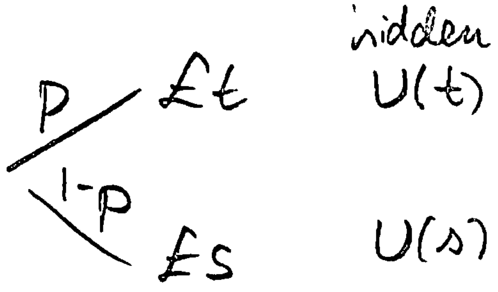
| Shape of CME fct & of utility, relationship, ex | W4-L3

Review CME: (subjective)

$b(p)(s,t)$: fix

$p \in [0,1]$ given



CME $m(p)$ is the maximal amount of money the person is prepared to forfeit to bet $b(p)(s,t)$.

$m(p)$ monotone (modest assumption)

Assume strictly monotone to make things easy. Then inverse exists.

Utility is the inverse of m : $U(y) = m^{-1}(y)$

Often used in mathematical models:

$$m(p) = p^{\alpha}, \quad \alpha > 0, \text{ so } U(y) = y^{1/\alpha}$$

In previous example, approx., $m(p) = p^2 \cdot c$, where c is some constant

Interpretation of CME: Compare $m(p)$ with the expected value of the bet $E[b(p,s,t)] = pt + (1-p)s$.

$m(p) < E[b(p,s,t)]$: prefers smaller but certain amount to expected but uncertain equivalent, i.e. willing to pay to remove risk
"risk averse"

$m(p) > E[b(p,s,t)]$: assigns amount higher than its expected value to the bet, i.e. pays to gamble
"risk seeking"

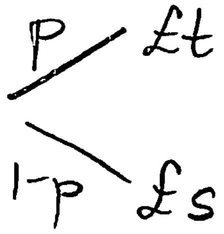
$m(p) = E[b(p,s,t)]$: "risk neutral"

W4-L3

The shape of utility :

fix s, t

bet $b(p)$:



Value of £t and £s is subjective, which can be expressed as $U(t)$ and $U(s)$

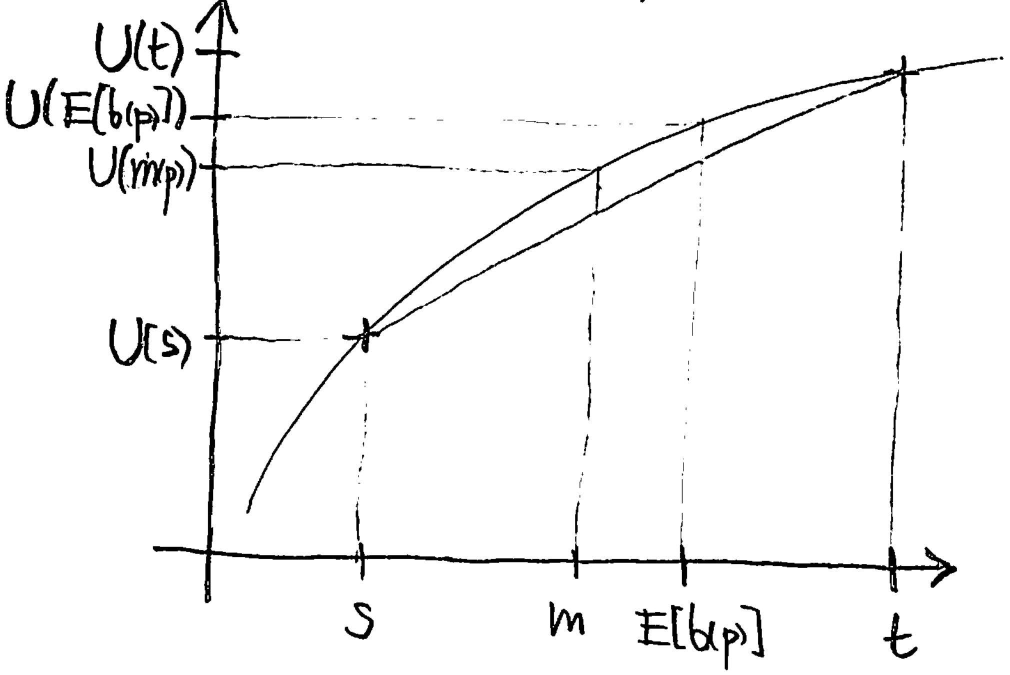
$E[b(p)] = p \cdot t + (1-p) \cdot s$

The utility U depends on the person, the time, the situation, the mood etc.

