APTS 2013/14: de-classifying spatial statistics

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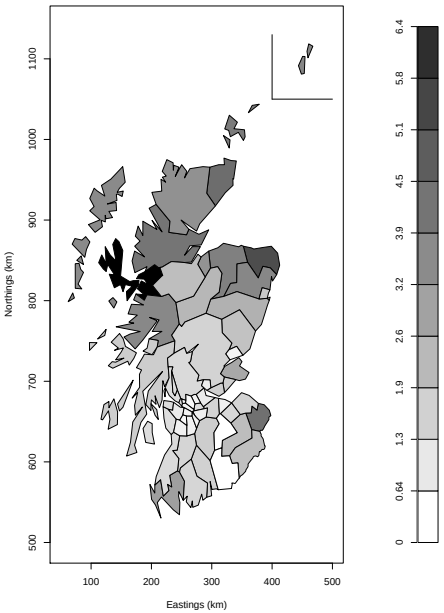
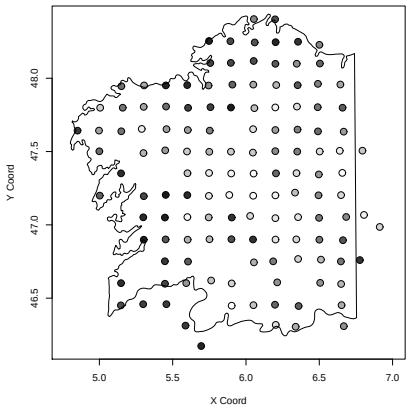
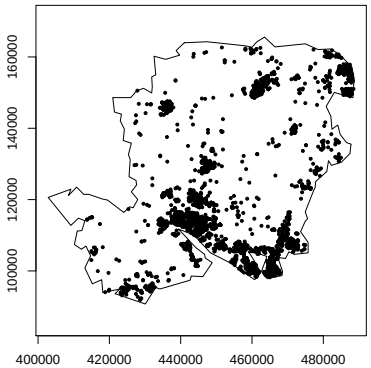
Spatial statistics according to Cressie



