ACII Short Talks

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Outline

- SMC
- Genealogies
- MCMC within SMC
- Distributed Computation by Dividing and Conquering

SMC

Distributions of Interest

Distributions of interest are, $\{\hat{\eta}_n\}$, where given η_1:

$$\hat{\eta}_n(dx) = \frac{G_n(x)\eta_n(dx)}{\int \eta_n(dx)G_n(x)}$$

$$\eta_{n+1}(dx) = \int \hat{\eta}_{n-1}(dx')M_n(x', dx)$$

for Markov kernels $\{M_n\}$ and non-negative potential functions $\{G_n\}$.

Canonical example: filtering in time series $G_n(x_n) = g(y_n|x_n)$ for observations $\{y_n\}$, likelihood g, and $M_n \equiv M$ the transition kernel of a dynamic model, then $\hat{\eta}_n(dx)$ is the law of X_n given $y_{1:n}$.

A Simple SMC Algorithm

1. Sample $X_1^1, \ldots, X_1^N$ iid from M_1
2. Set

$$\hat{\eta}_1^N = \frac{\sum_{i=1}^N G_1(X_1^i)\delta_{X_1^i}}{\sum_{j=1}^N G_1(X_1^j)}.$$

3. At Time $n > 1$:
   - 3.1 Select: Sample $A_{n-1}^1, \ldots, A_{n-1}^N$ iid from

$$\frac{\sum_{i=1}^N G_{n-1}(X_{n-1}^i)\delta_i}{\sum_{j=1}^N G_{n-1}(X_{n-1}^j)}$$

   - 3.2 Mutate: For $i = 1, \ldots, N$: sample X_n^i from $M_n(X_{n-1}^{A_{n-1}^i}, \cdot)$.
   - 3.3 Reweight: Set

$$\hat{\eta}_n^N = \frac{\sum_{i=1}^N G_n(X_n^i)\delta_{X_n^i}}{\sum_{j=1}^N G_n(X_n^j)}.$$

Theoretical Properties of SMC: Genealogy

Thanks to Dario Spanò.

The collection

$$\{A_n^i\}_{n>1,i=1,\ldots,N}$$

defines a genealogy.
Borrowing tools from population biology, and some novel techniques...the large-sample, large time limiting behaviour can be characterised.

Koskela et al. [2018], Brown et al. [2021a,b]

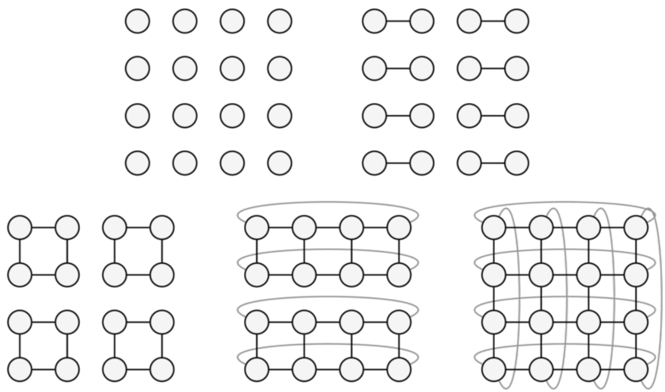
MCMC within SMC

- Replace the selection and mutation steps with MCMC.
- Generate $X_n^1, \dots, X_n^N$ by simulating a Markov chain of invariant distribution:

$$\Phi(a, x) = \frac{G_{n-1}(X_{n-1}^a) M_n(X_{n-1}^a, dx)}{\sum_{j=1}^N G_{n-1}(X_{n-1}^j)}$$

- An old idea [Berzuini et al., 1997] but only recently formally studied [Finke et al., 2020].

Dividing and Conquering: Decomposing Models



Idea

If "$\eta_1^N \approx \eta_1$" and "$\eta_2^N \approx \eta_2$" then "$\eta_1^N \times \eta_2^N \approx \eta_1 \times \eta_2$" + SMC.

Lindsten et al. [2017], Kuntz et al. [2021], Chan et al. [2021], Crucinio and Johansen [2022]

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