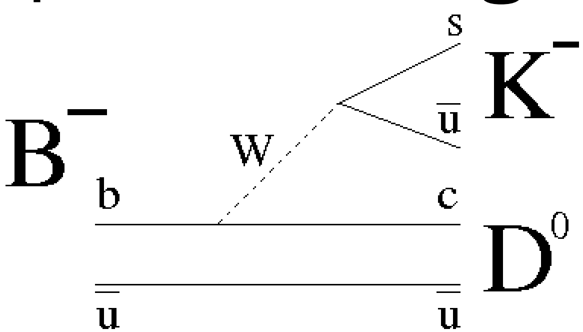
BELLE-CONF-0523

Clean measurement of γ

- A theoretically clean measurement of γ can be made using $B \to DK$ decays
- Reconstruct D mesons in states accessible to both D^0 and $\underline{D}^0$
  - interference between $b \to c\underline{u}s$ and $b \to u\underline{c}s$
  - relative weak phase is γ
  - various different D decays utilized
  - large statistical errors at present

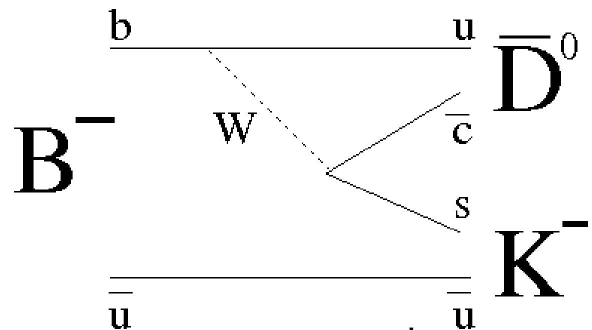
The Idea

- Two possible diagrams for $B^- \to DK^-$



$\propto V_{cb} V_{us}^*$

- colour allowed
- final state contains D^0



$\propto V_{ub} V_{cs}^*$

- colour suppressed
- final state contains $\overline{D}^0$

Relative magnitude of suppressed amplitude is r_B

Relative weak phase is $-\gamma$, relative strong phase is δ_B

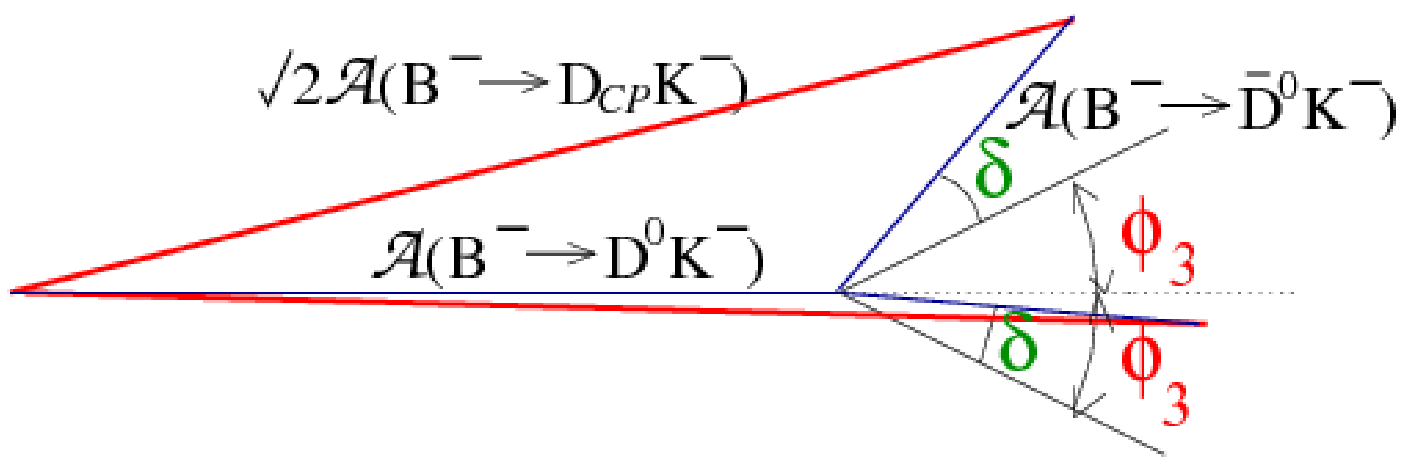
Need D^0 and $\overline{D}^0$ to decay to common final state

$D \to CP$ eigenstates

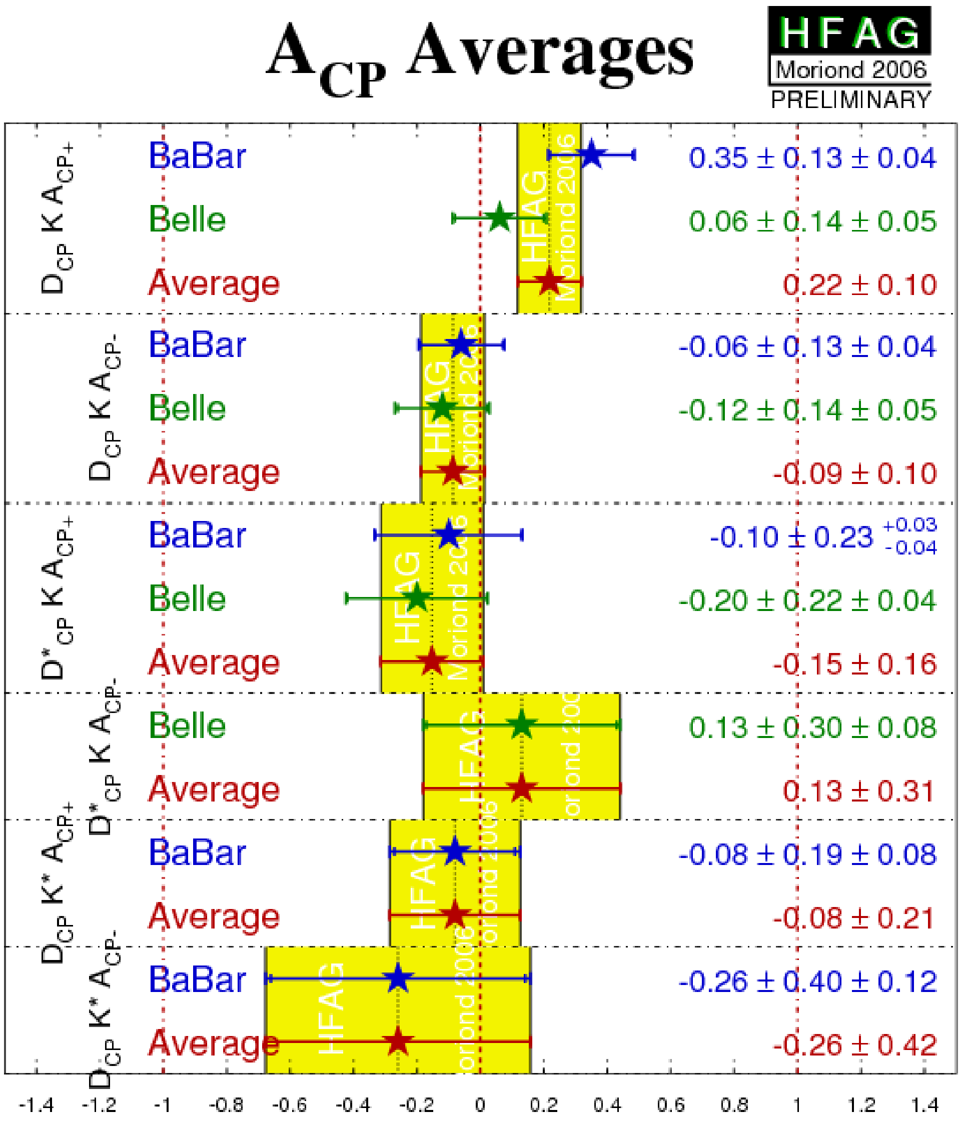
- Neglecting CP violation in charm decay

