Warwick Statistics

Risk and Predictability - Where Might Modern Mathematics Take Me?

Offer-holder Visit Day, March 2019

(Prof Bärbel Finkenstädt, Dr Jere Koskela, Dr Vicky Henderson)

Welcome to the offer-holders open day

Offer-holders for 3 degree courses:

- Data Science
- Mathematics and Statistics
- MORSE

… and parents or other accompanying persons!

The purpose of today

A varied programme of events, which we hope will:

- Inform you.
- Inspire you.
- Help you to make the decision that is right for you about which university offer to accept.

Schedule

11:00-11:50	Talk \Risk and Predictability | Where Might Modern Mathematics Take Me?" Opportunity for questions.
12:00-13:00	Lunch Information about Careers, Funding, Admissions and Wellbeing.
13:00-13:45	Talk “How to solve it? Examples from STEP and A-level papers", Opportunity for questions.
14:00-15:20	Campus tour led by current students / Small group meetings with academic staff
15:20-	Tea, and more information

Where might modern mathematics take me?

Some things to know:

- Mathematics - and especially Statistics - becomes much more interesting at university level.
- The demand for well-rounded maths graduates remains absolutely buoyant, everywhere in the world.
- Demand for our kind of maths, especially so!

Our kind of maths? Probability, statistics, operational research, mathematical finance, machine learning…

These are the most sought after areas of mathematics in the world at large.

In this talk we mention just a few of the exciting application areas for modern mathematics.

Destinations of our recent graduates

A wide range of:

- management consultancy
- investment banking
- medical, social or economic research
- academia
- market research
- ‘big data’ in commerce, science, government, . . .
- insurance and actuarial work
- software engineering
- engineering consultancy
- sport, entertainment

More details on employment statistics and careers in the flyer in your pack

Some recent student projects

- Forecasting Sleep Apnea
- Portfolio Management Under Uncertainty
- Evaluating changes in attitudes experience and accident risk in novice drivers
- Comparison of population based Monte Carlo methods
- Mobile Health Analysis
- Statistical inference of stochastic differential equations
- Game-theoretric modelling of cybersecurity
- Erdos-Kac theory and Mod-Poisson convergence
- Exponential random graphs modelling
- On the complexity and behaviour of crypto currencies compared to other markets

Statistical Inference for circadian pacemaking

Bärbel Finkenstädt (Statistics)

Francis Lévi (WMS, INSERM & Hôpital Paul Brousse, France)

Circadian Rhythm

The body's circadian rhythm, or **biological clock**, runs over a period of around 24 hours and affects not only sleep patterns but immune function and metabolism.

In response to environmental cues, such as light, a region of the hypothalamus called the suprachiasmatic nucleus **SCN**

- acts as a central timekeeper;
- sends out neuronal and hormonal signals that coordinate a molecular clock within every cell;
- orchestrates a network of peripheral clocks in nearly every organ and tissue of the body.

These cellular clocks are driven by the oscillating and interacting expression of at least 14 **core clock genes**.

The 2017 Nobel Prize in Physiology or Medicine was awarded to Jeffrey C. Hall, Michael Rosbash and Michael W. Young "for their discoveries of molecular mechanisms controlling the circadian rhythm" in fruit flies.

Chronotherapy (Chronomedicine)

Link between disruption of our circadian programme and increased risk to health

- chronic diseases including cancer, metabolic syndrome (obesity, diabetes)
- psychiatric disorders (depression, bipolar, schizophrenia, attention deficit).

Clock genes are important in cancer because they govern

- cell cycles,
- cell proliferation,
- cell death,
- DNA damage repair.

Synchronizing drug delivery with a patient's body clock can yield benefits.

Chronotherapy

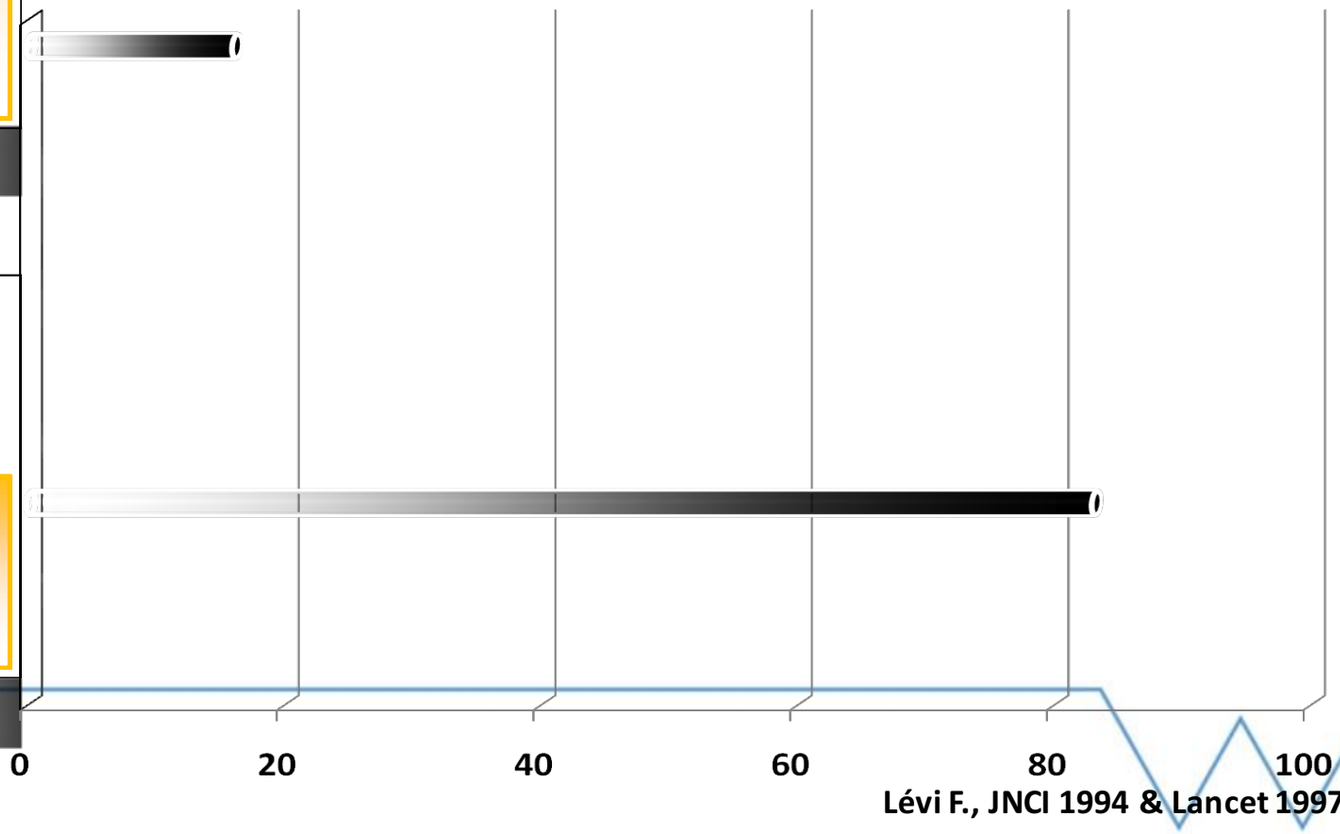
n=278

5FU over 120 h

CHRONOMODULATED
(night peak)

FLAT (constant rate)

incidence of severe stomatitis (% pts)