Before, we defined U through elicitation as inverse of m. Now, given U find m with $U(m(p)) = E[U(b(p))]$.

$E[U(b(p))] = p \cdot U(t) + (1-p)\, U(s)$ (linear interpol on y-axis)

(Expectation of bet with payoffs $U(t)$ and $U(s)$.)



$m - E[b(p)] =$ "risk premium"

$\forall p$ $m(p) < E[b(p)]$ means U is concave "risk averse"

$\forall p$ $m(p) = E[b(p)]$ means U is linear "risk neutral"

$\forall p$ $m(p) > E[b(p)]$ means U is convex "risk seeking"

W4-L3

Example : Fire insurance

value of house 100 (in £100 units)

value of house after fire 25

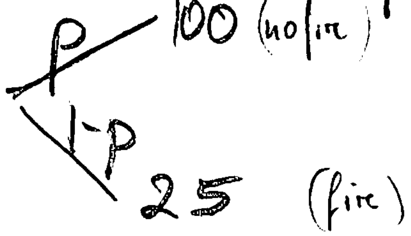
$p = 0.8$ for no fire

$q = 0.2$

$p = 1 - q$

(d₂) b : no insurance

$b(0.8, 25, 100)$



$$E[b] = 0.2 \cdot 25 + 0.8 \cdot 100 = 85 = EHV(d_2)$$

(d₁) Buy insurance (pay premium)

→ How much are you willing to pay for insurance?

(Owner's perspective) Assume $U(x) = \sqrt{x}$

Find m such that $U(m) = E[U(b)]$

risk averse (for owner, amount large)

$$E[U(b)] = 0.2 \cdot U(25) + 0.8 \cdot U(100)$$
$$= 0.2 \sqrt{25} + 0.8 \sqrt{100} = 0.2 \cdot 5 + 0.8 \cdot 10 = \underline{\underline{9}}$$

$$U(m) \overset{!}{=} 9 \Rightarrow m = 81$$

Hence, owner is willing to pay up to $100 - 81 = 19$ for insurance

$(19 = 15 + 4)$

risk premium $= m - E[b] = 81 - 85 = -4$

insurance premium $= -$ risk premium $= 4$

→ How much would insurer charge? $U(x) = x$

risk neutral (for insurers, amount small)

$E[\text{loss}] = 0.2 \cdot \text{damage} = 0.2 \cdot 75 = \underline{\underline{15}}$

Wants at least 15.

⇒ Interval for deal : [15, 19]

W4 - L3

About interpretation / terminology:

Owner's perspective: Risk avoiding.
Owner's wealth is not really 100, he only owns that house including the risk for fire. So, the owner's wealth is just E[6] = 85. Hence, the owner should already be willing to pay 15.
In addition, for the sake of removing uncertainty, the owner is willing to pay even more than 15 for insurance, in fact, the insurance premium of (up to) 4.

Insurer's perspective: Risk neutral.
Needs to ask for at least 15 to cover potential costs.
Due to owner's willingness to pay more, there is room for a deal. Any value between 15 and 19 should be an acceptable price for both of them.

Comment :