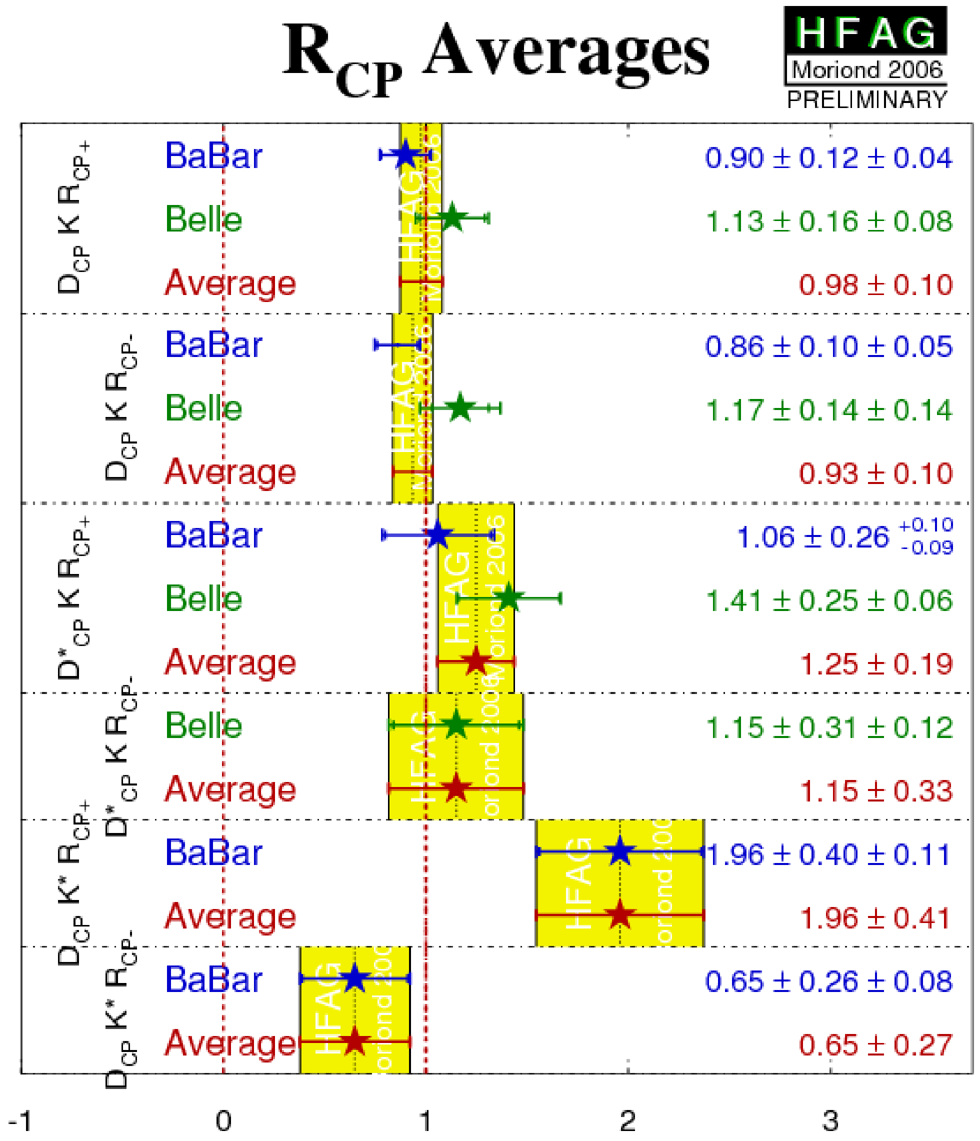
$$A(D^0 \to CP) = A(\overline{D}^0 \to CP)$$

- Possible states
  - CP even: (D_1) K^+K^-, $\pi^+\pi^-$, $K_s\pi^0\pi^0$
  - CP odd: (D_2) $K_s\pi^0$, $K_s\eta$, $K_s\eta'$, $K_s\rho^0$, $K_s\omega$, $K_s\varphi$



Experimental Summary

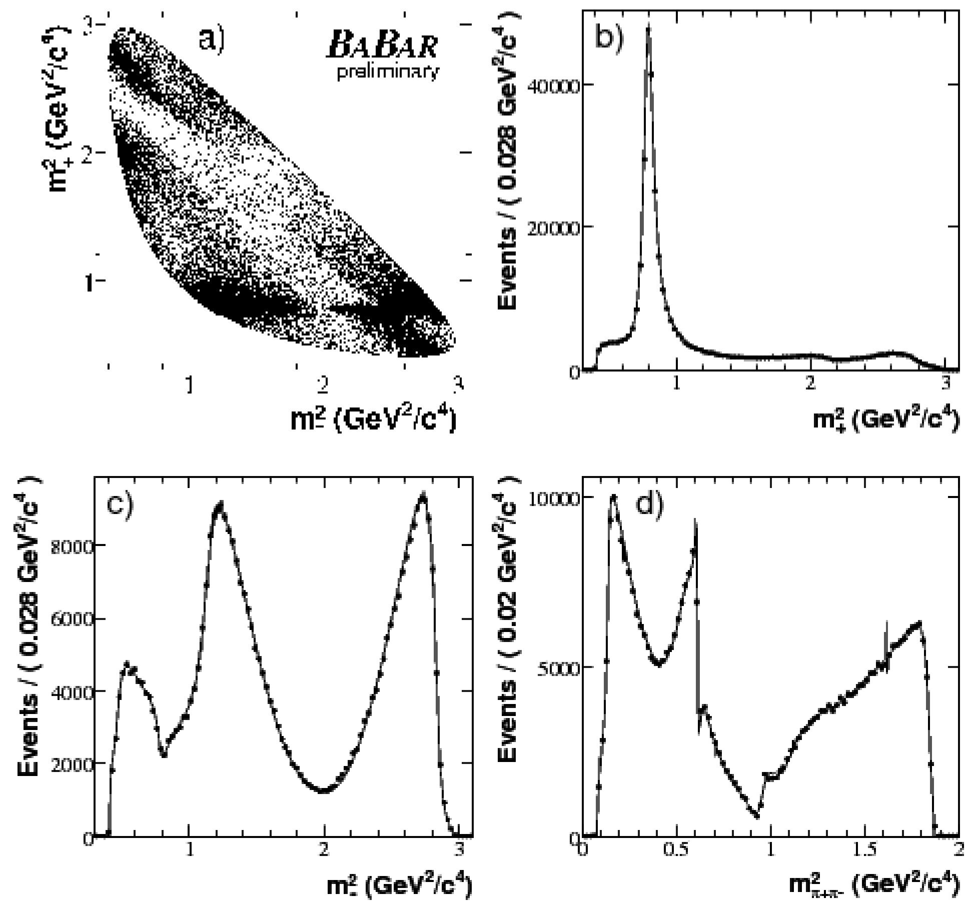




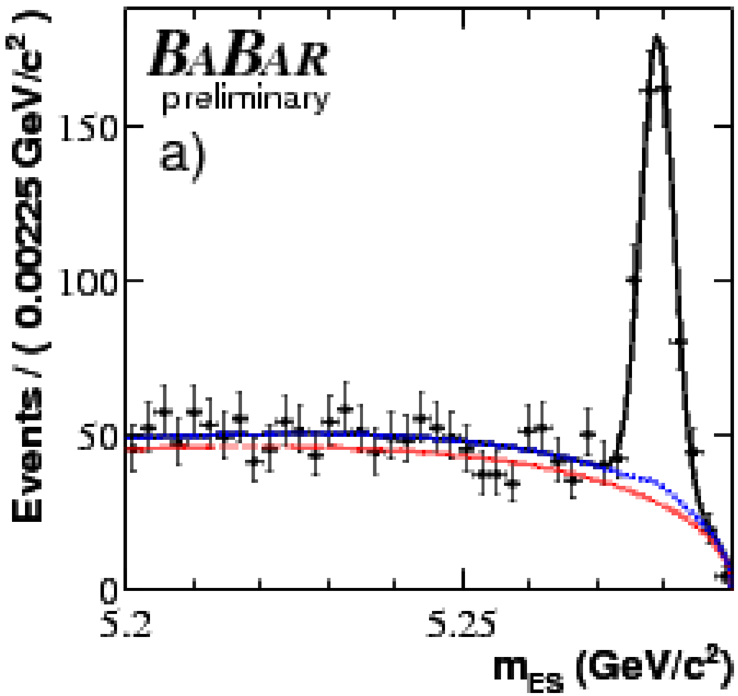
Multibody D decays – $D \to K_S \pi^+ \pi^-$

