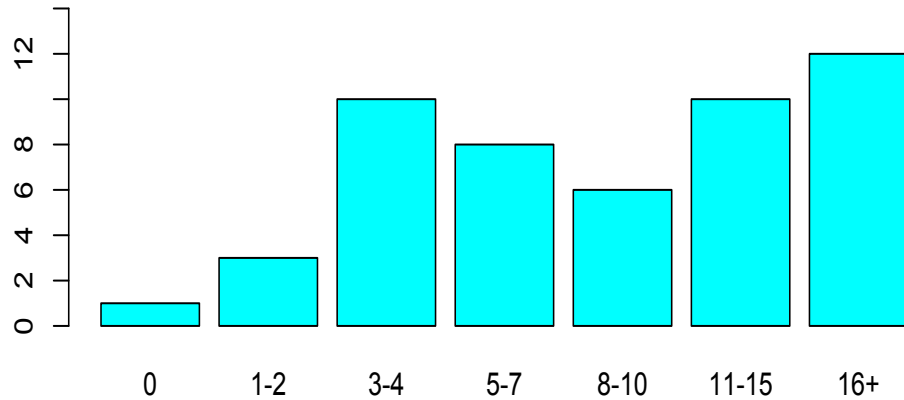


Availability bias (see W8 L3)

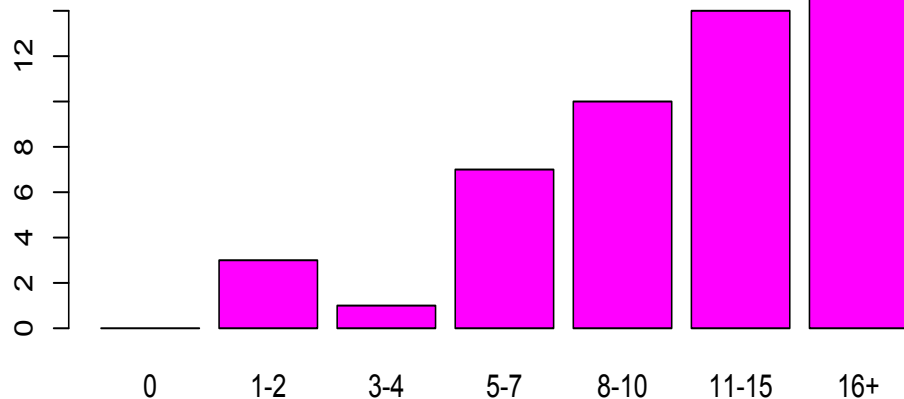
----n- group 51 students

----n-



----ing group 49 students

----ing



The *less restrictive condition* creates fewer words!

Violates normative rules of probability:

For A subset of B,

$$P(A) < P(B)$$

Reason: Increased efficiency of memory search in “- - - ing”-condition

Definitions: Heuristics & biases

Heuristic strategies:

- Shortcuts and approximations, especially if correct answers are not known or would take too long to construct.
- Using heuristics is useful and necessary, but may lead to

Context here: Probability (risk & prospect)

Perception, estimation and judgement

Resulting bias:

Differences arising from the use of heuristics or other non-normative strategies.

Question: What is the impact of knowledge and skills?

Experiment: K&T's Permutation study

Question 2 (in 2014 version)

Consider the two structures, (A) and (B), which are displayed below.

(A)

X X X X X X X X
X X X X X X X X
X X X X X X X X

(B)

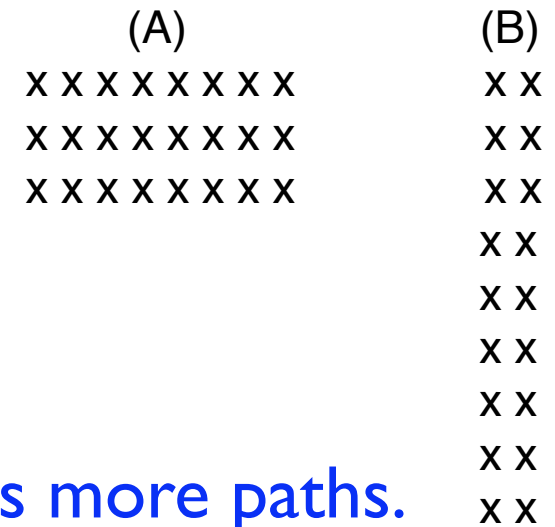
X X
X X
X X
X X
X X
X X
X X
X X
X X

A path in a structure is a line that connects an element in the top row to an element in the bottom row, and passes through one and only one element in each row.

