ST955 MSc Statistics Dissertation

Ranking in Schools Rugby

Ian Hamilton

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Contents

1 Abstract

In schools rugby union, matches take place based on geographical and historical ties, and thus do not have the features of a typical round robin tournament. Teams will only play a subset of the other teams in the tournament, giving fixture schedules of varying difficulty, varying size, and with no systematic home or away status. In this report we develop a ranking model that respects standard rugby union league scoring rules and accounts for relative schedule strength. In doing this we extend the literature beyond the known win, draw, loss scenario to account for a further two possible result outcomes, and additionally for a try bonus point. We investigate the model in the context of the English Premiership and Daily Mail Trophy, and use it to assess the current Daily Mail Trophy ranking methodology.

2 Introduction

Up until fairly recently there was no formal league competition for school rugby teams in the UK. In 2013 the Daily Mail expressed a desire to sponsor a league-based tournament, and so the Daily Mail Trophy was inaugurated. The Daily Mail Trophy covers teams who enter and who play at least five other participating schools. Currently the ranking is based on Merit Points, which are defined as the average number of League Points per match plus Additional Points, awarded in order to adjust for schedule strength.

League Points are awarded as:

4 points for a win
2 points for a draw
0 points for a loss
1 bonus point for losing by less than seven points
1 bonus point for scoring four or more tries

This is the standard scoring rule for rugby union leagues in the UK.

Additional Points in the Daily Mail Trophy are awarded based on the ranking of the current season's opponents in the previous season's tournament

Rank 1 to 25:	0.3
Rank 26 to 50:	0.2
Rank 51 to 75:	0.1
Otherwise:	0

So, for example, a team with eight fixtures qualifying for the Daily Mail Trophy, with one of those against a top 25 team, three against 26-50th placed teams, two against 51-75th placed teams, and the others against >75th placed teams, averaging 3.2 League Points per match, would get a total score of $3.2 + 1 * 0.3 + 3 * 0.2 + 2 * 0.1 = 4.1$.

This scoring rule is arbitrary and could readily come to perverse conclusions. For example, in an extreme scenario, a team A who had won all possible points from their matches and thus achieved a League Points average of 5, could be ranked below a team B who failed to win a single League Point but had played 17 of the previous season's top 25, gaining a Merit Points total of 5 and 5.1 respectively. It could even be the case that the schedule strength of team A, as measured by the positions of those opposition teams in this season's ranking, was more challenging than that of team B, and at the extreme it would even be theoretically possible that a team failing to gain a single League Point could win the entire tournament.

In 2017/18 season, the tournament consisted of 102 teams, each playing between five and twelve other participating teams, and playing 436 matches overall. The requirement for an adjustment for ability of opposition became very apparent after the first tournament in 2013/14. In this first season, instead of the current scoring rule, 0.1 additional points were awarded for every match played. This was designed to differentiate a team winning ten from ten over a team winning five from five, for example. The winning team was unbeaten but their fixture list was notably weaker than others, with only four of their twelve opponents having a winning record and having a cumulative record of P 115, W 44, D 3, L 58. In contrast, the second placed team, while losing one match narrowly (to the fourth ranked team), had a fixture list where six of their eleven opponents had a winning record and a cumulative record of P 97, W 52, D 5, L 40. While they did not play each other, they did have four opponents in common, three of which the second placed team beat more convincingly. In response to this, the scoring system was changed to the current one in order to account better for the strength of opposition. However there was further controversy in the 2015/16 season when, with this updated scoring system, the winning team lost four of their thirteen matches, with several other highly ranked teams unbeaten, including victories over teams that had beaten the tournament winner.

There is a deep literature on ranking based on pairwise comparisons. Much of this builds on the Bradley-Terry model which, in this setting, represents the probability that team i beats team j as

$$P(i > j) = \frac{\pi_i}{\pi_i + \pi_j}$$

where π_i may be thought of as representing the positive-valued ability or strength of team i. This was originally studied by Zermelo (1929) before being rediscovered by Bradley and Terry (1952), and has been further developed by Davidson (1970) to allow for ties; by Davidson and Beaver (1977) to allow for order effects, or, in this context, home advantage; and by Firth (2017) to allow for standard association football scoring rules (three for a win, one for a draw).

Here we are seeking to define a model that respects the standard league scoring rule in rugby union, but is able to account for the differences in schedule strength. More precisely, we seek to define a model where League Points earnt represent a sufficient statistic for ability. In doing so we may then consider the projected points per match for each team were they to complete a full round robin tournament at a neutral venue, and use this to rank them. It is important to understand therefore that what we are seeking to define here is a retrodictive rather than a predictive model. This is a concept familiar in North America where the KRACH "Ken's Rating for American College Hockey" model, devised by Ken Butler, is commonly used to rank collegiate and school teams in ice hockey and other sports, see Wobus (2007). KRACH is therefore the commonly used name for the Bradley-Terry model applied to these sports. While implementations vary, in Butler's original model, he also introduced a prior to the ability parameter, via what he referred to as a 'fictitious team', Whelan and Schlobotnik (2018), an idea to which we will return in this report.

An example is presented for further illustration. Consider seven teams and the results of three of those against the other four.

	Team D	Team E	Team F	Team G
Team A	-	-	W 40-0*	L 0-8
Team B	W 32-0*	L 14-15	L 14-15	-
Team C	W 20-19*	W 15-14	L 0-32*	-

where $*$ means that a try bonus point was scored by the winning team alone.

Let us also suppose that the results of Teams D-G in last season's tournament are such that there are additional points of 0.3 for playing Team D, 0.2 for playing Team E, 0.1 for playing Team F and nothing for playing Team G.

Let us consider the ranking of just Teams A-C. We get the following table based on these results, and using average League Points per game (LPPG) the Daily Mail Trophy methodology (DMT) and net score per game (NSPG), where NSPG for a team is the average of the difference between points scored and points conceded over all their matches.

	P	W	D	L	LPPG	Rank	DMT	Rank	NSPG	Rank
Team A	2	1	0	1	2.5	2	2.6	3	+16	1
Team B	3	1	0	2	2.3	3	2.9	2	+10	2
Team C	3	2	0	1	3	1	3.6	1	-10	3

We can see that these do not give consistent rankings for these three teams, and in particular, Team A is third under the DMT methodology, second under League Points per Game and first under NSPG.

Were we however to consider the unplayed matches here, we might reasonably predict, based on the known results, that Team A would be likely to beat Teams D and E, and Teams B and C would be likely to lose to

Team G, which would then place Team A first amongst these three teams on all rankings.