Lévi F., JNCI 1994 & Lancet 1997

Thoracic sensor

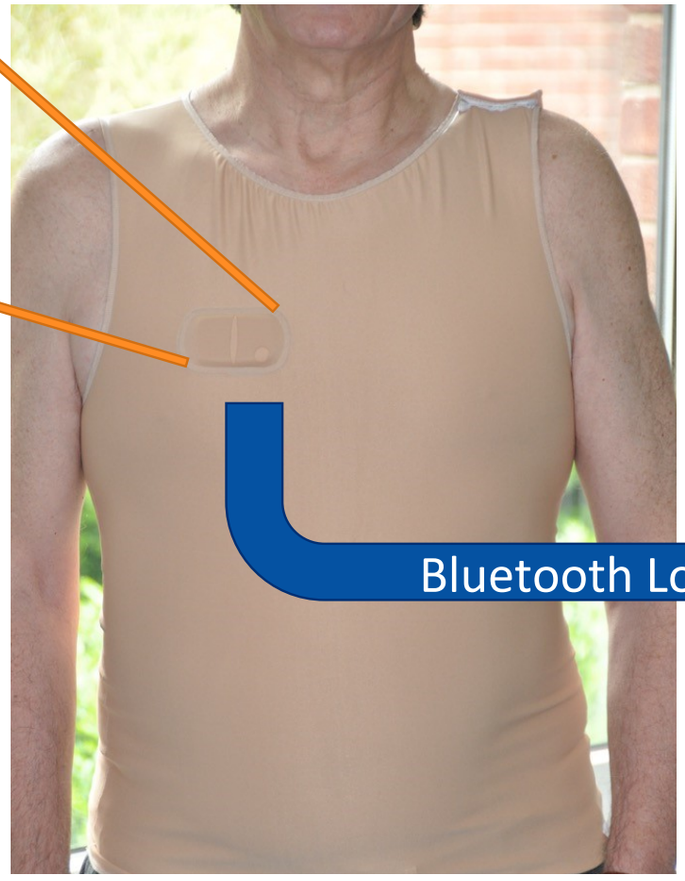
Skin temperature

1 MIN

Accelerations

3D-position

5 MINS



10 MINS

Bluetooth Low Energy

Collector

24 HOURS

GPRS

Server

Bluetooth scale

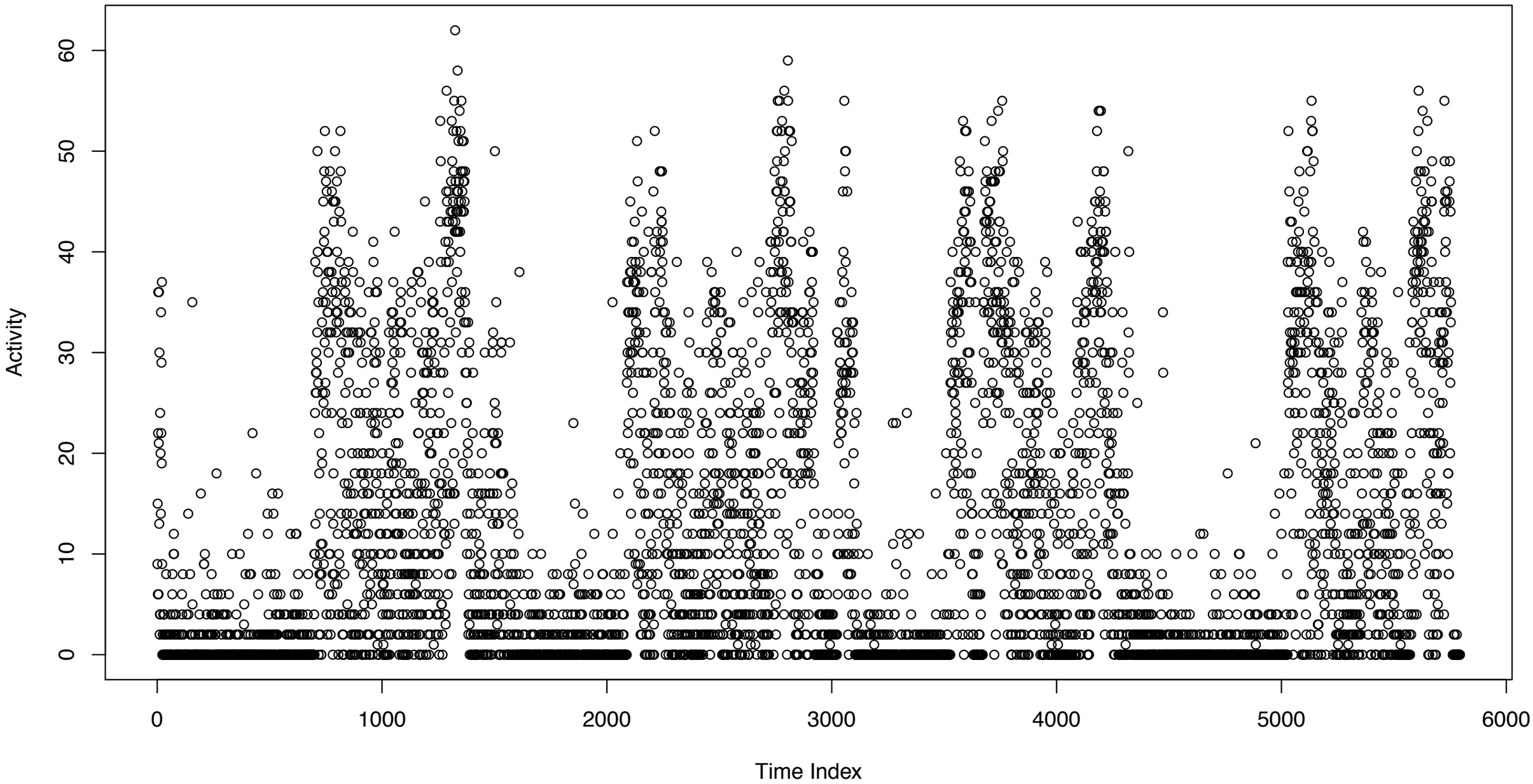
Body weight

24 HOURS

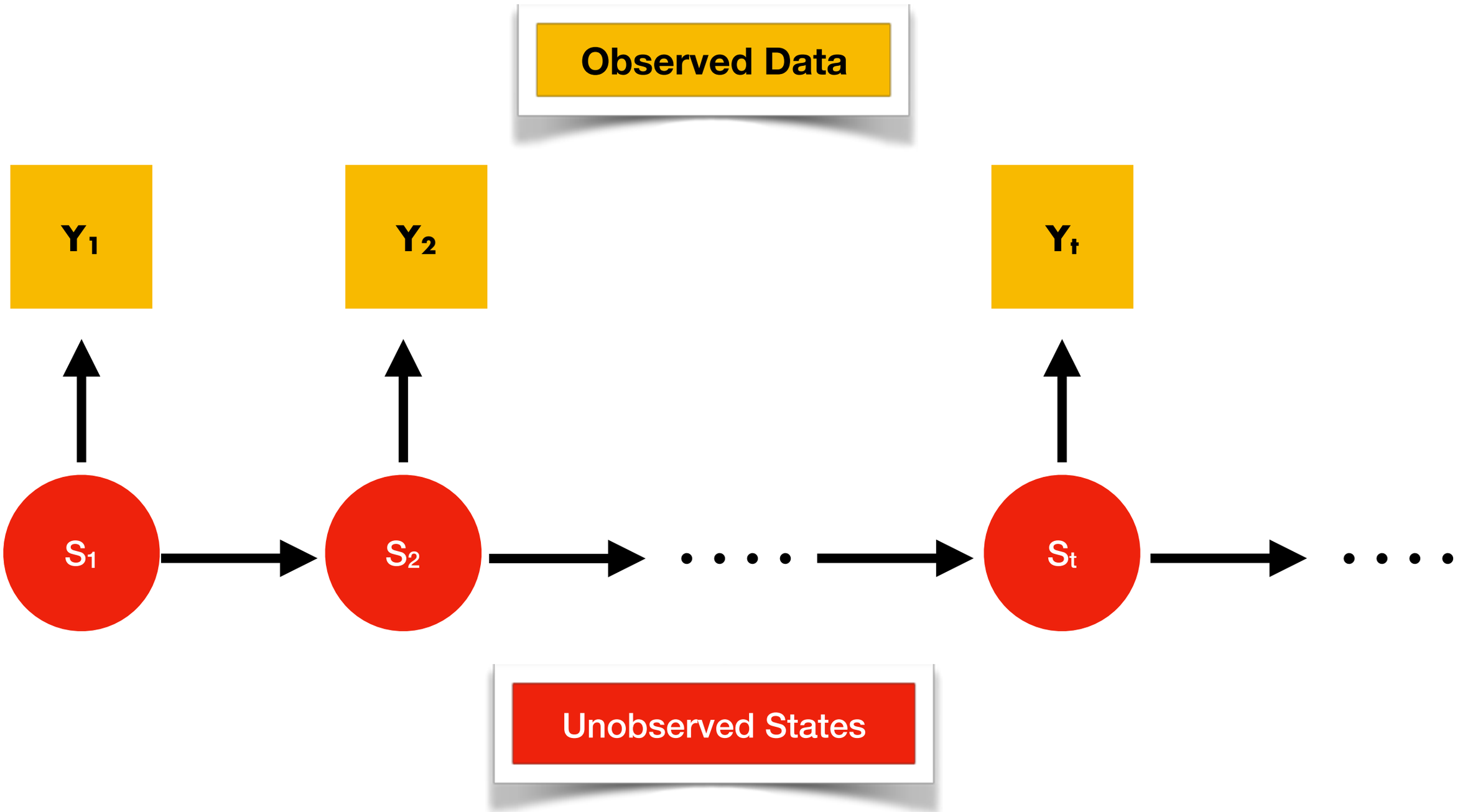
Bluetooth Low Energy

24 HOURS

Komarzynski S. et al., Real-time capture of inter- and intra-subject variations in human circadian coordination in healthy and cancerous persons at home, J Med Internet Res 2018 doi:10.2196/jmir.9779


Activity
0
10
20
30
40
50
60
0
1000
2000
3000
4000
5000
6000
Time Index

Hidden Markov Model



Data likelihood:

$$\begin{aligned} \mathrm{P}(Y^{(T)}) &= \sum_{s_1,\ldots,s_T=1}^{m} \mathrm{P}(S_1) \prod_{t=2}^{T} \mathrm{P}(S_t|S_{t-1}) \prod_{t=1}^{T} \mathrm{P}(Y_t|S_t) \\ &= \boldsymbol{\delta}\mathbf{P}(Y_1|S_1)\boldsymbol{\Gamma}\mathbf{P}(Y_2|S_2)\boldsymbol{\Gamma}.....\boldsymbol{\Gamma}\mathbf{P}(Y_T|S_T)\mathbf{1}' \end{aligned}$$

HMM is parametrized by:

1. Initial state distribution $\boldsymbol{\delta} \in \mathbb{R}^{1\times m}$
2. Conditional probability matrix $\mathbf{P}\left(Y_t|S_t\right) \in \mathbb{R}^{m\times m}$
3. Markov chain transition matrix $\boldsymbol{\Gamma} \in \mathbb{R}^{\mathbf{m}\times\mathbf{m}}$, with $\boldsymbol{\Gamma}_{j,k} = \mathrm{P}\left(S = k|S = j\right)$

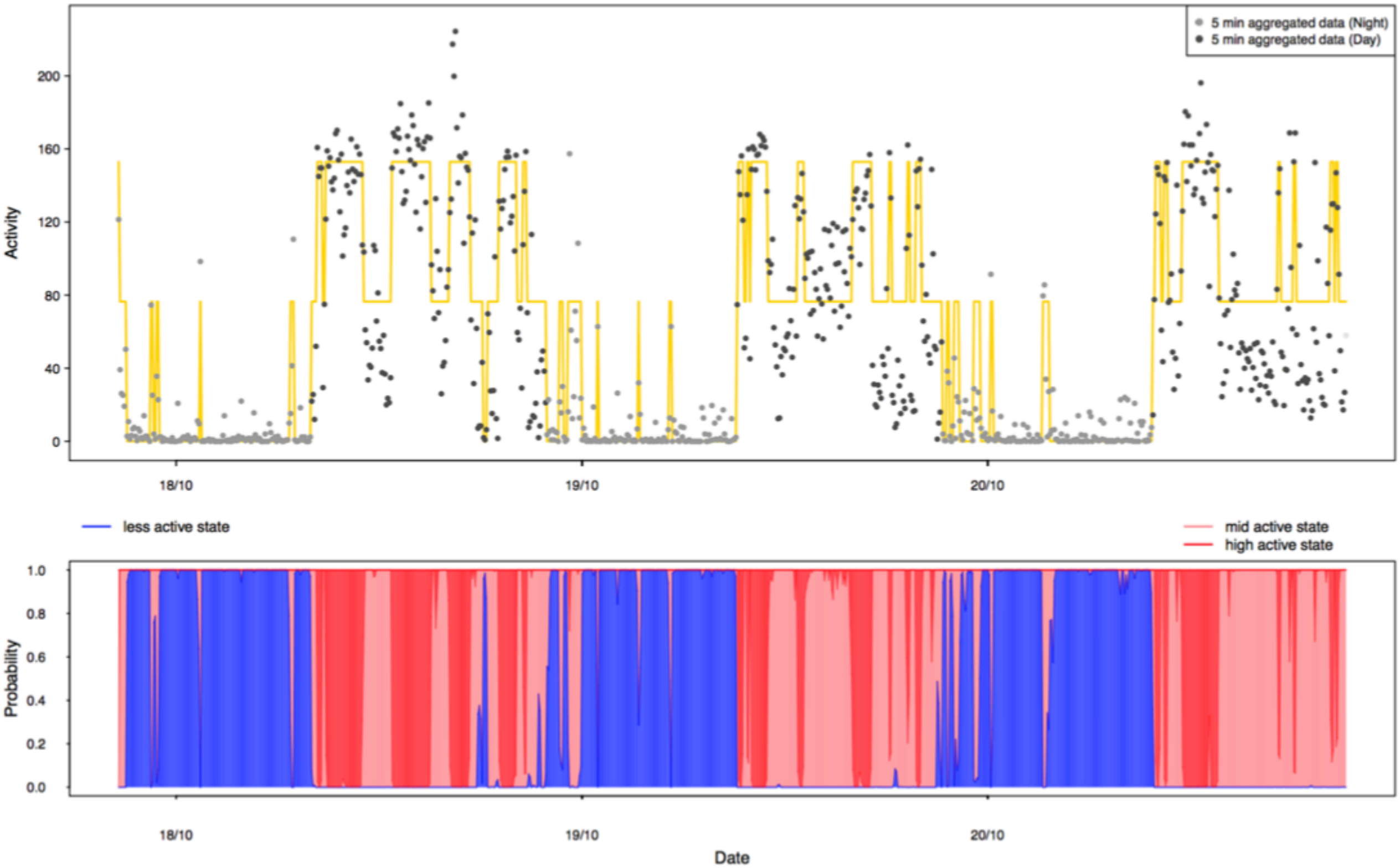
State 1: Inactive (IA)/Rest

State 2: Moderately active (MA)

State 3: Highly active (HA)

