Methodology: Sequence of coin tosses

Sequence #1

T HHHHT T T T HHHHT HHHHHHHHHT T THHT T HHHHHT T T T T T HHT HHT HHH T
T T HT T HHHHT HT T T HT T T HHT T T T HHHHHHT T T HHT T HHHT HHHHHT T T T
T HT T T HHT T HT T HHT T T HHT T T HH THHT HHT T T T T HHT HHHHHHT HT HT T
HT HT T HHHT T HHT HT HHHHHHHHT THT T HHHT HHT T HT T T T T T HHHT HHH

Sequence #2

T HT HT T T HT T T T T HT HT T T HT T HHHT HHT HT HT HT T T T HHT T HHT T HHH T
HHHT T HHHT T T HHHT HHHHT T T HT HT HHHHT HT T T HHHT HHT HT T T HHT H
HHT HHHHT T HT HHT HHHT T T HT HHHT HHT T T HHHT T T T HHHT HT HHHHT H
T,T HHT T T T HT HT HT T HT HHT T HT T THT T T T HHHHT HT HHHT T HHHHHT HH

How can you tell? Which features can you look at?

- number of heads, number of tails
- number of alternations
- numbers and lengths of runs

Methodology:
Expected number of runs in a sequence of coin tosses

$X_i \in \{0, 1\}$ $(i = 1, 2, \ldots, N)$

independent identically distributed

$P(X_i = 0) = P(X_i = 1) = \frac{1}{2}$

$Z_r =$ number of run of length r

Calculation (see blackboard) shows:

$$E[Z_r] = \left(\frac{1}{2}\right)^{r-1} + (N - r - 1)\left(\frac{1}{2}\right)^{r+1}$$

Methodology:
Another Aspect: Distribution of the longest head run

$X_i (i = 1, \ldots, n)$ 0-1 sequence of length n,
independent and identically distributed with $P(X_i = 1) = 0.5$

$R_n =$ length of the longest run of heads in n tosses

CDF $F_n(x) = P(R_n \leq x)$

$A_n =$ number of sequences of length n with longest run at most x

$F_n = 2^{-n} A_n$

Strategy:

- Partition the set of these sequences
- derive a recursive formula

Methodology: Distribution of the longest head run

Strategy:

- Partition the set of these sequences
- derive a recursive formula

Key idea:

To see how this works, consider the case in which the longest head run consists of three heads or fewer. If $n \leq 3$ then clearly $A_n(3) = 2^n$ since any outcome is a favorable one. For $n > 3$, each favorable sequence begins with either T, HT, HHT, or HHHT and is followed by a string having no more than three consecutive heads. Thus

$$A_n(3) = A_{n-1}(3) + A_{n-2}(3) + A_{n-3}(3) + A_{n-4}(3) \quad \text{for } n > 3.$$

Methodology: Distribution of the longest head run

To see how this works, consider the case in which the longest head run consists of three heads or fewer. If $n \leq 3$ then clearly $A_n(3) = 2^n$ since any outcome is a favorable one. For $n > 3$, each favorable sequence begins with either T, HT, HHT, or HHHT and is followed by a string having no more than three consecutive heads. Thus

$$A_n(3) = A_{n-1}(3) + A_{n-2}(3) + A_{n-3}(3) + A_{n-4}(3) \quad \text{for } n > 3.$$

Using the recursion, the values of $A_n(3)$ can easily be computed:

n	0	1	2	3	4	5	6	7	8	$\cdots$
$A_n(3)$	1	2	4	8	15	29	56	108	208	$\cdots$

$$A_0(3) = 2^0 = 1, \quad A_1(3) = 2^1 = 2, \quad A_2(3) = 2^2 = 4, \quad A_3(3) = 2^3 = 8,$$
$$A_4(3) = A_3(3) + A_2(3) + A_1(3) + A_0(3) = 1 + 2 + 4 + 8 = 15$$

Using the recursion, the values of $A_n(3)$ can easily be computed:

n	0	1	2	3	4	5	6	7	8	$\cdots$
$A_n(3)$	1	2	4	8	15	29	56	108	208	$\cdots$

Thus for, say, $n = 8$ tosses of a fair coin, the probability is $208/2^8 = 0.8125$ that the longest head run has length no greater than 3.

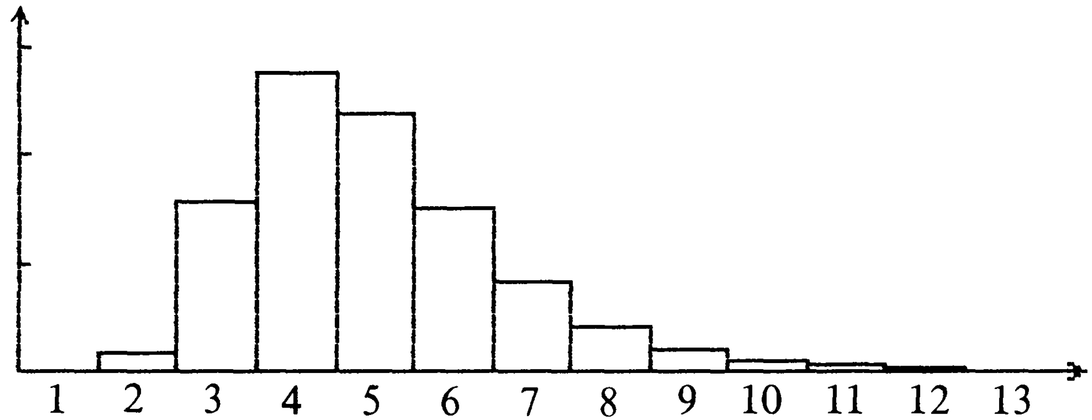
General case: head run length at most x

$$A_n(x) = \begin{cases} \sum_{j=0}^{x} A_{n-1-j}(x) & \text{for } n > x; \\ 2^n & \text{for } n \le x. \end{cases} \qquad (1)$$

