Kelvin-Helmholtz instability in the solar corona, solar wind and in geomagnetosphere.

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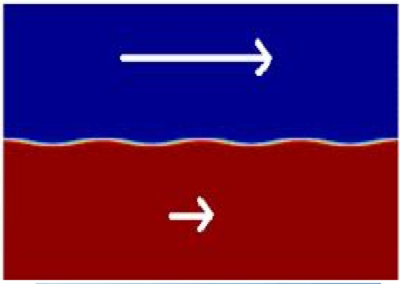
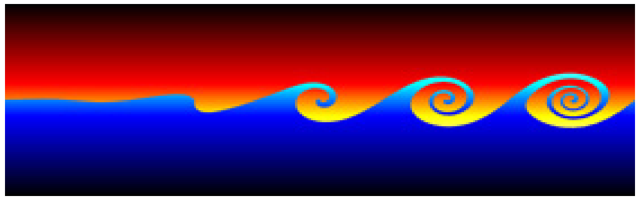
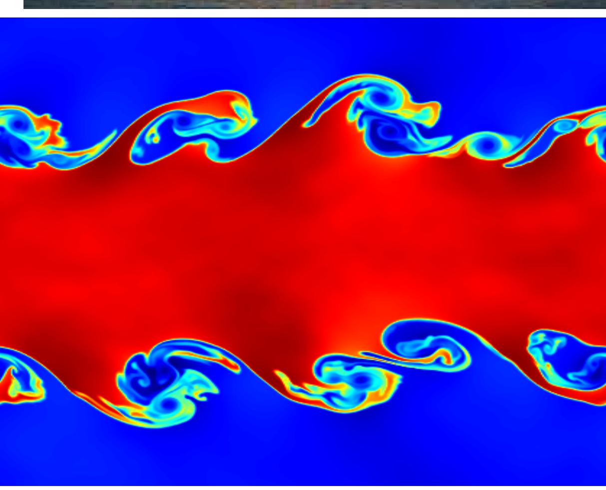
KHI is most is the most common and most strong instability in nature. Some examples.

White caps on the water, overturning wave





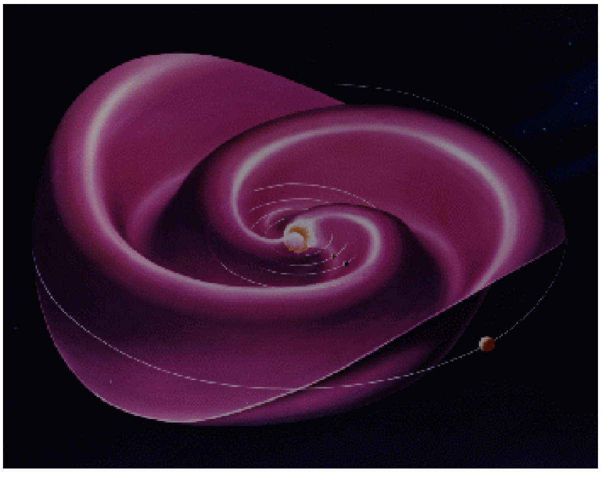



development of KHI of the jet with two boundaries or cylindrical jet gives formation of vortices on both jet borders - Karman road. Airplane and flamethrower jets disappear due to KHI development- diffusive widening.

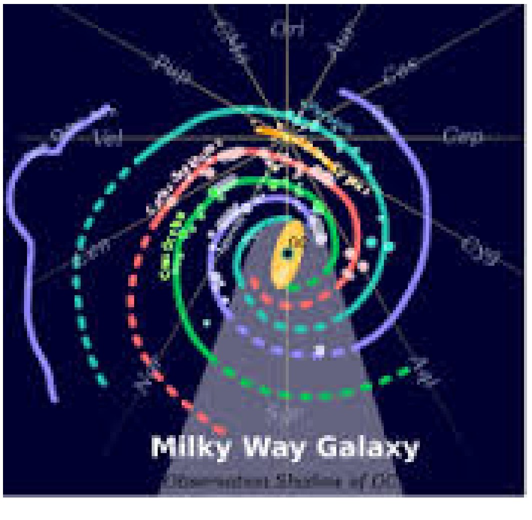
Pacific typhoon

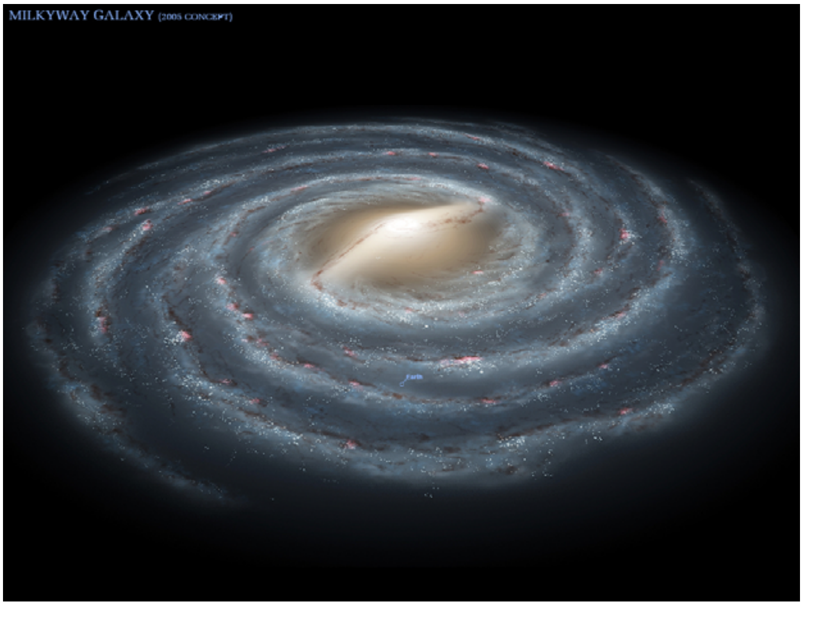
Atlantic hurricane

Spiral structure of atmospheric cyclones, heliospheric current layer and spiral arms of a galaxies - is possible KHI manifestation in rotational velocity shear-due to sharp change of angular velocity