- Very powerful approach
  - Interfering resonances on Dalitz plot enhance sensitivity
  - Need good description (model) of DP structure

Component	$Re\{a_r e^{i\phi_r}\}$	$Im\{a_r e^{i\phi_r}\}$	Fit fraction (%)
$K^*(892)^-$	-1.223 ± 0.011	1.3461 ± 0.0096	58.1
$K_0^*(1430)^-$	-1.698 ± 0.022	-0.576 ± 0.024	6.7
$K_2^*(1430)^-$	-0.834 ± 0.021	0.931 ± 0.022	3.6
$K^*(1410)^-$	-0.248 ± 0.038	-0.108 ± 0.031	0.1
$K^*(1680)^-$	-1.285 ± 0.014	0.205 ± 0.013	0.6
$K^*(892)^+$	0.0997 ± 0.0036	-0.1271 ± 0.0034	0.5
$K_0^*(1430)^+$	-0.027 ± 0.016	-0.076 ± 0.017	0.0
$K_2^*(1430)^+$	0.019 ± 0.017	0.177 ± 0.018	0.1
$\rho(770)$	1	0	21.6
$\omega(782)$	-0.02194 ± 0.00099	0.03942 ± 0.00066	0.7
$f_2(1270)$	-0.699 ± 0.018	0.387 ± 0.018	2.1
$\rho(1450)$	0.253 ± 0.038	0.036 ± 0.055	0.1
Non-resonant	-0.99 ± 0.19	3.82 ± 0.13	8.5
$f_0(980)$	0.4465 ± 0.0057	0.2572 ± 0.0081	6.4
$f_0(1370)$	0.95 ± 0.11	-1.619 ± 0.011	2.0
σ	1.28 ± 0.02	0.273 ± 0.024	7.6
σ'	0.290 ± 0.010	-0.0655 ± 0.0098	0.9



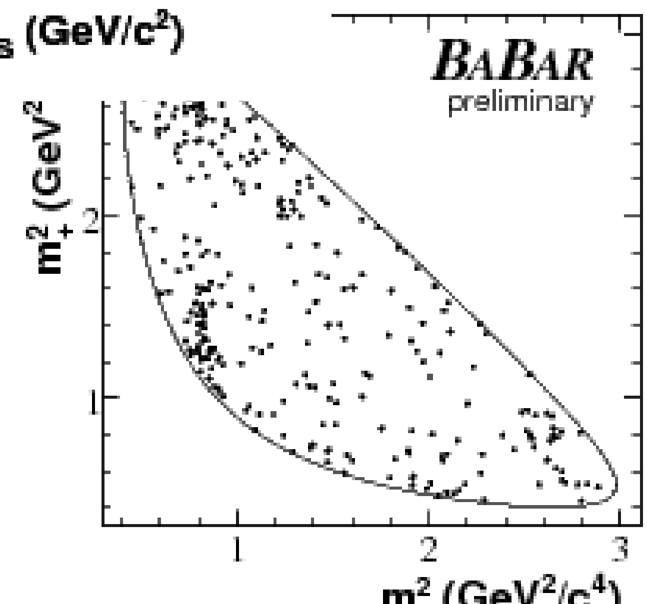
$B \to DK$, $D \to K_S \pi^+ \pi^-$ data