In which of the two structures are more paths?

How many paths do you think there are in each structure?

Mathematically correct answer:
both the same ($8^3=2^9=512$)



Common intuitive judgement:
(A) has more paths than (B)

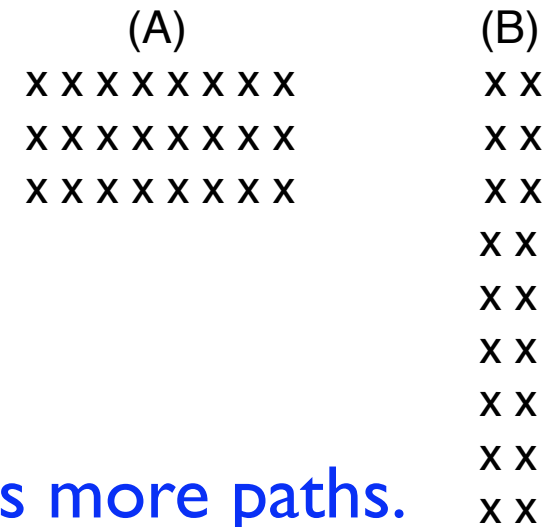
K&T's data: 46 out of 54 participants said (A) has more paths.

Explanation: Differential availability of paths in the two structures for a number for reasons including:

- most obvious paths are columns, and (A) has 8 while (B) has only 2
- paths in (A) are more distinctive and less confusable than those in (B)
- paths in (A) are shorter and hence easier and faster to visualise

Criticism?

Mathematically correct answer:
both the same ($8^3=2^9=512$)



Common intuitive judgement:
(A) has more paths than (B)

K&T's data: 46 out of 54 participants said (A) has more paths.

Explanation: Differential availability of paths in the two structures for a number for reasons including:

- most obvious paths are columns, and (A) has 8 while (B) has only 2
- paths in (A) are more distinctive and less confusable than those in (B)
- paths in (A) are shorter and hence easier and faster to visualise

Criticism: “same” not included - despite it being correct!

Test this on Warwick Students from Maths and Stats...

Question: Permutations

We added explicit answer choices adding the option “same” in half of the questionnaires (type a, **type b**) to see if this effects answers.

Question 2

Consider the two structures, (A) and (B), which are displayed below.

(A)

```
x x x x x x x x
x x x x x x x x
x x x x x x x x
```

(B)

```
x x
x x
x x
x x
x x
x x
x x
x x
x x
```

A path in a structure is a line that connects an element in the top row to an element in the bottom row, and passes through one and only one element in each row. In which of the two structures are there more paths? How many paths do you think there are in each structure?

More in (A) More in (B) **Same**

Estimated number of paths in (A): Estimated number of paths in (B):

Linda: Within-subject design (direct test)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

Linda is active in the feminist movement.

Linda is a psychiatric social worker.

Linda is a member of the League of Women Voters.

Linda is a bank teller.

Linda is an insurance salesperson.

Linda is a bank teller and is active in the feminist movement.

ST222@Warwick results:

	(A)	(B)	<i>same</i>
type a (<i>same</i> not given as answer choice):	44%	19%	38%
type b (<i>same</i> given):	33%	21%	46%

Interpretation:

- For type b, where the correct answer was an option, a (weak) majority picked it.
- For type a, where the correct answer not an option, a majority selected (A), but 38% of the respondents just added *the missing option!*
- For both types, the remaining answers showed a strong majority for type (A) in line with the availability heuristic. The relative proportions of remaining answer are:

type a: (A) 71% (B) 31%
type b: (A) 61% (B) 39%

Conjunction: Linda

T & K series of experiments involving making probability (or rank) judgments about people's profession based on short profiles.

