APTS Lab 2: Solutions *

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1. See `Lab2.R`.

2. We have

$$\int_{-\infty}^{\infty} y K_h(Y_i - y)\,dy = \int_{-\infty}^{\infty} (Y_i + hz)K(z)\,dz = Y_i.$$

Hence the estimator that results is

$$\hat{m}_h(x) = \frac{\int_{-\infty}^{\infty} \frac{y}{n} \sum_{i=1}^n K_h(X_i - x)K_h(Y_i - y)\,dy}{\frac{1}{n}\sum_{i=1}^n K_h(X_i - x)} = \frac{\sum_{i=1}^n K_h(X_i - x)Y_i}{\sum_{i=1}^n K_h(X_i - x)},$$

which is the local constant (Nadaraya–Watson) estimator.

3. Since the local constant estimator is

$$\hat{m}_h(x;0) = \frac{n^{-1}\sum_{i=1}^n K_h(X_i - x)Y_i}{\hat{s}_{0,h}(x)},$$

we have

$$\begin{aligned}\mathbb{E}\{\hat{m}_h(x;0)|X_1,\ldots,X_n\} - m(x) &= \frac{\hat{s}_{1,h}(x)}{\hat{s}_{0,h}(x)}m'(x) + \frac{\hat{s}_{2,h}(x)m''(x)}{2\hat{s}_{0,h}(x)} + o_P\Big(\frac{\hat{s}_{2,h}(x)}{\hat{s}_{0,h}(x)}\Big) \\ &= h^2\mu_2(K)m'(x)\frac{f'(x)}{f(x)} + \frac{1}{2}h^2\mu_2(K)m''(x) + o_P(h^2).\end{aligned}$$

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