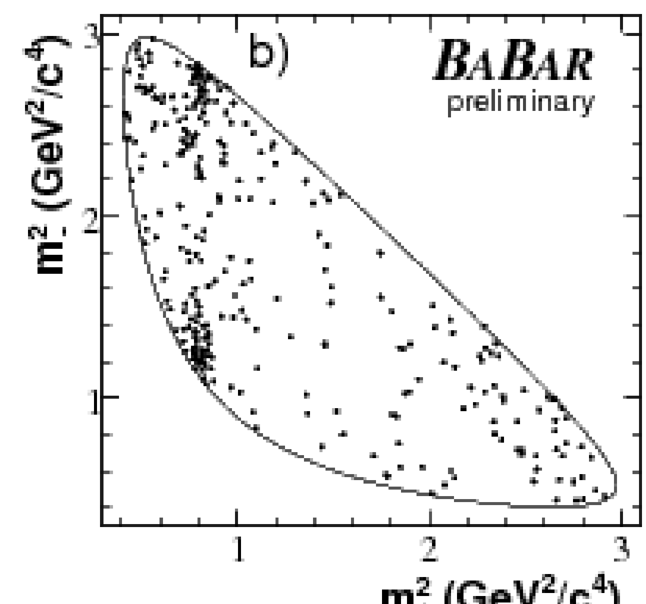


BABAR

hep-ex/0607104

Statistics limited

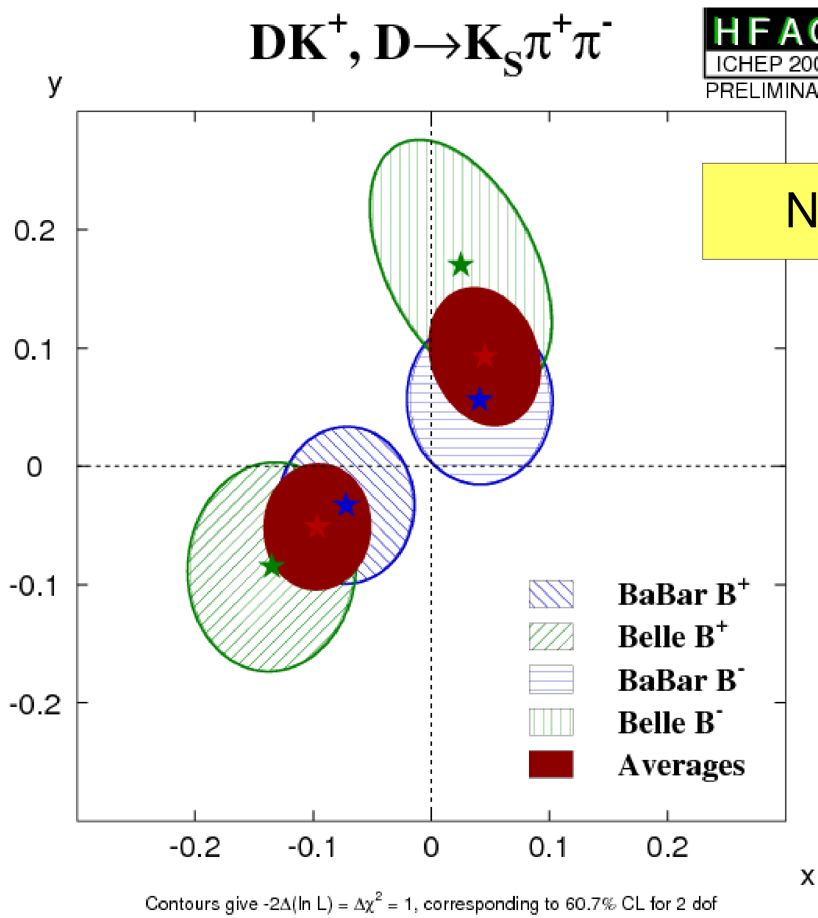


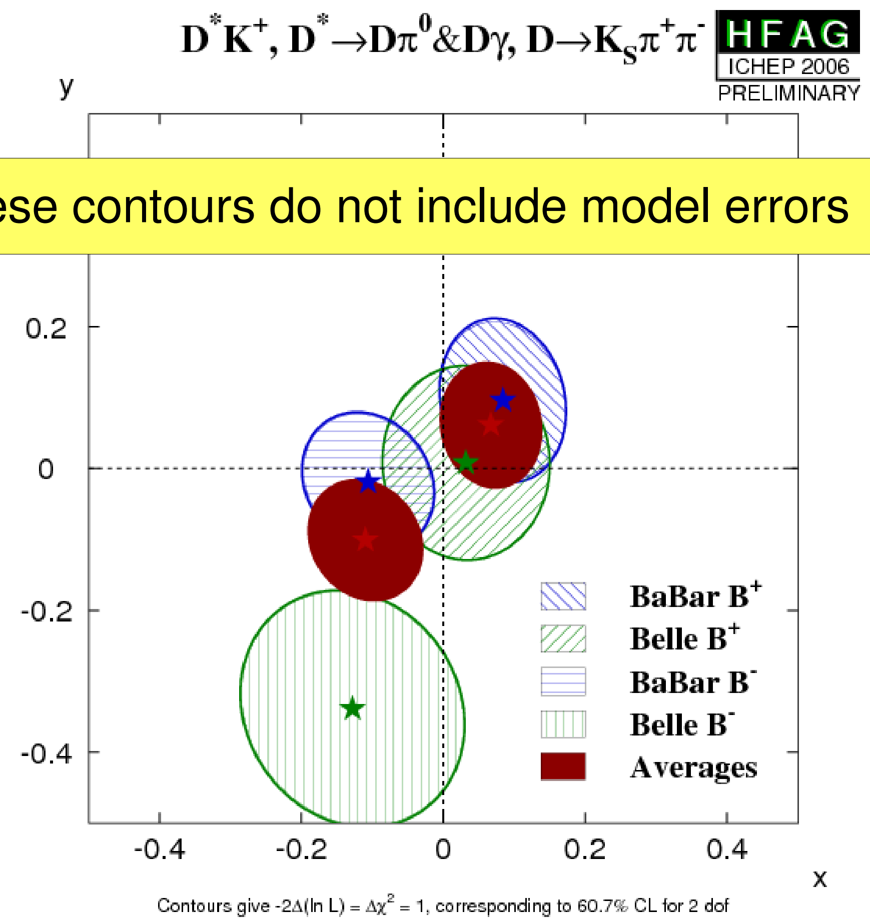


$B \rightarrow DK$, $D \rightarrow K_S\pi^+\pi^-$ results

Results presented in terms of "Cartesian parameters"

$$(x_\pm, y_\pm) = (\mathrm{Re}(r_B e^{i(\delta \pm \gamma)}), \mathrm{Im}(r_B e^{i(\delta \pm \gamma)}))$$



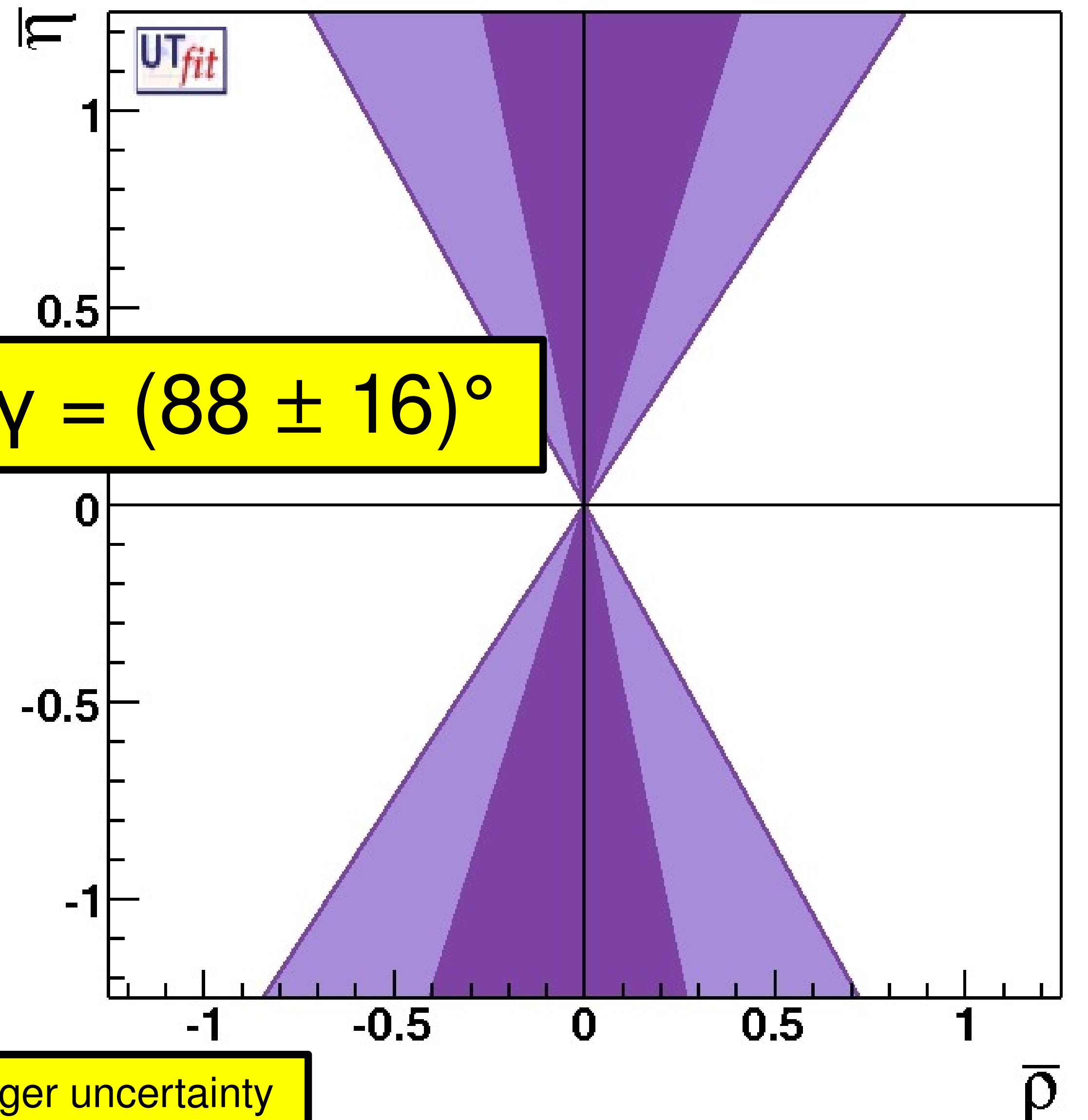


Note: these contours do not include model errors

Constraint from γ

Best constraint from combining all available results

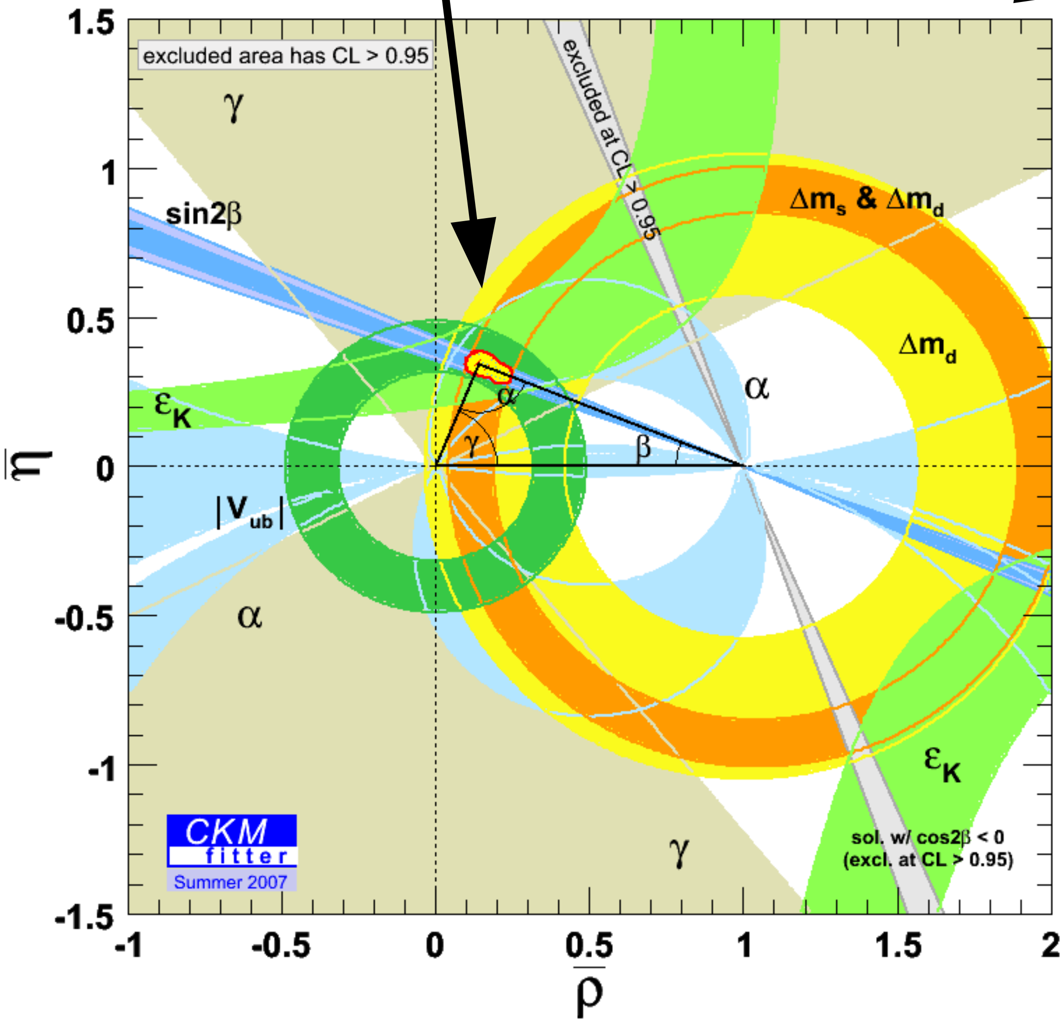
- $B \to DK$, $B \to D^{(*)}K$, $B \to DK^{(*)}$
- Different D decays
  - $D \to CP$ eigenstates
  - $D \to$ suppressed states
    - (eg. $K\pi$)
  - $D \to$ multibody states
    - (eg. $K_S \pi^+ \pi^-$)