Lattice data

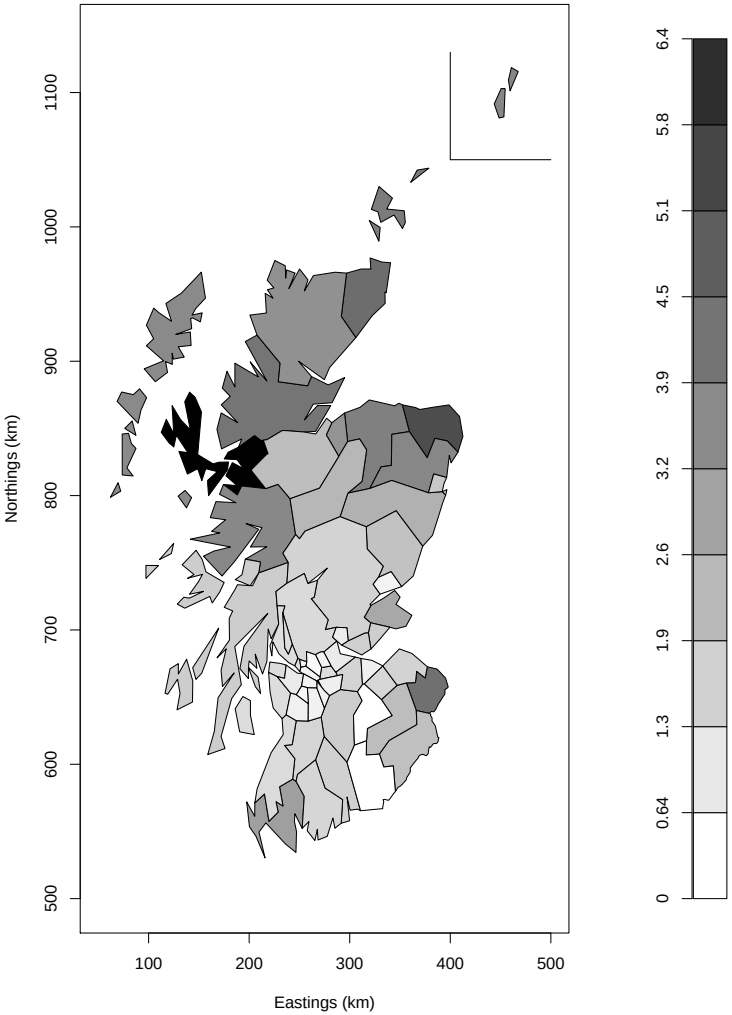


Geostatistics



Point processes

Lattice data

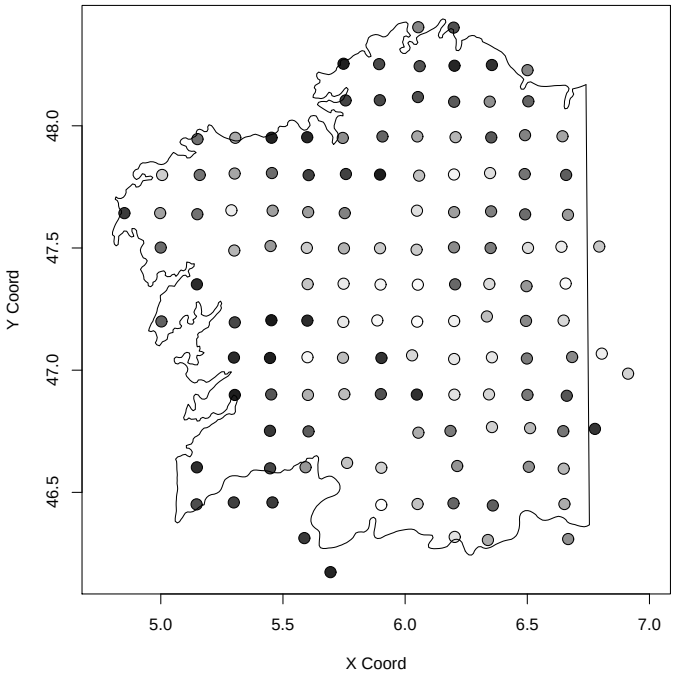


Data: outcomes $Y_i : i = 1, ..., n$

Model: Markov random field:
$[Y_i | \{Y_j : j \neq i\}] : i = 1, ..., n$

"...or indeed, for any multivariate distribution at all"
Hawkes, in discussion of Besag (1974)

Geostatistics

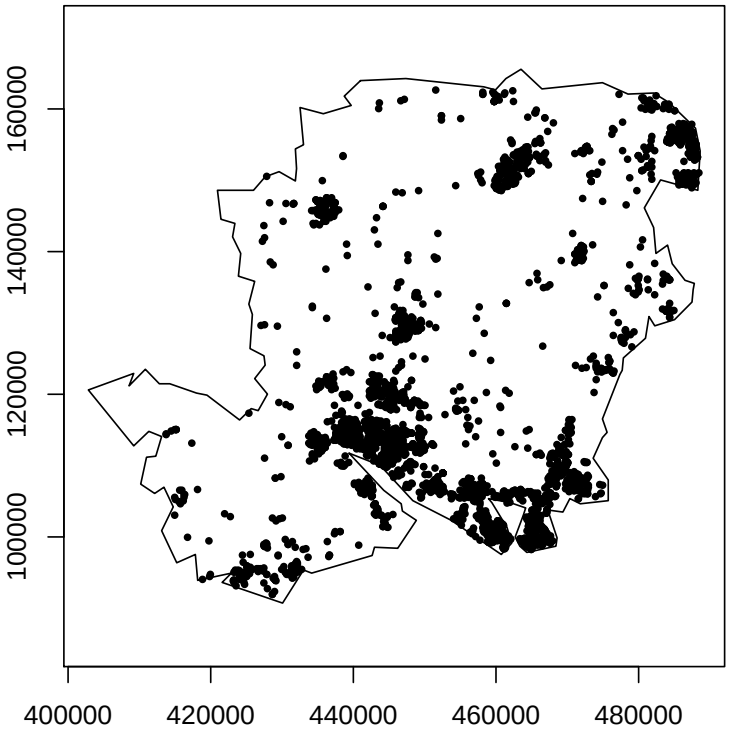


Data: outcome and locations $(Y_i, x_i) : i = 1, ..., n$ (variable n)

