ST222 2017 TEST ABOUT PART I - SOLUTIONS (WITH SHORT REASONS)

(1) Pick a coin from a bag that contains $n-1$ fair coins and one two-headed coin. You toss it three times.

- FALSE. Complement of $\{hhh\}$ is missing.
- 9. See W1-L1.

(2) Toss a fair coin 3 times. This can be described by the outcome space $\Omega = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$ and the algebra $\mathcal{F}$ consisting of all subsets of Ω. Let H_i $(i = 1:3)$ be the event that the coin lands heads on the ith toss and A the event that it shows two heads in total. Let B an event unknown to you.

FALSE $\mathcal{F}_1$ does not contain Ω.

FALSE $P(H_1 \cap A) \neq P(H_1) \cdot P(A)$.

TRUE $P(H_1 \cap H_2) = \frac{1}{2} \cdot \frac{1}{2} = P(H_1) \cdot P(H_2)$ etc.

(3) Let Ω be an outcomes space and $\mathcal{F}$ a σ-algebra on Ω. Let μ be a function on $\mathcal{F}$.

FALSE Only true if $|\mathcal{F}|$ is the power set, but not in general.

FALSE Not complete set of conditions.

FALSE The correct formula is $\mu(A_1|A_2) \cdot \mu(A_2) = \mu(A_2|A_1) \cdot \mu(A_1)$.

(4) Let Ω be an outcome space and $\mathcal{F}$ a σ-algebra on Ω. For $A \in \mathcal{F}$ and $M > 0$, let $b(M, A)$ be the bet that pays M if A occurs and 0 otherwise. What is the behavioural definition for the (subjective) probability of A?

(a) The minimum you would demand to offer that bet.

(5) Choose the optimal decision for three different possible outcomes with probabilities

$$p(\omega_1) = 1/2, \quad p(\omega_2) = p(\omega_3) = 1/4,$$

rewards $R(d_1, \omega_1) = £49, R(d_1, \omega_2) = R(d_1, \omega_3) = £25,$ $R(d_2, \omega_1) = £36, R(d_2, \omega_2) = £100, R(d_2, \omega_3) = £0,$ $R(d_3, \omega_1) = £81, R(d_3, \omega_2) = R(d_3, \omega_3) = £0$ and according to the following decision rules:

- Expected monetary value: d_1 d_2 d_3 Solution: d_2.
- Expected utility with $u(x) = \sqrt{x}$: d_1 d_2 d_3 Solution: d_1.

(6) You prefer a fifty-fifty chance of winning either £100 or £10 to a lottery in which you win £200 with a probability of 1/4, £50 with a probability of 1/4, and £10 with a probability of 1/2. You also prefer a fifty-fifty chance of winning either £200 or £50 to receiving £100 for sure. Which axiom do your preferences violate?

Independence.
Reason: The second preference described above gets inverted by mixing it with a 0.5 chance to win £10.

Continues on the back

(7) Let Ψ be an outcomes space with algebra $\mathcal{F}$, X a random variable on $(\Psi, \mathcal{F})$ representing the outcome and let D be a decision space. Let L be your loss function on $D \times \Psi$ and let P be your subjective probability $(\Psi, \mathcal{F})$. Give a formula for the expected monetary value decision strategy for an optimal solution d^*.

$$d^* = \text{argmax}_{d \in D} \mathrm{E}[L(d, X)]$$

FALSE d^* is unique. E.g. add idential copies of d^* to D.

Your friend has a similar model but with decision space $\tilde{D}$, loss function $\tilde{L}$ and subjective probability $\tilde{P}$.
$d^* = \tilde{d}^* \iff D = \tilde{D} \wedge L = \tilde{L} \wedge P = \tilde{P}$
FALSE The same solution doesn't imply the same question (though the inverse is true). The same decision can be optimal in different settings.

(8) Let $b(p, s, t)$ be the bet that pays out s with probability p and t with probability $1 - p$.
TRUE The CME for b is the value m such that $u(m) = E[u(b(p, s, t))]$.
TRUE A risk averse attitude corresponds to the case CME smaller than $E[b(p, s, t))]$.
TRUE A risk seeking attitude corresponds to a convex utility function.
For all explanations see Lecture notes W4 L3.

(9) A patient with severe chronic pain is offered surgery that will remove the pain completely with probability 80%, kill him with a probability of 4%, and has no effect in the remainder of cases. Assign the outcome *death* utility 0 and *no pain* utility 1. For *chronic pain* the patient's utility is 0.85 (elicited through comparison with a bet). How would the patient choose based on the expected utility principle? SURGERY
Calculating expected utility for surgery gives 0.936, which is larger than 0.85.
However, not everybody would agree that expected utility is the best principle to apply here, because the outcome death doesn't really scale. Alternatively, a worst case approach would lead to a different decision.

(10) Let $\mathcal{A}$ be an action space with a binary relation $\succ$. What are the names of the following properties?

- For all $x, y \in \mathcal{A},\ x \succ y \vee x \sim y \vee y \succ x.$ Name: Completeness
- For all $x, y, z \in \mathcal{A},\ \neg\, x \succ y \wedge \neg\, y \succ z \ \Rightarrow\ \neg\, x \succ z.$ Name: Negative transitivity

(11) You consider an offer to buy insurance for the price of c against the loss of a value v. From historical data it is estimated that the probability for such a loss to occur is about 1%, and the probability for a partial loss of $v/10$ is about 5%.

- For what values of c is the maximin decision to buy insurance?
  For any $c < v$. This is based on looking at the worst case and then compare the choices of buying or not buying insurance either way.
- Does this seem reasonable? Give a reason for your answer.
  This approach is driven by the extreme outcome of loosing v without taking into account that its probability is very low (0.01).

(12) In the St Petersburg game the prize is initially £1. A fair coin is tossed until head is shown, at which point the prize is paid out. Each time tail comes up the prize is doubled. Suppose the utility in the is bounded $u(x) \leq A$ for all $x \geq 0$. Show that the maximum utility of the game is bounded.

$$\frac{1}{2} \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i \cdot u(2^i) \leq \frac{1}{2} A \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i$$

The series converges to a finite value.

(13) Let $b(p, s, t)$ be the bet that pays out s with probability p and t with probability $1 - p$.

Sorry, that was a unintended copy of Question 8.