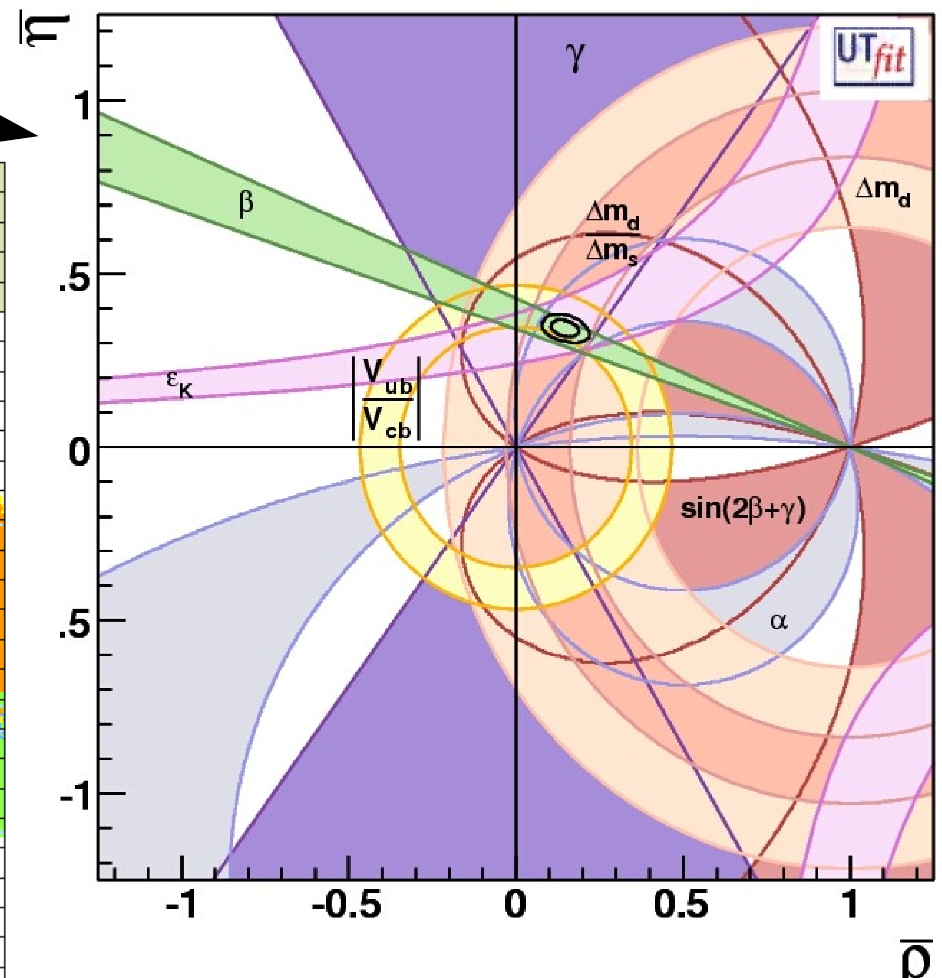


NB. Other statistical approaches give much larger uncertainty

Consistency of measurements with the KM mechanism

Different statistical approaches





... same answer

Dual Goals of CKM Metrology

- Unitarity Triangle parameters are fundamental in the Standard Model
  - We should measure them as well as possible
- Flavour provides an excellent arena to search for New Physics effects
  - History: Effects from higher scales seen
  - Most NP models predict inconsistencies with CKM Unitarity Triangle
- Certainly need to reduce current errors (~10%)

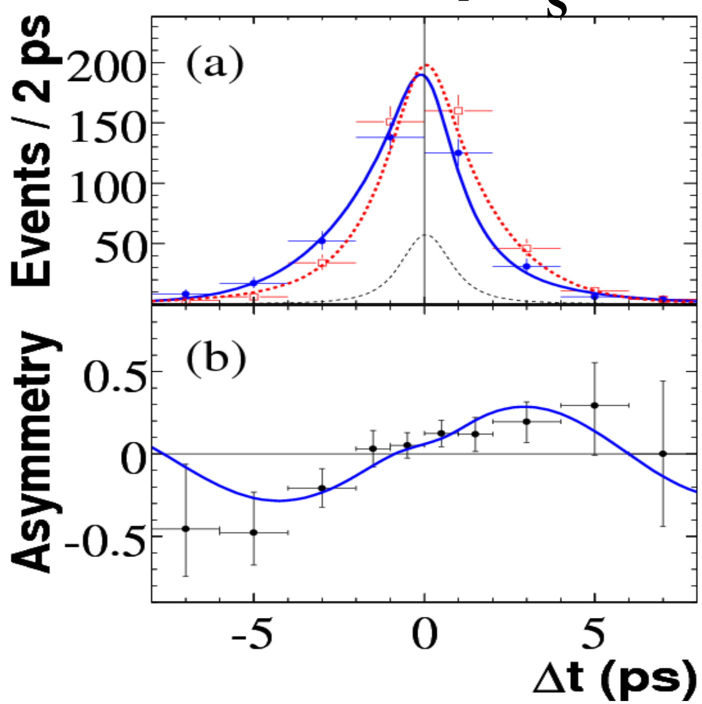
Searches for New Physics

- Massive, beyond SM, particles may contribute to B decay processes in loop diagrams
  - same true for kaon, charm & charged lepton physics
  - strong constraints in NP model building (flavour problem)
- Particularly interesting (not yet well tested) are $b \to s$
  - B_s mixing
  - $b \to sg$ (eg. time-dependence in $B^0 \to \phi K_S$, etc.)
  - $b \to s\gamma$ (eg. rates and moments, TDCPV in $B^0 \to K_S \pi^0 \gamma$)
  - $b \to sll$ (eg. FB asymmetry in $B \to K^* ll$)
  - $b \to s\nu\nu$ (also $s \to d\nu\nu$)