Model: spatially continuous stochastic process, $Y(x) : x \in \mathbb{R}^2$

Presumed scientific interest in $Y(x)$ at non-sampled locations

Point process



Data: outcomes $x_i \in A : i = 1, ..., n$ $(A \subset \mathbb{R}^2)$

Model: countably infinite set of points, $x_i \in \mathbb{R}^2 : i = 1, 2, ...$

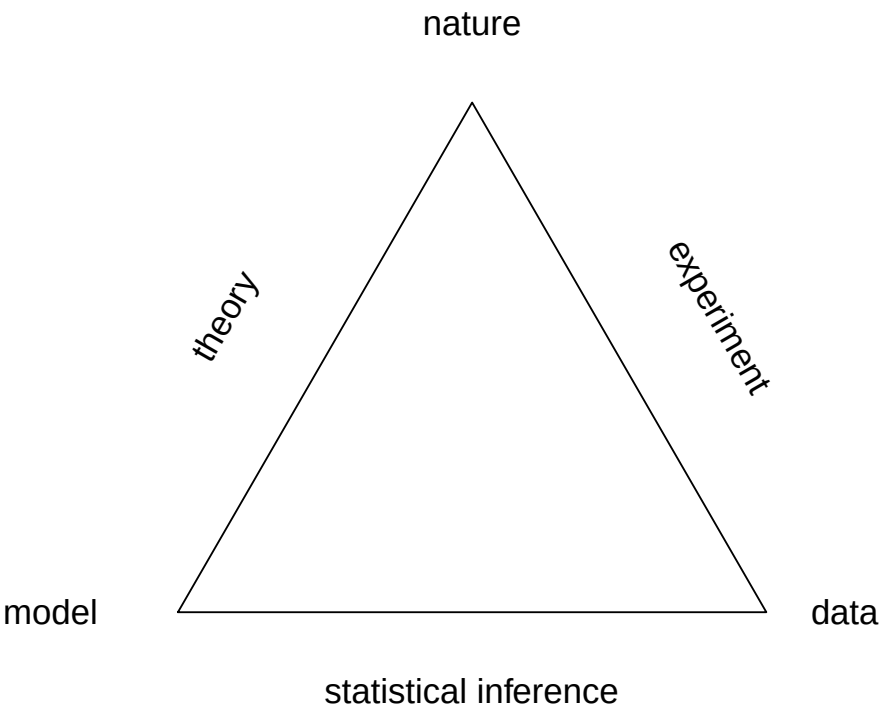
Locations are of scientific interest in themselves.

Spatial statistics according to Cressie

A classification of

- **processes?**
- **models?**
- **methods?**
- **data-formats?**

Analyse problems, not data



A statistical model is:

- **a device to answer a question**
- **a bridge between scientific theory and empirical evidence**
- **a framework to enable principled inference in the presence of uncertainty**


Programa Doenças Infecciosas
e Desigualdade Social:
Comunidade de Pau da Lima
Legenda
Escala 1: 700
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Leptospirosis cohort study: Pau da Lima

- subjects i at locations x_i, blood-samples taken at times $t_{ij} \approx 0, 6, 12, 18, 24$ months
- sero-conversion defined as change from zero to positive, or at least four-fold increase in concentration
- data consist of:
  - $Y_{ij} = 0/1 : j = 1, 2, 3, 4$ (seroconversion no/yes)
  - $r_i(t)$ known and hypothesised risk-factors

Longitudinal data, binary outcome $\Rightarrow$ GEE? GLMM?

