Optimal design:

From insurance policy to economic policy

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Consumption-investment-insurance problems

Humps by pricing

Separation of preferences

Humps by separation of preferences

Humps by habit

Stochastic mortality in a complete market

Stochastic mortality in an incomplete market

Equilibrium considerations/predictions and the absence of mortality markets

- Consumption-investment-insurance problems, Richard (JFE 1975)

$$
\begin{aligned}
dX(t) &= \underbrace{X(t)\,dG^{\pi}(t)}_{\text{capital gains}} + \underbrace{w(t)\,dt}_{\text{labor income}} - \underbrace{c(t)\,dt}_{\text{consumption}} - \underbrace{\widehat{\mu}(t)\,b(t)\,dt}_{\text{life insurance premium}} \\
V(t,x) &= \max_{c,b,\pi} E_{t,x}\left[\int_t^n \left(\underbrace{u(c(s))\,I(s)\,ds}_{\text{utility from consumption}} + \underbrace{u(X(s)+b(s))\,dN(s)}_{\text{utility from bequest}}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
c^*(t,x) &= \underbrace{f(t)}_{\text{consumption-to-wealth ratio}} \left(x + \underbrace{h(t)}_{\text{human capital}}\right) \\
x + b^*(t,x) &= \underbrace{g(t)}_{\text{price and risk aversion impact}} (x + h(t)) \\
\pi^*(t,x)\,x &= M(x + h(t))
\end{aligned}
$$

- Humps by pricing

$$\begin{aligned} dc(t) &= f(\text{'MPRs'})\, c(t)\, dt + ...c(t)\, dW^f(t) \\ d\pi(t) &= ... \\ db(t) &= ... \end{aligned}$$

$$\begin{aligned} f(\text{'MPRs'}) &= \frac{\widehat{\mu} - \mu}{1-\gamma} + ... \\ \widehat{\mu} &> \mu \text{ during younger years} \\ \widehat{\mu} &< \mu \text{ during older years} \end{aligned}$$

Ideas about how to dampen the volatility of consumption?

- Separation of preferences

Fahrenwaldt, Jensen, Steffensen (ssrn.com 2016)

$$V(t,x) = \underbrace{\int_t^n \underbrace{v}_{\text{EIS}} \underbrace{\left(u^{-1}\left(E_{t,x}\left[u\left(c\left(s\right)\right)\right]\right)\right)}_{\text{certainty equivalent}} ds}_{\text{global time aggregation}}$$

$$c^*(t,x) = f(t)\,(x + h(t))$$

- Humps by separation of preferences

Jensen, Steffensen (IME 2015)

$$V(t,x) = \int_t^n \underbrace{v}_{\text{EIS}} \left(w^{-1} \left(\begin{array}{c} \underbrace{w}_{\text{EBS}} \underbrace{\left(u^{-1} \left(E_{t,x} \left[u\left(c\left(s\right)\right) I\left(s\right) \right] \right) \right)}_{\text{certainty equivalent}} \\ + \underbrace{w}_{\text{EBS}} \underbrace{\left(u^{-1} \left(E_{t,x} \left[u\left(X\left(s\right) + b\left(s\right)\right) \frac{dN\left(s\right)}{ds} \right] \right) \right)}_{\text{certainty equivalent}} \end{array} \right) \right) ds$$

Patience + lack of cross-generational elasticity = humps

- Humps by habit

Kraft, Munk, Seifried, Wagner (ET 2017)

$$\begin{aligned} dh(t) &= c(t) - \beta(t)\, h(t) \\ u(c,h) &= \frac{1}{1-\gamma}(c-h)^{1-\gamma} \end{aligned}$$

$$dc(t) = ...c(t)\, dt + ...(c(t) - h(t))\, dt$$

Habit formation + impatience = humps

- Stochastic mortality in a complete market

Partly covered by Menoncin, Regis (IME 2017)

$$
\begin{aligned}
d\mu(t) &= ...dt + ...dW^{\mu}(t) \\
F(t,\mu) &= E_t^Q\left[e^{-\int_t^n r}\Phi(\mu(n))\right] \\
h(t) &= E_t^Q\left[\int_t^n e^{-\int_t^s r+\mu} a(s)\,ds\right]
\end{aligned}
$$

$$
\begin{aligned}
c^*(t,x,\mu) &= \underbrace{f(t,\mu)}_{\text{consumption-to-wealth ratio}}\left(x + \underbrace{h(t,\mu)}_{\text{human capital}}\right) \\
dc(t) &= f(\text{'MPRs'})\,c(t)\,dt + ...c(t)\,dW^f(t) + ...c(t)\,dW^{\mu}(t) \\
d\text{'Position in }F\text{'} &= ...
\end{aligned}
$$

- Stochastic mortality in an incomplete market

$$d\mu(t) = ...dt + ...dW^{\mu}(t)$$

$$\begin{aligned} h(t) &= \int_t^m e^{-\int_t^s r} a(s) \\ \text{or } h(t) &= \int_t^m e^{-\int_t^s r} (a(s) - c(s))\, ds \end{aligned}$$

$$\begin{aligned} c^*(t,x,\mu) &= \underbrace{f(t,\mu)}_{\text{consumption-to-wealth ratio}} \Big(x + \underbrace{h(t)}_{\text{human capital}}\Big) \\ dc(t) &= f(\text{'MPRs'})\, c(t)\, dt + ...c(t)\, dW^f(t) + ...c(t)\, dW^{\mu}(t) \end{aligned}$$

All systematic mortality risk is borne by the policy holder (pure VA, f)

- Equilibrium considerations/predictions and the absence of mortality markets

$$\text{MP financial R} = \frac{\alpha - r}{\sigma}$$

$$\text{MP unsystematic mortality R} = 0 \;\; (\widehat{\mu} = \mu)$$

$$\text{MP systematic mortality R} = 0$$

Zero market price on mortality risk and all risk is borne by the policy holder