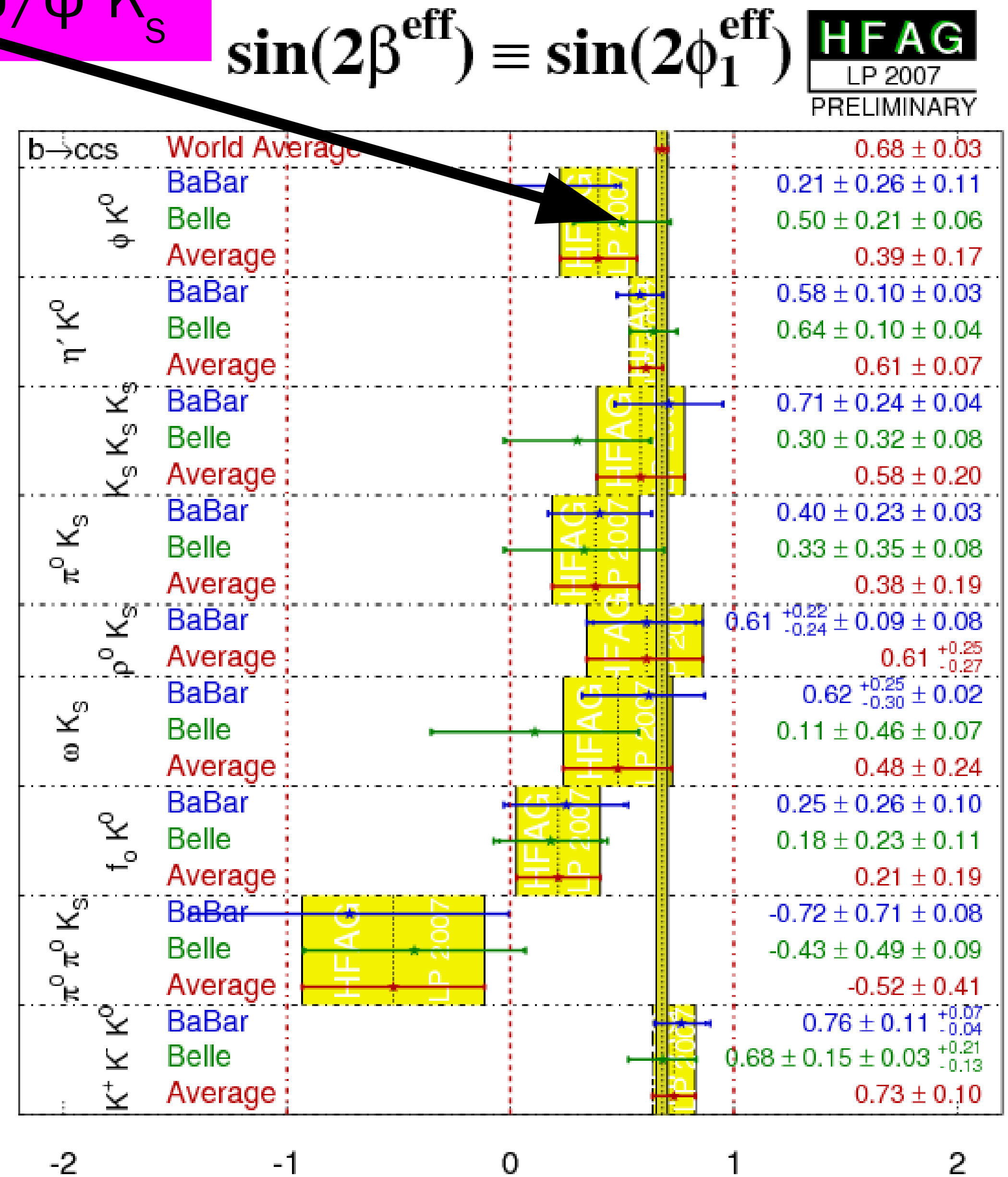
Discrepancies in hadronic $b \to s$ TDCPV

SM expectation : same as $B^0 \to J/\psi K_S$

BABAR - $B^0 \to \eta' K_S$



PRL 98 031801 (2007)



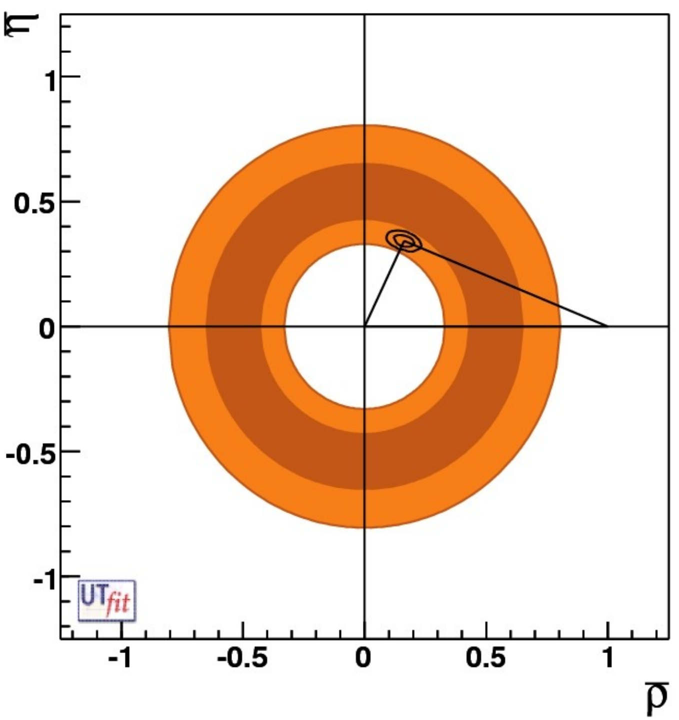
Improved & additional measurements essential

Discrepancies in leptonic B decays

- Interesting alternative approach to $|V_{ub}|$

$$\mathcal{B}(B^- \to \ell^- \overline{\nu}) = \frac{G_F^2 m_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

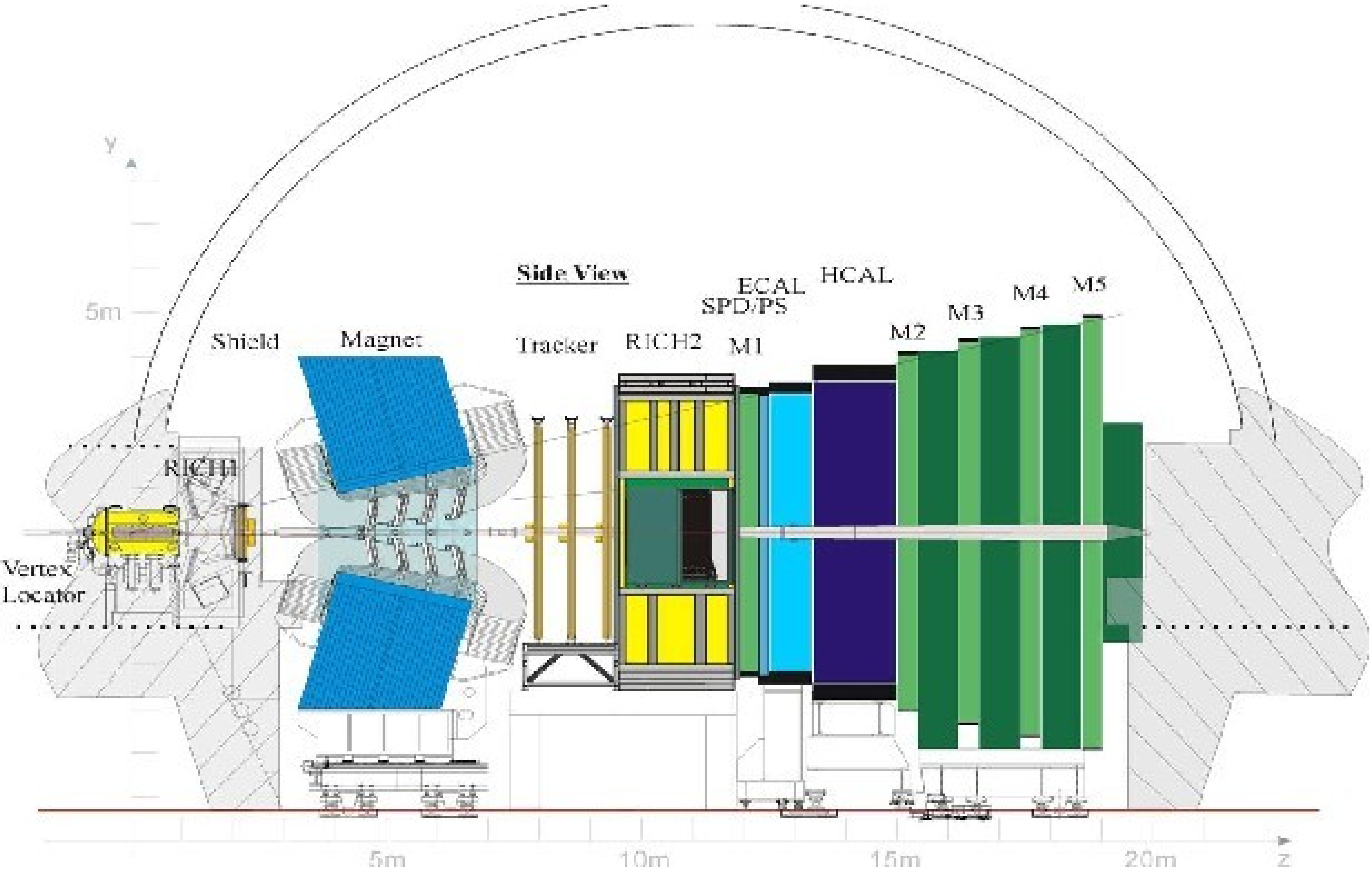
- Sensitive to corrections from new physics (charged Higgs)
- First evidence for this decay:
  - $B(B^- \to \tau^- \nu) = (1.8 \pm 0.5 \pm 0.5) \times 10^{-4}$
  - BELLE – PRL 97 (2006) 251802