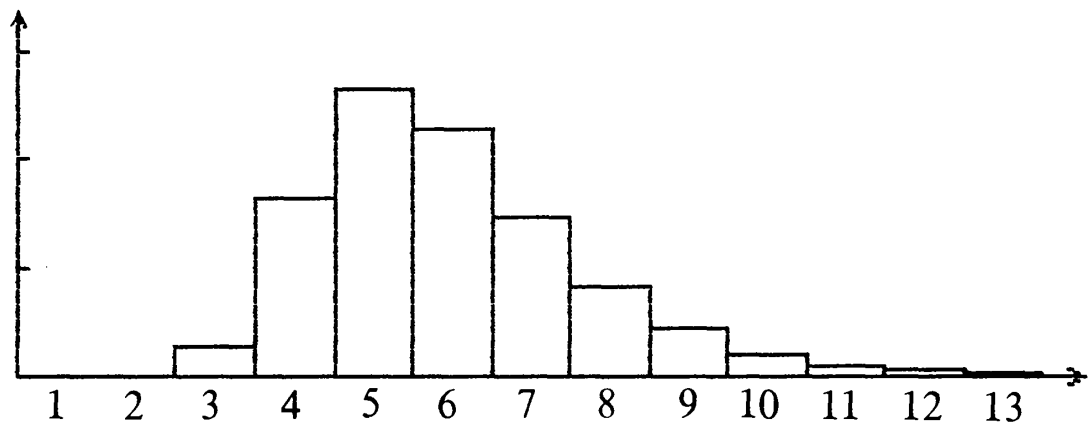
Note that for $n = 1, 2, 3, \ldots,$ the number $A_n(1)$ of sequences of length n that contain no two consecutive heads is the $(n+2)$nd Fibonacci number.

Distribution of longest run lengths (of heads) for larger n

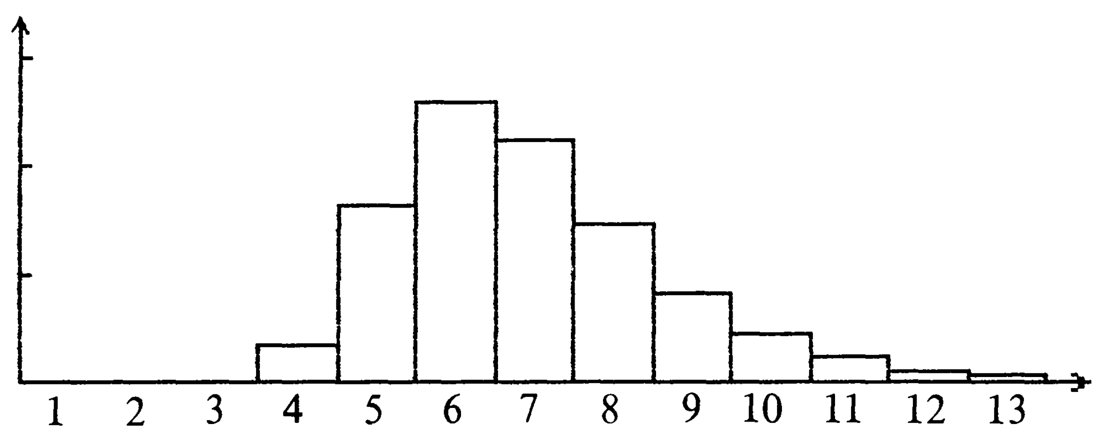
n=50



n=100



n=200



Human perception: Coin tossing

High density in heads or tails in repeated chain tossing. If a coin toss is repeated several times and the majority of the results consists of "heads", the assumption of local representativeness will cause the observer to believe the coin is biased toward "heads".

Try yourself. If you don't have any change to toss, use the online coin flip simulator at:

https://www.random.org/coins/

Part III: Normative theory versus descriptive theory

	Normative approaches	**Descriptive approaches**
Probabilistic judgement Uncertainty, risk	Subjective probability	Perceived probabilities and observed processing (axioms may not hold)
Decision theory Preferences, choices	Expected utility maximisation	Observed choice behaviour
Game theory Strategies, moves	Reward maximisation	Observed moves and motives

Human perception of probability: Gambler's fallacy

Gambler's fallacy:

The confidence that after a long run of one kind of outcome the other kind of outcomes are more likely.

In random sequences that are actually composed of independent events this is *wrong* (e.g. coin tossing, many games).

Explanation for this wrong belief:

Erroneous conceptualisation of the law of large numbers, the belief that *small samples should be representative* for the distribution (which is generally not true).

Methodology: Local representativeness heuristics

Local representativeness assumption means that there was a ***law of small numbers***, whereby small samples are perceived to represent their population to the same extent as large samples (Tversky & Kahneman 1971).

Specifically, this would mean:

- A small sample which appears randomly distributed reinforces the belief that the population is randomly distributed.
- A small sample with a skewed distribution would weaken this belief.

For independent random sequences, this is wrong, because they have no memory.