We may also note that Team C is ranked higher than Team B under the first two methodologies, and likely would continue to be so even using full round robin projected League Points per game. However were Team B to play Team C, we might expect Team B to win, based on historical scores rather than wins and losses, as indicated by the NSPG ranking. We are though explicitly looking for a model that is consistent with the scoring rule of the competition, which prizes result outcomes over scores, and hence ranking Team C above Team B is consistent with our approach.

While a retrodictive model with the desired sufficient statistic for team abilities would provide a statistically sound method for ranking the teams, it suffers from some drawbacks in the context of the Daily Mail Trophy:

1. such a methodology is likely to be very technical for a generalist audience and therefore there is a lack of transparency

2. the method relies on the connectivity of the participating teams in a number of ways. If the teams are not a connected set then the method will be unable to give a definitive ranking. Even if a ranking is achieved by the end of the tournament, due to the relatively sparse and geographically-based nature of the fixture list, this may not be achieved until relatively late on, giving no ranking for a reasonable proportion of the season. It can also mean that significant weight comes to rest on particular results if those are ones particularly important for the connectivity of the set of teams

It is worth considering therefore what features a desirable scoring system in this context might have. Some considerations, in no particular order, are that it should:

1. be such that the top-ranked team should not be obviously wrong in the opinion of a large proportion of stakeholders in the tournament - coaches, players, parents, administrators etc.

2. be possible to identify top-ranked teams in general and how far apart they are, so that a team may have an idea of how far from top position it is

3. be such that all other relative rankings should not be perceivable as unreasonable by a large proportion of the tournament stakeholders

4. be dependent on the results of only one season

5. be such that any participating school could win

6. be consistent in the sense that if they were applied to a full round robin they would achieve the same ranking as using League Points

7. be such that there is not a requirement for additional fixtures beyond the regular fixture list

8. be transparent in the sense that it is readily explicable to a generalist audience
9. allow for a ranking from early in the season
10. take into account the relative strength of opposition faced

The present system fails on at least one of these, namely the dependence on only this season's results. Meeting all of these is quite a tall order, and so it is likely that some of these would have to be relaxed. We may also consider the first condition a little obvious. However it is not clear that the current tournament scoring rule achieves this, as seen with the previously mentioned 2015/16 season, and the hypothetical example of the unbeaten and losing teams. Given that there is no concept of relegation, and no prizes for positions other than first, it is not unreasonable to think of criterion 1 as having greater weight than the more general criterion 3.

The current construct of League Points per game plus Additional Points to reflect the strength of opposition faced meets the requirements of transparency, and is one with which participants are familiar. However the calibration of these additional points is currently arbitrary. We will use the model developed to investigate the effectiveness of the current methodology.

3 Model

We shall consider the development of the model in a number of parts. We will capture the information from the result; we will look at the try bonus point; we will introduce a parameter for home advantage; we will consider how this may be represented and solved as a log-linear model; and in the final part we will consider extending to allow for a prior distribution of abilities. We also make short notes on an intuitive way to interpret the ability parameters derived and on the concept of mean ability.

3.1 Result outcome

While the result outcome is commonly presented as a standard win, draw, loss plus a losing bonus point, we may think of this equivalently as five possible result outcomes - wide win, narrow win, draw, narrow loss, wide loss. We wish to maintain League Points as a sufficient statistic for our ability parameters. While it may not be immediately obvious how we would do that, the work of Davidson (1971) and Firth (2017) would suggest that we could consider taking the power of a team's ability in the probability of each outcome to be the points earnt by that team under that outcome. This would suggest a representation of the five non-normalised result probabilities as

$$
\begin{aligned}
P(\text{team } i \text{ beats team } j \text{ by wide margin}) &\propto \frac{(1-\rho-\lambda)}{2}\pi_i^4 \\
P(\text{team } i \text{ beats team } j \text{ by narrow margin}) &\propto \frac{\lambda}{2}\pi_i^4\pi_j \\
P(\text{team } i \text{ draws with team } j) &\propto \rho\pi_i^2\pi_j^2 \\
P(\text{team } j \text{ beats team } i \text{ by narrow margin}) &\propto \frac{\lambda}{2}\pi_i\pi_j^4 \\
P(\text{team } j \text{ beats team } i \text{ by wide margin}) &\propto \frac{(1-\rho-\lambda)}{2}\pi_j^4
\end{aligned}
$$

where π_i represents the ability of team i, and ρ and λ are paremeters to be determined.

We choose this structural parametrisation since, taking the conventional standardisation of the abilities that the mean ability is 1, as in Ford (1957), we can then interpret ρ as the probability of a draw between two teams of mean ability, and λ as the probability of a narrow result outcome (win or loss) in a match between two teams of mean ability.

Define $\kappa = \frac{\lambda}{(1-\rho-\lambda)}$ and $\nu = \frac{2\rho}{(1-\rho-\lambda)}$ for computational ease. We may then define the non-normalised probabilities more neatly as

$$
\begin{aligned}
P(\text{team } i \text{ beats team } j \text{ by wide margin}) &\propto \pi_i^4 \\
P(\text{team } i \text{ beats team } j \text{ by narrow margin}) &\propto \kappa\pi_i^4\pi_j \\
P(\text{team } i \text{ draws with team } j) &\propto \nu\pi_i^2\pi_j^2 \\
P(\text{team } j \text{ beats team } i \text{ by narrow margin}) &\propto \kappa\pi_i\pi_j^4 \\
P(\text{team } j \text{ beats team } i \text{ by wide margin}) &\propto \pi_j^4
\end{aligned}
$$

The parameters $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_m), \kappa, \nu$ may be estimated by method of maximum likelihood, where m is the number of teams in the tournament.