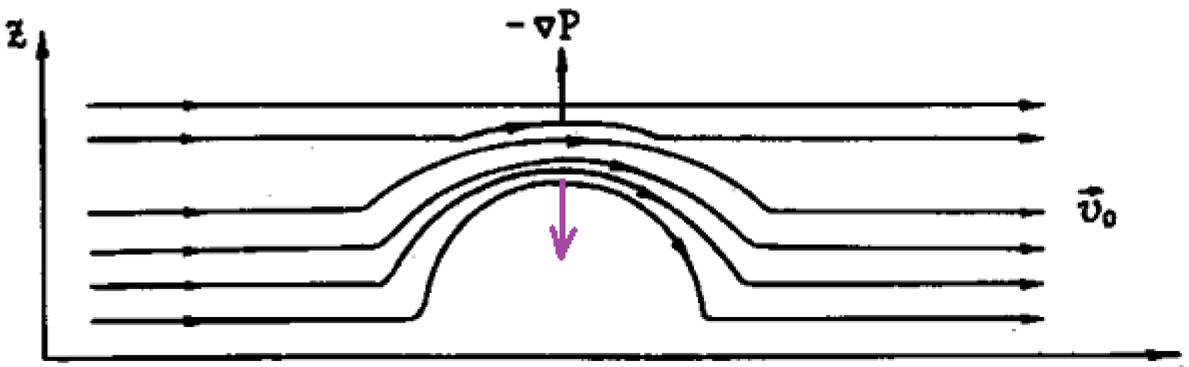
Heliosphere



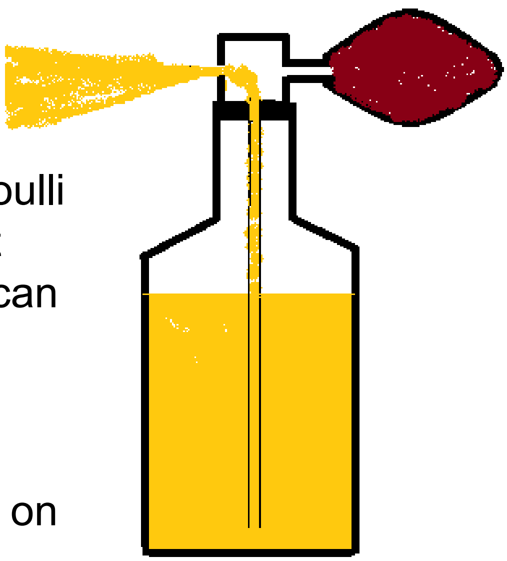


Galaxy spiral arms

Kelvin-Helmholtz instability **(K-H I)** develops on a tangential discontinuity (**TD**) - thin boundary between two flows having different velocities.
Example: wind instability at the surface of a sea.
Also it is called *the velocity shear instability.*
It was described by **Helmholtz (1868)** and **Kelvin (1871).**
Nature of instability: initiation of wing lift by concentration of streamlines over random boundary displacement. Increase of dynamic pressure $\rho V^2/2$ according to Bernoulli theorem: $P+\rho V^2/2+\rho gz = const$
causes decompression over the displaced boundary $\Delta P = -\Delta\rho\upsilon^2/2 = -\rho\boldsymbol{\upsilon}\cdot\boldsymbol{\Delta\upsilon}$
which forces initial displacement to grow more, $md\mathbf{V}z/dt = \mathbf{F}_z = -\nabla P = \rho(\boldsymbol{\upsilon}\cdot\nabla)\boldsymbol{\upsilon} > 0$
so initial convexity (and concavity also) increases with time.



surface capillar and Maxwell tension forces



Another close example - the effect of pulverizer. When pumping of pear on the neck of the vial, a fast flow of air appears, which also due to Bernoulli theorem causes the hydrodynamic vacuum, that sucks out a cologne from a bottle. Resulting jet can be unstable- as a flame-thrower jet.

What forces can stabilize KHI?

1) capillary action - the surface tension tends to compensate for the increase in accidental bump on the surface of the liquid; The level of the non-wetting liquid in the capillary is lowered.
2) similar stabilizing effect has a longitudinal magnetic field - at a bending of border, the magnetic field line is stretched like a bowstring, causing a restoring force.

Main results of the K-H I linear theory

Kelvin-Helmholtz instability **(K-H I)** develops on a tangential discontinuity (**TD**) - thin boundary between two flows having different velocities. In real conditions because of finite width of TD in common case one needs to consider instability of velocity shear layer of finite width D>0. But such a task is hard for analytical description. Last one is given by **approximation of TD,** considering instability of plane Z=0 with zero thickness between two homogeneous self-spaces **I** (Z<0) and **II** (Z>0). At TD all parameters change their values. The velocity $\mathbf{V}_0$ and magnetic field $\mathbf{B}_0$ vectors change their values and directions, being in the plane parallel to the boundary ($V_z=B_z=0$).

Consider perturbations on equilibrium parameters in form:

$$f_1(x,y,z,t) \propto f(z)\exp\{i(k_x x + k_y y - \omega t)\}$$

Linearization of MHD equations gives for vertical displacement ς and total pressure perturbation $\Pi_1 = P_1 + \mathbf{B_1}\cdot\mathbf{B_0}/4\pi$

$$\Pi_1 = \rho_0(\Omega/\chi)^2 \frac{d\varsigma}{dz}, \qquad (1)$$

$$\frac{d\Pi_1}{dz} = \rho_0\Omega^2\varsigma$$

here $\chi^2 = k^2 - \varpi^4/[\varpi^2(c_s^2 + a^2) - (\mathbf{k}\cdot\mathbf{a})^2 c_s^2]$ (2)

$\Omega^2 = \varpi^2 - (\mathbf{k}\cdot\mathbf{a})^2, \varpi = \omega - \mathbf{k}\cdot\mathbf{v}_0(z),$ $\mathbf{k} = \{k_x, k_y, 0\} - wave$ vector,

$\omega = \mathrm{Re}(\omega) + i\gamma$ - frequency.

For $\mathrm{Im}(\omega) = \gamma > 0$ there is an instability: perturbations grow in time

$f_1(t) \propto \exp\{\gamma\cdot t\}$ and an imaginary part of a frequency is called a growth rate. To have dispersion dependence $\omega = \omega(k)$ we need to solve the system (1). Boundary conditions are obtained by integrating of equations (1) for a finite shear layer and taking: $\lim\limits_{D\to 0}\int (1)dz.$. It gives continuity of full pressure and vertical displacement: $\Pi_I(z=0) = \Pi_{II}(z=0),$

$$\varsigma_I(z=0) = \varsigma_{II}(z=0). \qquad (3)$$

Damping decision of (1) is

$$f_1(x,y,z,t) \propto \exp\{-\chi\,|z| + i(\mathrm{kx} + \mathrm{ky} - \mathrm{wt})\} \quad (4)$$

Substitution of (4) in conditions (3) gives a dispersion equation of TD. This equation for finite values of magnetosound Mach number $M_{ms} = \Delta\upsilon/(2c_m) \neq 1$ is not derived analytically in common case (here $\Delta\upsilon = |\mathbf{\upsilon}_{\mathbf{II}} - \mathbf{\upsilon}_{\mathbf{I}}|$ -velocity jump, $c_m = \sqrt{(c_s^2 + a^2)}$ - magnetosound velocity, c_s and a- are sound and Alfven velocities).