Historical event: Monte Carlo

Monte Carlo Casino, August 18, 1913:

- Game of roulette, the ball fell in black 26 times in a row
- Probability for this is 1/67,108,863
- Gamblers lost millions for francs betting *against* black believing the *streak was causing an imbalance in the randomness of the wheel*
- Assumed that it had to be followed by a *streak of red*

Examples and non-examples of gambler's fallacy

- Joseph Jagger at Monte Carlo
- Black Jack
- Childbirth
- Evolutionary explanation
- Reverse gambler's fallacy

Practical applications: Detection of gambler's fallacy

Decision-Making under the Gambler's Fallacy: Evidence from Asylum Judges, Loan Officers, and Baseball Umpires *(NBER Working Paper No. 22026), D Chen, TJ Moskowitz, and K Shue*

Individuals have a slight bias against deciding the same way in successive cases in a number of areas:

- **Asylum judges in the US:** Odds that a judge rejects an asylum seeker are 3.3 percentage points higher if the judge has approved the previous case, all else being equal.
- **Loan officers in India:** Officers were eight percentage points less likely to approve the loan currently under review if they had approved the previous loan.
- **Baseball:** Umpires were 1.5 percentage points less likely to call a strike if the previous pitch was a called strike.

http://www.nber.org/digest/jun16/w22026.html

Example: Hot hand

Belief in hot hand:

The confidence that after a long run of one kind of outcome it's likely to obtain more of these.

Has occurred in descriptions of sports (basketball) and gambling (e.g. roulette). In random sequences that are actually composed of independent events this is wrong.

Contradiction? Hot hand vs gamber's fallacy

Hot hand belief can be seen as opposite fallacy of the gambler's fallacy.

Leading potentially to opposite conclusions.

There are many ways in which you can get something wrong, so that is not a contraction.

Whether/which people apply any of these depends on context and personality etc.

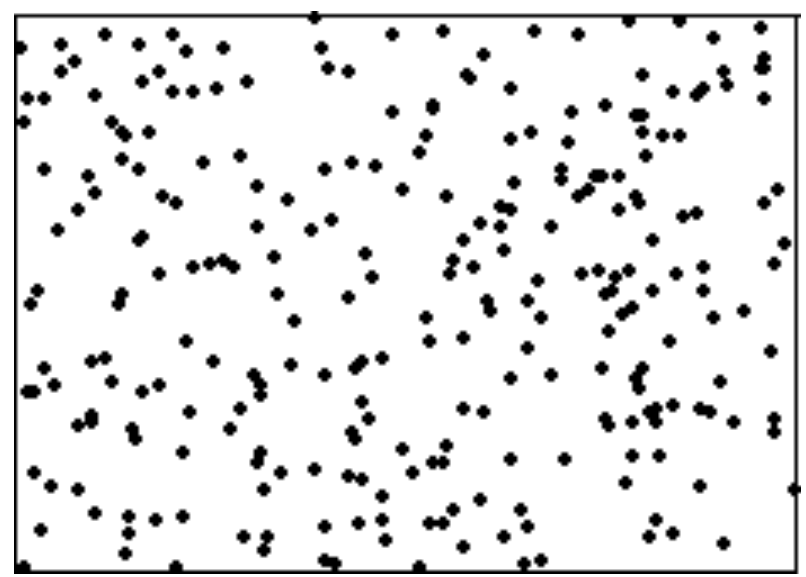
Look at more fallacies…

Concept: Clustering illusion

The tendency to erroneously consider the inevitable "streaks" or "clusters" arising in small samples from random distributions to be statistically significant.

Explanation: Underestimation of the amount of variability likely to appear in a small sample of random or semi-random data.

Examples:
Hot hand in basketball,
Seeing structure in
Poisson point patterns



Gilovich, Thomas; Robert Vallone & Amos Tversky (1985). "The hot hand in basketball: On the misperception of random sequences". *Cognitive Psychology* **17**: 295–314.

Example: Perception of randomness

From a study with over 800 Warwick UG students across subjects (2012)

"You are given a non-transparent box containing a large number of identical marbles, half are black ***(B)*** *and half are white* ***(W)****. Take out a marble and note its colour. Put it back and give the box a little shake. Take out another marble and note its colour. Do this repeatedly.*

*Write down a colour sequence (****B*** *or* ***W****) of* ***10*** *marbles you might have observed."*

Answer version 1: ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

Answer version 2: Use attached small notepad

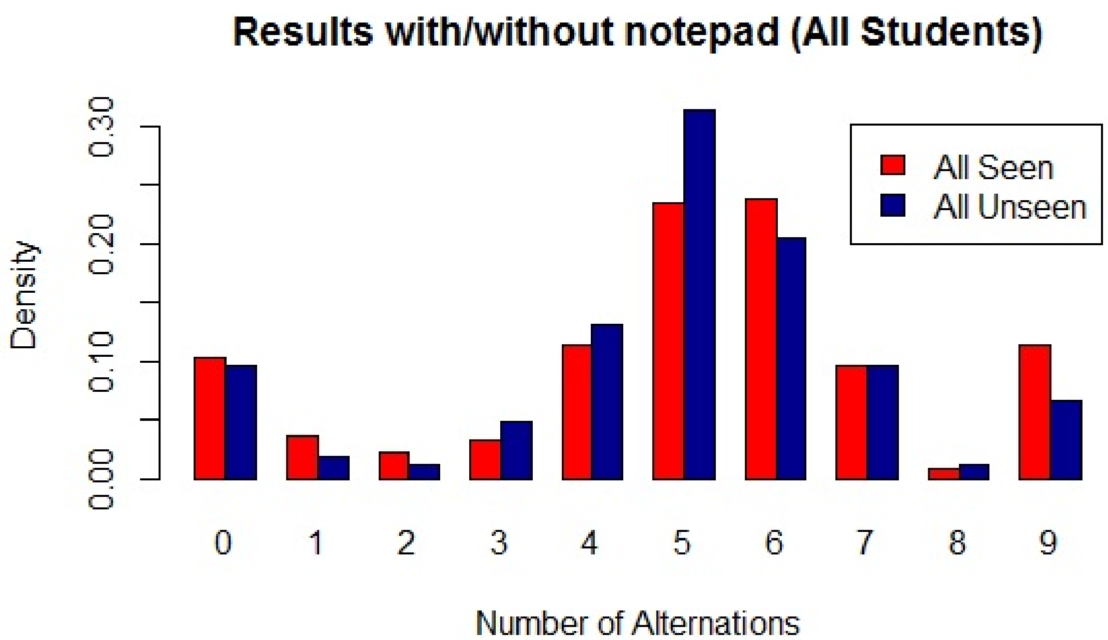
What is the number of alternations? What do they put first?