Let w_{ij}, n_{ij}, d_{ij} be the number of wide wins of i over j, narrow wins of i over j, and draws between i and j respectively, and $r_{ij} = w_{ij} + n_{ij} + d_{ij} + n_{ji} + w_{ji}$ be the total matches between i and j. Let $W = \{w_{ij} : i, j = 1, \ldots, m\}, N = \{n_{ij} : i, j = 1, \ldots, m\}$, and $D = \{d_{ij} : i, j = 1, \ldots, m\}$. Then we may express the likelihood as

$$
L(\boldsymbol{\pi}, \kappa, \nu; W, N, D) = \prod_{i<j} \frac{(\pi_i^4)^{w_{ij}}(\kappa\pi_i^4\pi_j)^{n_{ij}}(\nu\pi_i^2\pi_j^2)^{d_{ij}}(\kappa\pi_i\pi_j^4)^{n_{ji}}(\pi_j^4)^{w_{ji}}}{(\pi^4 + \kappa\pi^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4 + \pi_j^4)^{r_{ij}}}
$$

If we then define the 'score' for team i as $s_i = 4\sum_j (w_{ij} + n_{ij}) + 2\sum_j d_{ij} + \sum_j n_{ji}$, and let $n = \sum_i \underset{<}{} \sum_j n_{ij}$ be the total number of narrow wins and $d = \sum_i \underset{<}{} \sum_j d_{ij}$ the total number of draws then

$$L(\boldsymbol{\pi}, \kappa, \nu; W, N, D) = \frac{\kappa^n \nu^d \prod_{i=1}^{m} \pi_i^{s_i}}{\prod_{i<j} (\pi^4 + \kappa\pi^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4 + \pi_j^4)^{r_{ij}}}$$

and the statistic $(\boldsymbol{s}, n, d)$ is a sufficient statistic for $(\boldsymbol{\pi}, \kappa, \nu)$.

3.2 Try bonus point outcome

Having established a model for the result outcome, we seek to extend that to incorporate the try bonus point. In doing this we wish to maintain 'score' as a sufficient statistic for team ability. A nice feature for this particular problem is that it allows the try bonus point to give us information on the relative ability of the teams. This may allow us to garner differentiating information on team ability even where we have more than one team with a 100% winning record (there were seven such teams in the 2017/18 tournament). Approaching it in this way leads to a number of possible models. We will look at three:

1. try bonuses for each team independent of the ability of the opposition team and the result, dependent only on the ability of the team themselves
2. try bonus point dependent on ability of the team and the opposition, but independent of the result
3. try bonus point dependent on the result, the team themselves and the ability of the opposition

3.2.1 Independent of ability of opposition and result

For this model we model each team's try bonus point as having non-normalised probability

$$P(\text{team } i \text{ gains try bonus point}) \propto \xi\pi_i$$
$$P(\text{team } i \text{ does not gain try bonus point}) \propto 1 - \xi$$

Defining in this way ξ is the probability that a team of mean ability gains a try bonus.

This model is very simple, introducing a single extra parameter, and gives us a sufficient statistic in line with the League Points scoring rule. However it is a notable simplification to assume that a team's ability to score a try bonus is not influenced by the ability of the opposition. Taking a fully independent approach to the try bonus would also reduce the information we may draw on for assessing teams' abilities. This could be particularly relevant in the event of there being more than one unbeaten team, whereby including a dependence on the opposition allows us to draw information if an unbeaten team's opposition is able to gain a try bonus.

3.2.2 Dependent on ability of opposition, independent of result

For this model we consider the four potential outcomes dependent on the ability of opposition but independent of the result, with non-normalised probabilities

$$
\begin{aligned}
P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) &\propto \eta\pi_i\pi_j \\
P(\text{only team } i \text{ gains try bonus point}) &\propto \zeta\pi_i \\
P(\text{only team } j \text{ gains try bonus point}) &\propto \zeta\pi_j \\
P(\text{neither team gains try bonus point}) &\propto 1 - \eta - 2\zeta
\end{aligned}
$$

Here η represents the probability that in a match between two teams of mean ability, both gain a try bonus, and ζ the probability that team i alone gains a try bonus. Since they are of the same mean ability, ζ is also the probability that team j alone gains a try bonus.

This model also benefits from simplicity, introducing just two additional parameters, and gives us a sufficient statistic in line with the tournament scoring rule. However it would seem reasonable to suggest that given the abilities of two teams, the result of their match gives us further relevant information on the probability of a bonus point. And so we consider a third set of models.

3.2.3 Dependent on ability of opposition and result

For these models we consider the bonus point outcome conditional on the result. We are now conditioning on the five potential result outcomes (wide win, narrow win, draw, narrow loss, wide loss). The symmetry of the narrow win and narrow loss, and wide wine and wide loss mean we need only consider three conditional parametrisations: wide result, narrow result, draw.

Let us assume that i is the winning team then we parametrise as follows for the wide result

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \eta_{wb}\pi_i\pi_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \zeta_{ww}\pi_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \zeta_{wl}\pi_j$$
$$P(\text{neither team gains try bonus point}) \propto 1 - \eta_w - \zeta_{ww} - \zeta_{wl}$$

where in the case of a wide result in a match between two teams of mean ability, η_{wb} represents the probability that both gain a try bonus, ζ_{ww} the probability that the winner alone gains a try bonus, and ζ_{wl} the probability that the loser alone gains a try bonus.

Similarly for the narrow result with the same assumption we have

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \eta_{nb}\pi_i\pi_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \zeta_{nw}\pi_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \zeta_{nl}\pi_j$$
$$P(\text{neither team gains try bonus point}) \propto 1 - \eta_n - \zeta_{nw} - \zeta_{nl}$$

where in the case of a narrow result in a match between two teams of mean ability, η_{nb} represents the probability that both gain a try bonus, ζ_{nw} the probability that the winner alone gains a try bonus, and ζ_{nl} the probability that the loser alone gains a try bonus.

In the case of a draw we can take advantage of symmetry to define just two further parameters

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \eta_d\pi_i\pi_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \zeta_d\pi_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \zeta_d\pi_j$$
$$P(\text{neither team gains try bonus point}) \propto 1 - \eta_d - 2\zeta_d$$

where in the case of a draw in a match between two teams of mean ability, η_d represents the probability that both gain a try bonus, and ζ_d the probability that team i (or equivalently team j) gains a try bonus.

In this version we therefore have an additional eight parameters for specifying our model. We could reduce this while maintaining a dependence on result by noting that a narrow win or loss implies that the teams are separated by only one score and so concluding that a narrow result one way or the other has little extra information, and subsuming that condition within our parametrisation of the draw. This would mean that we had only five parameters for defining the try outcome. However compared to the two for the more substantial result outcome this still seems high, and due to the small frequencies (possibly zero) involved, estimation of, for example, ζ_{wl} could be poor.

3.2.4 Independent offensive and defensive ability

One potential criticism of all these approaches is that a team's ability to earn a try bonus point is likely to be dependent upon its own attacking ability and the opposition's defensive ability and independent of its own defensive ability and the opposition's attacking ability. In using the overall ability of the teams as our parameter in these models we lose this real world sense. It is possible to create such a model that continues to respect League Points as a sufficient statistic. However a model that incorporated separable offensive and defensive abilities would have a very large number of additional parameters, one per team, and so in the context of a tournament of 102 teams and 436 matches, such a model would not be appropriate. As such, we do not consider this as part of our analysis here. For completeness however a version of it is presented in the Appendix.