Approximation of incompressible medium: Density change is absent ($d\rho/dt = 0$, $\mathbf{div}\vec{\mathbf{\upsilon}} = 0$,) and index of exponential damping (2) is maximal: (=k)- generation of surface perturbations fast damping from boundary.

Dispersion equation of subsound "incompressible" TD:

$$\omega=\{\mathbf{k}[(\rho\mathbf{V})_{\mathrm{I}}+(\rho\mathbf{V})_{\mathbf{2}}]\pm i\{\rho_1\rho_2(\mathbf{k\Delta V})^2-$$
$$-(\rho_1+\rho_2)[(\mathbf{kB}_1)^2+(\mathbf{kB}_2)^2]/4\pi+G\}^{1/2}\}/(\rho_1+\rho_2).$$

$G=kg(\rho_2-\rho_1)$-addition due to Raleigh –Taylor effect in the gravity field $\mathbf{g}$

$$If\ \ \rho = const, B = const : \gamma = \mathrm{Im}(\omega) = \sqrt{(\mathbf{k}\cdot\mathbf{\Delta\upsilon})^2 - 4(\mathbf{k}\cdot\mathbf{a})^2};$$

$$for : \mathbf{k}\,||\,\mathbf{\upsilon}\,||\,\mathbf{B}....\gamma > 0..if..\Delta\upsilon > 2a$$

Influence of $\mathbf{B_0}||\mathbf{V_0}$ parallel magnetic field: from the equation of motion

$$\partial\mathbf{\upsilon}/\partial t + (\mathbf{\upsilon}\cdot\nabla)\mathbf{\upsilon} - (\mathbf{B}\cdot\nabla)\mathbf{B}/(4\pi\rho) = -\rho^{-1}\cdot\nabla(P + B^2/8\pi)$$

Maxwell tensions are directed opposite to Reynolds tensions and tend to stabilize TD (above mentioned analogy with bow string stretching. Perturbations perpendicular to magnetic field move field lines free without stretching and do not cause recovery force of Maxwell tensions.

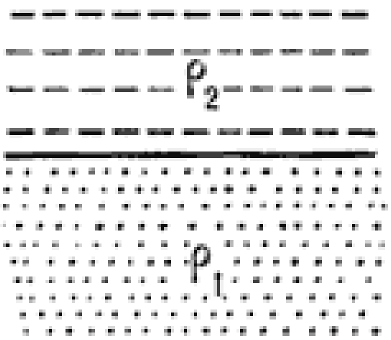
For near - sound velocity jump **(**M~1**)** one should take into account stabilizing effect of **compressibility of medium**. **Landau (1944)**, hydrodynamics (**B**=0), $\mathbf{k}\,||\,\mathbf{V}$, $\gamma = 0$ for $M > M_L = \Delta\upsilon_L/2c_s = \sqrt{2}$

Syrovatsky (1954):$\gamma \neq 0$ for oblique perturbations $\cos\varphi = \mathbf{k}\cdot\mathbf{\upsilon}/k\upsilon < M_L^{-1}$

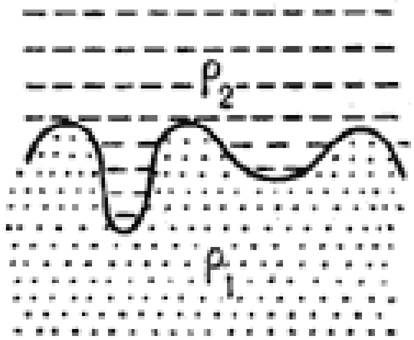
Parker (1964), MHD,**k** || **V** $||$ **B:** for $c_s - \delta < \Delta\upsilon < c_s + \delta$ - very narrow range, not satisfied in SW.

***Weaknesses of TD-approximation**:*

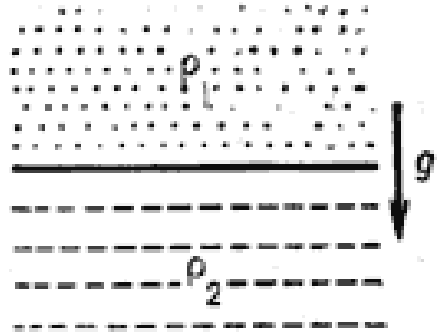
1. short-wave boundary of instability is unknown;
2. effect of compressibility and density jump is overestimated.



initial state of
unstable equilibrium
· heavy liquid is
over the light

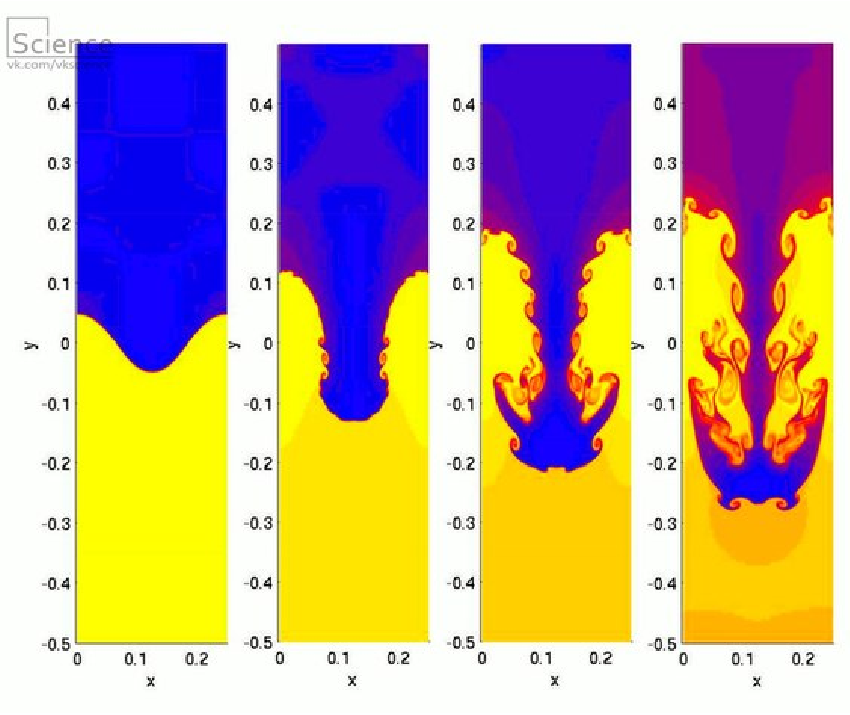


RTI
development –
vertical mixing



final state-
heavy liquid is
under the light

- Instability of the water over the oil



Instability of shear layer of finite width: D=2d;