Source: MMORSE projects Mohan, Streather, Yip, supervised by Brettschneider 2012-2014

Example: Perception of randomness

What is the number of alternations?

Empirical distribution in study [N > 800]:



Theoretical answer:

Expected value of alternations in 10 independent fair Bernoulli trials is 4.5.
(Calculate that using indicators!)

Unimodal, some extreme values (0,1), mean about 5.
Difference between seen/unseen mainly in the centre, not significant.

Discussion: Overly alternating is consistent with previous findings.
Small difference between seen/unseen, though our sequences are shorter.

Part III: Normative theory versus descriptive theory

	Normative approaches	**Descriptive approaches**
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Game theory Strategies, moves	Reward maximisation	Observed moves and motives

Example: Perception of random sequences

What did they put first?

About 90% put B first.

Possible reason: "Black and White" is a standing expression and was used in the description of the experiment.

Explanation: ***Anchoring bias***

Information received at first dominates thinking.

Empirical studies from the literature: Anchoring bias

Group A

Is the Mississippi River more or less than 70 miles long? How long is it?

Group B

Is the Mississippi River more or less than 2000 miles long? How long is it?

Karen Jacowitz and Daniel Kahneman, *Measures of Anchoring in Estimation Tasks,* Pers Soc Psychol Bull November 1995 vol. 21 no. 11 1161-1166

Empirical studies from the literature: Anchoring bias

Group A

Is the Mississippi River more or less than 70 miles long? How long is it?

Mean answer: 300

Group B

Is the Mississippi River more or less than 2000 miles long? How long is it?

Mean answer: 1500

Karen Jacowitz and Daniel Kahneman, *Measures of Anchoring in Estimation Tasks,* Pers Soc Psychol Bull November 1995 vol. 21 no. 11 1161-1166

Anchoring bias: Priming influences answers.

	A given	B given	A estim.	B estim.
Mississippi (mi)	70	2000	300	1500
Everest (ft)	2000	45500	8000	42550
Meat (lbs/year)	50	1000	100	500
SF to NY (mi)	1500	6000	2600	4000
Tallest Redwood (ft)	65	550	100	400
UN Members	14	127	26	100
Female Berkeley Profs	25	130	50	95
Chicago Population (mil.)	0.2	5.0	0.6	5.05
Telephone Invented	1850	1920	1870	1900
US Babies Born (per day)	100	50000	1000	40000

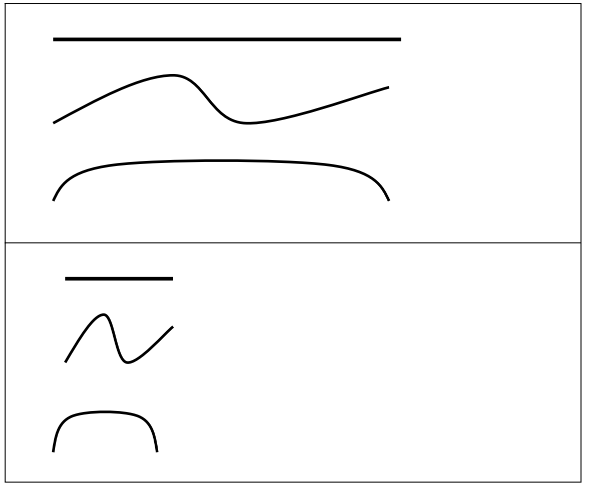
Anchoring bias with unrelated information

Participants:

Seventy-one Stanford University undergraduates participated to fulfill part of a course requirement. The experiment consisted of two questionnaires in a packet of approximately 20 unrelated one-page questionnaires. Packets were randomly ordered and then distributed in class, and participants were given a week to complete the entire packet.

Design, stimuli, and procedure:

Participants were presented with a set of three horizontal lines and were asked to replicate the lines as best as they could without using a ruler. The three lines were a straight line, a wavy line, and an inverted u. Participants in the short-anchor condition replicated 1-in. long lines, while participants in the long-anchor condition replicated 3.5-in. lines.



Anchors aweigh: A demonstration of cross-modality anchoring and magnitude priming, Daniel M. Oppenheimer , Robyn A. LeBoeuf , Noel T. Brewer, Cognition (2007)

On the next page, participants were presented with an ostensibly unrelated judgment task in which they were asked to estimate various quantities. The target quantity, the length of the Mississippi River, was always asked about first (only a simple question about how long it is, without the phrase "…is about … long" from the previous experiment). Several decoy questions followed to prevent participants from guessing the hypothesis.

Six participants who gave estimates falling more than 3.5 standard deviations from the mean were excluded as outliers.

Results and discussion:

- participants with **short** lines: average estimate of 72 miles
- participants with long lines: average estimate of 1224 miles

This difference was statistically significant.

Participants who had been anchored by copying long lines reliably estimated the river to be longer than those anchored with short lines. In other words, not only can anchoring occur when no explicit comparison is made between an anchor and a target (cf. Wilson et al., 1996), it can even arise across modalities.

Variation of this experiment

Participants:

Ninety-eight individuals recruited from arbitrarily chosen intersections in San Francisco participated in exchange for a candy bar.

Task:

Estimate the average temperature in Honolulu in July in degrees Fahrenheit.

Results:

- participants with long lines: average estimate of 87.5 degrees
- participants with short lines: average estimate of 84.0 degrees

Results were statistically significant.

Despite being from incompatible dimensions (length, temperature).

Anchoring bias in calculations

Group A

Within 5 seconds, estimate the product: $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Group B

Within 5 seconds, estimate the product: $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$

First sequence median guess: 2250.