3.2.5 Try bonus outcome summary

All these models are viable in the sense of a team's score as defined by League Points representing a sufficient statistic. However the fully dependent models introduce a large number of structural parameters. For the purpose of this analysis we will restrict ourselves to the fully independent and partially independent try bonus models of sections 3.2.1 and 3.2.2 respectively. For the rest of this document we will refer to them as Model 1 and Model 2 respectively, with the model number referring to the number of additional structural parameters introduced with each of the models. We will consider how we may compare them in section 3.9.

3.3 Combining Result and Try outcomes

Here we give a full explanation of the combination of the result and try bonus outcomes using Model 2, taking try bonus modeled as dependent on opposition, but independent of result outcome. Further details of the other models are presented in the Appendix.

For ease of manipulation, define $\theta = \frac{\eta}{\zeta}$ and $\phi = \frac{1-\eta-2\zeta}{\zeta}$ so that our non-normalised probabilities for the try bonus outcomes become

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \theta\pi_i\pi_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \pi_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \pi_j$$
$$P(\text{neither team gains try bonus point}) \propto \phi$$

Let bb_{ij}, b_{ij}, z_{ij} be the number of matches between i and j where both gain a try bonus, number of matches between i and j where team i gains a try bonus but team j does not, and the number of matches between i and j where neither team gains a try bonus respectively. Let $BB = \{bb_{ij} : i, j = 1, \ldots, m\}, B = \{b_{ij} : i, j = 1, \ldots, m\}$, and $Z = \{z_{ij} : i, j = 1, \ldots, m\}$. Then we get a likelihood function

$$L(\boldsymbol{\pi}, \kappa, \nu, \theta, \phi; W, N, D, BB, B, Z) = \prod_{i<j} \frac{(\pi_i^4)^{w_{ij}} (\kappa\pi_i^4\pi_j)^{n_{ij}} (\nu\pi_i^2\pi_j^2)^{d_{ij}} (\kappa\pi_i\pi_j^4)^{n_{ji}} (\pi_j^4)^{w_{ji}}}{(\pi^4 + \kappa\pi^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4 + \pi_j^4)^{r_{ij}}} \frac{(\theta\pi_i\pi_j)^{bb_{ij}} (\pi_i)^{b_{ij}} (\pi_j)^{b_{ji}} (\phi)^{z_{ij}}}{(\theta\pi_i\pi_j + \pi_i + \pi_j + \phi)^{r_{ij}}}$$

If we then define the 'score' for team i as $s_i = 4\sum_j (w_{ij} + n_{ij}) + 2\sum_j d_{ij} + \sum_j n_{ji} + \sum_j (b_{ij} + bb_{ij})$, and let $bb = \sum_i \sum_{<} \sum_j bb_{ij}$ be the total number of matches where both teams score a try bonus and $z = \sum_i \sum_{<} \sum_j z_{ij}$ the total number of matches where neither scores a try bonus then

$$L(\boldsymbol{\pi}, \kappa, \nu, \theta, \phi; W, N, D, BB, B, Z) = \frac{\kappa^n \nu^d \theta^{bb} \phi^z \prod_{i=1}^{m} \pi_i^{s_i}}{\prod_{i<j} (\pi_i^4 + \kappa\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4 + \pi_j^4)^{r_{ij}} (\theta\pi_i\pi_j + \pi_i + \pi_j + \phi)^{r_{ij}}}$$

and the statistic $(\boldsymbol{s}, n, d, bb, z)$ is a sufficient statistic for $(\boldsymbol{\pi}, \kappa, \nu, \theta, \phi)$.

3.4 Home advantage

We could choose to parametrise the home advantage in a number of ways but following the example of Davidson and Beaver (1977) and others, we will take a single scaling factor across all teams and matches. We choose to do this by applying a scaling parameter to the home team and its reciprocal to the away team.

That is to say, suppose team i is the home team, team j the away team, and τ is our home advantage scaling parameter then we may express the non-normalised probabilities for the result outcome as

$$
\begin{aligned}
P(\text{team } i \text{ beats team } j \text{ by wide margin}) &\propto \tau^4\pi_i^4 \\
P(\text{team } i \text{ beats team } j \text{ by narrow margin}) &\propto \kappa\tau^3\pi_i^4\pi_j \\
P(\text{team } i \text{ draws with team } j) &\propto \nu\pi_i^2\pi_j^2 \\
P(\text{team } j \text{ beats team } i \text{ by narrow margin}) &\propto \frac{\kappa\pi_i\pi_j^4}{\tau^3} \\
P(\text{team } j \text{ beats team } i \text{ by wide margin}) &\propto \frac{\pi_j^4}{\tau^4}
\end{aligned}
$$

and for the try bonus point as

$$
\begin{aligned}
P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) &\propto \theta\pi_i\pi_j \\
P(\text{only team } i \text{ gains try bonus point}) &\propto \tau\pi_i \\
P(\text{only team } j \text{ gains try bonus point}) &\propto \frac{\pi_j}{\tau} \\
P(\text{neither team gains try bonus point}) &\propto \phi
\end{aligned}
$$

Note that in this context the probabilities λ, ρ, ζ, and η, that may be derived directly from κ,ν,θ, and ϕ, should now be interpreted as those relating to matches between two teams of mean ability played at a neutral venue i.e. where $\tau = 1$.