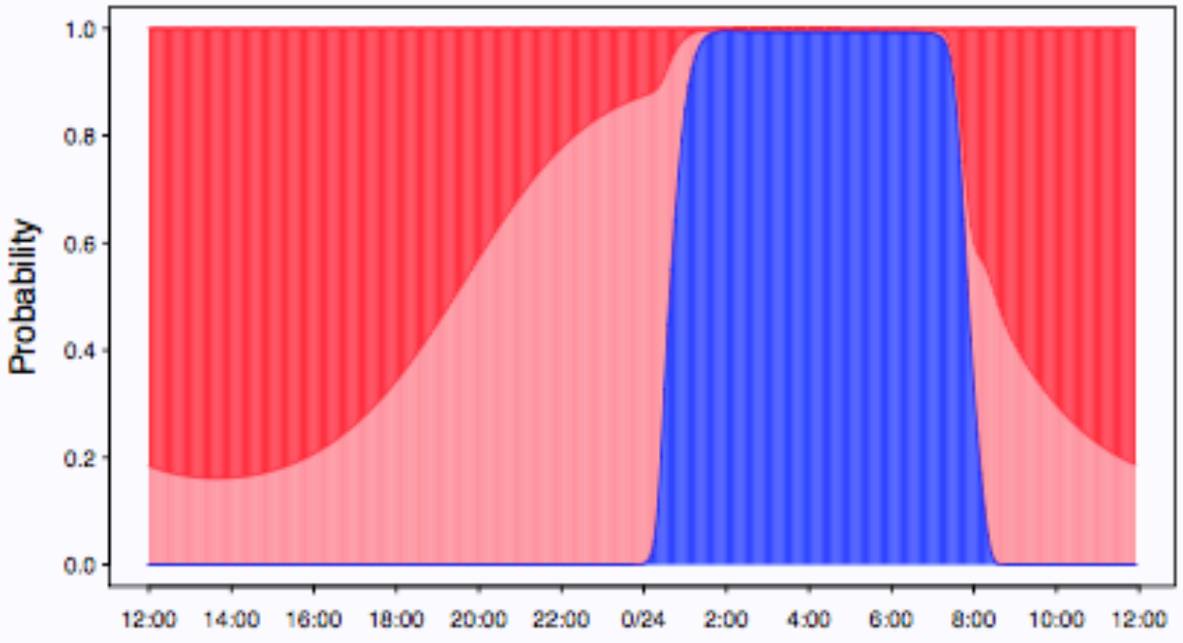
Transition matrix

$$\hat{\mathbf{\Gamma}} = \begin{pmatrix} 0.945 & 0.055 & 0.000 \\ 0.065 & 0.859 & 0.076 \\ 0.000 & 0.140 & 0.860 \end{pmatrix}$$


5 min aggregated data (Night)
5 min aggregated data (Day)
Activity
200
160
120
80
40
0
18/10
19/10
20/10
less active state
mid active state
high active state
Probability
1.0
0.8
0.6
0.4
0.2
0.0
18/10
19/10
20/10
Date