Second sequence median guess: 512.

Correct answer: 40,320.

Example: Framing effect

Key example from seminal paper on the framing effect:
The Framing of Decisions and the Psychology of Choice. Amos Tversky; Daniel Kahneman.
Science, New Series, Vol. 211, No. 4481.

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is **expected to kill 600 people**. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

Which of the two programs would you favour?

So then the researchers asked the following version of the same question:

If Program C is adopted 400 people will die.

If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Which of the two programs would you favour?

Problem **1** *[N* = 1521*]*:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved. [72 percent]
If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. [28 percent]
Which of the two programs would you favour?

Problem 2 [N = 1551]:

...
If Program C is adopted 400 people will die. [22 percent]
If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. [78 percent]
Which of the two programs would you favour?

Amos Tversky; Daniel Kahneman, *The Framing of Decisions and the Psychology of Choice,* Science, New Series, Vol. 211, No. 4481. (Jan. 30, 1981), pp. 453-458.
http://links.jstor.org/sici?sici=0036-8075%2819810130%293%3A211%3A4481%3C453%3ATFODAT%3E2.0.CO%3B2-3
Available also eg. at psych.hanover.edu/classes/cognition/papers/tversky81.pdf

If Program A is adopted, 200 people will be saved. [72 percent]
If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. [28 percent]

If Program C is adopted 400 people will die. [22 percent]
If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. [78 percent]

Interpretation: ***Framing effect, risk aversion***

People behave risk-averse in the saving-lives formulation. They want to have certainty about saving lives.

In contrast, they behave risk-seeking in the loosing-lives formulation.
The sure loss of 400 people (D) is not acceptable to them.

However, according to EUT it should all be the same!

Survey in Week 1 of this module in 2015

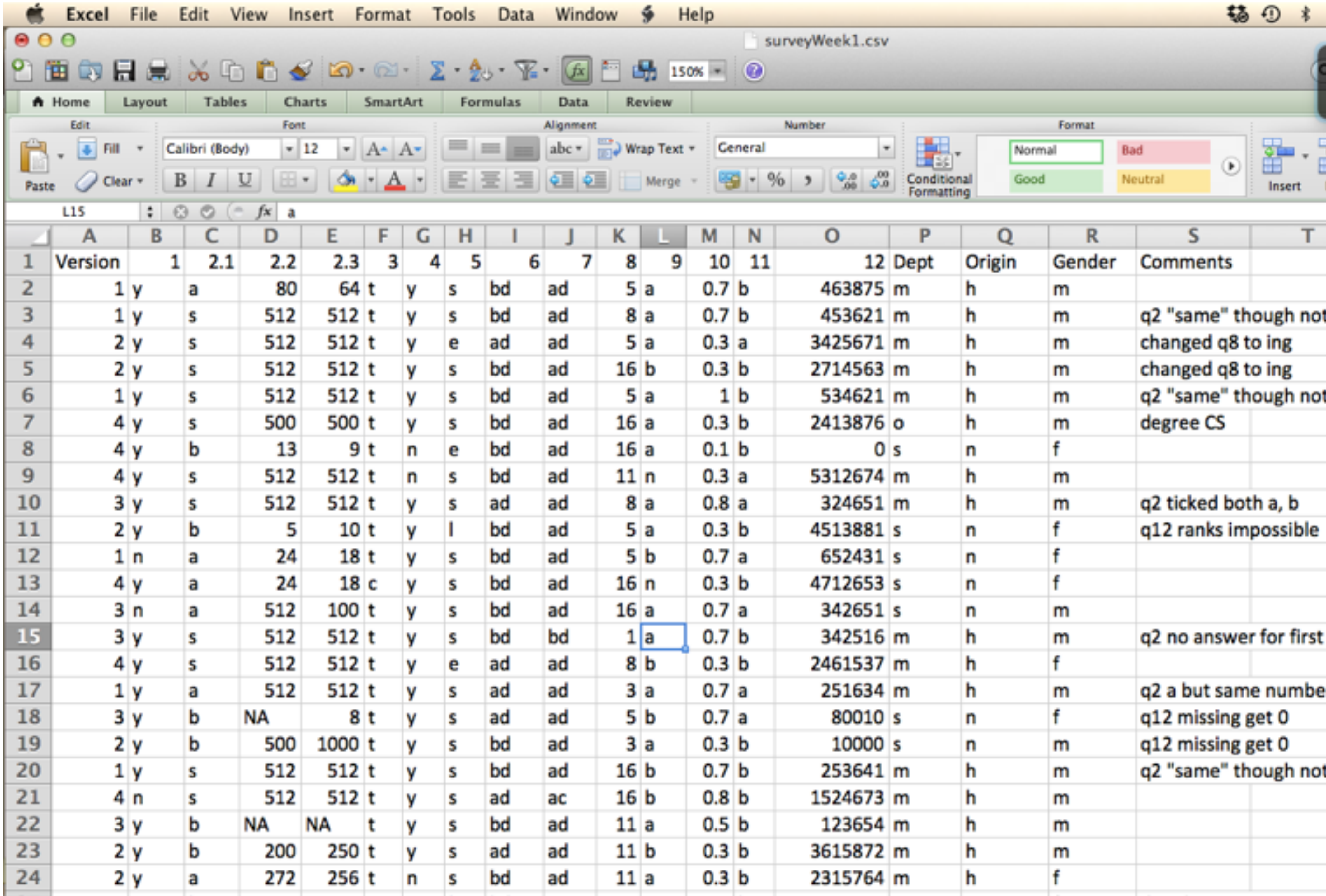
ST222 **Lecturer: Dr Julia Brettschneider**

This is a collection of questions about decision making in a variety of situations. This is not a test. The intention is to give you some concrete experience with making decisions, so the methodology we study will become more meaningful.

