THE UNIVERSITY OF WARWICK

FIRST YEAR EXAMINATION: Mock Examination Paper

GAMES, DECISIONS AND BEHAVIOUR

Time Allowed: **2 hours**

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

Calculators are not needed and are not permitted in this examination.

ANSWER 3 QUESTIONS.

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1. a) Consider a game which involves rolling two normal, unbiased 6-faced dice (one of which is red and the other blue, so we can distinguish between them). You may assume that the dice are "fair" in the sense that each number from 1 to 6 is equally likely to be obtained if one of them is rolled.

(i) What is the sample space (the set of possible outcomes), Ω, for this experiment?

(ii) If we are only interested in three possible classes of outcome:

- Both numbers are odd.
- Both numbers are even.
- One number is odd and the other even (in either order).

then what is the smallest algebra which we could employ?

(iii) What are the atoms of this algebra?

(iv) What is the associated probability mass function?

(v) How big is the algebra used in this experiment – how many elements does it contain? How big is the largest algebra over Ω... which is it easier to work with?

b) Joe owes £100,000 to the Mafia. They want the money immediately. He has £60,000. He has the opportunity to place a bet in a casino in which with probability $\frac{1}{2}$ he will make a profit of £40,000 and with probability $\frac{1}{2}$ he will lose £60,000.

(i) What is his EMV decision?

(ii) Does that coincide with common sense?

(iii) What utility function would you advise Joe to use in this circumstance?

(iv) What is the expected utility of the two possible decisions? And what is the expected utility decision?

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2. a) The diagnosis of a certain disease D is not straight forward. However, there is a certain task T at which people without the disease outperform people with the disease. People without D succeed at T with a probability of π, while people with the disease end up guessing corresponding to a success probability of 0.5. One way of diagnosing D is to have a person perform T a number of times and count the number r of successes. Let p bet the prevalence of D (i.e. the probability someone drawn at random from the reference population has D).

Immediate treatment (d_1) for D can be given at cost C, regardless of whether or not a person has D. It is a harmless intervention. Deferred treatment (d_2) is also harmless, but comes at cost $\beta\, C$ and is only given to a person if the person has.

You are working on mathematical models to advice doctors in this decision. From a financial point of view, the decision reduces to a relationship between the expected losses $\overline{L}_i = \mathbb{E}[L(d_i)]$ for d_i $(i = 1, 2)$. Immediate treatment will be performed if $\overline{L}_1 \leq \overline{L}_2$, otherwise it will be deferred. For simplicity, the loss function L consists of just the costs described above.

(i) Write down the formula for the loss function and calculate the expected losses $\overline{L}_i$ $(i = 1, 2)$ as functions of the parameters p, π, β, n and the number r of correctly performed tasks.

(ii) Derive a formula that expresses the condition $\overline{L}_1 \leq \overline{L}_2$ as a conditions on r.

(iii) Calculate the threshold for r more explicitly for $p = 0.2, \pi = 0.9, \beta = 10$ and $n = 4$.

b) Consider a zero sum game with the following payoff matrix (for player 1; remember player 2 has payoffs corresponding to the negative of those of player 1 in a zero sum game):

	δ_1	δ_2	δ_3
d_1	0	5/6	1/2
d_2	1	1/2	3/4

(i) What is player 1's maximin mixed strategy?

(ii) What is player 2's maximin mixed strategy?

(iii) What is the value of this game?

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3. a) Consider a game with the following payoff matrix:

	α	β	γ	δ
a	(4,5)	(1,4)	(1,5)	(3,3)
b	(5,6)	(3,4)	(2,5)	(4,7)
c	(3,7)	(6,10)	(8,11)	(2,8)
d	(4,8)	(8,5)	(10,4)	(3,4)

Using iterated elimination of dominated strategies show that there is a single strategy which the players should, under the assumption that *rationality is common knowledge*, adopt deterministically.

b) Consider the elicitation of preferences below.

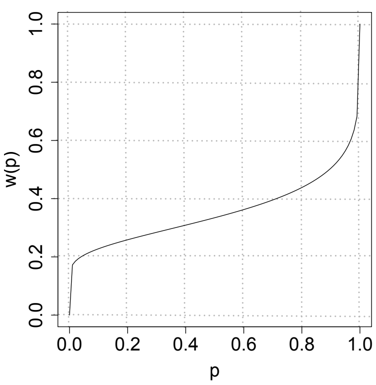
I Which of the following situations would you prefer?
A: Certainty of receiving 1 million.
B: 10% chance of 5 million; 89% chance of 1 million; 1% chance of nothing.

II Which of the following situations would you prefer?
C: 11% chance of 1 million; 89% chance of nothing.
D: 10% chance of 5 million; 90% chance of nothing.

Empirical studies have shown that most people prefer A to B and D to C.

(i) Why do these preferences contradict expected utility theory (EUT)?

(ii) Model and explain these preferences using prospect theory. For the value function v assume $v(0) = 0, v(1) = 1$ and $v(5) = 2$ (defined on millions of the currency unit). For the probability weighting function use the Prelec function, $w(p) = \exp(-\delta(-log(p))^{\gamma})$ $(p \in [0, 1])$, with parameters $\delta = 1.2$ and $\gamma = 0.25$.



p	w(p)
0.00	0.0
0.01	0.2
0.10	0.2
0.11	0.2
0.89	0.5
0.90	0.5
1.0	1.0

(rounded)

(iii) Explain which aspect of the probability distortion expressed by w leads to a modification of preferences when using prospect theory rather than EUT. Include a crucial quantitative observation about w to make your point.

4. a) You are a 4th year student doing a Master's dissertation project on cognitive heuristics and biases of the type studied extensively by Tversky and Kahneman in their research program in the 1970s to 1980s. Part of your work consists in running your own empirical studies on this. You are given the opportunity to study this empirically by conducting a short questionnaire based survey in an ST222 lecture.

Among other things, you want to study how prior exposure to numerical quantities affects the estimate of an unrelated numerical quantity. You plan to give all of the students the same estimation task, but expose half of them to a different quantity then the other half. Design a suitable experiment by working through the following steps.

(i) Formulate the hypothesis you want to test.

(ii) Select a numerical quantity you will ask the students to estimate.

(iii) How would you split them into two groups (in practical terms)?

(iv) Which prior exposures would you give them, and how?

(v) Name two difficulties that may arise during the conduction of the survey. *Use no more than 10 words each.*

(vi) Briefly describe two essential limitations of your study and how they would effect the results. *Use no more than 20 words each.*

b) Let Ω be a space of outcomes and $\mathcal{F}$ be an algebra of subsets of Ω. Let $A, B \in \mathcal{F}$ be two sets of events with $A \subset B$. One of your flat mates holds the belief that $P(A) > P(B)$. Can you construct a Dutch book against him? Detail one way how you can do that.

c) Let w be a probability weighting function and v be a value function in a prospect theory model. Two prospects (x_i, p_i) $(i = 1, 2)$ are equally preferable if their values $v(x_i)\, w(p_i)$ $(i = 1, 2)$ are equal.

Let $\lambda > 0$. A probability weighting function w satisfies the λ-*reduction invariance* property if the following is true:

The prospects $((x, p), q)$ and (x, r) are equally preferable, if and only if the prospects $((x, p^\lambda), q^\lambda)$ and (x, r^λ) are equally preferable.

Prove that any Prelec function $w(p) = \exp(-\delta(-log(p))^\gamma)$ $(p \in [0, 1])$ with parameters $\delta, \gamma > 0$ is λ-reduction invariant for all $\lambda > 0$.