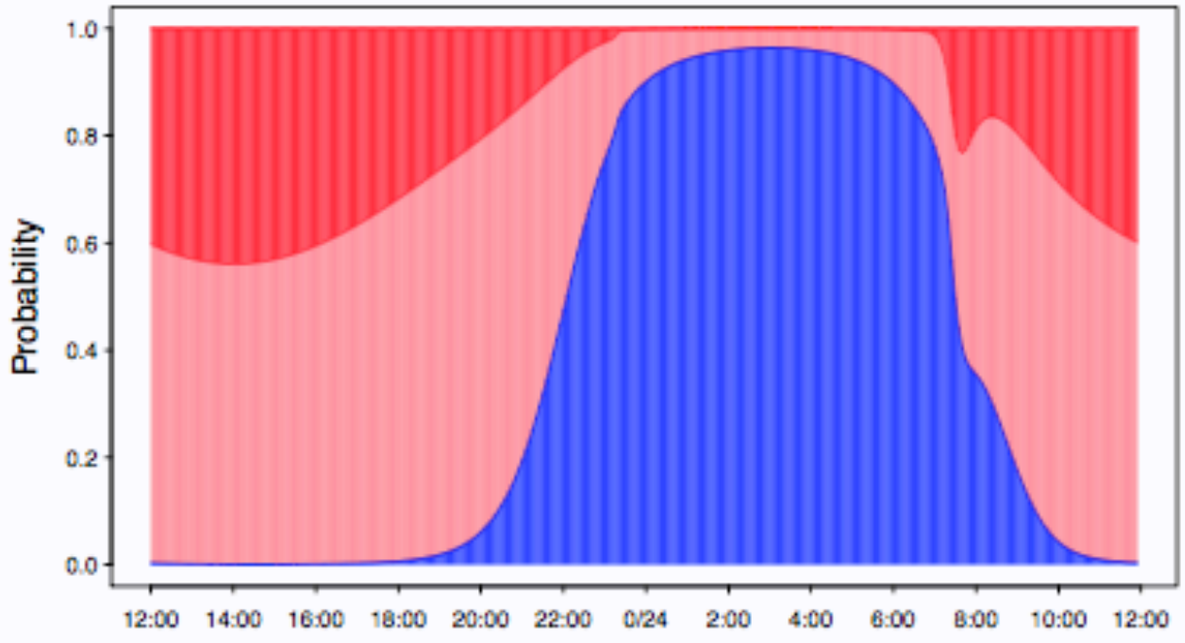
Subject 9

Rest Amount: 7.16 hours; Center Rest Time: 4:16; Rhythm Index: 0.96



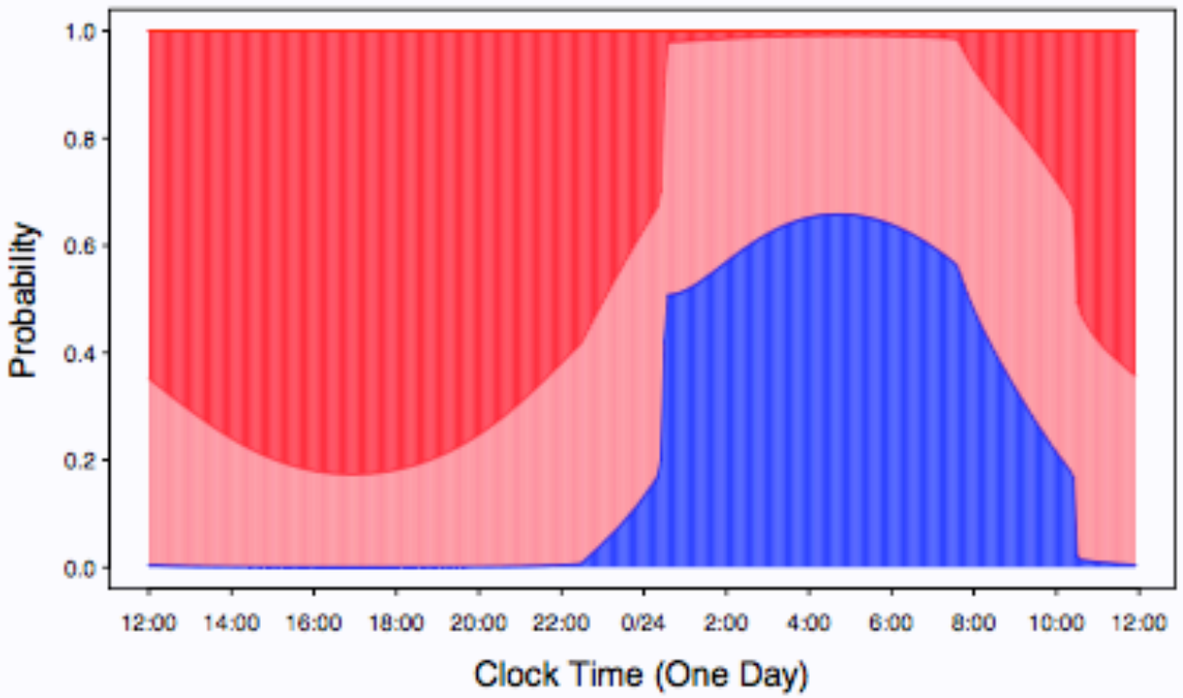
Subject 42

Rest Amount: 9.43 hours; Center Rest Time: 2:53; Rhythm Index: 0.81



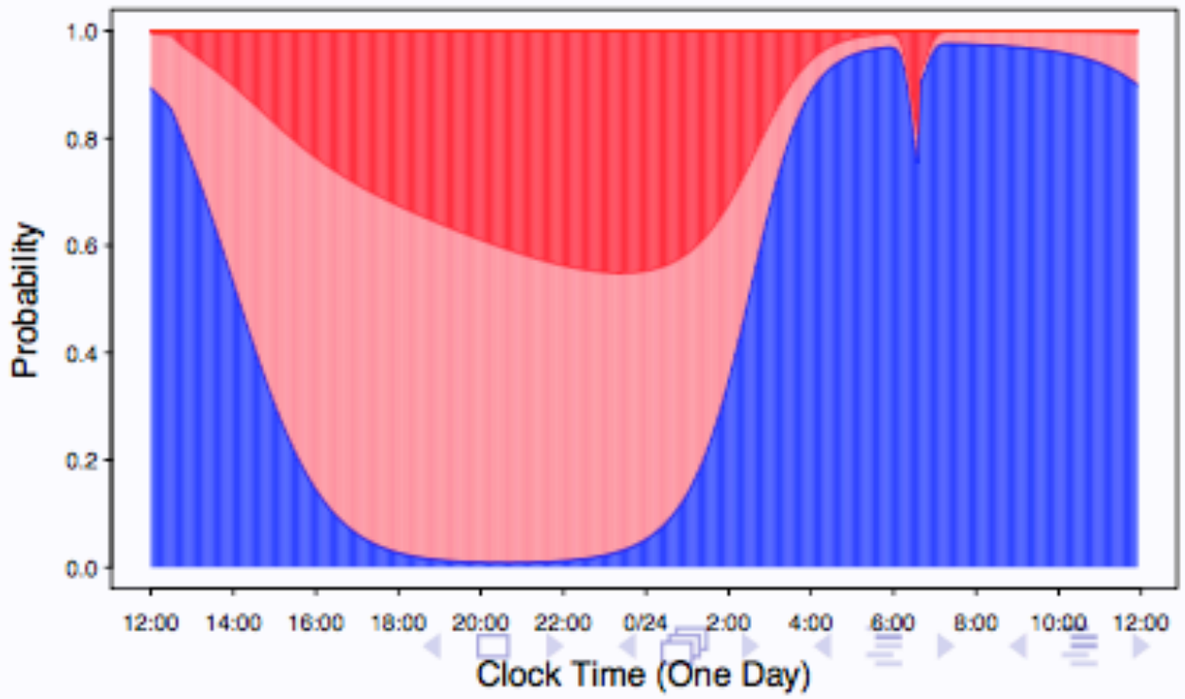
Subject 20

Rest Amount: 5.49 hours; Center Rest Time: 4:48; Rhythm Index: 0.53

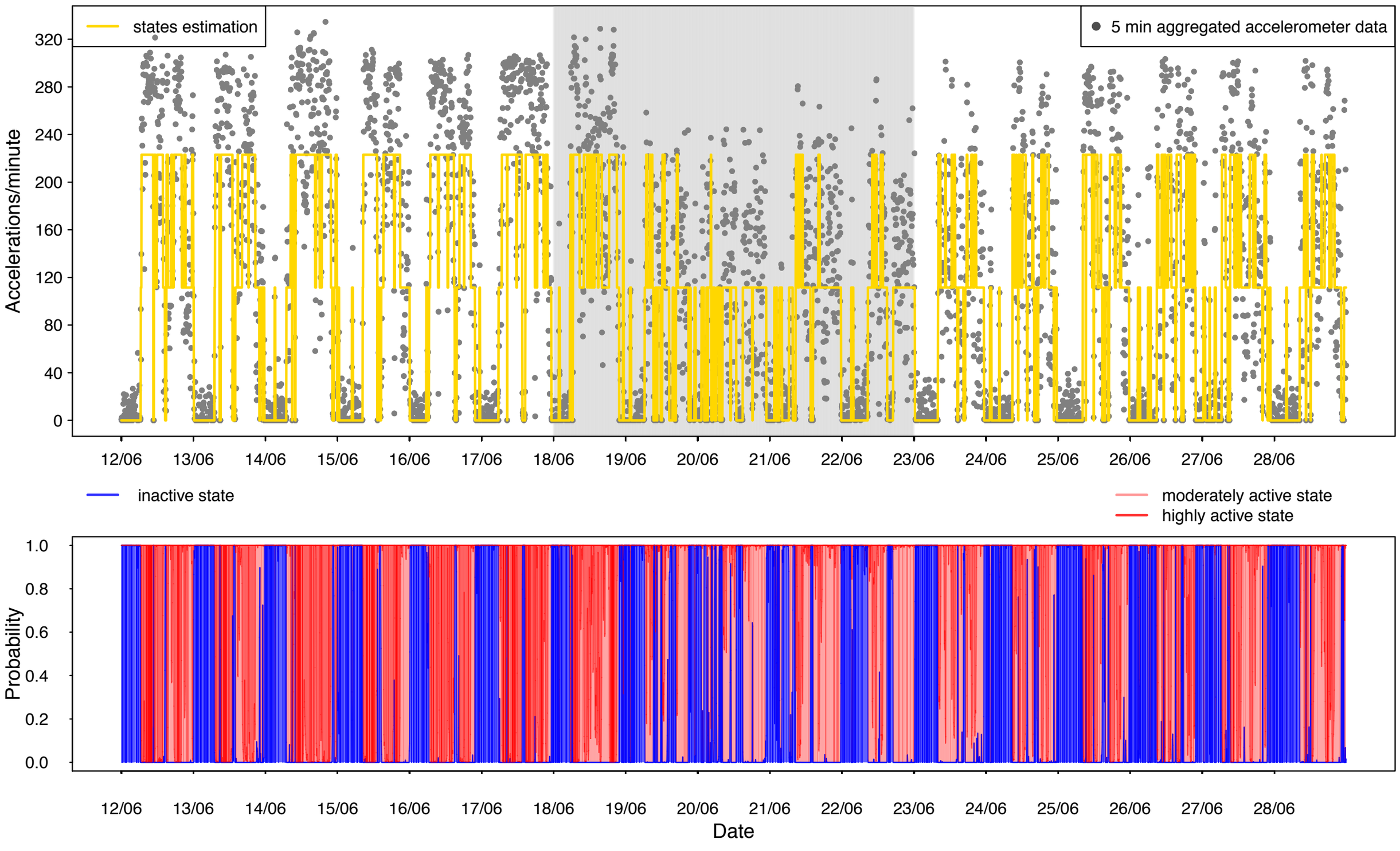


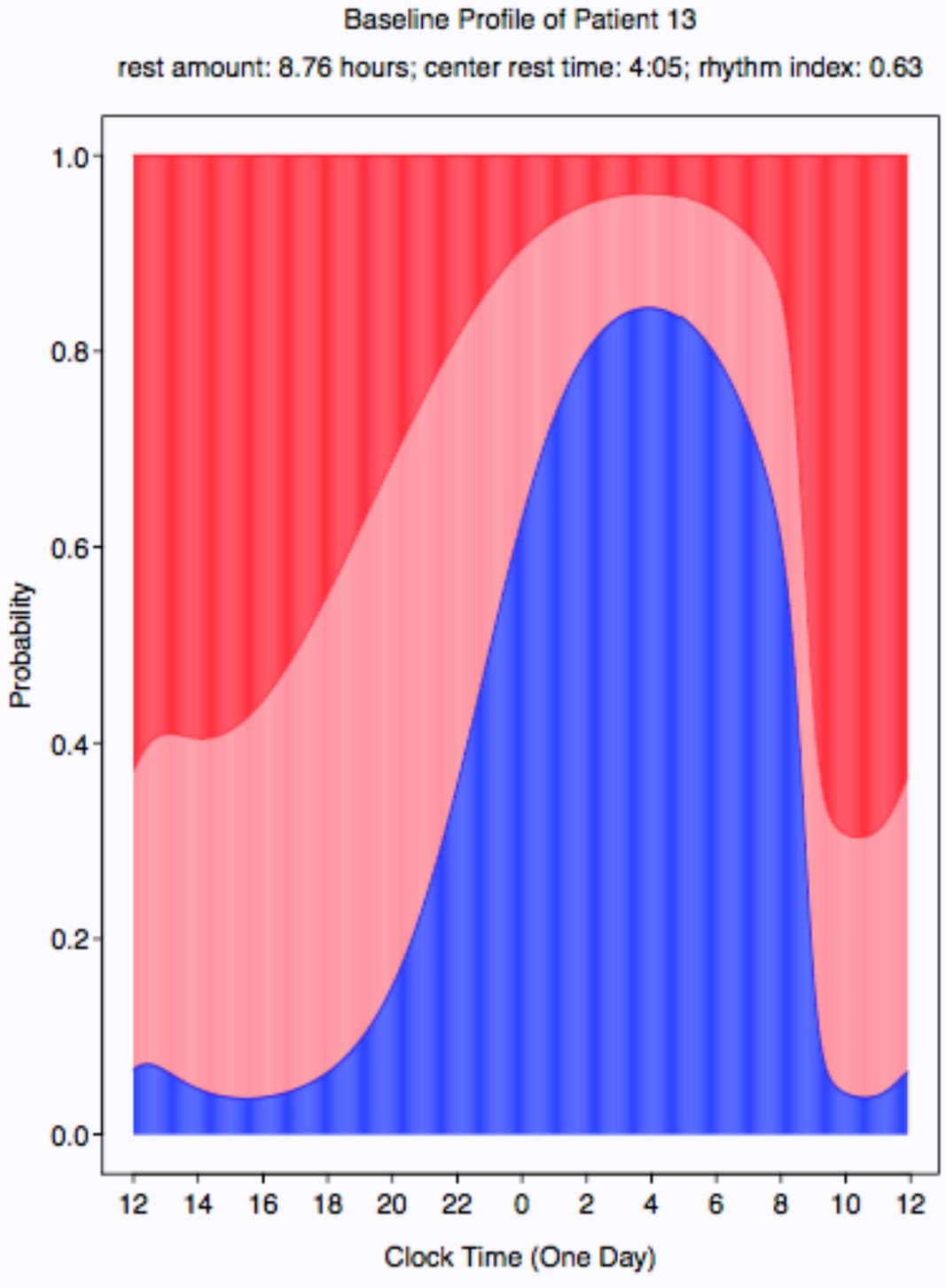
Subject 21

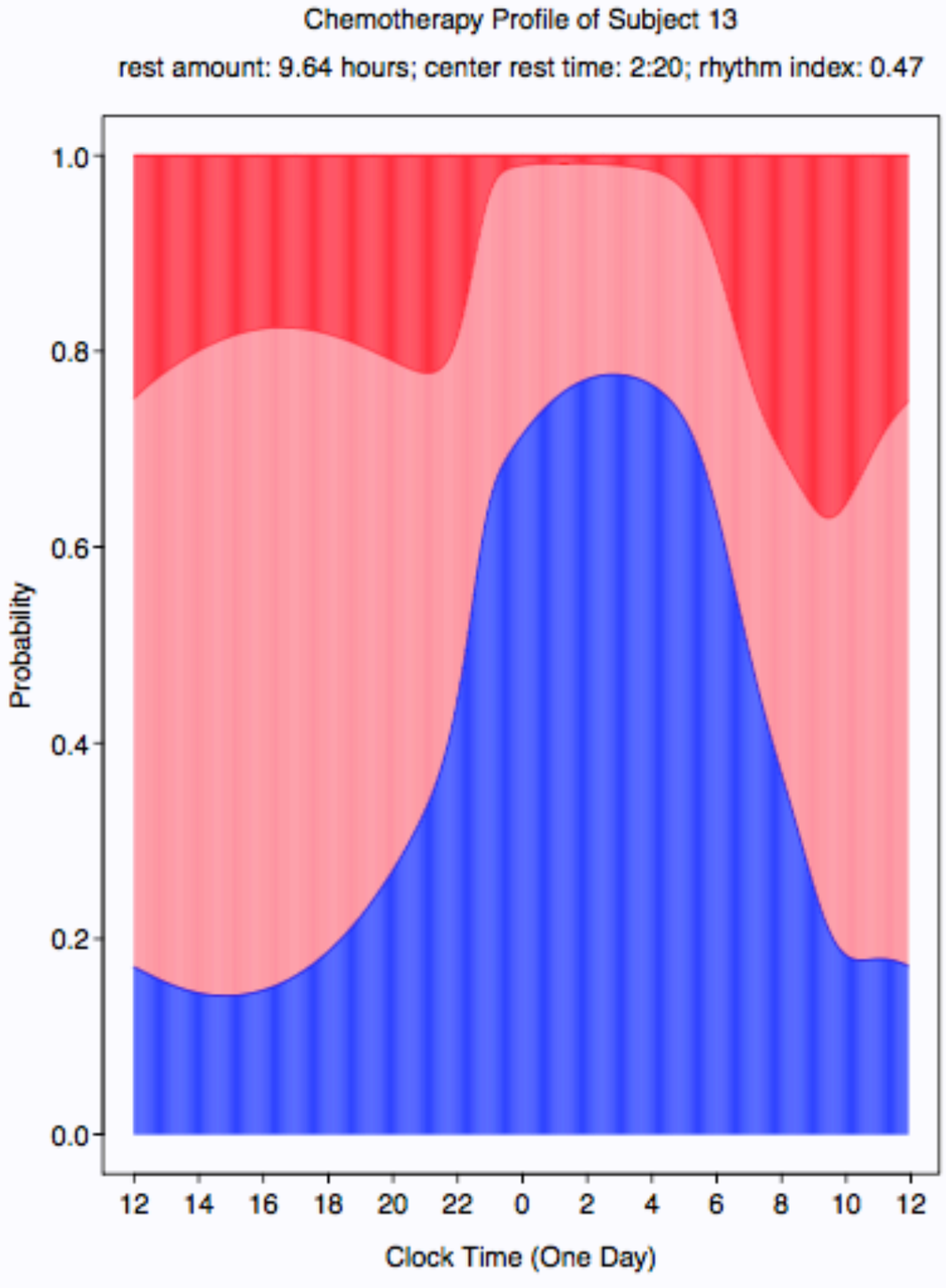
Rest Amount: 11.39 hours; Center Rest Time: 8:17; Rhythm Index: 0.79



Local Decoding of Patient 13



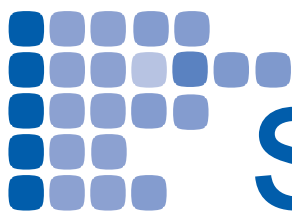

Baseline Profile of Patient 13
rest amount: 8.76 hours; center rest time: 4:05; rhythm index: 0.63
Probability
1.0
0.8
0.6
0.4
0.2
0.0
12 14 16 18 20 22 0 2 4 6 8 10 12
Clock Time (One Day)


Chemotherapy Profile of Subject 13
rest amount: 9.64 hours; center rest time: 2:20; rhythm index: 0.47
Probability
1.0
0.8
0.6
0.4
0.2
0.0
12 14 16 18 20 22 0 2 4 6 8 10 12
Clock Time (One Day)

Q: Where modern mathematics might take me?
A: Here.