Please answer the questions quietly on your own and return this sheet in about 20 min. The questions will later be posted on the module website, so you can discuss answers with your class mates and friends.

ST222@Warwick: 12 questions, some in two versions

ST222@Warwick: The data file



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	Version	1	2.1	2.2	2.3	3	4	5	6	7	8	9	10	11	12	Dept	Origin	Gender	Comments
2	1	y	a	80	64	t	y	s	bd	ad	5	a	0.7	b	463875	m	h	m	
3	1	y	s	512	512	t	y	s	bd	ad	8	a	0.7	b	453621	m	h	m	q2 "same" though not
4	2	y	s	512	512	t	y	e	ad	ad	5	a	0.3	a	3425671	m	h	m	changed q8 to ing
5	2	y	s	512	512	t	y	s	bd	ad	16	b	0.3	b	2714563	m	h	m	changed q8 to ing
6	1	y	s	512	512	t	y	s	bd	ad	5	a	1	b	534621	m	h	m	q2 "same" though not
7	4	y	s	500	500	t	y	s	bd	ad	16	a	0.3	b	2413876	o	h	m	degree CS
8	4	y	b	13	9	t	n	e	bd	ad	16	a	0.1	b	0	s	n	f	
9	4	y	s	512	512	t	n	s	bd	ad	11	n	0.3	a	5312674	m	h	m	
10	3	y	s	512	512	t	y	s	ad	ad	8	a	0.8	a	324651	m	h	m	q2 ticked both a, b
11	2	y	b	5	10	t	y	l	bd	ad	5	a	0.3	b	4513881	s	n	f	q12 ranks impossible
12	1	n	a	24	18	t	y	s	bd	ad	5	b	0.7	a	652431	s	n	f	
13	4	y	a	24	18	c	y	s	bd	ad	16	n	0.3	b	4712653	s	n	f	
14	3	n	a	512	100	t	y	s	bd	ad	16	a	0.7	a	342651	s	n	m	
15	3	y	s	512	512	t	y	s	bd	bd	1	a	0.7	b	342516	m	h	m	q2 no answer for first
16	4	y	s	512	512	t	y	e	ad	ad	8	b	0.3	b	2461537	m	h	f	
17	1	y	a	512	512	t	y	s	ad	ad	3	a	0.7	a	251634	m	h	m	q2 a but same numbe
18	3	y	b	NA	8	t	y	s	ad	ad	5	b	0.7	a	80010	s	n	f	q12 missing get 0
19	2	y	b	500	1000	t	y	s	bd	ad	3	a	0.3	b	10000	s	n	m	q12 missing get 0
20	1	y	s	512	512	t	y	s	bd	ad	16	b	0.7	b	253641	m	h	m	q2 "same" though not
21	4	n	s	512	512	t	y	s	ad	ac	16	b	0.8	b	1524673	m	h	m	
22	3	y	b	NA	NA	t	y	s	bd	ad	11	a	0.5	b	123654	m	h	m	
23	2	y	b	200	250	t	y	s	ad	ad	11	b	0.3	b	3615872	m	h	m	
24	2	y	a	272	256	t	n	s	bd	ad	11	a	0.3	b	2315764	m	h	f	

Question 5: Judging sample variation

Question from Kahneman & Tversky's 1970s program on probability judgement

Question 5 - type a (type b)

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50%, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys. Which hospital do you think recorded more such days?

The larger hospital   The smaller hospital   About the same (within 5% of each other)

Kahneman D & Tversky A, *Subjective probability: A judgement of representativeness.* Cognitive Psychology, 3 (1972), 430-454

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The larger hospital The smaller hospital About the same (within 5% of each other)

Correct answer: The smaller hospital.
Reason: Smaller samples are more variable. Hence they record more days with over 60% boys.

Kahneman D & Tversky A, *Subjective probability: A judgement of representativeness.* Cognitive Psychology, 3 (1972), 430-454

Question 5: Judging sample variation

Question 5 - type a (type b)

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The larger hospital   The smaller hospital   About the same (within 5% of each other)

Original study was on Stanford UG students without training in proba/stats: They answered mostly wrong

Are trained Warwick UG students better?

############################ Question 5
Q5: D[,8] (which hospital?)

```
> table(D[a,8])
 e  l  s
 2  2 47
```

Type a question

larger	smaller	equal	
3.9%	**92.2%**	3.9%	# though "equal" option was not available!

```
> table(D[b,8])
 e  l  s
 7  3 39
```

Type b question

larger	smaller	equal
6.1%	**79.6%**	14.3%

Question 8: Word frequencies

Question from Kahneman & Tversky's 1970s program on probability judgement

Question 8 - type a (n)

In four pages of a novel (about 2,000 words), how many words would you expect to find that have the form _ _ _ _ _ **n** _ ? Indicate your best estimate by circling one of the values below:

0 1-2 3-4 5-7 8-10 11-15 16+

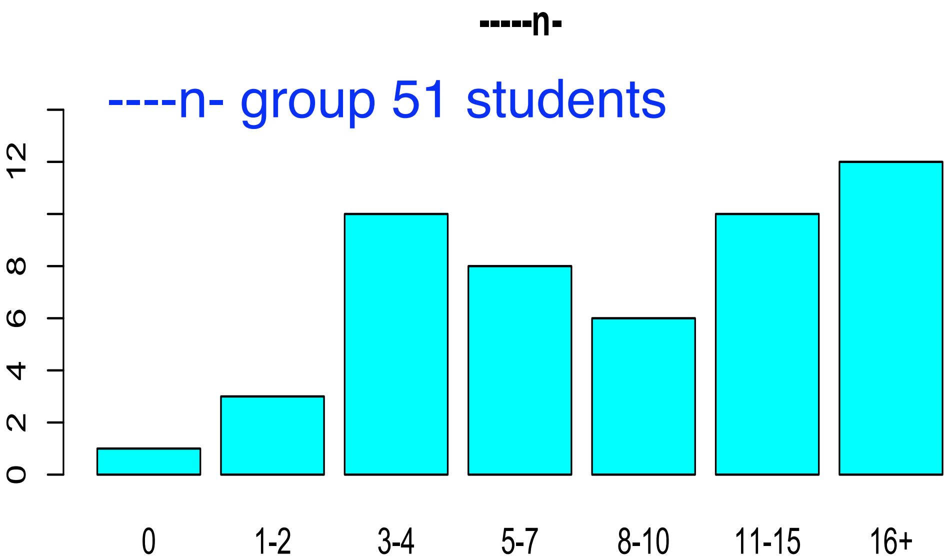
Question 8 - type b (ing)

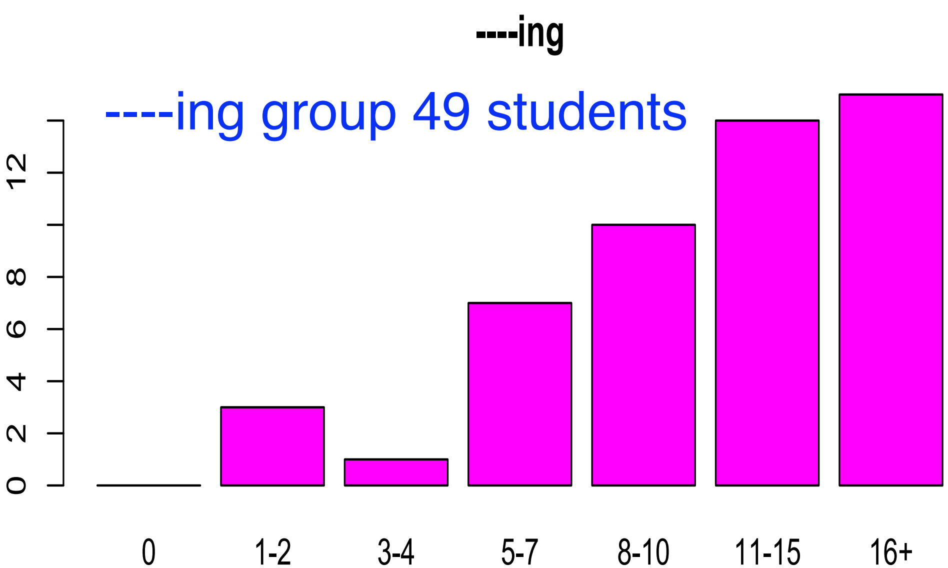
In four pages of a novel (about 2,000 words), how many words would you expect to find that have the form _ _ _ _ **i n g** (seven-letter words that end with "ing")? Indicate your best estimate by circling one of the values below:

0 1-2 3-4 5-7 8-10 11-15 16+

Kahneman D & Tversky A, *On the psychology of prediction.* Psychological Review 80, *237-51*.

ST222@Warwick:





The *less* restrictive condition creates fewer words!

Violates normative rules of probability:

For A subset of B,

$P(A) < P(B)$

Is this normal? Why?

Confirms result form the literature:

Judging frequency (question as above)

----n-: median 2.3 ---ing: median 6.4

Creating as many as possible words in 60 sec:

----n-: median 4.7 ---ing: median 13.4

Similar results obtained comparing word groups -----l- and -----ly

Latter classes produced more words despite being contained in former!

What are explanations for this incoherence?

Availability heuristics:

Increased efficiency of memory search offsets reduced extension of target class.

Example: Searching for "-ing" may lead to the words "timing", "resting", "drawing", "going", "talking" faster than searching for "-n-"

Example: Allais paradox

First experiment:

S1: 1M for sure
R1: 5M with 0.10, 1M with 0.89, 0M with 0.01

Second experiment:

S2: 1M with 0.11, 0M with 0.89
R2: 5M with 0.10, 0M with 0.90

What is better?

Allais, M. (1953), *Le Comportement de l'Homme Rationnel devant le Risque: Criticue des Postulats et Axiomes de l'Ecole Americaine,* Econometrica 21: 503 – 546.

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Allais conjecture:

S1 > R1: certain outcome
S2 < R2: huge difference in gain
(small difference in proba)

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Example: Allais paradox

First experiment:

S1: [1M, 1.00]
R1: [5M, 0.10], [1M, 0.89], [0M, 0.01]

Second experiment:

S2: [1M, 0.11], [0M, 0.89],
R2: [5M, 0.10], [0M, 0.90],

Allais conjecture about preferences:

S1 > R1: certain outcome
S2 < R2: huge difference in gain
(small difference in proba)

Mathematically equivalent to

S1': [1M, 0.89], [1M, 0.11]
R1': [1M, 0.89], [0M, 0.01], [5M, 0.10]

S2': [0M, 0.89], [1M, 0.11]
R2': [0M, 0.89], [0M, 0.01], [5M, 0.10]

If E[u(S1')] > E[u(R1')]
then E[u(S2')] > E[u(R2')]
(addends cancel out)

INCONSISTENT with what?