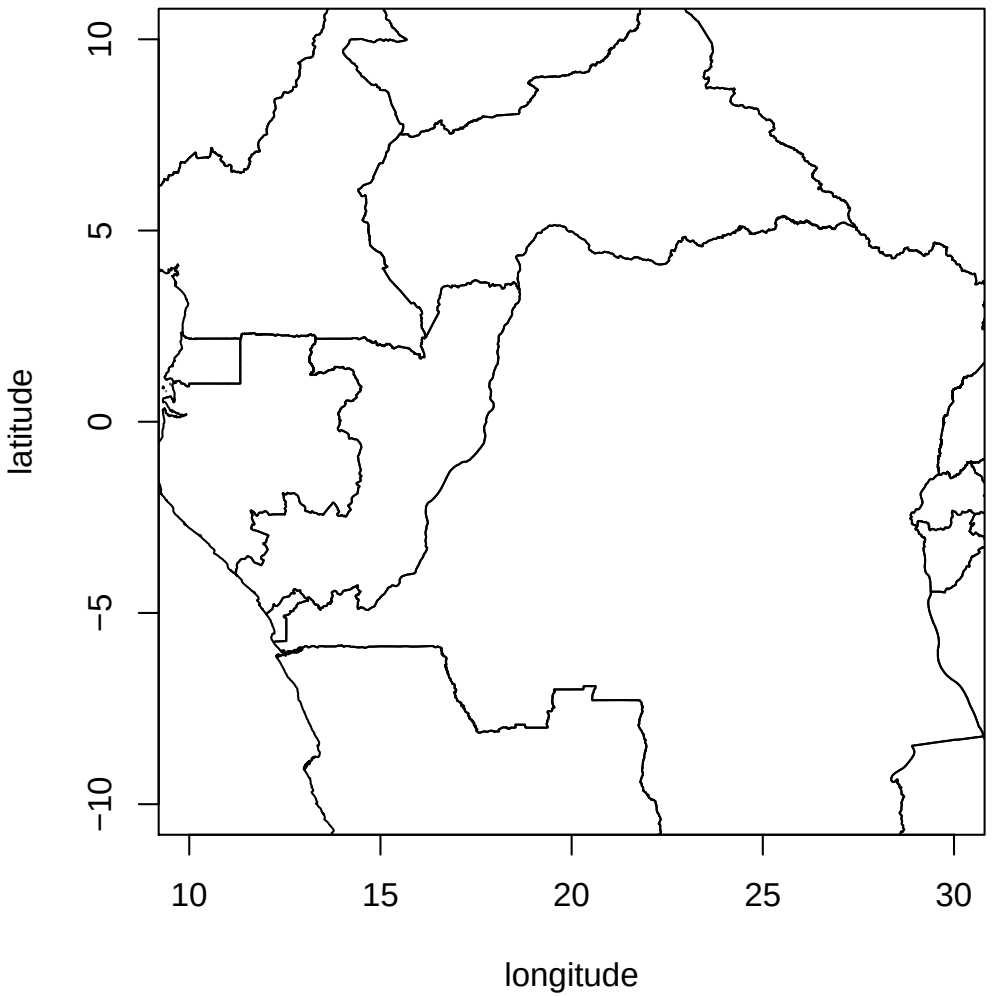
Leptospirosis cohort study: model formulation

- $Y_{it} = 1 \Leftrightarrow$ at least one infection event
- model infection events as person-specific, inhomogeneous Cox processes,

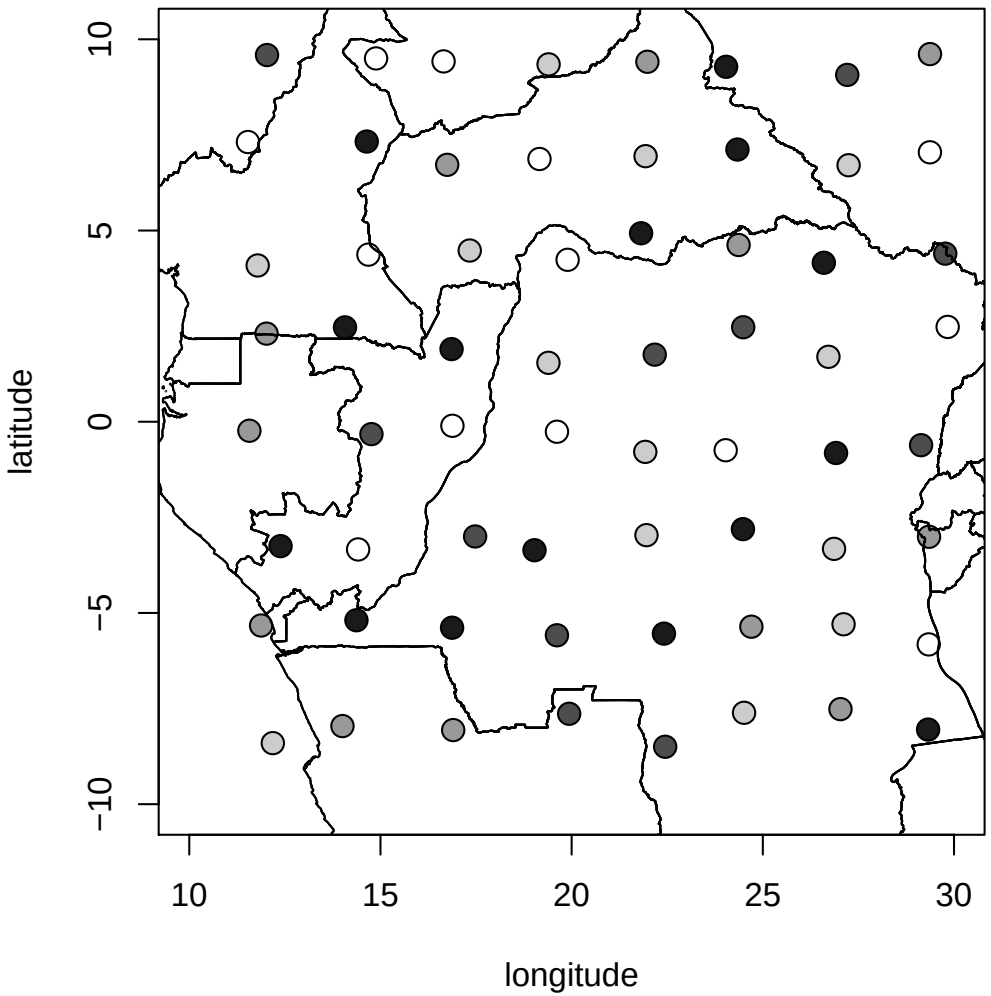
$$\Lambda_i(t) = \exp\{r_i(t)'\beta + U_i + S(x_i)\}$$