Jere Koskela

Offer holder visitor day, 18-19.03.2019



Warwick
Statistics

Where modern mathematics might take me?

– 2007

2008 – 2011

Where modern mathematics might take me?

– 2007

2008 – 2011

Summer 2010

Where modern mathematics might take me?

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 V}{\partial S_t^2}+rS_t\frac{\partial V}{\partial S_t}-rV=0.$$

Credit: https://www.gfmag.com/topics/global-banking/5-black-scholes-merton-and-algorithms

Where modern mathematics might take me?

– 2007

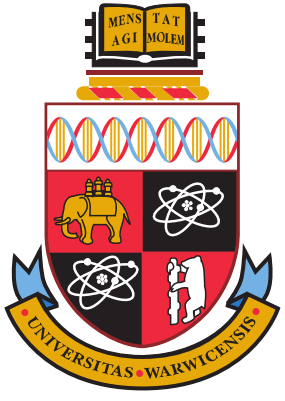
2008 – 2012

Summer 2010 & 2011

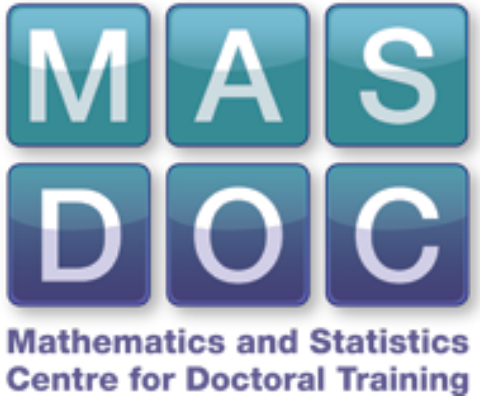
Where modern mathematics might take me?

Skorokhod Embeddings and
Their Applications

Jere Koskela



Where modern mathematics might take me?



2012 – 2016

2016 – 2017

2017 –

Where modern mathematics might take me?

?

How you *really* make financial decisions

Psychologists have uncovered a wealth of **behavioural biases** in the way we make decisions under uncertainty.
We are **not** rational !

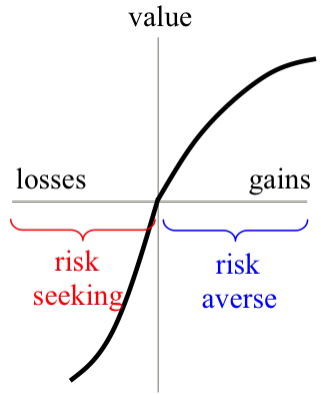
- Major banks (eg. *Barclays*) and investment managers (eg. *BlackRock*) have behavioural teams
- Government has a *Behavioural Insights Team* to provide policy recommendations
- *Thinking Fast and Slow*, D Kahneman (Nobel Prize, 2002)
- *Nudge*, R. Thaler (Nobel Prize, 2017)
- BBC2 *Horizon* programme "How you *really* make decisions"

How do Mathematics, Statistics and Probability contribute?

- *Identify* potential biases — *Analyze* data & *design* statistical tests
- *Develop* stochastic models to capture human behaviour under biases: to *explain* and *predict* how we might behave — in particular, in a dynamic setting

Experimental and Empirical Evidence suggests....

Tend to prefer a certain £500
to a 50% chance of £1000
risk averse over gains
But prefer a 50% chance of
losing £1000 to a certain loss
of £500 **risk seeking over losses**



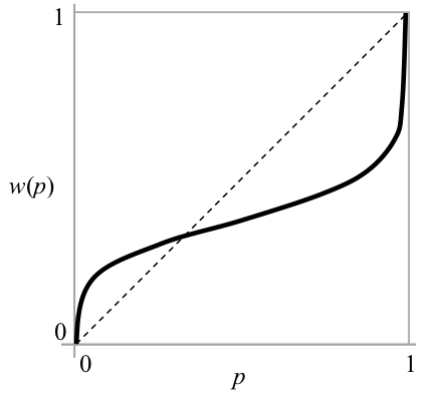
Averse to gambles such as (£110,
50%; −£100, 50%) **loss averse**
Use **reference points**, mental accounts, framing
Delay realization of losses (relative to gains) - *disposition effect*

Why do people buy lottery tickets and insurance?

Tend to prefer a
$\frac{1}{1000}$ chance of £5000 to a certain £5

But prefer a certain loss of
£5 to a $\frac{1}{1000}$ chance of losing £5000

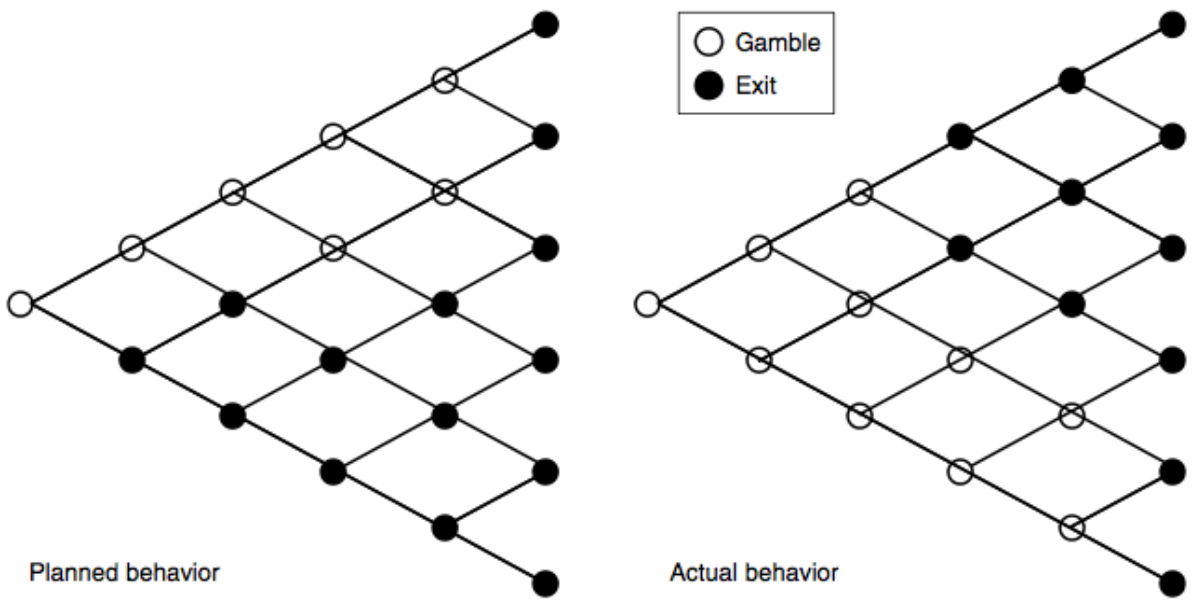
We tend
to **over-weight** small probabilities



The National Lottery
Life changing®

Dynamic Prospect Theory Models I

- Prospect theory was developed by Kahneman and Tversky (1979, 1992)
- We incorporate PT and especially probability weighting into stochastic trading models.
- *Time-inconsistent behaviour* emerges.



Dynamic Prospect Theory Models II

If we now make the grid finer and finer (continuous time) then....

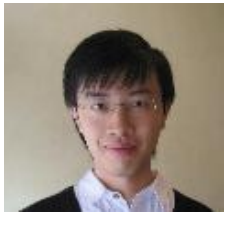
- if we assume investors can "stick to their plans" then model can explain:
  - Stop-loss strategies in markets
  - Preference for right skewness
  - Disposition effect
- ...the naive investor **never stops** & gambles until the bitter end..... Casino gambling.....

