Zero sum R, $S = -R$

		δ_1	δ_2	δ_3	δ_4	δ_5
	d_1	4	5	6	4	4
5	d_2	4	2	3	4	4
4	d_3	2	4	5	5	5
				3	2	1 redundant

2 $\delta_1 \leq \delta_4$, $S = -R$, so δ_1 better

3 $\delta_2 \leq \delta_3$, $S = -R$, so δ_2 better

4 $d_1 > d_3$

5 $d_1 \geq d_2$

Remaining:		δ_1	δ_2
	d_1	4	5

Player I has no choice
Player II plays δ_1

Strategy (d_1, δ_1) rewards: $(4, -4)$

Ex Pick-a-Hand game

Player II's moves:
1 coin in his left hand (L)
2 coins in his right hand (R).

Player II hides coin(s)
Player I chooses

$$A = \begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} L \\ R \end{array} & \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \end{array}$$

- no dominant moves
- not separable (see below)

Reward (for I)
For II : $-A$

- Player II could min loss by putting 1 coin in left hand (L). Max loss = 1.
  Reasonable, but player I could guess, so Incentive to play R?
  All depends on information available
  Apriori, can only guarantee max loss of 1

- Player I may want to max gain by using R, but if Player II guesses that will get 0.
  A priori, can only guarantee won't lose anything.

Want strategies that do not depend on available information!

Key idea : introduce probability distribution for moves "mix strategy

Player I $\quad P(L) = 1-p, \quad P(R) = p$

Expected rewards? Player II pure strategies (for now)

Player II plays R (pure)

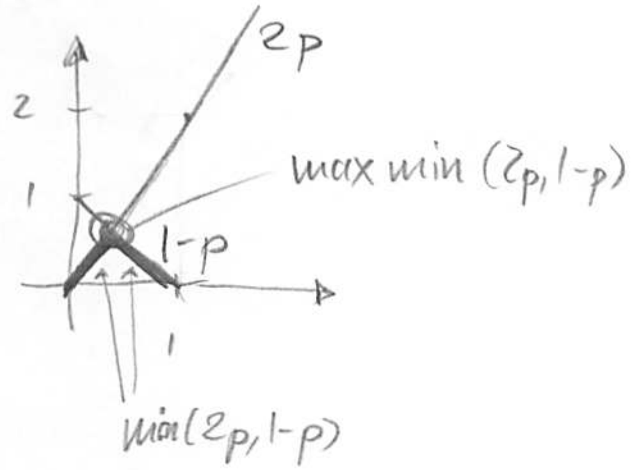
$\Rightarrow$ Expected gain for Player I $= (1-p)\cdot 0 + p \cdot 2 = 2p$

Player II plays L (pure)

$\Rightarrow$ Expected gain for Player II $= (1-p)\cdot 1 + p\cdot 0 = 1-p$

key idea | If II knew p, he would play $\min(2p, 1-p)$.

I knows this and therefore chooses p that maximises $\min(2p, 1-p)$



$$2p = 1-p$$
$$\Leftrightarrow \quad p = 1/3$$

expected loss for II

$= \min(2\cdot\frac{1}{3}, 1-\frac{1}{3})$

$= 2/3$

Hence, I using mixed strategy with $p = \frac{1}{3}$ ensures expected payoff $\underline{\underline{2/3}}$ (compare to 0)

Same argument for II playing mixed with $P(L) = 1-q$, $P(R) = q$

Player I pure R:

$\Rightarrow$ expected gain $= (1-q)\cdot 0 + q\cdot 2 = 2q$

Player I pure L:

$\Rightarrow$ expected gain $= (1-q)\cdot 1 + q\cdot 0 = 1-q$

$$\max_{q} \min(2q, 1-q)$$

$\Rightarrow q = \frac{1}{3}$, expected gain for I $= \underline{\underline{\frac{2}{3}}}$

Value of the game is $2/3$.

(Also fair price to play is $2/3$)