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Perturbation Theory for particle scattering

1) $i\frac{\partial\psi}{\partial t} = [H_0(x) + \lambda V(t,x)]\psi$

Note: $H_0(x)\psi_m(x) = E_m\psi_m(x)$

$i\frac{\partial}{\partial t}\left[\sum_n C_n(t)\phi_n(x)e^{-iE_nt}\right] = [H_0(x) + \lambda V(t,x)]\sum C_n(t)\phi_n(x)e^{-iE_nt}$

$= i\left[\sum\frac{\partial C_n}{dt}\phi_n(x)e^{-iE_nt} + \cancel{(-iE_n)C_n(t)\phi_n(x)e^{-iE_nt}}\right]$
$= \cancel{\sum E_n C_n(t)\phi_n(x)e^{-iE_nt}} + \lambda V(t,x)\phi_n(x)e^{-iE_nt}C_n(t)$

Clearly the E_n terms cancel out; left with
$i\sum_n\frac{dC_n}{dt}\phi_n(x)e^{-iE_nt} = \lambda V(t,x)\phi_n e^{-iE_nt}C_n(t)$

Multiply by ϕ_f^*
$i\sum_n\frac{dC_n}{dt}\phi_f^*(x)\phi_n(x)e^{-iE_nt} = \lambda\sum_n\phi_f^*(x)V(t,x)\phi_n(x)e^{-iE_nt}C_n(t)$

Use normalisation ie if $f=n$
$i\sum_f\frac{dC_f}{dt}|\phi_f^*\phi_f|e^{-iE_ft} = \lambda\sum_n\phi_f^*(x)V(t,x)\phi_n(x)e^{-iE_nt}C_n(t)$
$\to 1$
$i\sum_f\frac{dC_f}{dt} = \lambda\sum_n C_n(t)e^{i(E_f-E_n)t}\phi_f^*(x)V(t,x)\phi_n(x)$

Integrate over all space
$\frac{dC_f}{dt} = -i\lambda\sum_n C_n(t)e^{i(E_f-E_n)t}\int\phi_f^*(x)V(t,x)\phi_n d^3\underline{x}$

∴ the differential equation

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2) Use some shorthand notation; $E_f - E_m = W_{fm}$
and $\int\phi_f^*V\phi_m d^3x = V_{fm}(t)$

Note: $\frac{dC_f^{s+1}}{dt} = -i\sum_m V_{fm}e^{iW_{fm}t}C_m^s(t)$

so $C_n(t) = C_n^0(t) + \lambda C_n^1(t) + \lambda^2C_n^2(t)$

Use recursive formula for each term

zeroth

$$\frac{dC_F^0}{dt} = 0 \text{ ; Hence } C_F^0(t) = \text{constant} = \delta_{Ki}$$

(Kroencker delta)
due to 'free' no interaction!

1st

$$\frac{dC_F^1}{dt} = -i \sum_m V_{F\mu} e^{i\omega_{F\mu}t} C_\mu^0(t)$$

but $C_\mu^0(t) = \delta_{\mu i}$

$$= -i \sum_\mu V_{F\mu} e^{i\omega_{F\mu}t} \delta_{\mu i} = -i \sum_\mu V_{Fi} e^{i\omega_{Fi}t}$$

$$C_F^1(t) = -i \int_{-\infty}^{t} V_{Fi} e^{i\omega_{Fi}t} dt$$

2ND

$$\frac{dC_F^2}{dt} = -i \sum_\mu V_{F\mu} e^{i\omega_{F\mu}t} C_\mu^1(t)$$

$$= (-i)^2 \sum_\mu V_{F\mu} e^{i\omega_{F\mu}t} \int_{-\infty}^{t'} V_{\mu i} e^{i\omega_{\mu i}t} dt'$$

$$C_F^2 = (-i)^2 \int_{-\infty}^{t} dt\, V_{F\mu} e^{i\omega_{F\mu}t} \int_{-\infty}^{t'} V_{\mu i} e^{i\omega_{\mu i}t'} dt'$$

Hence

$\cancel{C_F = \delta_{Ki} - i\lambda \sum_m V_F}$

$$\cancel{C_F = \delta_{Ki} - i\lambda \int_{-\infty}^{t} V_{Fi} e^{i\omega_{Fi}t} dt - \lambda^2 \int_{-\infty}^{t'} dt' V_{F\mu} e^{i\omega_{F\mu}t'} \int_{-\infty}^{t''} V_{\mu i} e^{i\omega_{\mu i}t''} dt''}$$

Using $C_F(t) = C_F^0(t) + \lambda C_F^1(t) + \lambda^2 C_F^2(t)$

$$C_F(t) = \delta_{Ki} - i\lambda \int_{-\infty}^{t} V_{Ki} e^{i\omega_{Ki}t} dt - \lambda^2 \int_{-\infty}^{t'} dt' V_{F\mu} e^{i\omega_{F\mu}t'} \int_{-\infty}^{t''} V_{\mu i} e^{i\omega_{\mu i}t''} dt''$$

not really derivation I was looking for 1

wanted to subst. expansion to result of 1 and compare order-by-order

Problem Sheet 4

in this problem sheet
m, n are both written as 4-indices (despite being non-greek!)