Profiles of parameters are described by smooth functions:

$$v(z) = u(1 + \tanh(z/d))$$

$$\rho(z) = \rho_{00}\{1 + R[1+\tanh(z/d)]\}/(1+R);$$

$$B_0(z) = b_{00}\backslash\{1 - \delta[1 + \tanh(z/d)]\}/(1 - \delta);$$

It is considered a task on eigen function and eigen values of system (1), satisfying to smooth exponential damping (4) of perturbations far from shear layer:

$$f_1(x, y, z, t) \propto \exp\{-\chi |z| + i(\mathrm{kx} + \mathrm{ky} - \mathrm{wt})\} \qquad (4)$$

$$z \to R, \ \lim\{d/dz \ln f_1(z)\} + \chi = 0$$

New results of taking into account of boundary finite thickness.
For usually considered submagetosound case: $M_{ms} = \Delta V/(2c_m) < 1$ "incompressible" approximation is applicable.
For perpendicular configuration **k||V⊥ B** dependence of dimensionless growth rate from dimensionless wavelength kd is shown in Figures 1.

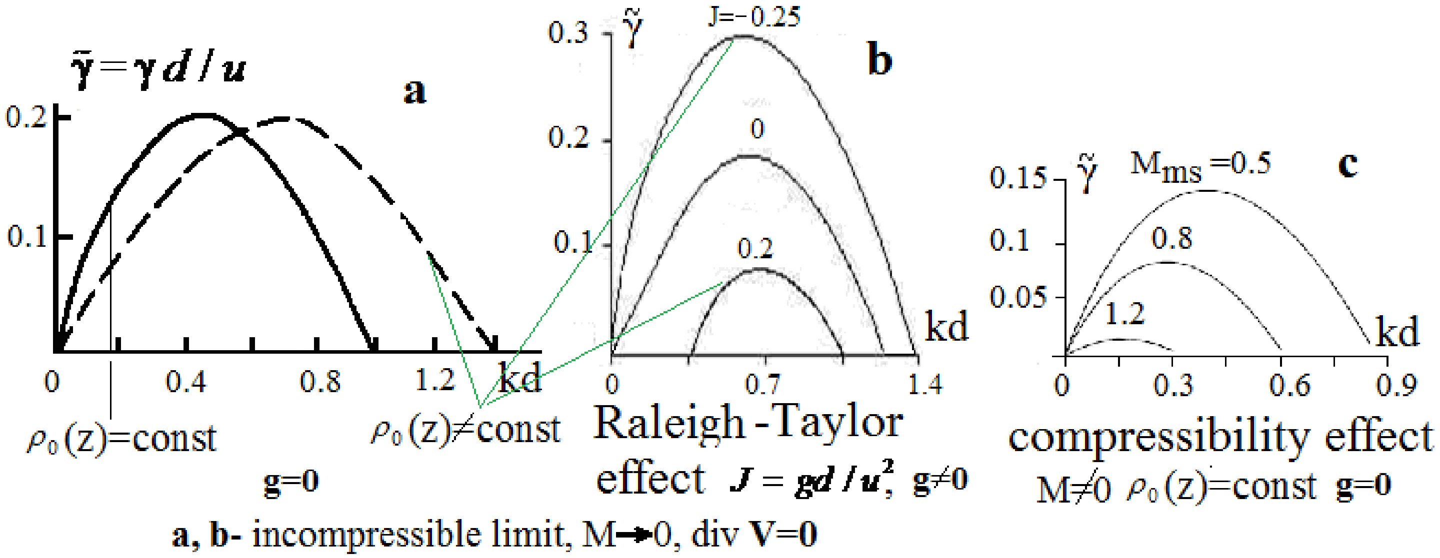
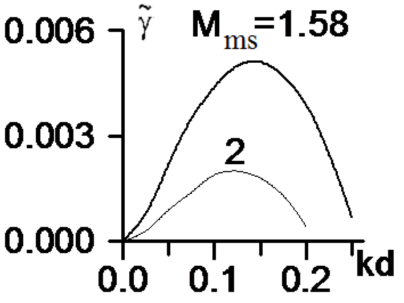


Fig.1 Normalized growth rate $\tilde{\gamma} = \gamma d/u$ *as a function of wave number kd for longitudinal perturbations and perpendicular magnetic field* $\mathbf{k} \parallel \mathbf{V}_0 \perp \mathbf{B}_0$ *Morozov and Mishin (1981), Mishin and Morozov (1983); Mishin 1993, 2003).* Parallel magnetic field $\mathbf{k} \parallel \mathbf{V}_0 \parallel \mathbf{B}_0$ reduces the growth rate and narrows the range of instability *(Chandra, 1973)*, like the compressibility effect, firstly studied by Blumen et al, 1970,1975). However, magnetic pressure P_B reduces value M_{ms}.

$$M_{ms} = \frac{u}{c_s}\frac{c_s}{\sqrt{c_s^2 + a^2}} = \frac{M}{\sqrt{1 + 2P_B/(\Gamma P_0)}} = \frac{M}{\sqrt{1 + 2/(\Gamma\beta)}}$$

Decrease of M_{ms}- compressibilty effect means increasing growth rate.I.e. perpendicular magnetic field can amplify instability!

