ST222 2017 TEST ABOUT PART II

(1) In a two person game, let $D = \{d_1, \ldots, d_n\}$ and $\Delta = \{\delta_1, \ldots, \delta_m\}$ be the moves available to Player I and Player II and R and S be their payoff matrices.

- A move $d* \in D$ dominates all other moves if for all $d \in D$ with $d \neq d^*$ and for all $\delta \in \Delta$, $R(d^*, \delta) \geq R(d, \delta)$.
  TRUE FALSE

- A move $d_* \in D$ is dominated if for all $d \in D$ with $d \neq d_*$ and for all $\delta \in \Delta$, $R(d_*, \delta) \leq R(d, \delta)$.
  TRUE FALSE

- The game is called separable if there are r_1 and r_2 such that for all $i = 1, \ldots, n$ and $j = 1, \ldots, m$, $R(d_i, \delta_j) = r_1(d_i) + r_2(\delta_j)$.
  TRUE FALSE

- Assume the game is separable with r_1, r_2 as above. If $d^* \in D$ and $r_1(d^*) \geq r_1(d)$ for all $d \in D$ and there is $j \in \{1, \ldots, m\}$ with $R(d^*, \delta_j) \leq R(d^*, \delta_{j'})$ for all $j^{'} \in \{1, \ldots, m\}$, then Player I should not play d^*.
  TRUE FALSE

- If the game is separable, then $m = n$.
  TRUE FALSE

- The game is called zero-sum game if $\sum_{j=1}^{m} R(d_i, \delta_j) = 0$ for all $i = 1, \ldots, n$.
  TRUE FALSE

- If the game is a zero-sum games, then $m = n$.
  TRUE FALSE

- If the game is a zero-sum games, then there is no equilibrium.
  TRUE FALSE

- The matrix
$$\begin{bmatrix} 17 & 0 \\ 0 & -17 \end{bmatrix}$$
  defines a separable zero-sum game. TRUE FALSE

(2) Consider the situation in the previous questions and also assume it is a zero sum game. Use $\overline{x} = (x_1, \ldots, x_n)$ and $\overline{y} = (y_1, \ldots, y_m)$ to denote mixed strategies.

- Player I's maximin mixed strategy is the $\overline{x}$ that maximises
$$\max_{\overline{x}} \min_{\overline{y}} V_1 = \sum_{i=1}^{n} \sum_{j=1}^{m} x_i R(d_i, \delta_j) y_j$$
  TRUE FALSE

- Player II's maximin mixed strategy is the $\overline{x}$ that minimises
$$\min_{\overline{y}} \max_{\overline{x}} V_2 = \sum_{i=1}^{n} \sum_{j=1}^{m} x_i R(d_i, \delta_j) y_j$$
  TRUE FALSE

- $V_1 = V_2$
  TRUE FALSE

(3) Consider the game defined by the matrix below.

$$\begin{bmatrix} (1,1) & (0,-1) & (1,1) & (-1,-1) & (1,0) \\ (0,0) & (1,0) & (0,1) & (1,0) & (0,1) \\ (-1,1) & (1,-1) & (-1,0) & (0,-1) & (0,0) \end{bmatrix}$$

- Does this game have a discriminant strategy? If yes, which one? If not, why not?

- Find the equilibria.

(4) Consider the zero-sum game defined by the matrix below.

$$\begin{bmatrix} 1 & 4 & 7 & 8 & 2 \\ 2 & 5 & 8 & 7 & 2 \\ 3 & 6 & 9 & 6 & 2 \end{bmatrix}$$

- Does this game have a discriminant strategy? If yes, which one? If not, why not?

- Find the equilibria.