- $P(Y_{it} = 1|\Lambda_i(\cdot)\} = 1 - \exp\left\{-\int_{t_{i,j-1}}^{t_{ij}} \Lambda_i(u)du\right\}$

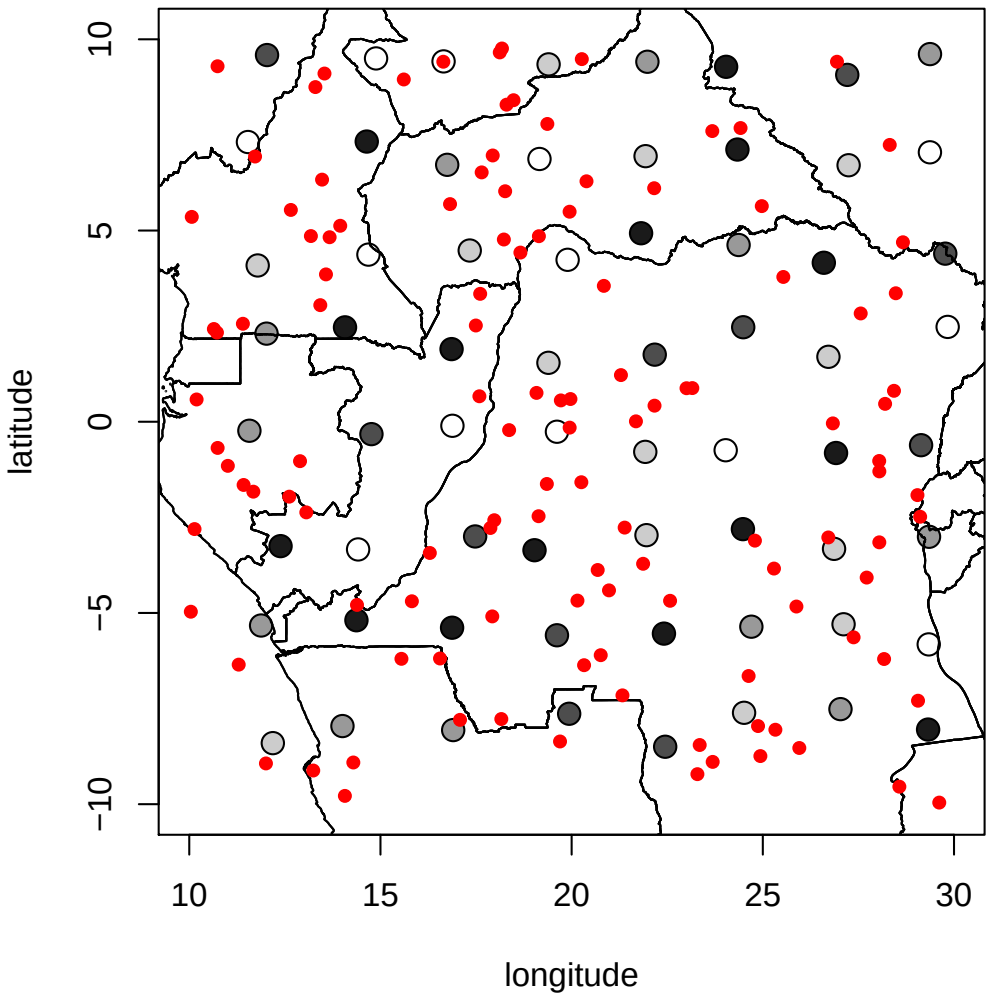
Data-synthesis



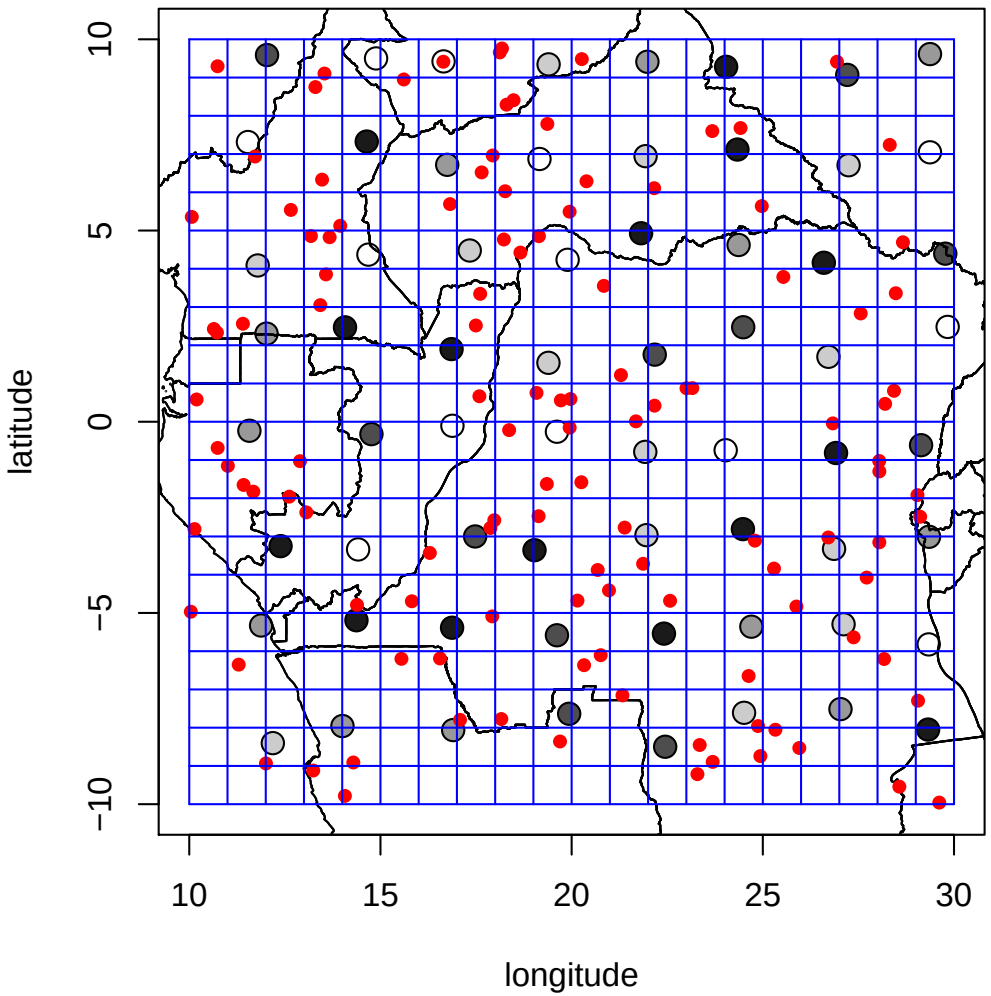
Data-synthesis



Data-synthesis



Data-synthesis



Data-synthesis

$\mathbf{S} =$ state of nature
$\mathbf{Y} =$ all relevant data
$\mathbf{T} =$ $\mathcal{F}(\mathbf{S}) =$ target for prediction

Model: $[\mathbf{S}, \mathbf{Y}] = [\mathbf{S}][\mathbf{Y}|\mathbf{S}]$

Predictive inference: $[\mathbf{S}, \mathbf{Y}] \Rightarrow [\mathbf{T}|\mathbf{Y}]$

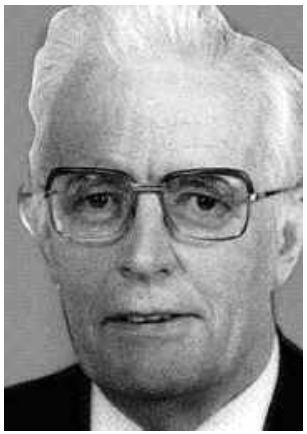
Statistics and scientific method

"Far better an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise."

John Tukey (1915–2000)

"...the importance of making contact with the best research workers in other subjects and aiming over a period to establish genuine involvement and collaboration in their activities."

Sir David Cox (b 1924)



And I leave you with...

- **the role of modelling**

  "We buy information with assumptions"

  Coombs (1964)

- **choice of model/method should relate to scientific purpose.**

  "Analyse problems, not data"

  PJD

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