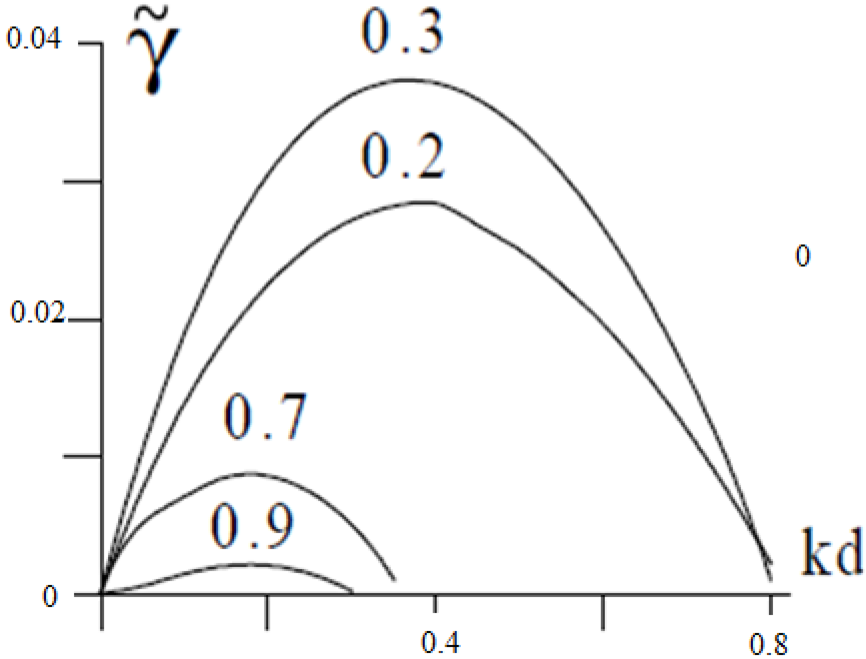


Influence of the perpendicular magnetic field $\mathbf{k} \parallel \mathbf{V}_0 \perp \mathbf{B}_0$ *on the dependence of the growth rate ˜ (kd) of supersonic longitudinal disturbances when M=u/c_s = 2. Curves 1 and 2 correspond to the following values of the parameter:* β *= 100(M_{ms} = M = 2)*
And β *= 2 (M_{ms} = 1.58). In TD approximation stability is for M>1.41. More temperature is more growth rate- unlike to instability of permeant flux.*

Role of oblique disturbances in the instability of a supersonic shear flow

In the foregoing discussion we have considered the longitudinal disturbances (**k** parallel to flow velocity $\mathbf{v}_0$, or **k** = **k**||). However, within the TD approximation, Syrovatsky (1954) showed that in the case of a supersonic velocity difference, even if the Landau stability criterion (Landau, 1944) is satisfied: *M>1.41* (which holds for longitudinal disturbances), the oblique disturbances, for which the wave vector **k** in the plane (x, y) is directed at an angle to the velocity vector $\mathbf{v}_0$), can be unstable. This is because the projection of the flow velocity upon the phase velocity direction can become smaller than the effective sound velocity c_m, which would lead to a decrease of the “wave” Mach number $\widetilde{M}_{ms} = (u / c_{ms}) \cos\varphi = M_{ms} \cos\varphi$. So, for M>1.4 KHI can develop. This result obtained by Syrovatsky (1954) in TD approximation, was developed in velocity shear layer instability (for finite width D>0) by Blumen et al., 1970, 1975 in ordinary hydrodynamics. These authors also showed existence of new radiative mode of KHI for supersonic perturbations. They have small growth rate ($\tilde{\gamma} = \gamma d / u$ <0.01) but slow dumping in space because index of dumping (2) for them is getting small. So, such instability can play an important role in supersonic shear flows which usually are considered to be stable. Their results were developed by Mishin and Morozov 1981, 1983, …2005 for MHD (with external $\mathbf{B}_0$) with accounting of nonhomogeneity of all plasma parameters for supermagnetosonic shear flows in space physics.

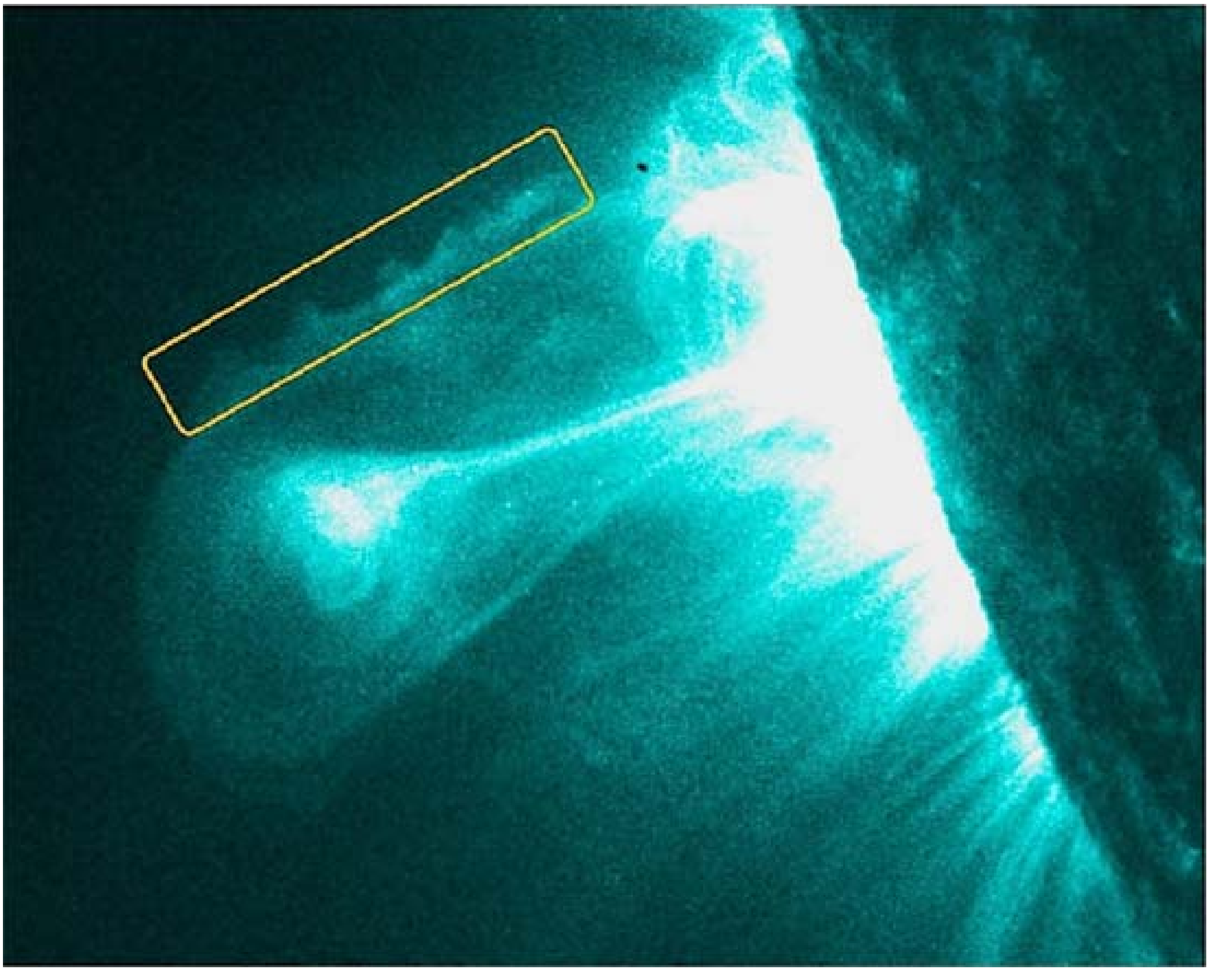
Supersonic MHD KHI on oblique perturbations