In order to work with the home advantage we adjust our notation to allow the subscript notation to now define home and away teams rather than who wins or gains points. So taking i as the home team and j as the away team, define the notation for the number of each result outcome as

hw_{ij}	home win by wide margin
hn_{ij}	home win by narrow margin
d_{ij}	draw
an_{ij}	away win by narrow margin
aw_{ij}	away win by wide margin

and for the number of each try outcome as

hb_{ij}	home try bonus only
ab_{ij}	away try bonus only
bb_{ij}	both try bonus

zb_{ij} zero try bonus

Then our likelihood becomes

$$L(\boldsymbol{\pi},\kappa,\nu,\theta,\phi,\tau;W,N,D,BB,B,Z)=\prod_{i=1}^{m}\prod_{j=1}^{m}\frac{(\tau^4\pi_i^4)^{hw_{ij}}(\kappa\tau^3\pi_i^4\pi_j)^{hn_{ij}}(\nu\pi_i^2\pi_j^2)^{d_{ij}}(\frac{\kappa\pi_i\pi_j^4}{\tau^3})^{an_{ij}}(\frac{\pi_j^4}{\tau^4})^{aw_{ij}}}{(\tau^4\pi^4+\kappa\tau^3\pi^4\pi_j+\nu\pi_i^2\pi_j^2+\frac{\kappa\pi_i\pi_j^4}{\tau^3}+\frac{\pi_j^4}{\tau^4})^{r_{ij}}}\frac{(\theta\pi_i\pi_j)^{bb_{ij}}(\tau\pi_i)^{hb_{ij}}(\frac{\pi_j}{\tau})^{ab_{ij}}(\phi)^{zb_{ij}}}{(\theta\pi_i\pi_j+\tau\pi_i+\frac{\pi_j}{\tau}+\phi)^{r_{ij}}}$$

where W,N,D,BB,B,Z have the same meaning as before but may be re-expressed in terms of the new notation, and r_{ij} is now the total number of matches where i is the home team and j the away team.

Let h be the difference in points scored by home teams and away teams

$$\begin{aligned}h&=\sum_i\sum_j 4(hw_{ij}+hn_{ij})+2d_{ij}+an_{ij}+bb_{ij}+hb_{ij}-\sum_i\sum_j 4(aw_{ij}+an_{ij})+2d_{ij}+hn_{ij}+bb_{ij}+ab_{ij}\\&=\sum_i\sum_j 4(hw_{ij}-aw_{ij})+3(hn_{ij}-an_{ij})+(hb_{ij}-ab_{ij})\end{aligned}$$

then we may express the likelihood as

$$L(\boldsymbol{\pi},\kappa,\nu,\theta,\phi,\tau;W,N,D,BB,B,Z)=\frac{\kappa^n\nu^d\theta^{bb}\phi^z\tau^h\prod_{k=1}^{m}\pi_k^{s_k}}{\prod_{i=1}^{m}\prod_{j=1}^{m}(\tau^4\pi_i^4+\kappa\tau^3\pi_i^4\pi_j+\nu\pi_i^2\pi_j^2+\frac{\kappa\pi_i\pi_j^4}{\tau^3}+\frac{\pi_j^4}{\tau^4})^{r_{ij}}(\theta\pi_i\pi_j+\tau\pi_i+\frac{\pi_j}{\tau}+\phi)^{r_{ij}}}$$

and the statistic $(\boldsymbol{s},n,d,bb,z,h)$ is a sufficient statistic for $(\boldsymbol{\pi},\kappa,\nu,\theta,\phi,\tau)$.

Taking the log we have log-likelihood

$$\begin{aligned}\log L(\boldsymbol{\pi},\kappa,\nu,\theta,\phi,\tau;W,N,D,BB,B,Z)&=n.\log\kappa+d.\log\nu+bb.\log\theta+z.\log\phi+h.\log\tau+\sum_{i=1}^{m}s_i.\log\pi_i\\&-\sum_{i}\sum_{<\ j}r_{ij}\log(\tau^4\pi_i^4+\kappa\tau^3\pi_i^4\pi_j+\nu\pi_i^2\pi_j^2+\frac{\kappa\pi_i\pi_j^4}{\tau^3}+\frac{\pi_j^4}{\tau^4})\\&-\sum_{i}\sum_{<\ j}r_{ij}\log(\theta\pi_i\pi_j+\tau\pi_i+\frac{\pi_j}{\tau}+\phi)\end{aligned}$$

We may find estimates for these parameters through a maximum likelihood approach.

3.5 Log-linear representation

As the form of the log-likelihood suggests and following Fienberg (1979) the estimation of these parameters may be simplified by using a log-linear model. We consider two approaches.

3.5.1 Fully elucidated outcomes

Let x_{ijkl} denote the observed count for the number of matches with home team i, away team j, result outcome k, and try bonus outcome l. Furthermore let m_{ijkl} be the expected value corresponding to x_{ijkl}. The log-linear version of the model can then be written as

$$log\, m_{ijkl} = \alpha_{ij} + \alpha_{ijk\cdot} + \alpha_{ij\cdot l}$$

where α_{ij} is a normalisation parameter, and $\alpha_{ijk\cdot}$ and $\alpha_{ij\cdot l}$ represent those parts due to the result outcome and try outcome respectively. That is

$$\alpha_{ijk\cdot} = \begin{cases} 4\delta_i + 4\sigma & \text{if home win by wide margin} \\ 4\delta_i + \delta_j + \beta_n + 3\sigma & \text{if home win by narrow margin} \\ 2\delta_i + 2\delta_j + \beta_d & \text{if draw} \\ \delta_i + 4\delta_j + \beta_n - 3\sigma & \text{if away win by narrow margin} \\ 4\delta_j - 4\sigma & \text{if away win by wide margin} \end{cases}$$

$$\alpha_{ij\cdot l} = \begin{cases} \delta_i + \delta_j + \gamma_{bb} & \text{if both home and away try bonuses} \\ \delta_i + \sigma & \text{if home try bonus only} \\ \delta_j - \sigma & \text{if away try bonus only} \\ \gamma_{zb} & \text{if no try bonus for either side} \end{cases}$$

We use the gnm package in R to give us maximum likelihood estimators for $(\boldsymbol{\delta}, \beta_n, \beta_d, \gamma_{bb}, \gamma_{zb}, \sigma)$.

We can then find our required parameter set $(\boldsymbol{\pi}, \kappa, \nu, \theta, \phi, \tau)$ by setting $\pi_i = \exp(\delta_i)$, $\kappa = \exp(\beta_n)$, $\nu = \exp(\beta_d)$, $\theta = \exp(\gamma_{bb})$, $\phi = \exp(\gamma_{zb})$, $\tau = \exp(\sigma)$.

We note that a number of matches in the Daily Mail Trophy are played at a neutral venue (17 in the 2017/18 season). In these instances, sigma is set to zero, analagous to setting τ to 1.

3.5.2 Independent result and try outcomes

In Models 1 and 2, the result and try outcomes were considered independently and so we may seek to consider the log-linear representation similarly. That is we may solve simultaneously

$$log\, m_{ijk\cdot} = \alpha_{ij} + \alpha_{ijk\cdot}$$
$$log\, m_{ij\cdot l} = \alpha'_{ij} + \alpha_{ij\cdot l}$$

where $m_{ijk\cdot}$ and $m_{ij\cdot l}$ are the expected number of matches with home team i, away team j and result outcome k and try bonus outcome l respectively, $\alpha_{ijk\cdot}$ and $\alpha_{ij\cdot l}$ have the same definitions as previously, but the normalisation parameters α_{ij} and α'_{ij} will take different values.