What Research can I do as an Undergraduate? And Beyond?

Fourth year Integrated Masters (MMORSE) student **Nikesh Lad** analyzed individual investor behaviour with a very large dataset - 158,000 accounts over a five year period.

Third year student **Rosie Ferguson** did an 8 week **URSS** project in Summer 2015.

Recent PhD student **Alex Tse** (now at Cambridge) worked on prospect theory models in financial trading.



Behavioural Biases in Financial Decision Making

Nikesh Lad | Department of Statistics | Supervisors: Dr J. Brettschneider, Dr V. Henderson

Introduction

Traditional economic theory postulates that investors are "wealth maximisers". However, emotion and psychological factors influence our decisions. Behavioural finance attempts to fill the void of phenomena in stock markets that cannot be described plausibly in models based on rationality.

Project aim:

- Investigate individual investor behaviour using real trading data.
- Explore whether the propensity to sell a stock is positively related to whether the stock has attained its historical high price.

Literature

Descriptive theory

- Heuristics: a mechanism or strategy which people use (often unconsciously) to reduce the complexity of tasks.
  - Often leads to biases, e.g. framing and availability.
- Loss aversion: refers to the asymmetric motives people have to strongly prefer avoiding losses to acquiring gains.
- Disposition effect: a paradox where investors tend to "sell winners too early and rise losers too long."

Theoretical models

- Prospect theory: value function on the domain of gains and losses.
  - Replaces expected utility with probability weighting function.
  - Reflects the human tendency to overweight small probabilities and underweight high probabilities.

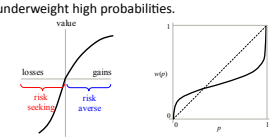


Figure 1: (a) value function (b) probability weighting function

Literature describes two broad categories of investor behaviour.

1. Time-consistent models

Threshold models: optimal strategy is to sell the stock the first time it reaches a threshold level; property known as time-consistency.

- Example – realization utility.

2. Non time-consistent models

Regret models: investors observe the maximum price of a stock and 'gamble for resurrection.' Wait until the stock price reaches this historical high price again before selling – will not sell below this price.

Data

Use trading data from a US discount brokerage firm (Odean, 1998).

- January 1991 to December 1996.
- 78,000 unique households collectively with 158,034 accounts.
- Filter data for trades common stocks; leaves 10,373 stocks.
- A random sample of 10,000 households is taken for analysis.

Data has three main demographic categories: active trader, affluent households and general households.

Analysis

Holding times

Investigate three different holding times to develop a picture of investor behaviour.

Buy-to-sell – how long does an investor hold a stock for?

- Gamma curve fits the features of distribution well, verified by goodness-of-fit tests. Represents waiting time until the *r*th event.
  - Event: the investor faces a *sell* versus *hold* decision.
- Interpret the shape parameter as characterising the investors level of patience which determines their waiting time.
- Would expect the shape parameter for active traders to be less than for affluent or general households.

Holding time	Median (days)	Mean (days)	Shape
Buy-to-sell	169	342	0.768
Active trader	163	312	0.707
General household	218	356	0.819
Affluent household	298	427	1.027
Maximum-to-sell	49	167	0.428
Buy-to-maximum	63	175	0.486

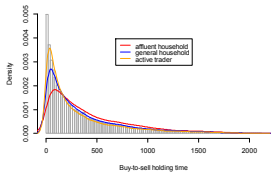


Figure 2: histogram of buy-to-sell holding time with demographic densities

Maximum-to-sell – does the observance of a maximum price increase propensity to sell?

- 42.1% of stock trades have maximum-to-sell holding time of less than 28 days.

Consider holding time *relative* to the buy-to-sell holding time.

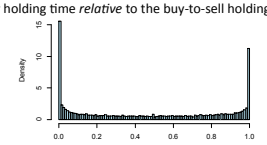


Figure 3: histogram of the distribution of scaled maximum-to-sell holding time

Produces interesting result, after noticing maximum price investors:

- Found to be selling stocks very promptly
- Found to be waiting a long time to sell; here maximum price happens very shortly after stock purchase.

Buy-to-maximum – how long does the investor wait to observe a historical high price?

- The longer the investor waits to realise a maximum price, the higher the median return, see Figure 6.

Return analysis

Analyse the returns of a stock trade, defined as

$$return = \frac{sell\ price - buy\ price}{buy\ price}.$$

- Large proportion of investors making *small* gains or losses, with 30.2% of trades with returns between –0.1% and 0.1%.
- Distribution of returns is leptokurtic with a large positive skew, distribution is not Normally distributed.

Demographic	Active	General	Affluent
Median return (%)	0.035	0.056	0.049

- A scaled-*t* distribution is found to provide an adequate fit.

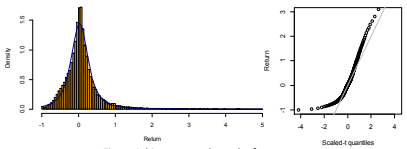


Figure 4: histogram and q-q plot for returns

Difference between maximum and sell price

Define the maximum price as the highest price that occurs since the stock was purchased and until the stock was sold (note that the maximum price can occur at the sell time itself).

- Investors typically observed to sell at a price just below the maximum price of the stock trajectory, since the stock was bought.
- 61.1% of stock trades sold within a price range of $0 to $5 below the maximum price.
- 12.6% of stock trades sold at the maximum price itself.

Relationships between returns and holding times

- Positive returns are best realised when the investor observes the maximum price and reacts promptly.

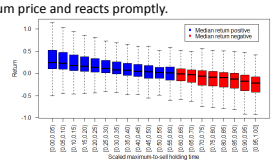


Figure 5: boxplots of returns versus scaled maximum-to-sell holding time

- Median return is negative if the investor waits roughly less than 20 days to observe the maximum price.

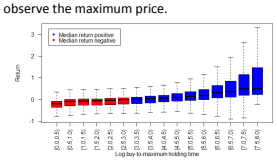


Figure 6: boxplots of returns versus buy-to-maximum holding time

Conclusions

A representation of a typical investor.

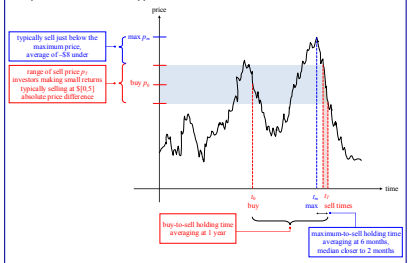


Figure 7: illustrative stock price trajectory

Propensity to sell seems to be higher if the investor observed a historical high price of the stock price trajectory and is dependent on a number of factors:

- Whether the stock is making a positive or negative return.
  - Selling occurs at a prompter rate for positive returns.
- If the maximum price occurs at a time which is not close to when the stock was purchased (Figure 5).
- The longer the investor waits to realise a maximum price, the higher the median return – greater chance of experiencing maxima of greater magnitudes (Figure 6).
- The type of investor.
  - On average, active traders have shorter buy-to-sell holding times and yield lower returns.
  - Consistent with idea that active investment strategies can underperform passive strategies.

Is this behaviour time consistent?

- Not in the classical sense – large proportion of investors are selling stocks just below the maximum price and *not* the first time the price reaches some pre-determined level.

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Questions?

What next?

11:00-11:50	Talk \Risk and Predictability | Where Might Modern Mathematics Take Me?" Opportunity for questions.
Now	Lunch Information about Careers, Funding, Admissions and Wellbeing.
13:00-13:45	Talk “How to solve it? Examples from STEP and A-level papers", Opportunity for questions.
	(Alternative event for accompanying persons: Welfare talks)
14:00-15:20	Campus tour led by current students / Small group meetings with academic staff
15:20-	Tea, and more information