(1) $$C_f^{\,2}(t) = (-i)^2 \sum_m \int_{-\infty}^{t} dt'\, V_{fm}\, e^{i(E_f - E_m)t'} \int_{-\infty}^{t'} V_{mi}\, e^{i(E_m - E_i)t''} dt''$$

$$\int_{-\infty}^{t'} V_{mi}\, e^{i(E_m - E_i)t''} dt'' = V_{mi} \frac{\left[e^{i(E_m - E_i)t''} \right]_{-\infty}^{t'}}{E_m - E_i}$$

$$= \frac{V_{mi} \left[e^{i(E_m - E_i)t'} \right]}{i(E_m - E_i)}$$

$$C_f^{\,2}(t) = (-i)^2 \sum_m \int_{-\infty}^{t} dt'\, V_{fm}\, e^{iE_f} \cancel{e^{-iE_m}} \frac{V_{mi}}{i(E_m - E_i)} \cancel{e^{iE_m}} e^{-iE_i}$$

$$C_f^{\,2} = -\sum_m \int_{-\infty}^{t} dt' \frac{V_{fm}\, e^{i(E_f - E_i)}\, V_{mi}}{i(E_m - E_i)}$$

Using if t is very long time,
$$2\pi\, \delta(E_f - E_i) = \int_{-\infty}^{\infty} e^{i(E_f - E_i)}$$

$$= -2\pi i\, \delta(E_f - E_i) \sum_{m \neq i} \frac{V_{fm} V_{mi}}{(E_i - E_m)}$$

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2) Take same approach as above

$$\frac{dC_f^{\,3}}{dt} = -i \sum_m V_{fm}\, e^{i\omega_{fm} t}\, C_m^{\,2}(t)$$

$$= (-i)^3 \sum_m V_{fm}\, e^{i(E_f - E_m)t} \int_{-\infty}^{t'} \sum_n dt''\, V_{mn}\, e^{i(E_m - E_n)t''} \int_{-\infty}^{t''} dt''' \sum_i V_{ni}\, e^{i(E_n - E_i)t'''}$$

Integrate such that
$$C_f^{\,3}(t) = (-i)^3 \int_{-\infty}^{t} \sum_m V_{fm}\, e^{i(E_f - E_m)t'} dt' \int_{-\infty}^{t'} dt''\, V_{mn}\, e^{i(E_m - E_n)t''}$$
$$\times \int_{-\infty}^{t''} dt'''\, V_{ni}\, e^{i(E_n - E_i)t'''}$$

Do the last integral
$$\int_{-\infty}^{t''} dt'''\, V_{ni}\, e^{i(E_n - E_i)t'''} = V_{ni} \left[\frac{e^{i(E_n - E_i)t'''}}{i(E_n - E_i)} \right]_{-\infty}^{t''}$$

$$= V_{ni} \left[\frac{e^{i(E_n - E_i)t''}}{i(E_n - E_i)} \right]$$

CF is now $= (-i)^3 \int_{-\infty}^{t} V_{fm} e^{i(E_f-E_m)t'} dt' \int_{-\infty}^{t'} dt'' V_{mn} e^{i(E_m-E_n)t''} \left[\frac{V_{ni} e^{i(E_n-E_i)t''}}{i(E_n-E_i)}\right]$

Now the secondary integral

$$\sum_n \sum_i \int_{-\infty}^{t'} dt'' V_{mn} e^{i(E_m-E_n)t''} \left[\frac{V_{ni} e^{i(E_n-E_i)t''}}{i(E_n-E_i)}\right]$$

$e^{iE_nt''}$ terms cancel in this integral

$$\sum_n \sum_i \int_{-\infty}^{t'} dt'' V_{mn} e^{i(E_m-E_i)t''} \frac{V_{ni}}{i(E_n-E_i)}$$

$$= \frac{V_{mn} V_{ni}}{(E_n-E_i)} \int_{-\infty}^{t'} e^{i(E_m-E_i)t''} dt''$$

$$= \frac{V_{mn} V_{ni}}{i(E_n-E_i)} \left[\frac{e^{i(E_m-E_i)t'}}{i(E_m-E_i)}\right]$$

CF is now $\frac{(-i)^3}{(i)^2} \int_{-\infty}^{t} V_{fm} e^{i(E_f-E_m)t'} dt' \frac{e^{i(E_m-E_i)t'} V_{mn} V_{ni}}{(E_n-E_i)(E_m-E_i)}$

Em terms cancel (in expo e^{iE_mt}

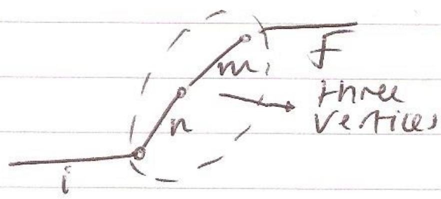
$$C_F = \frac{(-i)^3}{(i)^2} \int_{-\infty}^{t} \frac{V_{fm} V_{mn} V_{ni}}{(E_n-E_i)(E_m-E_i)} dt' e^{i(E_f-E_i)t'}$$