3.6 A more intuitive measure

Having estimated the parameters, we may use them to estimate the outcome probabilities. This allows us also to estimate the projected points per match for team i PPPM_i by summing the projected points per match were team i to play each of the other teams once at a neutral venue.

$$\text{PPPM}_i = \frac{1}{m-1} \sum_{j \neq i} p'_{ijkl}.m'_{ijkl}$$

where m'_{ijkl} and p'_{ijkl} are respectively the probability and the points accruing to team i, of result outcome k and try bonus outcome l in a match between teams i and j at a neutral venue (we use the dash notation here to differentiate from our earlier use where the ordered ij subscript denoted i as the home team).

Since in this model the result and try outcome are independent then

$$\text{PPPM}_i = \frac{1}{m-1} \left[\sum_{j \neq i} p'_{ijk\cdot}.m'_{ijk\cdot} + \sum_{j \neq i} p'_{ij\cdot l}.m'_{ij\cdot l} \right]$$

with $p'_{ijk\cdot}$ and $p'_{ij\cdot l}$ represent the points due to team i from the result and try outcomes respectively, and $m_{ijk\cdot}$ and $m_{ij\cdot l}$ the corresponding probabilities.

We have that

$$p'_{ijk\cdot}.m'_{ijk\cdot} = \frac{4\pi_i^4 + 4\kappa\pi_i^4\pi_j + 2\nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4}{\pi_i^4 + \kappa\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4 + \pi_j^4}$$

and differentiating with respect to π_i we get

$$\frac{\partial}{\partial \pi_i}(p'_{ijk\cdot}.m'_{ijk\cdot}) = \frac{\pi_j^2(4\nu\kappa\pi_i^5\pi_j+4\nu\pi_i^5+9\kappa^2\pi_i^4\pi_j^3+9\kappa\pi_i^4\pi_j^2+16\kappa\pi_i^3\pi_j^3+16\pi_i^3\pi_j^2+\nu\kappa\pi_i^2\pi_j^4+4\nu\pi_i\pi_j^4+\kappa\pi_j^6)}{(\pi_i^4+\kappa\pi_i^4\pi_j+\nu\pi_i^2\pi_j^2+\kappa\pi_i\pi_j^4+\pi_j^4)^2} \geq 0$$

with this equal to zero if and only if π_j is zero, and so $\frac{\partial}{\partial \pi_i}\left(\sum_{j\neq i} p'_{ijk\cdot}.m'_{ijk\cdot}\right)$ is strictly greater than zero, since not all the π_j may be zero.

Similarly

$$p'_{ij\cdot l}.m'_{ij\cdot l} = \frac{\theta\pi_i\pi_j + \pi_i}{\theta\pi_i\pi_j + \pi_i + \pi_j + 1}$$

and differentiating with respect to π_i we get

$$\frac{\partial}{\partial \pi_i}(p'_{ijk\cdot}.m'_{ijk\cdot}) = \frac{\pi_j + 1}{(\theta\pi_i\pi_j + \pi_i + \pi_j + 1)^2} \geq 0$$

and so likewise $\frac{\partial}{\partial \pi_i}\left(\sum_{j\neq i} p'_{ij\cdot l}.m'_{ij\cdot l}\right)$ is strictly greater than zero.

Since the derivative $\frac{\partial}{\partial \pi_i}(\text{PPPM}_i)$ is strictly positive then a team ranked higher based on ability π_i will also be ranked higher based on projected points per match PPPM_i and vice versa, and so this may be used as an alternative measure.

This is convenient as it gives an intuitive meaning to the ranking measure and allows us to transparently compare an intermediate season state to the end of season state of a full round robin tournament, a feature we will utilise in section 3.9.

3.7 Adding a prior

One potential criticism of the models proposed so far is that they give no additional credit to a team that has achieved their results against a large number of opponents as compared to a team that has played only a small number. This is an intuitive idea in line with those discussed by Efron and Morris (1977) in the context of shrinkage with respect to ability evaluation in sport. It is particularly an issue in the context of the Daily Mail Trophy where the number of fixtures may vary considerably. This is recognised in the Daily Mail Trophy rules in two ways. First, the Additional Points methodology explicitly rewards teams playing more matches against sufficiently highly rated opponents. Second, it excludes registered teams from the league who fail to play at least five other registered teams. Note however that their results are still included in the tournament. This is to avoid the scenario where dramatic changes take place at the end of the season when it becomes apparent that some teams have not completed a sufficient number of matches.

An obvious way to consider such a concept in the context of these models is to apply a prior distribution to the ability parameters. According to Whelan and Schlobotnik (2018), this is an idea considered by Butler in the development of the KRACH model. In some scenarios, one might consider applying asymmetric priors based on, for example, previous seasons' results. This may be appropriate if we were seeking to use this as a pure prediction model, for example. Even then, given the large variation in ability that can exist from one season to the next in a school team, a result of the enforced turnover of players, then this may not be advisable. Since one of our expressed criteria is that we would like a method that is only dependent on the current season's results, and since the purpose is explicitly to give additional credit to teams playing more matches, then it is sensible here to apply a symmetric prior to all teams.

This may be achieved through the consideration of a dummy 'team 0', against which each team plays two matches. From one match they 'win' and gain a point and from the other they 'lose' and gain nothing. The weight of these matches may then be adjusted through the 'weight' parameter within the gnm function in R in order to give a greater or lesser weight to this prior. This effectively adds the same value to the 'score' s_i, as defined in section 3.1, associated with each of the π_i. As the weight increases, the degree to which this weight dominates the part of the 'score' derived from match results increases.

Including a prior has two other distinct advantages. One is that it ensures that the set of teams is connected, which addresses some of the second set of concerns highlighted in the introduction, though it might arguably exacerbate the first, namely that the model lacks transparency for a general audience. The second is that it enables us to ensure that we have a finite mean for the ability parameters, which in turn enables us to reinterpret our structural parameters as the more intuitive probabilities that we originally introduced.