The Future of Flavour Physics

- B sector
  - LHC will produce copious amounts of B mesons (and also D mesons, B&D baryons, τ leptons, ...)
  - LHCb is dedicated B physics experiment
  - Difficulties
    - triggering interesting events
    - maintaining manageable trigger rate
    - reconstructing neutral particles

LHCb detector

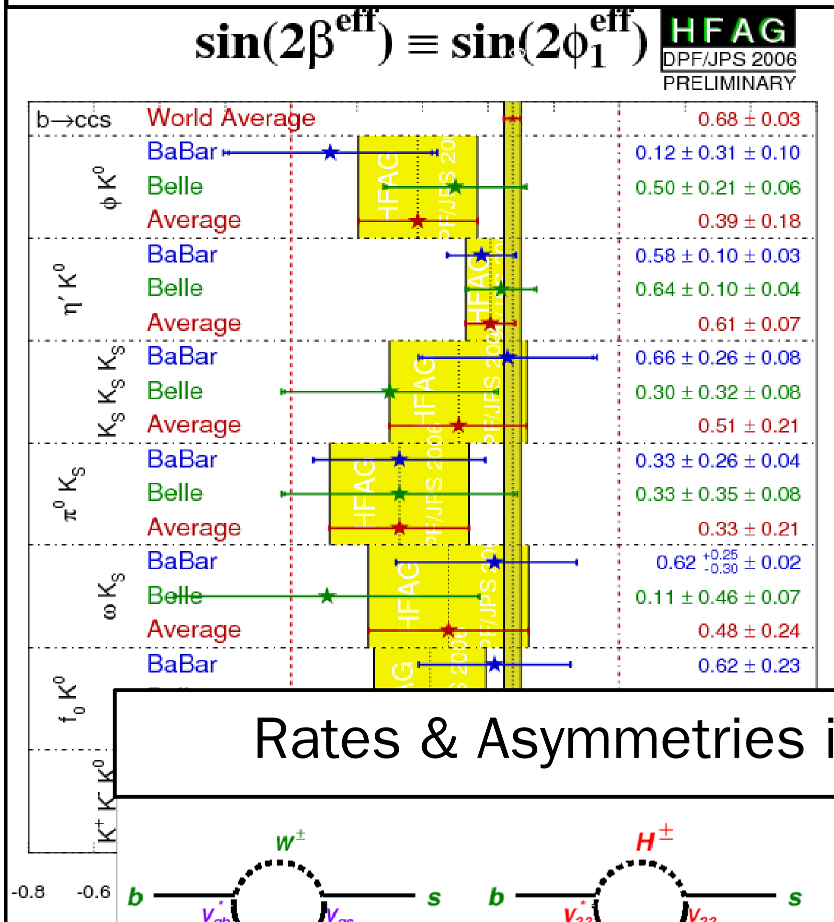


LHCb & Super Flavour Factory

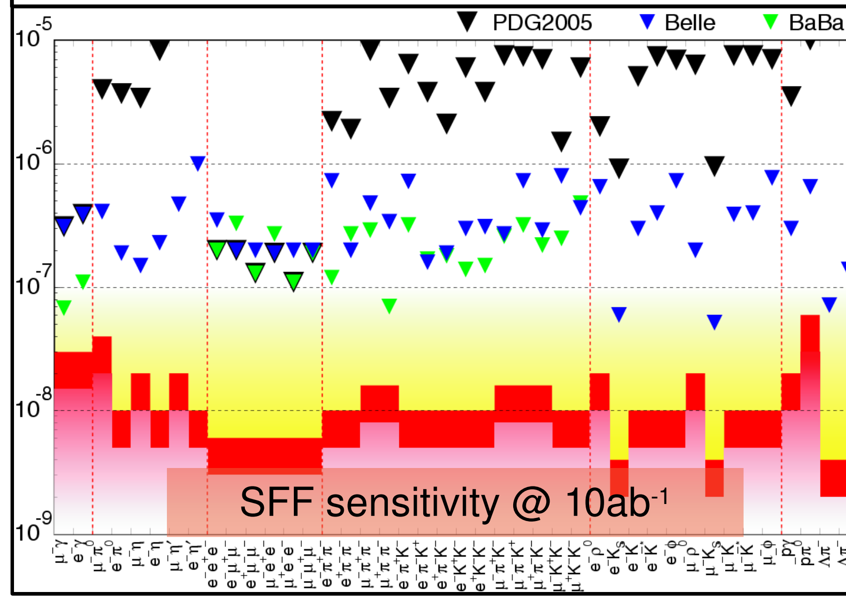
- LHCb will provide essential information on numerous important modes that cannot be studied elsewhere
  - eg. $B_s \to J/\psi\phi$, $B_s \to D_s^+D_s^-$, $B_s \to K^+K^-$, $B_s \to D_s^+K^-$, etc.
  - ATLAS and CMS can also contribute for, eg. $B_{s,d} \to \mu^+\mu^-$
- However, there are certain channels that are impossible for LHCb
  - modes with neutrals/neutrinos/hard trigger topologies
  - need a "Super Flavour Factory"