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Linda: Within-subject design (direct test)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

Linda is active in the feminist movement.

Linda is a psychiatric social worker.

Linda is a member of the League of Women Voters.

Linda is a bank teller.

Linda is an insurance salesperson.

Linda is a bank teller and is active in the feminist movement.

Linda: Within-subject design (direct test)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

F *Linda is active in the feminist movement.*

Linda is a psychiatric social worker.

Linda is a member of the League of Women Voters.

B *Linda is a bank teller.*

Linda is an insurance salesperson.

B & F *Linda is a bank teller and is active in the feminist movement.*

Linda: Rules of probability versus empirical evidence

B & F *Linda is a bank teller and is active in the feminist movement.*

F *Linda is active in the feminist movement.*

B *Linda is a bank teller.*

Normative rules of probability:

$$B \cap F \subseteq B \quad \text{implies} \quad P(B \cap F) \leq P(B)$$

$$B \cap F \subseteq F \quad \text{implies} \quad P(B \cap F) \leq P(F)$$

Empirical findings:

Very dominant response (86% in initial study) pattern is to rank

$$P(B) < P(B \cap F) < P(F)$$

Why? What do people think? Created many years of discussions...

Linda: Replication and variation

Results confirmed many times under various conditions:

- Addressing potential issues of the design:
e.g. between-subject (indirect) and within-subject (direct)
- Alternative stories
- Different levels of statistical sophistication
- Even ST222@Warwick!

Linda: Between-subject design (indirect test)

Half of participants receive type a questionnaire, other half type b.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

Linda is a psychiatric social worker.

Linda is an insurance salesperson.

B & F *Linda is a bank teller and is active in the feminist movement.*

Linda is a member of the League of Women Voters.

type a

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

F *Linda is active in the feminist movement.*

Linda is a psychiatric social worker.

B *Linda is a bank teller.*

Linda is an insurance salesperson.

Linda is a member of the League of Women Voters.

type b

Linda: Meet also Bill!

Bill is 34 years old. He is intelligent, but unimaginative, compulsive, and generally lifeless. In school, he was strong in mathematics but weak in social studies and humanities.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Bill is a physician who plays poker for a hobby.

Bill is an architect.

A *Bill is an accountant.*

J *Bill plays jazz for a hobby.*

Bill surfs for a hobby.

Bill is a reporter.

A & J *Bill is an accountant who plays jazz for a hobby.*

Bill climbs mountains for a hobby.

Linda: Level of statistical sophistication

Experiment was conducted on subjects with different levels of statistical training. Surprisingly, this had negligible effect.

Table 3.1 Tests of the conjunction rule in likelihood rankings

Subjects	Problem	Direct test				Indirect test		
		V	R (A & B)	R (B)	N	R (A & B)	R (B)	Total N
Naive	Bill	92	2.5	4.5	94	2.3	4.5	88
	Linda	89	3.3	4.4	88	3.3	4.4	86
Informed	Bill	86	2.6	4.5	56	2.4	4.2	56
	Linda	90	3.0	4.3	53	2.9	3.9	55
Sophisticated	Bill	83	2.6	4.7	32	2.5	4.6	32
	Linda	85	3.2	4.3	32	3.1	4.3	32

Note. V = percentage of violations of the conjunction rule; R (A & B) and R (B) = mean rank assigned to A & B and to B, respectively; N = number of subjects in the direct test; Total N = total number of subjects in the indirect test, who were about equally divided between the two groups.

