ST222 2017 TEST ABOUT PART I

(1) Pick a coin from a bag that contains $n-1$ fair coins and one two-headed coin. You toss it three times.

- $\Omega = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$ is an outcome space and $\mathcal{F} = \{\emptyset, \Omega, \{hhh\}\}$ is an algebra for this experiment. TRUE FALSE

- You have observed three heads. A friend offers you a bet. If it is the two-headed coin, he gives you a chocolate coin, and otherwise you give him one. For which n is this bet fair? 5 7 9

(2) Toss a fair coin 3 times. This can be described by the outcome space $\Omega = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$ and the algebra $\mathcal{F}$ consisting of all subsets of Ω. Let H_i $(i = 1:3)$ be the event that the coin lands heads on the ith toss and A the event that it shows two heads in total. Let B an event unknown to you.

TRUE FALSE $\mathcal{F}_1 = \{\emptyset, \{hhh\}, \{hht, hth, htt, thh, tht, tth, ttt\}\}$ is an algebra for this experiment.

TRUE FALSE H_1 and A are independent.

TRUE FALSE H_i $(i = 1, \ldots, 3)$ are pairwise independent.

(3) Let Ω be an outcomes space and $\mathcal{F}$ a σ-algebra on Ω. Let μ be a function on $\mathcal{F}$.

TRUE FALSE If $|\Omega| = n$ then $|\mathcal{F}| = 2^n$.

TRUE FALSE μ is called *coherent* if $\mu(\Omega) = 1$ and, for all $A \in \mathcal{F}$, $\mu(A^c) = 1 - \mu(A)$.

TRUE FALSE If μ is a probability, then for all $A_1, A_2 \in \mathcal{F}$ with $\mu(A_1) > 0, \mu(A_2) > 0$,

$$\mu(A_1|A_2) \cdot \mu(A_1) = \mu(A_2|A_1) \cdot \mu(A_2).$$

(4) Let Ω be an outcome space and $\mathcal{F}$ a σ-algebra on Ω. For $A \in \mathcal{F}$ and $M > 0$, let $b(M, A)$ be the bet that pays M if A occurs and 0 otherwise. What is the behavioural definition for the (subjective) probability of A?

(a) The minimum you would be prepared to pay for playing that bet.

(b) The minimum you would demand to offer that bet.

(5) Choose the optimal decision for three different possible outcomes with probabilities

$$p(\omega_1) = 1/2, \quad p(\omega_2) = p(\omega_3) = 1/4,$$

rewards $R(d_1, \omega_1) = £49, R(d_1, \omega_2) = R(d_1, \omega_3) = £25$, $R(d_2, \omega_1) = £36, R(d_2, \omega_2) = £100, R(d_2, \omega_3) = £0$, $R(d_3, \omega_1) = £81, R(d_3, \omega_2) = R(d_3, \omega_3) = £0$ and according to the following decision rules:

- Expected monetary value: d_1 d_2 d_3

- Expected utility with $u(x) = \sqrt{x}$: d_1 d_2 d_3

(6) You prefer a fifty-fifty chance of winning either £100 or £10 to a lottery in which you win £200 with a probability of 1/4, £50 with a probability of 1/4, and £10 with a probability of 1/2. You also prefer a fifty-fifty chance of winning either £200 or £50 to receiving £100 for sure. Which axiom do your preferences violate?

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(7) Let Ψ be an outcomes space with algebra $\mathcal{F}$, X a random variable on $(\Psi, \mathcal{F})$ representing the outcome and let D be a decision space. Let L be your loss function on $D \times \Psi$ and let P be your subjective probability $(\Psi, \mathcal{F})$. Give a formula for the expected monetary value decision strategy for an optimal solution d^*.

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TRUE FALSE d^* is unique.

Your friend has a similar model but with decision space $\tilde{D}$, loss function $\tilde{L}$ and subjective probability $\tilde{P}$.
TRUE FALSE $d^* = \tilde{d}^* \iff D = \tilde{D} \wedge L = \tilde{L} \wedge P = \tilde{P}$

(8) Let $b(p, s, t)$ be the bet that pays out s with probability p and t with probability $1 - p$.
TRUE FALSE The CME for b is the value m such that $u(m) = E[u(b(p, s, t))]$.
TRUE FALSE A risk averse attitude corresponds to the case CME smaller than $E[b(p, s, t))]$.
TRUE FALSE A risk seeking attitude corresponds to a convex utility function.

(9) A patient with severe chronic pain is offered surgery that will remove the pain completely with probability 80%, kill him with a probability of 4%, and has no effect in the remainder of cases. Assign the outcome *death* utility 0 and *no pain* utility 1. For *chronic pain* the patient's utility is 0.85 (elicited through comparison with a bet). How would the patient choose based on the expected utility principle? SURGERY NO SURGERY

(10) Let $\mathcal{A}$ be an action space with a binary relation $\succ$. What are the names of the following properties?

- For all $x, y \in \mathcal{A}$, $x \succ y \vee x \sim y \vee y \succ x$. Name: . . . .
- For all $x, y, z \in \mathcal{A}$, $\neg\, x \succ y \wedge \neg\, y \succ z \;\Rightarrow\; \neg\, x \succ z$. Name: . . . .

(11) You consider an offer to buy insurance for the price of c against the loss of a value v. From historical data it is estimated that the probability for such a loss to occur is about 1%, and the probability for a partial loss of $v/10$ is about 5%.

- For what values of c is the maximin decision to buy insurance?
- Does this seem reasonable? Give a reason for your answer.

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(12) In the St Petersburg game the prize is initially £1. A fair coin is tossed until head is shown, at which point the prize is paid out. Each time tail comes up the prize is doubled. Suppose the utility in the is bounded $u(x) \leq A$ for all $x \geq 0$. Show that the maximum utility of the game is bounded.

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(13) Let $b(p, s, t)$ be the bet that pays out s with probability p and t with probability $1 - p$.

TRUE FALSE The CME for b is the value m such that $u(m) = E[u(b(p, s, t))]$.

TRUE FALSE A risk averse attitude corresponds to the case CME bigger than $E[b(p, s, t))]$.

TRUE FALSE A risk seeking attitude corresponds to a convex utility function.