

1. (a)
  - i. Mostly well done with full credit. Some students did partly mix up decision space and outcome space by including in the outcome space three options rather than two. However, the third option (*no gain nor loss*) is the result of a decision (*not invest in stocks*) rather than anything that has to do with random outcomes.
  - ii. Those who attempted this part of the question often gave a detailed answer gaining full credit. However, some people just wrote very superficial statements. Some answers used the balls-in-bag bet approach, but did not address the issue of precision.
  - iii. Most people used initial amount and the change ( $\mathcal{L}C + \mathcal{L}100, \mathcal{L}C - \mathcal{L}200$ ) to find the threshold  $p_0$ . However, for EMV the initial value is irrelevant, so using just the difference in value ( $\mathcal{L}100, -\mathcal{L}200$ ) was also accepted. However, some people ended up using  $\mathcal{L}C$  in  $d_1$  but not in  $d_2$ , which gave an incorrect answer, so they lost a point. Some lost a point for confusing  $\mathcal{L}C + \mathcal{L}100$  with  $100\mathcal{L}C$ . There was a minor typo in the question: It should have read  $\mathcal{L}C > \mathcal{L}200$  instead of  $\mathcal{L}C > \mathcal{L}100$ , which was announced in the class. The calculation was not actually effected, but as a result of the announcement some people ended up changing other constants in the question. None of this mattered to the calculation either, and nobody lost any points here or in the next part for using different constants.
  - iv. Most people who used the initial value  $\mathcal{L}C$  out in the previous question realised that due the nonlinearity of the utility function they could not do so here. Others lost a point for leaving  $\mathcal{L}C$  out. Some lost a point for confusing  $p \log(\mathcal{L}C)$  with  $\log(p)\mathcal{L}C$ .
- (b)
  - i. Half the class gave a proper definition requiring the existence of functions or constant for an certain representation of the matrix, but the other half was rather sloppy and lost points for saying something very vague.
  - ii. A lot of people gave the right answer, but some gave no proof (not even a reference to any results seen in the context of the lecture) resulting in loss of points. Just saying “linear algebra says” does not count as a proof either. Some gave a beautiful complete derivation and got full credit. Some did not make whether their condition is necessary or sufficient or both and lost a point.
- (c)
  - i. A lot of people guess the fallacy given in the solution (conjunction fallacy). The term “Linda fallacy” (stemming from the example that made this fallacy famous) was equally excepted, but we discourage using this as the rest of the world my not understand. Other people argued along the lines of reason based thinking, which was also accepted. However, some people seemed to just gave some fallacy name that did not seem to fit and omitted explained why they thought it would, so we could not give credit for that. Explanations about the underlying probability confusions were usually correct, though some people confused intersection ( $\cap$ ) and union ( $\cup$ ). Other did use  $A$  and  $B$  for events without connecting them to events in the question.
  - ii. Most people gave the right answer. However, frequent errors included somehow involving  $P(A \cup B)$  (corresponding to OR) or  $P(A) + P(B)$ , which do not occur in the context of this question.
2. (a)
  - i. This was difficult for most students even though it was the special case of an exercise sheet problem. Many people forgot to consider marginal terms. Another frequent error was to not multiply by 2 to account for runs of either heads or tails. Some solutions just

counted the number of heads or tails without ensuring they are consecutive (i.e. runs). Some used the right formula for the expectation but confused length of the run with number of runs. Some did not properly convert a probability into the expectation of the corresponding indicator function ( $P(A) = E[1_A]$ ).

- ii. Some of the answers were too vague. We did expect some notion of the possibility of quantitative probabilistic judgements, even though no specific suggestion for a test with formula or distributions was expected.
- (b)
- i. Many students forgot to answer the second part of this question (condition under which this belief is incorrect).
  - ii. A fair number of people only stated the law of large numbers (LLN) while not answering the actual question how an incorrect application of it would lead to the gambler's fallacy. If the LLN was correctly reproduced for this context half of the credit was still given.
  - iii. A lot of great ideas here (sports, exams, busses, rainy days, judges, stocks etc).
  - iv. Many people gave the name of this effect/fallacy though it wasn't asked for ("hot hand") and discuss the validity of that in more detail; a point was given. However, the question was actually about how the contradicting principles could exist in parallel, which only few students actually addressed. bullet crucial points that shed
- (c)
- i. Very few people noticed that  $\delta_5$  is dominated by  $\delta_4$ . This was not required, but make the solution a bit shorter. Some people stated that  $\delta_4$  is dominated by  $\delta_5$ , which is incorrect, because the matrix in a zero-sum games refers to player 1 and these moves are player 2's so the matrix changes sign. This lead to an incorrect answer all the way through this question, but only 1 point was deducted overall.
  - ii. Usually done well. Some people used very bad scales making it hard to read their graph. Some did not understand how to select the maximin point and picked a different one.
  - iii. Usually answered correctly (adjusting for errors made earlier).
3. (a)
- i. We can classify answers to this question into three categories: the first one is people who answered correctly to the question (a good percentage but not high as expected); the second one of people doing right calculations but getting the wrong answers, in this group many students did not take into account the fact that if Lola wins in the first round she raises 3500 Marks and she can bet 3600 Marks (not only the 3500 just won, it is clearly written "the stake is returned"); the group (quite a few students) who used a summation from zero to infinity or they considered three rounds, not understanding that "If 20 comes up in each round" meant "If 20 comes up in each of the 2 rounds".
  - ii. Some students did not answer this question correctly because they took into account only one period. It is true that EMV suggests not to enter also if Lola had to play for only one period, but you should have taken into account the entire game presented in the first point. In general I awarded full marks if correct calculations were carried on using wrong answers from the first question.
  - iii. In general this question was answered correctly. Common sense suggests entering the game. Only a few students did not answer correctly making comments related to the