A for the characteristic that does not fit the previous story
(Bill: accountant, Linda: bank teller)

B for the characteristic that does fit the previous story
(Bill: jazz, Linda: feminist)

ST222'14@Warwick: Within-subject design

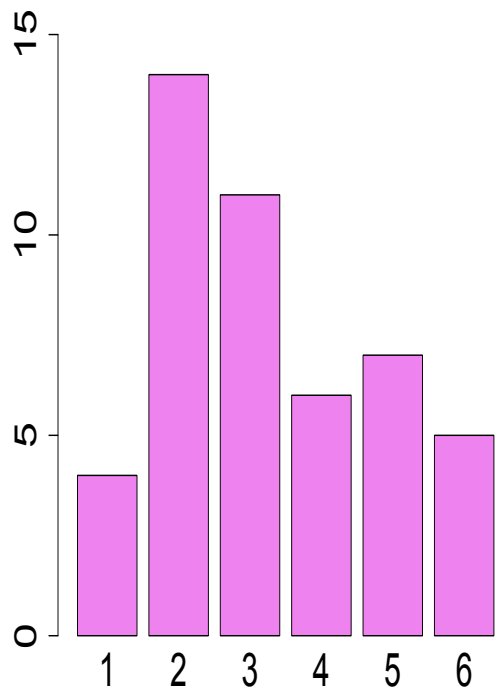
mean rank B & F = 3.3,
rescaled: $3.0 * 7/6 = 3.85$