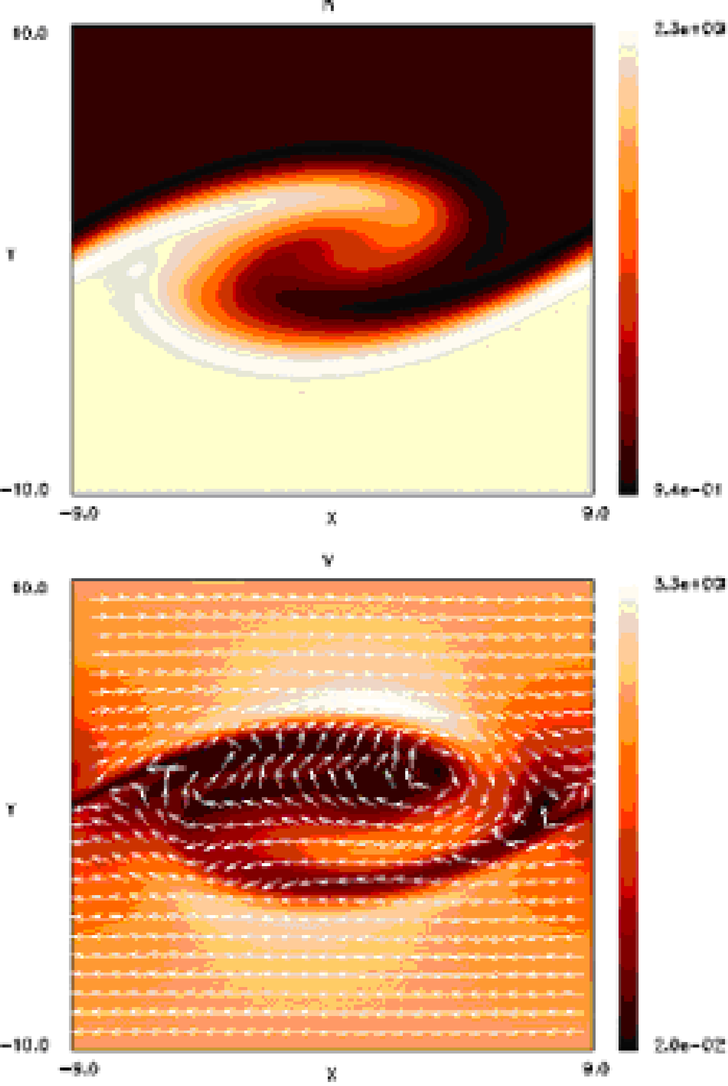
Influence of $\cos\phi$ on the dependence of the growth rate $\tilde{\gamma}(kd)$ for the magnetic field aligned with the flow $\mathbf{v}_0$, or $\mathbf{B}_0 = \mathbf{B}_{0\|}$ when $M = 2$, $\beta = 1.5$, $M_{ms} = 1.49$, and $\rho = const$. The value of $\cos\phi$ is shown above the curves.

For the parameters selected, the magnetosonic Mach number $M_{ms} = 1.49$ ($M_a = \sqrt{5}$). Since the phase velocity $v_{ph} \propto \cos\phi$, the "Mach wave" number $\tilde{M} = \tilde{\omega}/(kc_m)$ obeys the relationship: $\tilde{M} \propto \cos\phi$ (see Squire's transformation in Blumen, 1970). Hence a decrease of $\cos\phi$ is accompanied by a decrease of $\tilde{M}$, and when $\cos\phi \le 0.7$ there occurs a transition to subsonic disturbances $\tilde{M} \le 1$. This implies not only an increase of the growth rate but also an abrupt decrease of the imaginary part of the coefficient χ, as well as a significant displacement of the boundary α_b toward shorter wavelengths.

Manifestations KHI in spac
e.**K-H I** in Solar corona



Foullon et al (Astrophys. J. Letts. 729:L8 (4pp), 2011 March1)
Fast coronal mass ejecta erupting from the Sun, with KH waves detected on its northern flank.
The *Solar Dynamics Observatory/Atmospheric Imaging Assembly* (SDO/AIA) image, shown in solar centered X (increasing toward west) vs Y (increasing toward north) coordinates, is taken in 131 Å channel. The overlaid rectangular region of interest indicates the northern flank region, where substructures, corresponding to the presumed KH waves, are detected against the darker coronal background.



K-H I in Solar corona

Ofman and Thompson (Astrophys. J. Letts. 734:L11(5pp),2011 June10) observed vortex-shaped features in coronal images from the *Solar Dynamics Observatory* associated with an eruption starting at about 2:30 UT on 2010 April 8. The series of vortices were formed along the interface between an erupting (dimming) region and the surrounding corona. They ranged in size from several to 10 arcsec and traveled along the interface at 6–14 km s−1. The features were clearly visible in six out of the seven different EUV wave bands of the Atmospheric Imaging Assembly. They identified the event as the first observation of KHI in the corona in EUV. The interpretation is supported by linear analysis and by a nonlinear 2.5-D model of KHI. The instability is driven by the velocity shear between the erupting and closed magnetic field of the coronal mass ejection and plays an important role in energy transfer processes in coronal plasma.

Key words: Sun: activity – Sun: corona – Sun: coronal mass ejections (CMEs) – Sun: UV radiation

KHI in solar wind

Experimental results.