Like before, assume t is very large, such that

$$\int_{-\infty}^{t} dt' e^{i(E_f-E_i)t'} = 2\pi\delta(E_f-E_i)$$

$$C_F = \sum_{n \neq m} \frac{-2\pi i \, \delta(E_f-E_i) V_{fm} V_{mn} V_{ni}}{(E_n-E_i)(E_m-E_i)}$$

4 like



c) spin 0 particle, described by KG. If no interaction, then free KG $(\partial_\mu \partial^\mu + m^2)\phi = 0$

=>

We introduce interaction by using minimal substitution such that $\partial^\mu \to \partial^\mu - ieA^\mu$;

Use this in the Klein Gordon equation

$$[(\partial^\mu - ieA^\mu)(\partial_\mu - ieA_\mu) + m^2]\phi = 0$$
$$(\partial^\mu \partial_\mu - ie\partial^\mu A_\mu - ieA^\mu \partial_\mu - e^2 A^\mu A_\mu + m^2)\phi = 0$$

Can seperate into 'free' and 'potential'

$$[\underbrace{\partial^\mu \partial_\mu + m^2}_{= \text{free}} - ie\underbrace{\partial^\mu A_\mu - ieA^\mu \partial_\mu - e^2 A^\mu A_\mu}_{\text{potential}}]\phi$$

Hence $(\partial^\mu \partial_\mu + m^2)\phi = -V\phi$

Hence $V = -ie(\partial^\mu A_\mu + A^\mu \partial_\mu) - e^2 A^\mu A_\mu$

but as EM interaction strength is given as $\frac{e^2}{4\pi} \sim 1/137$ approximate V!

$$\underline{V \approx -ie(\partial^\mu A_\mu + A^\mu \partial_\mu)}$$

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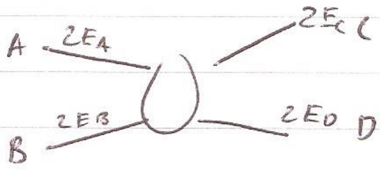
Problem Sheet 5

5) INVARIANT AMPLITUDES to Observable Quantities

$$dW = \frac{|M_{fi}|^2 (2\pi)^4 \delta^4(p_f - p_i)}{V^{ni}} \prod_{f=1} \frac{d^3 p_f}{(2E_f)(2\pi)^3}$$



For cross-section, $d\sigma = \frac{dW}{\mathcal{L}}$

where $\mathcal{L}$ = luminosity! In this case have $2E_A$ particles at A and $2E_B$ particles at B, each term has velocity v_a, v_b.

$$\text{Hence } \mathcal{L} = 4E_a E_b (v_a + v_b)$$
$$= 4(E_a E_b)\left[\frac{p_a}{E_a} + \frac{p_b}{E_b}\right] = 4|\vec{p}|\left[E_a + E_b\right]$$
$$= 4|\vec{p_i}|\left[P_a + P_b\right]$$

this is effectively the volume 'cleared out' by each particle, hence alter δ^n and the phase spaces

$$d\sigma = \frac{|M_{fi}|^2 (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D)}{4|p_i|[p_A + p_B]} \frac{d^3 p_c}{(2\pi)^3 (2P_C)} \frac{d^3 p_d}{(2\pi)^3 (2P_D)}$$

In Centre of Mass frame, $P_A + P_B = P_C + P_D$ ← this does not remove integral

$$d\sigma = \frac{|M_{fi}|^2}{64\pi^2 [p_A + p_B]} \delta^4(p_A + p_B - p_C - p_D) \frac{d^3\vec{p}}{P_C P_D}$$

Go to polar coordinates $d^3\bar{p} = |p_a|^2 d\bar{p}\, d\Omega$; using $E^2 = |p^2| + m^2$, $d(E_{\cancel{A}} + E_{\cancel{D}}) = \frac{|p_{\cancel{a}}| dp}{E_D} + \frac{|p| dp}{E_C}$

$$= |p| dp \left[\frac{1}{E_D} + \frac{1}{E_C}\right]$$
$$= |p|\left[\frac{E_D + E_C}{E_C E_D}\right]$$

Hence $|p|^2 d|p| d\Omega = \frac{|p| d(E_c + E_d) d\Omega}{E_C + E_D}$ ~ this reduces δ function = 1

BUT! $E_C + E_D = P_A + P_B$; note $|P_A| = |P_C|$!

Hence

$$d\sigma = \frac{|M_{fi}|^2}{64\pi^2} \frac{1}{(P_a + P_b)^2} \frac{|\vec{P_c}|}{|\vec{P_A}|}$$

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one should integrate over one ~~th~~ 3-momenta with help of δ-function, then express rest in polar coordinates and do one more integral with help of remaining part of δ-function