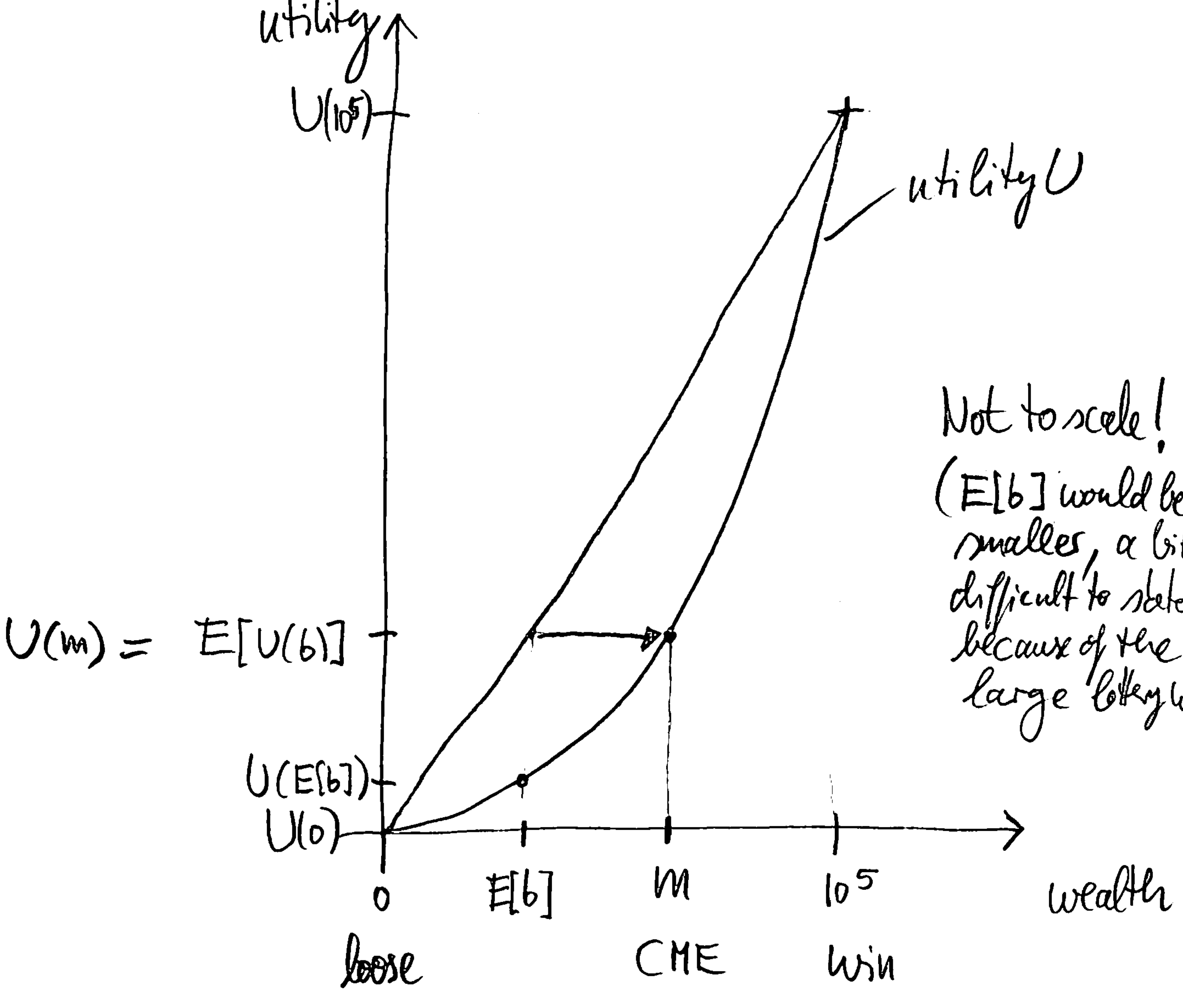
Example: Lottery $b(p,M) \begin{cases} \xrightarrow{p} £M \\ \xrightarrow{1-p} £0 \end{cases}$

Very small proba. p to win very large amount £M (for a small ticket price). For example, $M = 10^5$ and $p = 10^{-8}$. Bet $b(10^{-8}, 10^5) = b$

$$E[b] = p \cdot M = 10^{-8} \cdot 10^5 = 10^{-3}$$

$$E[U(b)] = p\,U(M) + (1-p) \cdot U(0)$$

Assumptions: $U(x) = x^2$, initial wealth 0
(risk seeking) (simplification)



Not to scale!
(E[b] would be smaller, a bit difficult to sketch because of the large lottery win)

W4—L3

$E[U(b)] = 10^{-8} \cdot (10^5)^2 + (1-10^{-8}) \cdot 0^2 = 10^2$

Find m such that $U(m) = E[U(b)]$

That means $m^2 = 10^2$, hence $m = 10$

A person with this utility (risk seeking) is willing to pay up to £10 to play this lottery.

$$\begin{aligned} \text{Risk premium} &= CME - EMV(b) \\ &= m - E[b] \\ &= 10 - 10^{-3} \\ &= 10 - 0.001 = 9.999 \end{aligned}$$

This person's risk premium is £9.999.

(In contrast, a person with risk neutral attitude $U(x) = x$ would only pay $E[b]$. Hence, his risk premium would be 0.)