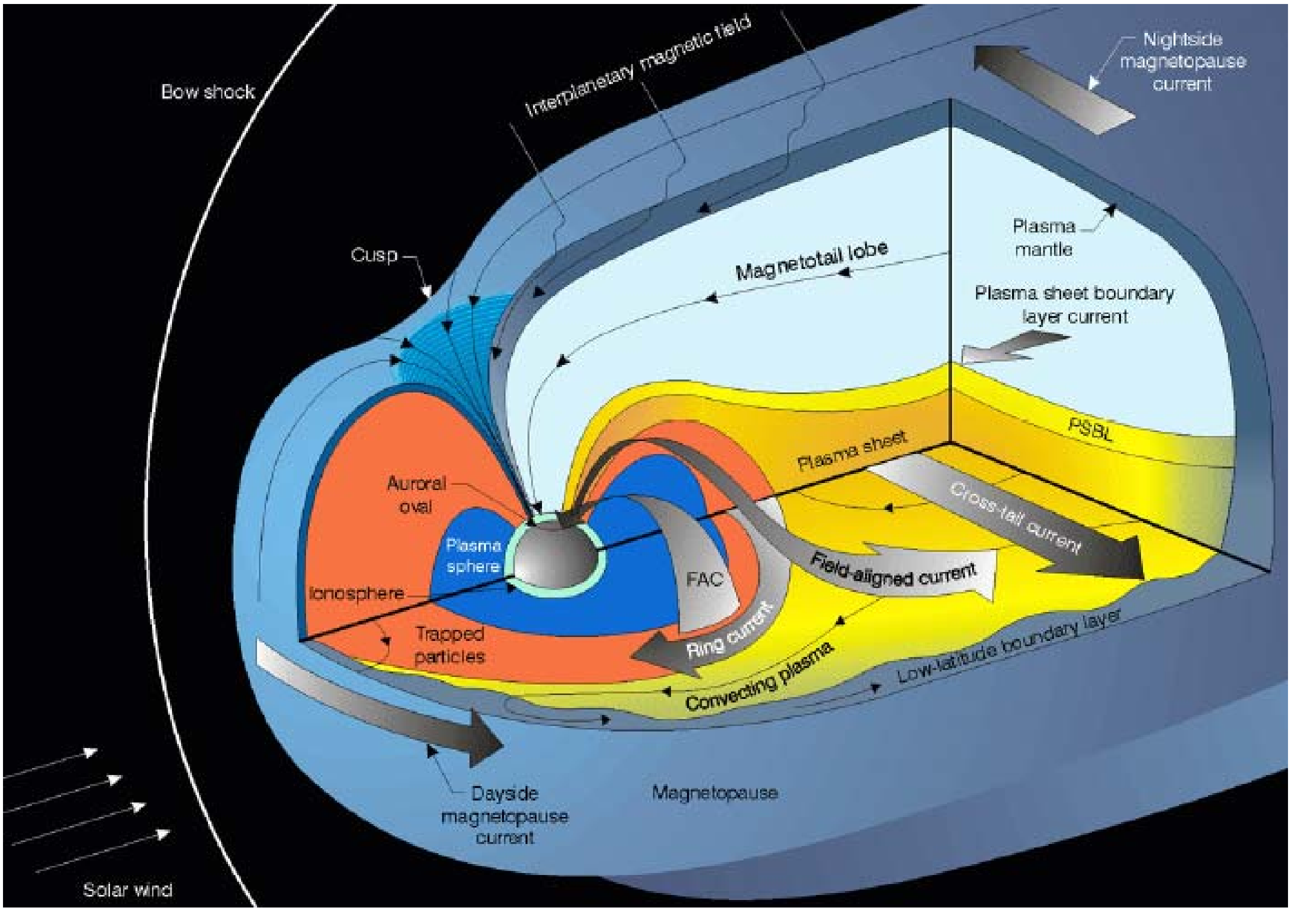
1) Near the Earth there are observations of magnetosound waves in solar wind. These waves cannot come from the Sun because of their fast transformation in Alfven waves. So, it should be mechanism of their generation in the solar wind, not near the Sun.
2) In solar wind near the Earth supersonic velocity shear flows with finite thickness are observed. Because KHI in the TD approximation here is known to be stable (Parker, 1964)-his condition of instability $a < \Delta V < c_s$, is not satisfied in SW.
So, there is problem of their explanation because of absence of usual viscosity able to provide rather big observable value of the width of shear layers between fast and slow flows. Such shear layers have to collapse due to the presence of a large normal component of the velocity with respect to the interfaces.

These both problems were explained by Korzhov, Mishin and Tomozov (1984), considered KHI of supermagnetosonic jet with two boudaries. They had shown that for observable conditions, KHI develops rather effectively on oblique perturbations in frequency range of observed magnetosonic waves (10^{-2}-10^{-4} Hz). Also we evaluated the value of effective anomalous viscosity resulted from KHI development. Its value

$$\nu_{an} \cong \gamma \cdot D^2 \cong \tilde{\gamma} \cdot \Delta v \cdot D.$$

is enough to overcome the kinematic steepening and explain observable velocity shear layer thickness and
the viscosity is sufficient to explain the existence of the finite width of the layers to overcome the kinematic steepening due to anomalous diffusion resulted from the instability evolution.

KHI on the magnetospheric boundary



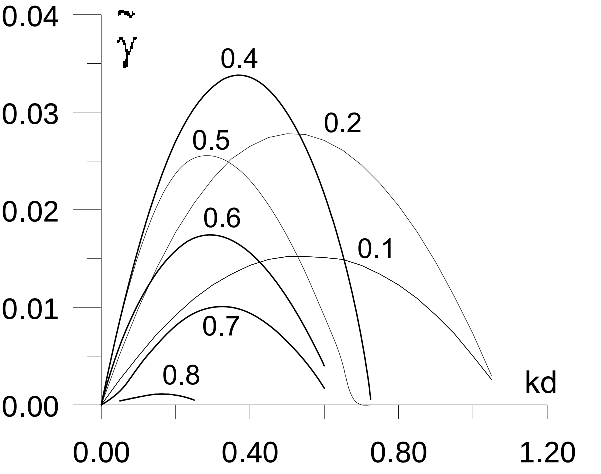
Magnetosphere is flown around by SW with subsonic velocities at dayside and with supersonic velocity at its flanks. At dayside KHI is possible in subsonic regime at low-latitude boundary layer, where $\mathbf{V}_{\perp}\mathbf{B}_{E}$. However strong stabilizing effect can be occurred by the magnetic field of the magnetosheath. The most good situation is for small B there, that is to be for radial IMF.

B_y (azimuthal) IMF causes the dawn- dusk asymmetry of the magnetosheath magnetic field distribution and KHI growth rate. KHI gives generation of surface (fast decreasing from the boundary) MHD waves in the range of geomagnetic pulsations.

