The Hazards of Extremism

- Maximin and maximax solutions may sometimes be acceptable.
- But they aren't stable: what if you introduce another possible outcome with probability $\epsilon \ll 1$?
- However small ϵ is, this outcome could be the only one you base you decision upon.
- But, in decision problems, you work with an idealisation in which you haven't really considered *every* possible outcome.
- This seems rather inconsistent.

Utility of Opportunity / Certain Monetary Equivalence

- If there is a problem with using EMV it is this: it assumes that we value a probability p of receiving some reward r as being of the same value as receiving a reward pr with certainty.
- Would you rather have £10^8 with certainty or a probability of 10^{-9} of having £10^{17}?
- We see that EMV might make sense for moderate probabilities and moderate sums, but it doesn't match our real preferences in general.
- It is useful to think how much a probability p of receiving a reward r is *worth* to us: we call this the *utility* of such a bet.

Some Notation

- Let A, B and C be random outcomes (i.e. particular rewards with some probability or nothing otherwise).
- Write $A \succ B$ if A is preferred to B.
- Write $A \sim B$ if A and B are equally preferable.
- Write $A \succeq B$ if A is at least as good as B.
- For some $t \in (0, 1)$, let $tA + (1 - t)B$ denote outcome A occurring with probability t and B with probability $1 - t$.

Axioms of Preference

If a collection of preferences obey the following:

1. Completeness: For any A, B one of the following holds:

$$A \succ B \qquad A \sim B \qquad A \prec B$$

2. Transitivity:

$$A \succeq B, B \succeq C \Rightarrow A \succeq C$$

3. Independence: if $A \succ B$ then, for any $t \in [0,1)$:

$$(1-t)A + tC \succ (1-t)B + tC$$

4. Continuity: If $A \succ B \succ C$, there exists $\rho \in (0,1)$ such that:

$$\rho A + (1-\rho)C \sim B$$

Then that collection of preferences is considered rational.

Utility Functions

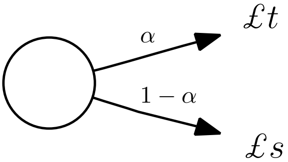
- If the axioms from the previous slide are satisfied. . .
- The preferences can be encoded in a *utility function*, U.
- This function maps the (monetary) value of each outcome to a real number.
- Maximising the *expectation* of the utility in a decision problem makes decisions compatible with the preferences.

  See next week for correspondence between utility function and preferences.

Eliciting Utilities

If preferences are to be represented by utilities, we must be able to determine utility functions.

This bet:



has CME value

$$m = f(\alpha).$$

Certainty Monetary Equivalent

Risk neutral: m=E[bet]

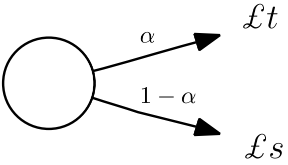
Risk averse: m<E[bet]

Risk seeking: m>E[bet]

Eliciting Utilities

If preferences are to be represented by utilities, we must be able to determine utility functions.

This bet:

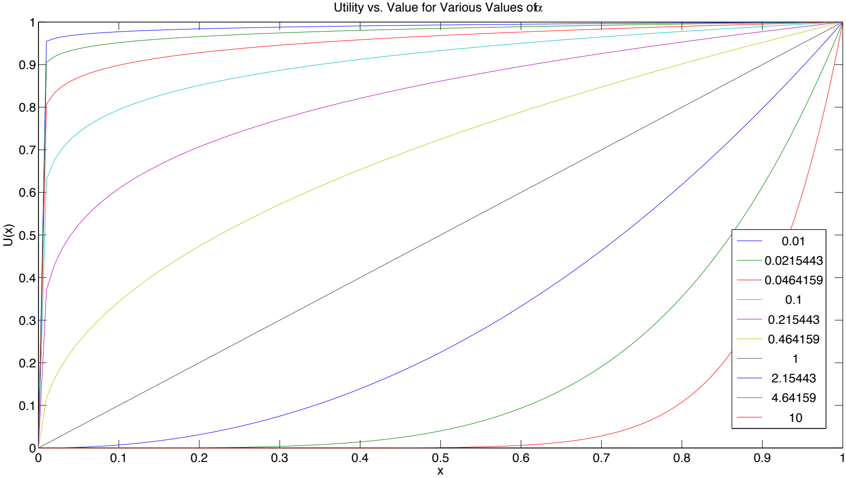


has CME value

$$m = f(\alpha).$$

- What m would you accept not to benefit from the bet shown?
- This is a function of α.
- The utility of m is $U(m) = f^{-1}(m)$.

A Family of Utilities



$$U(x) = x^{\alpha} \qquad \alpha > 0$$

The EMV Decision Rule

- If we calculate the expected loss for each decision, we obtain a function of our decision:

$$\bar{L}(d) = \mathbb{E}\left[L(d, X)\right] = \sum_{x \in \mathcal{X}} L(d, x) \times p(x)$$

- The *expected monetary value* strategy is to choose $d^\star$, the decision which minimises this expected loss:

$$\boxed{d^\star = \arg\min_{d \in D} \bar{L}(d)}$$

Expected Monetary Utility decision rule:

Replace L by U(L)

Example (Insurance)

- You must decide whether to pay c to insure your possessions of value v against theft for the next year:

$$d = \{\text{Buy Insurance}, \text{Don't Buy Insurance}\}$$

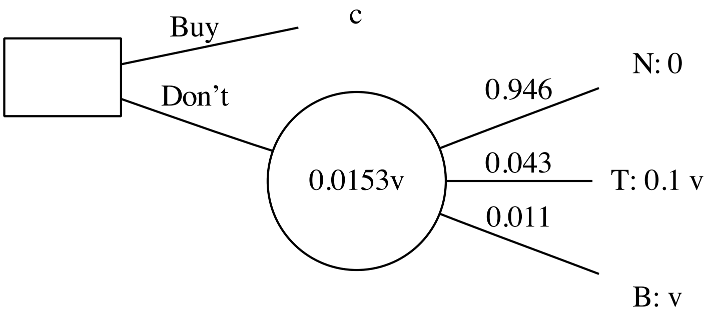
- Three events are considered possible over that period:

$$x_1 = \{\text{No thefts.}\} \qquad x_2 = \{\text{Small theft, loss } 0.1v\}$$
$$x_3 = \{\text{Serious burglary, loss } v\}$$

- Our loss function may be tabulated:

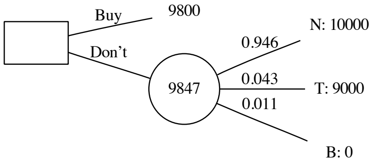
$L(d,x)$	x_1	x_2	x_3
Buy	c	c	c
Don't Buy	0	$0.1v$	v

In more complex examples, we should label the random events (say N for no robbery, T for small theft and B for burglary…

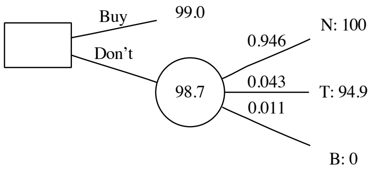


Example (The Utility of Insurance)

EMV:



EMU:



- Consider the insurance example.
- The first figure shows the EMV position: the insurer would prefer you to insure; you'd prefer not to.
- The second shows the EMU position with

$$U(x) = \sqrt{x}$$

  You prefer to insure.
- EMV makes sense for the insurer; EMU for you.

Making Decisions

We've covered the making of decisions:

1. Determine possible chance events and elicit probabilities.
2. Enumerate the possible actions.
3. Determine preferences via utility.
4. Choose actions to maximise expected utility.
5. Return to elicitation if necessary.

Now, we move on to games…