$P(B) < P(B \cap F) < P(F)$
contradicting normative rules

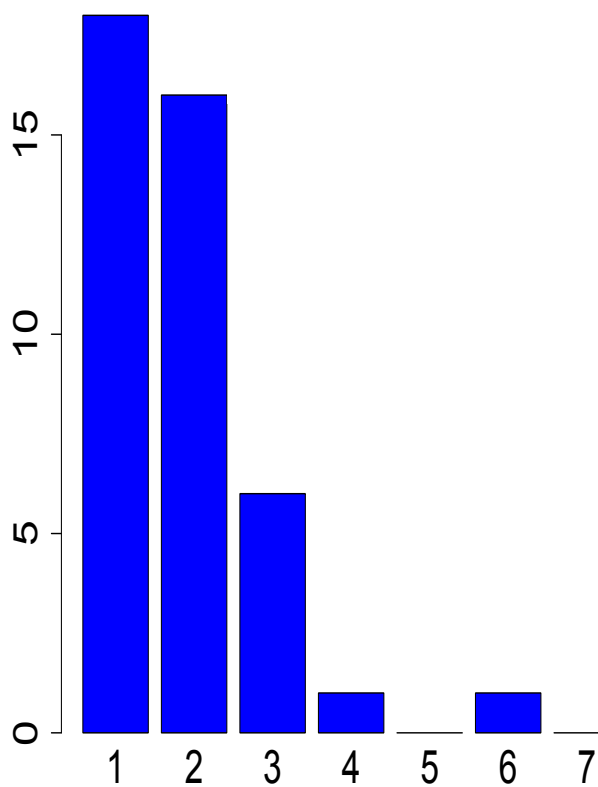
mean rank F = 1.9

mean rank B = 6.1

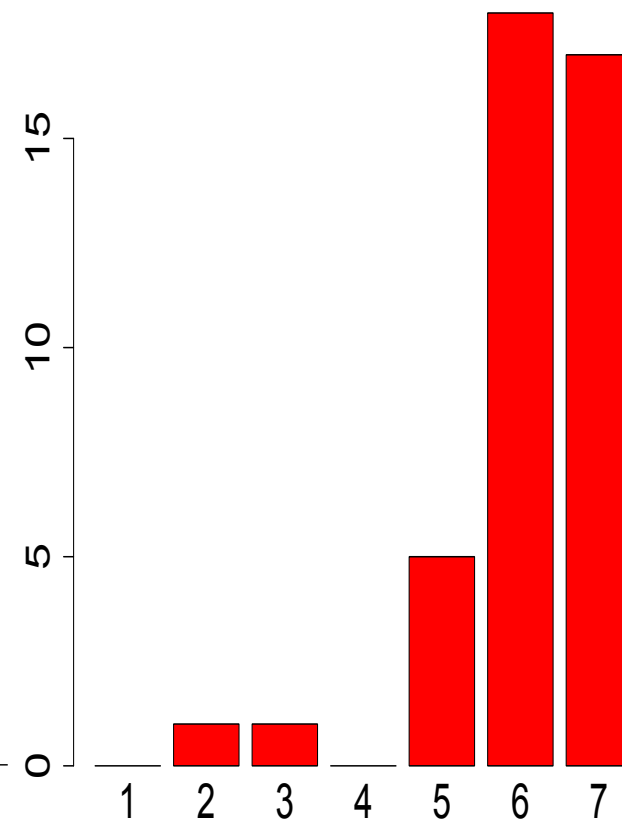
Feminist and Bank Teller



Feminist



Bank Teller



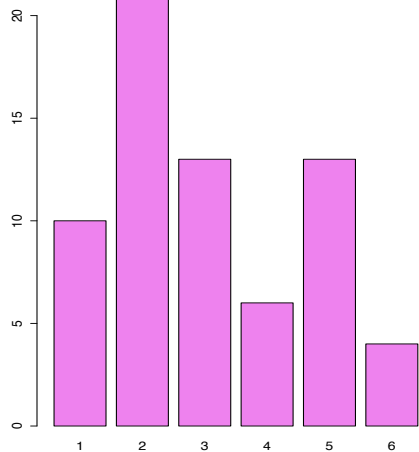
ST222'15@Warwick: Between-subject design

$$P(B) < P(B \cap F) < P(F)$$

contradicting normative rules

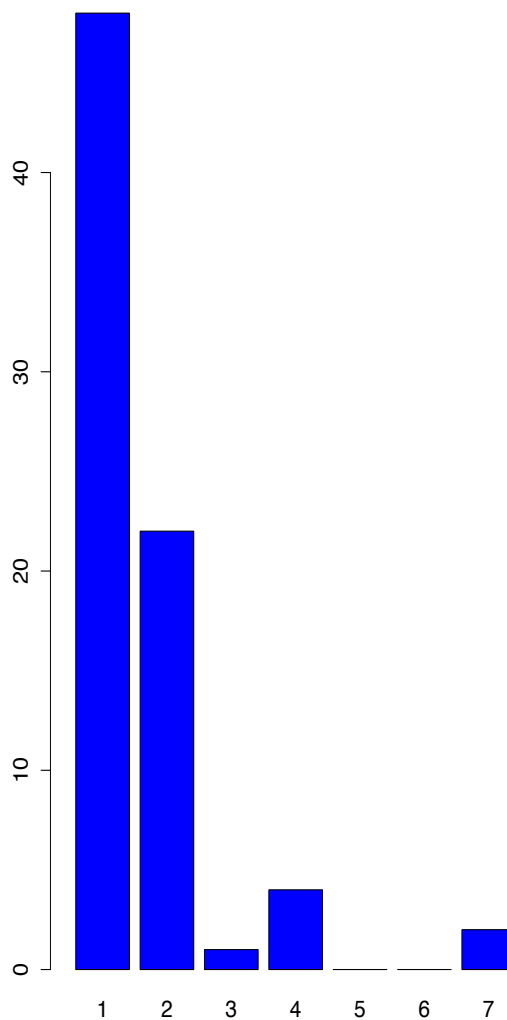
mean rank B & F = 3.0,
rescaled: $3.0 * 7/6 = 3.5$