Development of instability here can be amplified for a short intervals by solar wind impulses that are created by incidence of SW inhomogeneities on the magnetosphere. For these 1 min-intervals R-T I can give impulsive generation fo surface perturbations on dayside magnetopause not only at low latitudes. This KH+RT instability can give also penetration of plasma inside the magnetopause (Mishin, 1981, 1993).

In general, long-term instability on the dayside magnetopause is difficult to excite because of the low flow velocity and large longitudinal Alfven speed.

KHI at distant magnetotail boundary



Instability of the boundary layer of the distant tail for the velocity difference $\Delta v = 500$ km/s (M(z=0)=4.5) as a function of cosφ. The case of a flow along the magnetic field **B||V0**.

KHI develops on oblique perturbations at magnetotail boundary not only at low latitudes- at all its surface.

Its frequency range is the same as 10-min registered waves there and on the nightside Earth. Low velocity of oblique perturbations is good for their nonlinear development. Growth rate is enough to explain wide boundary layers existence here and to provide energy and impulse transfer from the SW into the magnetosphere as it supposed in viscous mechanism of their interaction (Mishin and Morozov 1983, Mishin 2003, 2005).

CONCLUSION

In collisionless space plasma, KHI at subsonic tangential discontinuities (TD) generate low frequency surface waves fast damping in space, not causing any diffusion of TDs. At supersonic TD, KHI generate waves slow damping outward. Their evolution results in the flow turbulization, forming an anomalous transport coefficients and corresponding diffusive TD broadening, and plays the most important role in their dynamics in all space plasma. Accounting only longitudional perturba-tions, Miura,1992;1999 concluded about ineffectiveness of supersonic KHI on the geotail boundary. After that, majority of authors continue to ignore the oblique disturbances , possibly because of problems of 3D modelling. Therefore they simulate KHI only at subsonic velocity shear layers in all space obtaining diffusion effects as a result of artificial numerical viscosity, inherent in their algorithms. However, in space plasma realistic collisional viscosity and conductivity are absent.

Literature

Chandrasekhar, S., *Hydrodynamic and hydromagnetic stability*. Clarendon Press, Oxford, 1962.

Michalke, A., On the inviscid instability of the hyperbolic tangen profile, *J. Fluid. Mech*, **19**, 689-702, 1964.

Chandra, K., Hydromagnetic stability of plane heterogeneous shear flow. J. *Phys. Soc. Japan*, **34**, 539-546, 1973.

Mishin, V.V., On the MHD instability of the Earth's magnetopause and it's geophysical effects, *Planet. Space Sci*., **29**, 359-363, 1981.

Morozov, A.G., Mishin, V.V., Influence of the magnetospheric boundary layer structure on Kelvin--Helmholtz instability, *Sov. Phys.-Geomagnetizm i aeronomiya*, **21**, 1044-1049, 1981.

Miura, A., Pritchett, P.l. Nonlocal analysis of the Kelvin-Helmholtz instability in compressible plasma, *J. Geophys. R*es., **87**, 7431-7444, 1982.

Mishin, V.V., Morozov, A.G., On the effect of oblique disturbances on Kelvin-Helmholtz instability at magnetospheric boundary layers and in solar wind, *Planet. Space Sci*., **31**, 821-828, 1983.

Korzhov, N.P., Mishin, V.V., Tomozov V.M., On the role of plasma parameters and the Kelvin-Helmholtz instability in viscous interaction of solar wind streams. *Planet. Space S*ci., **32**, 1169-1178, 1984.

Mishin, V.V. On the predominance of oblique disturbances in the supersonic shear layer instability of the geomagnetic tail boundary.
Nonlinear Processes in Geophysics. 2003**.** Vol.10, №4-5. P.351-361.

Leonovich A.S., and V.V. Mishin. Stability of magnetohydrodynamic shear flows with and without bounding walls. J. Plasma Physics 2005. Vol. 71, part 5, pp. 645-664.

Mishin, V.V. Velocity Boundary Layers in the Distant Geotail and the Kelvin-Helmholtz Instability. Planet. Space Science. 2005, Vol. 53. P. 157-160**.**

Blumen, W., Shear layer instability of a inviscid compressible fluid, *J. Fluid Mech*., **46**, 763-775, 1970.
Blumen, W., Drazin, P.G., Billings, D.F., Shear layer instability of a inviscid compressible fluid, Part 2, *J. Fluid Mech*., **71**, 305-316, 1975.
Drazin, P.G., Davey, A., Shear layer instability of a inviscid compressible fluid, Part 3, *J. Fluid Mech*., **82**, 255-260, 1977.
Belmont, G., Chanteur, G., Advances in magnetopause Kelvin-Helmholtz instability studies, *Phys*. *Scr*., **40**, 124-128, 1989.
Miura, A., Kelvin-Helmholtz instability at the magnetospheric boundary: dependence on the magnetosheath sonic Mach number, *J. Geophys. Res*., **97**, 10655-10675, 1992.
Raleigh J.W.S. *The theory of sound*. **2**, Chap. XXI, Dover, New York, 1945 (reprint of second edition 1894).
Moskvin, Yu.L., Frank-Kamenetsky, D.A., Geomagnetic micropulsations, and the solar wind, *Sov. Phys.-Geomagnetizm i aeronomiya*, **7**,144-149, 1964.
Syrovatsky, S.I., Instability of tangential discontinuities in compressible medium, *Sov. Phys. -JETP*, **27**, 121-123, 1954.
Sibeck, D.G., Slavin, J.A., Smith, E.J., ISEE-3 magnetopause crossings: evidence for the Kelvin-Helmholtz instability, *Magnetotail physics*, edited by T. Yz. Lui, 73-76, J. Hopkins Univ. Press, Baltimore, 1987.
Bame, S.J., Anderson, R.C., Asbridge, J.R., Baker, D.N., Feldman, W.C., Gosling, J.T., Hones, E.W., McComas, D.J., Zwickl, R.D., Plasma regimes in deep geomagnetic tail: ISEE 3, *Geophys*. *Res. Let.*,**10**, 912-915, 1983.
Goldstein, M.l., Roberts, D.A., Matthaeus, W.H., Magnetohydrodynamic turbulence in the solar wind, *Annual Rev. Astron. Ast*rophys., **33**, 283-325, 1995.
Miura, A., Self-organization in the two-dimensional magnetohydrodynamic transverse Kelvin-Helmholtz instability, *Geophys. Res*., **104**, 395-411, 1999.