fact that casinos make money out of people hope but that was not the expected answer. In general, students said that not gambling did not follow common sense because it was the only opportunity for Lola to save her boyfriend. There were some nice comments among the answers. Comments included “it depends on how much she cares about him”, “mafia will kill him” (there were no references to mafia). A couple of students transformed the boyfriend into “her husband”.

- iv. Almost every students answered correctly. Somebody transformed probability of the bet instead of suggesting a transformation of the utility function, that is wrong.
  - v. Almost every students answered correctly.
- (b)
- i. In general people got at least one point. The correct answer was “play”, provided  $p > 0$ . However, one point out of two was awarded to students who simply answered “play always”.
  - ii. In general, it was answered correctly. One point for clearly defining EMV (almost everyone did it). One point for deriving the condition needed to make “play” the best strategy.
  - iii. In general people got at least one point. The correct answer was “do not play, provided  $p < 1$ . However, one point out of two was awarded to students who simply answered “do not play”.
- (c)
- i. Well answered in general. Probably 99% success rate.
  - ii. Quite a few students answered this question properly. If sensible explanations were provided people were awarded at least a point, in general. Three comments are worth adding: it was not enough saying that a long sequence of occurrences of the same colour is deemed as unlikely by decision makers; it is not always true that a sequence of 5 rows has an higher probability of a sequence of 6 rows (think about  $(2/3)^6$  and  $(1/3)^5$ , here you could say that being the two sequences one the “continuation” of the other you could claim that the sixth row reduced probability of the long one with respect to the short one but some of you simply claimed that a sequence of 5 rows has an higher probability of a sequence of 6 rows; the answer that  $RGRRR$  was contained in  $GRGRRR$  was accepted if phrased in words (since I see that you mean that one is the continuation of the other) while it led to miss a point if written as  $RGRRR \subseteq GRGRRR$ , since it is true the opposite.
4. (a)
- i. Almost all students dealt with cases A and B correctly. Case C was generally well done, but not everyone dealt with the fact that there were two stages, and that the expected values were 25% those in part B.
  - ii. Again, this was well answered for parts A and B. Very few students gave the correct principle behind case C, but the mark was awarded whenever the student highlighted the difference from part B.
  - iii. Some students answered in terms of general risk-averse/risk-seeking strategies, others took a more algebraic approach. Most saw that whatever the utility function, the same choice should to be made in all three cases.
- (b)
- i. There was one mark for the definition, which most students remembered and one mark for the demonstration that it held. The clearest answers considered case-by-

- case. There were five cases to consider, and full marks were given as long as students gave at least three of them.
- ii. As above, there was one mark for the definition, which most students remembered and one mark for the demonstration that it held. The clearest answers considered case-by-case. Students who attempted to show this using complication and/or statements and make implications for all cases together tended to get in a muddle. There were four cases to consider, and full marks were given as long as at least two of them had been considered.
  - iii. This was the least well done part of Question 4. Many did not attempt it at all. One mark was available for the definition, and was awarded even if there was a small mistake in its presentation (for example a swapped  $x$  and  $z$ , or the inequalities the wrong way round). Many of those who gave a definition did not attempt to prove or disprove whether it held for lexicographical order. Those who did either provided a specific counter-example or conditions on  $x_1$ ,  $y_1$  and  $z_1$  that would generate a class of counter-examples.
- (c)
- i. The vast majority of students recognised that this required an application of Bayes' Theorem along with the Law of Total Probability. There was some confusion from the 90% mentioned in the question. Some students took this as the value of  $\mathbb{P}(\text{not drug user})$ , others as the value of  $\mathbb{P}(\text{positive test}|\text{drug user})$  - neither of which is correct. Most students, however, were able to state what needed to be calculated and plug in the correct constituent probabilities, but a surprisingly large number then made arithmetical slips leading to an incorrect final answer (for which one mark was docked).
  - ii. There was one mark for the correct answer 'base rate neglect' or an explanation of that fallacy. Vague answers, such as 'they got the conditional probability the wrong way round' were not accepted. There was a fair bit of guess work here - there were a varied of fallacies named that had nothing to do with the scenario.