Example: Allais paradox

First experiment:

S1: [1M, 1.00]
R1: [5M, 0.10], [1M, 0.89], [0M, 0.01]

Second experiment:

S2: [1M, 0.11], [0M, 0.89],
R2: [5M, 0.10], [0M, 0.90],

Allais conjecture about preferences:

S1 > R1: certain outcome
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(small difference in proba)

Mathematically equivalent to

S1': [1M, 0.89], [1M, 0.11]
R1': [1M, 0.89], [0M, 0.01], [5M, 0.10]

S2': [0M, 0.89], [1M, 0.11]
R2': [0M, 0.89], [0M, 0.01], [5M, 0.10]

If E[u(S1')] > E[u(R1')]
then E[u(S2')] > E[u(R2')]
(addends cancel out)

INCONSISTENT with expected utility theory, independence axiom

Example: Allais paradox

First experiment:

S1: 1M for sure
R1: 5M with 0.10, 1M with 0.89, 0M with 0.01

Second experiment:

S2: 1M with 0.11, 0M with 0.89
R2: 5M with 0.10, 0M with 0.90

Allais conjecture:

S1 > R1: because certain outcome is preferred
S2 < R2: huge difference in gain
(small difference in proba)

Empirical evidence

confirms Allais conjecture

Numerous studies using hypothetical, monetary and health outcomes

Example: Allais paradox

First experiment:

S1: 1M for sure
R1: 5M with 0.10, 1M with 0.89, 0M with 0.01

Second experiment:

S2: 1M with 0.11, 0M with 0.89
R2: 5M with 0.10, 0M with 0.90

Allais explanation for incoherence: Preferences are *not independent.*
10% of getting 5M carries 1% risk of getting nothing (feeling disappointed), in contrast to sure gain of 1M (feeling of certainty, being in control).

See later: can't be saved with using utility on payoffs.

How do Warwick UG students answer this question?

Question 6: Allais paradox

Question 6

You are asked to choose between the following 2 gambles below. Circle your preference.

[S1] A. A 100% chance of receiving $1 million.

[R1] B. A 10% chance of receiving $5 million, an 89% chance of receiving $1 million, and a 1% chance of receiving nothing.

After you have made your choice, you are asked to choose between the following two gambles. Circle your preference.

[S2] C. An 11% chance of receiving $1 million, and an 89% chance of receiving nothing.

[R2] D. A 10% chance of receiving $5 million, and a 90% chance of receiving nothing.

ST222@Warwick (details next slide):
About half of this class behaved consistent with EUT preferring R1 and R2 over S1 and S2. That means, you value certainty about outcomes less then typical subjects in existing studies.

Question 6

You are asked to choose between the following 2 gambles below. Circle your preference.

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[S2] C. An 11% chance of receiving $1 million, and an 89% chance of receiving nothing.

[R2] D. A 10% chance of receiving $5 million, and a 90% chance of receiving nothing.

```
################ Question 6
# Q6: D[,9] (Allais)

table(D[,9])
ac  ad  bc  bd
 5  43   3  49     (out of 100 total)
```

ST222@Warwick:

S1>R1 & S2<R2 43%
Like Allais predicted

S1<R1 & S2<R2 49%
Consistent(!) with EUT

Question 7: Ellsberg paradox

Suppose you have an urn containing 30 red balls and 60 other balls that are either black or yellow. (You don't know how many black or how many yellow balls there are, but that the total number of black balls plus the total number of yellow equals 60.) The balls are well mixed so that each individual ball is as likely to be drawn as any other. You are given a choice between the two gambles below. Circle the one you prefer.

A. You receive £100 if you draw a red ball.

B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state, you are given the choice between the two gambles below. Circle the one you prefer.

C. You receive £100 if you draw a ball that is not black.

D. You receive £100 if you draw a ball that is not red.

Good question: What is *original state*?! Question text from literature... They do not mean the exact physical arrangement of the balls, but refer to the *state* in which *each individual ball is as likely to be drawn as any other.*

30 red balls, 60 other balls that are either black or yellow.

A. You receive £100 if you draw a red ball.
B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state.

C. You receive £100 if you draw a ball that is not black.
D. You receive £100 if you draw a ball that is not red.

How to approach this?

Prefer A>B since proportion of red balls is known.

Alternatively, make (implicit) assumptions about proportions black/yellow, e.g. 30/30.

Ellsberg: Assume you settle on A>B. Then you should choose D>C for the same reason (preference for known probability).

Empirical studies show that a strong majority of people do indeed have these preferences (A>B, D>C).

30 red balls, 60 other balls that are either black or yellow.

A. You receive £100 if you draw a red ball.
B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state.

C. You receive £100 if you draw a ball that is not black.
D. You receive £100 if you draw a ball that is not red.

What does Expected utility theory (EUT) say?

Let M=u(£100), 0=u(£0).

E[u(A)] = 30/90 * M E[u(B)] = Black/90 * M

E[u(C)] = (30+60-Black)/90 * M E[u(D)] = 60/90 * M

E[u(A)] - E[u(B)] = (30-Black)/90 * M

E[u(C)] - E[u(D)] = (30+60-Black-60)/90 * M = (30-Black)/90 * M

EUT says A>B is equivalent to C>D.

See also exercise sheet 4

30 red balls, 60 other balls that are either black or yellow.

A. You receive £100 if you draw a red ball.
B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state.

C. You receive £100 if you draw a ball that is not black.
D. You receive £100 if you draw a ball that is not red.

What do ST222 students at Warwick say:

############ Question 7

ac	ad	bc	bd
12%	81%	2%	5%
8%	84%	3%	4%

ad bc contradict EUT
ac bd compatible with EUT

100 of ST222'14@Warwick
76 of ST222'15@Warwick

ST222@Warwick: Huge majority behaved as predicted by Ellsberg, i.e. they are **not** following expected utility theory (EUT).

Compare: Allais paradox and Ellsberg paradox

Allais paradox:
Different levels of uncertainty regarding the ***outcomes***.
All probabilities are known. They have different levels, including even probability of 1 (certainty).

Certainty effect:
Prefer the option that offers certain win to avoid disappointment of no win at all (even if probability very small).

Ellsberg paradox:
Uncertainty regarding the ***probabilities*** that govern the outcomes.
Specifically, the amounts of black and yellow balls are not given.

Ambiguity aversion:
Preference for known risks over unknown risks.