Feminist and Bank Teller



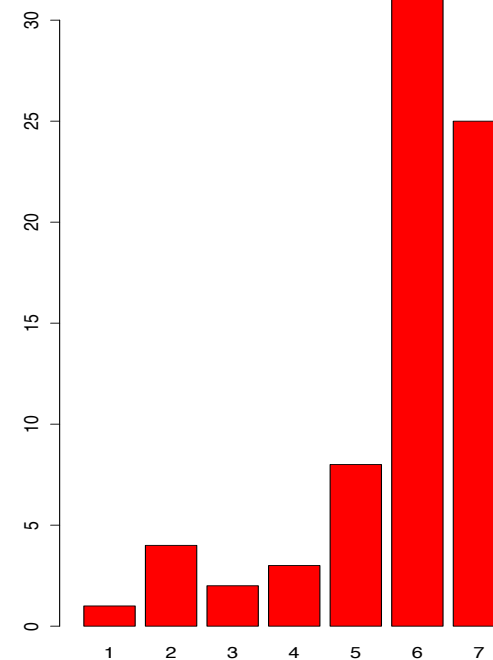
mean rank F = 1.7

Feminist



mean rank B = 5.9

Bank Teller



Concept: Conjunction fallacy

Definition

The *conjunction fallacy* is a behavioural bias reflecting the belief that the probability of the joint event A & B is bigger than the probability of one of the individual events.

More generally, it reflects that specific conditions are more probable than a single more general one.

Explanation

Depends on context. For the Linda experiment, T & K argue that it is due to the representativeness heuristic:

People think: *B is less typical for Linda than B&F, and B&F is less typical for Linda than F*

People conclude: *B is less probable than B&F, and B&F is less probable than F*

Conjunction: Dice playing

Replace explicit mentioning of probabilities by a reward in game.

Consider a regular six-sided die with four green faces and two red faces. The die will be rolled 20 times and the sequences of greens (G) and reds (R) will be recorded. You are asked to select one sequence, from a set of three, and you will win \$25 if the sequence you chose appears on successive rolls of the die. Please check the sequence of greens and reds on which you prefer to bet.

1. RGRRR
2. GRGRRR
3. GRRRRR

125 undergraduates at UBC and Stanford, monetary rewards.
65% of the subjects chose sequence 2.

Tversky A and Kahneman D, *Extentional versus intuitive reasoning: The conjunction fallacy in probability judgement*. Psychological Review 90 (1983), 293-315.

Conjunction: Dice

1. RGRRR
2. GRGRRR
3. GRRRRR

125 undergraduates at UBC and Stanford, monetary rewards.
65% of the subjects chose sequence 2. More than sequence 1!

Normative for probability:

GRGRRR is the conjunction of RGRRR and another event,
hence *less likely* to be observed than just RGRRR:

$$P(\text{GRGRRR}) \leq P(\text{RGRRR})$$

Subjects perceive GRGRRR are *more representative* of the die and
wrongly concluded it was *more probably*.

Tversky A and Kahneman D, *Extentional versus intuitive reasoning: The conjunction fallacy in probability judgement*. Psychological Review 90 (1983), 293-315.

Conjunction fallacy: Forecasting of events

- T & K's questionnaire at the *Second International Congress on Forecasting* in July of 1982
- **Subjects:** 115 professional analysts, employed by industry, universities, or research institutes
- Two *different* experimental groups
- Asked to rate the probability of two different statements
- Scale: <0.01%, 0.1%, 0.5%, 1%, 2%, 5%, 25%
- Each group seeing only one statement (indirect design)

"A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

"A Russian invasion of Poland, and a complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

Results: Estimates of probability were low for both statements, but significantly *lower* for the first group than the second ($p < 0.01$ by Mann-Whitney).

"A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

"A Russian invasion of Poland, and a complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

Results: Estimates of probability were low for both statements, but significantly *lower* for the first group than the second ($p < 0.01$ by Mann-Whitney).

Also tried variations (and obtained similar results), e.g.:

"A 30% drop in the consumption of oil in the US in 1983"

"A dramatic increase of oil prices and a 30% drop in the consumption of oil in the US in 1983"

"A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

"A Russian invasion of Poland, and a complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

Results: Estimates of probability were low for both statements, but significantly *lower* for the first group than the second ($p < 0.01$ by Mann-Whitney).

Discussion: Conjunctions involving a (hypothetical) cause are particularly prone to fallacies. Why? People may intuitively assess the probability of the effect given the *cause* rather than the joint probability, e.g.:

$P(\text{suspension of US-SU relationship} \mid \text{Russian invasion of Poland})$

$P(\text{suspension of US-SU relationship} \ \& \ \text{Russian invasion of Poland})$