The selection of a prior in the context of the Daily Mail Trophy is something that we look at in section 4.2, but it is useful to try to understand more exactly what we are attempting to achieve through a non-negligible prior. Suppose we have a set of teams of equal but above average ability. If the prior is too weak then one of these teams having played fewer games is likely to be ranked highest out of the group through natural variation making a 100% record more probable on a small number of matches than a large number. If, on the other hand, we make the prior too strong then a team having played a small number of matches will have much less ability to differentiate itself from the mean prior ability, which is below its natural ability, than a team of the same ability playing more matches. This is a particularly pertinent issue for schools rugby where some schools play rugby for only one academic term and others for two, and so the opportunity to play a higher number of matches is not equal.

3.8 Mean ability

Choosing to constrain our ability parameters by ascribing a mean ability of one is desirable as it allows us to give intuitive meaning to other parameters. We could do this in a number of ways. We will briefly look at two.

3.8.1 Arithmetic mean

One way would be to fit the model with no constraint and afterwards apply a scaling factor to achieve an arithmetic mean of 1. That is let μ be the arithemtic mean of the abilities π_i derived from the model

$$\mu = \frac{1}{m}\sum_{i}^{m}\pi_i$$

Then by setting $\pi'_i = \pi_i/\mu$ we have mean ability 1 for the π'_i. In order to preserve the correct probabilities having done this, we also need to consider our structural parameters $\kappa, \nu, \theta, \phi$ and τ. Substituting in $\mu\pi'_i$ we have for the result outcome non-normalised probabilities of

$$P(\text{team } i \text{ beats team } j \text{ by wide margin}) \propto \mu^4\tau^4{\pi'}_i^4$$
$$P(\text{team } i \text{ beats team } j \text{ by narrow margin}) \propto \mu^5\kappa\tau{\pi'}_i^4{\pi'}_j$$
$$P(\text{team } i \text{ draws with team } j) \propto \mu^4\nu{\pi'}_i^2{\pi'}_j^2$$
$$P(\text{team } j \text{ beats team } i \text{ by narrow margin}) \propto \mu^5\kappa\tau{\pi'}_i{\pi'}_j^4$$
$$P(\text{team } j \text{ beats team } i \text{ by wide margin}) \propto \mu^4{\pi'}_j^4$$

and for the try bonus outcome

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \mu^2\theta{\pi'}_i{\pi'}_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \mu{\pi'}_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \mu{\pi'}_j$$
$$P(\text{neither team gains try bonus point}) \propto \phi$$

Dividing through by μ^4 in the result outcome and by μ in the try outcome we can see that we can maintain the correct probabilities by taking our structural parameters as $\theta' = \mu\theta$, $\phi' = \phi/\mu$ and $\kappa' = \mu\kappa$, and that ν and τ remain unchanged.

3.8.2 A generalised mean

We may also set the mean ability to one by using a non-zero prior. Implementing in this way has an additional subtler methodological benefit as described by Firth (2018). Let us consider the projected points per match for a dummy 'team 0' that achieves one 'win' and one 'loss' against each other team in the tournament, as described in section 3.7. If zero points are awarded for a 'loss', and, without loss of generality, one point is awarded for a 'win' then

$$\text{PPPM}_0 = \frac{1}{m}\sum_{i=1}^{m}\frac{\pi_0}{\pi_0+\pi_i}$$

We are free to set the ability of π_0 as we wish, since it is not a real participant in the tournament and so we choose to set $\pi_0 = 1$. Intuitively since it has an equal winning and losing record against every team we might expect them to be the mean team and we have then set that ability to one as required. More formally we have that

$$\frac{1}{2} = \frac{1}{m}\sum_{i}^{m}\frac{1}{1+\pi_i}$$

and so rearranging we have

$$\frac{m}{\sum_{i}^{m}\frac{1}{1+\pi_i}} - 1 = 1$$

and by defining a generalised mean by the function on the left hand side of this equation we have our required mean of one for the ability parameters.

This is particularly beneficial in the context of a tournament such as the Daily Mail Trophy, because it is quite possible that a team will have achieved full points and so the ability parameter may be infinite. If this were the case then it would not be possible to achieve a mean of one using, for example, an arithmetic mean and scaling the other parameters appropriately, meaning we could no longer make our intuitive interpretations from the other parameters. This method will work for any non-zero weights given to these priors, and so even if we choose not to use a prior for the purposes of improving the representativeness of the model, we may set the prior weights arbitrarily small and benefit from this feature. We thus choose to use this method in the analysis below. In practical terms we may set this mean ability to 1 by excluding 'team 0' from the design matrix, which has the effect of setting $\delta_0 = 0$ and hence $\pi_0 = 1$.

3.9 Model selection

We have presented a number of alternatives for the model to be used. All these meet the crucial criterion of having League Points as a sufficient statistic. While these are explicitly retrodictive rather than predictive models, within the set of plausible models it is reasonable to look at their predictive ability in comparing them. We may also consider the sensitivity of ability parameters to the structural parameters and the stability of those. In order to assess the first of these it would be helpful to have a tournament where we may compare the intermediate ability parameter estimates to those at the end of a full round robin tournament. The Daily Mail Trophy, and schools rugby in general, does not support full round robin tournaments. For both purposes it would be helpful to have more than the three season's worth of clean results data that we have for the Daily Mail Trophy. And so we look at results from the English Premiership, the top league of professional English rugby union. Using the English Premiership allows us to use a well-validated data set stretching back a number of seasons for a rugby union tournament that exists in a round robin format. It also enables us to consider what sensible values for the structural parameters might be. Data for this have been sourced from http://en.espn.co.uk/premiership/rugby/series/index.html, scraped using code supplied by Alan Mizrahi (personal communication).

The use of a non-negligible prior is motivated primarily by differences in schedule size, particularly when those schedules may be against very different strength of opposition, and not closely connected. This is a feature of a tournament of 102 teams, where teams play only between five and twelve matches, often predominantly against teams who are close in terms of historic ability and geography. It is difficult therefore to replicate that from the Premiership data. We could investigate this by taking random sets of results from the Premiership, with consequently different numbers of matches for each team, and looking at the predictive ability based on those. We could make this more sophisticated by biasing the randomisation so that matches against similar strength teams are more likely to be chosen. We could also consider allowing the same match to be used multiple times within the ability estimation. However it would still seem that the purpose of using a non-negligible prior is particularly exposed to the differences in these data sets. We will therefore return to the use of a prior only in the context of the Daily Mail Trophy itself.