Super Flavour Factory Key Measurements

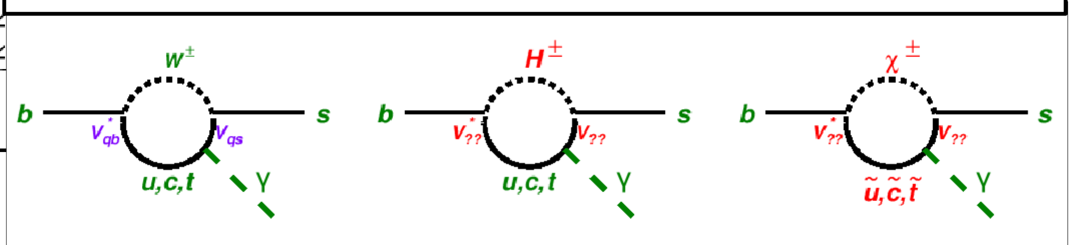
CP Violation in Hadronic b→s



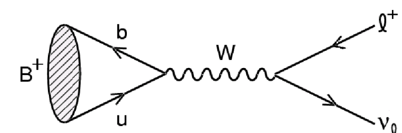
Lepton Flavour Violation in τ Decay

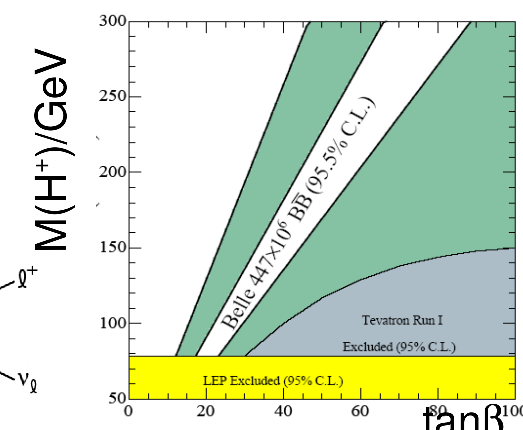


Rates & Asymmetries in b→sγ



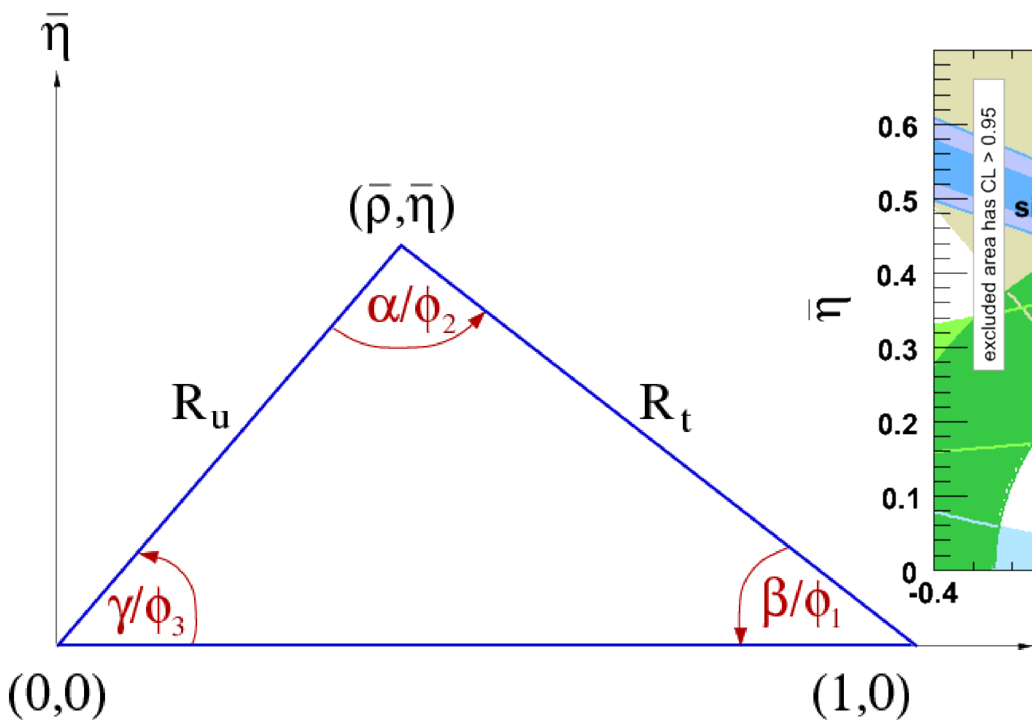
B → τν

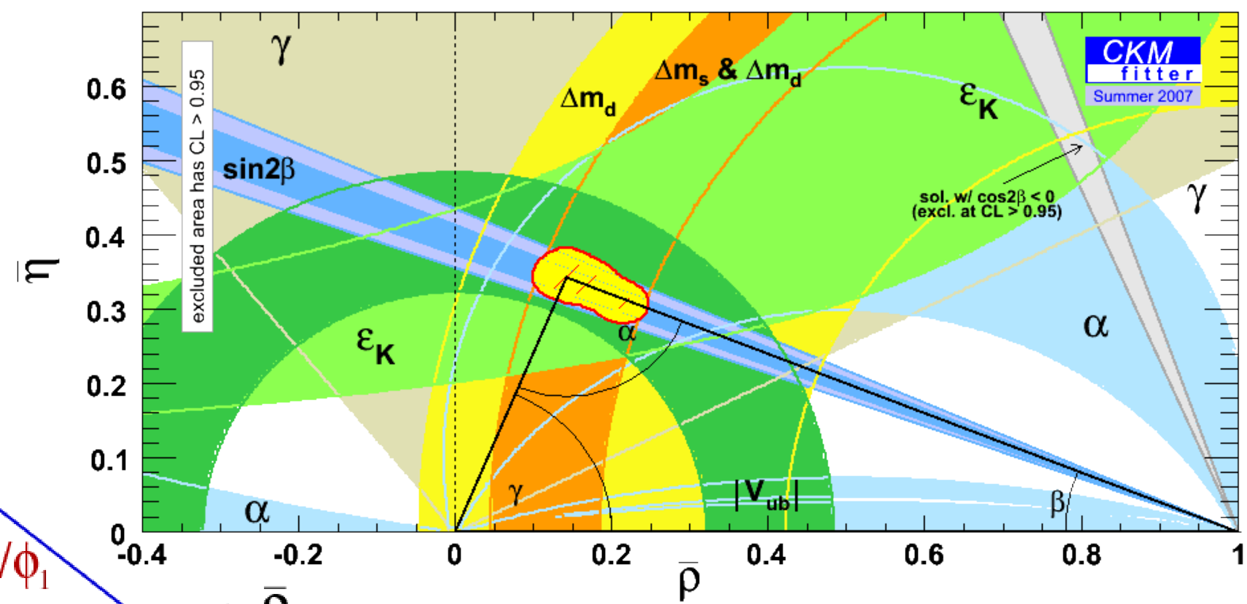




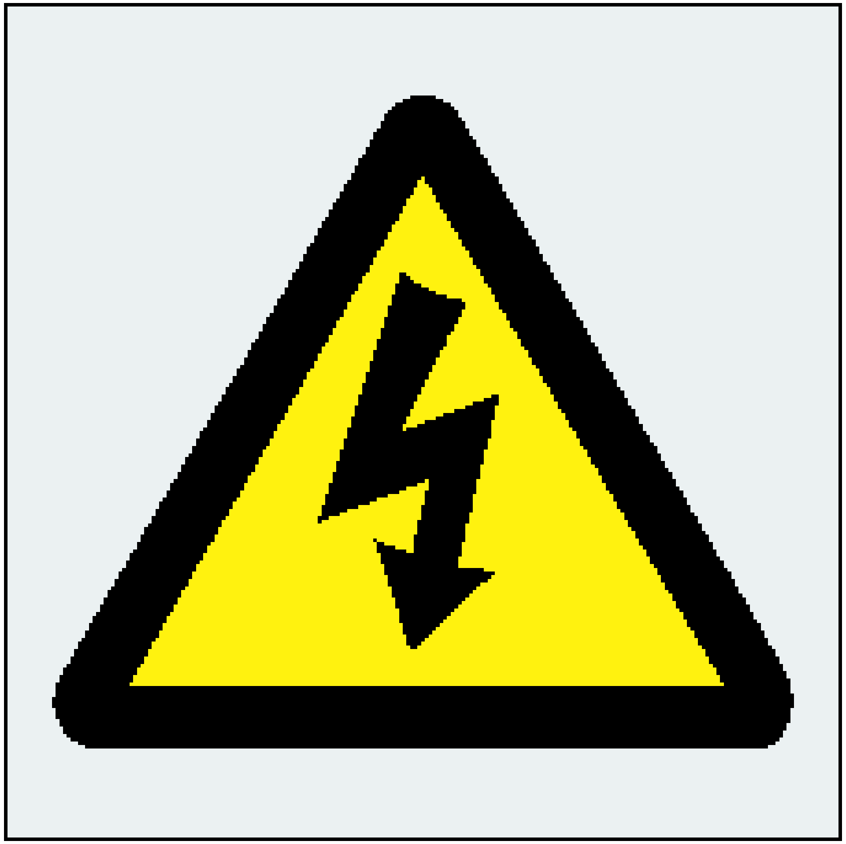
Summary

- Enormous progress by B factories to measure Unitarity Triangle and constrain flavour sector
- Still at the beginning of the programme ...
  - LHCb, KEKB upgrade, Super*B*, LHCb upgrade, ...





Back Up



Jarlskog

- All unitarity triangles have the same area
  - $A = J/2$
  - J is the Jarlskog invariant
  - $J = c_{12}\, c_{23}\, c_{13}^{\;2}\, s_{12}\, s_{23}\, s_{13} \sin\delta \sim 4\; 10^{-5}$
    - invariant measure of CP violation in the quark sector
  - $|J| = \mathrm{Im}(\, V_{ij}\, V_{kl}\, V_{kj}^{\;*}\, V_{il}^{\;*})$, for any choice of ijkl ($i \neq k$; $j \neq l$)
  - J related to commutator of mass matrices
    - $[M,M'] = iC \quad \det(C) = -2\, F\, F'\, J$
    - $F = (m_t - m_c)(m_t - m_u)(m_c - m_u)$
    - $F' = (m_b - m_s)(m_b - m_d)(m_s - m_d)$

Neutral B mixing parameters

- Recall: $q/p = -(\Delta m - \frac{1}{2}i\Delta\Gamma)/2(M_{12} - \frac{1}{2}i\Gamma_{12})$

  $(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2)$ $\Delta m\Delta\Gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}^*)$

- In the neutral B system $\Delta m \gg \Delta\Gamma$

  $\Delta m \sim 2|M_{12}|$ $\Delta\Gamma \sim 2\mathrm{Re}(M_{12}\Gamma_{12}^*)/|M_{12}|$ $q/p \sim -|M_{12}|/M_{12}$

- $|M_{12}|$ from mixing diagram

  $\Rightarrow$ $q/p \sim e^{-2i\beta}$ (in the usual phase convention)