3.9.1 Parameter sensitivity and stability

One could reasonably expect that given the absence of major changes to the sport that the structural parameters of the model will be consistent from season to season, with these values becoming clearer when looked at over a multi-season period. We would also generally like to be able to set these a priori within the calculations of ability, since when dealing with the intraseason calculations it may be, for example, that there have been no draws and so the draw parameter may not achieve a value and we will thus not be able to

	Min	Max	Mean
κ	0.655	1.345	0.994
ν	0.032	0.232	0.172
ψ	0.116	0.522	0.281
τ	1.089	1.221	1.180

	Interpretation	Min	Max	Mean
τ	Home advantage	1.089	1.221	1.180
λ	P(narrow result)	0.389	0.554	0.471
ρ	P(draw)	0.010	0.056	0.041
ξ	P(team gains bonus)	0.104	0.343	0.212

Table 1: Summary of Model 1 structural parameter values for English Premiership 2010/11-2017/18

	Min	Max	Mean
κ	0.411	1.339	0.917
ν	0.032	0.233	0.172
θ	0.054	0.525	0.227
ϕ	1.880	6.687	4.707
τ	1.090	1.220	1.167

	Interpretation	Min	Max	Mean
τ	Home advantage	1.090	1.220	1.167
λ	P(narrow result)	0.269	0.551	0.444
ρ	P(draw)	0.010	0.076	0.043
ζ	P(single team bonus)	0.114	0.233	0.152
η	P(both gain bonus)	0.007	0.103	0.039

Table 2: Summary of Model 2 structural parameter values for English Premiership 2010/11-2017/18

calculate PPPM for the teams, or the results may be otherwise non-representative of reasonable expectations of these structural parameters. And so we begin by looking at the structural parameters derived by running the models on the last eight years of full season Premiership data. Prior weights are set to 0.000001. Tables 1 and 2 present a summary of these. Full results are to be found in the Appendix.

Both models show reasonable consistency across seasons. It is perhaps notable that the interpretable try bonus parameters, relating to the prevalence of try bonuses, have been increasing over time. Speculatively this may be associated with a transition in general towards more attacking open rugby. This might suggest that we should use try bonus parameters at the upper end of the historical range. However this is merely speculation and so we choose to use the mean of the last eight seasons for all the structural parameters.

We would now like to look at the sensitivity of the ability parameters to these structural parameters. Were the ability parameters to vary greatly based on these structural parameters then it would raise concerns about the model's capability in estimating relative ranking. In order to do this we need to use results from an intermediate point in the season. As has been noted previously it is not possible to choose a point in the Premiership season that would make it reasonably reflective of the Daily Mail Trophy in terms of both proportion of full round robin matches played as well as connectivity. However we may choose a point that balances not having too many matches played, which would diminish the value of having an intermediate ranking measure as the ranking converges to that given by League Points, and not having too few, which could mean that the measure could not reasonably be expected to work. We choose for this purpose to look at the tournament after the completion of five of the total of twenty-two rounds, giving us reasonable differences in schedule strength but enough matches to make reasonable assessments.

Here we show the results as applied to the 2018 season, with each parameter individually set to the maximum and minimum values seen over the eight season period and the abilities estimated for each. This is done for

Team	Mean	κ Min	Max	ν Min	Max	ψ Min	Max	τ Min	Max
Saracens	4.053	4.059	4.049	4.064	4.048	4.042	4.048	4.023	4.063
Northampton	3.923	3.916	3.929	3.934	3.918	3.940	3.918	3.932	3.915
Newcastle	3.896	3.897	3.895	3.907	3.891	3.894	3.891	3.905	3.890
Bath	3.502	3.501	3.503	3.514	3.496	3.507	3.496	3.534	3.484
Exeter	2.603	2.605	2.603	2.612	2.600	2.614	2.600	2.659	2.677
Leicester	2.598	2.592	2.601	2.606	2.594	2.595	2.594	2.468	2.574
Sale	2.133	2.134	2.131	2.136	2.131	2.126	2.131	2.213	2.101
Harlequins	1.796	1.800	1.794	1.795	1.797	1.790	1.797	1.782	1.797
Gloucester	1.731	1.730	1.731	1.728	1.732	1.735	1.732	1.576	1.792
London Irish	1.428	1.441	1.418	1.421	1.431	1.429	1.431	1.542	1.426
Wasps	1.406	1.397	1.414	1.399	1.409	1.400	1.409	1.417	1.359
Worcester	0.198	0.198	0.198	0.191	0.202	0.198	0.202	0.205	0.194

Table 3: Model 1 PPPM sensitivity to structural parameters, English Premiership 2017/18 Round5

Team	Mean	κ Min	Max	ν Min	Max	θ Min	Max	ϕ Min	Max	τ Min	Max
Saracens	4.046	4.057	4.041	4.056	4.041	4.060	4.035	4.069	4.037	4.020	4.059
Northampton	3.927	3.915	3.923	3.938	3.923	3.945	3.911	3.928	3.927	3.932	3.919
Newcastle	3.897	3.899	3.892	3.907	3.892	3.904	3.891	3.905	3.893	3.904	3.889
Bath	3.506	3.506	3.501	3.518	3.501	3.529	3.487	3.525	3.501	3.533	3.484
Leicester	2.609	2.598	2.605	2.618	2.605	2.615	2.609	2.601	2.613	2.660	2.674
Exeter	2.581	2.582	2.577	2.590	2.577	2.551	2.604	2.555	2.589	2.472	2.578
Sale	2.143	2.146	2.141	2.147	2.141	2.141	2.144	2.145	2.141	2.210	2.102
Harlequins	1.794	1.801	1.795	1.793	1.795	1.790	1.799	1.798	1.793	1.781	1.796
Gloucester	1.711	1.709	1.712	1.708	1.712	1.696	1.725	1.693	1.717	1.580	1.791
Wasps	1.429	1.446	1.432	1.422	1.432	1.430	1.427	1.450	1.437	1.545	1.419
London Irish	1.422	1.412	1.425	1.415	1.425	1.421	1.423	1.409	1.411	1.412	1.364
Worcester	0.199	0.200	0.203	0.192	0.203	0.199	0.199	0.200	0.199	0.205	0.194

Table 4: Model 2 PPPM sensitivity to structural parameters, English Premiership 2017/18 Round5

both Model 1 and Model 2. The results are shown in Tables 3 and 4 with the teams ordered by rankings based on projected points per match, PPPM, assessed with the non-perturbed structural parameters.

We see very minimal sensitivity to the structural parameters in either model, and no impact on ranking. While this is only a single season the magnitudes seen in these results and the lack of outliers strongly suggest that we would be likely to see the same were we to conduct the same exercise on other season's results. As such, these results demonstrate no reason to be concerned about using either Model 1 or Model 2. There is also no reason based on this to prefer one model over the other.

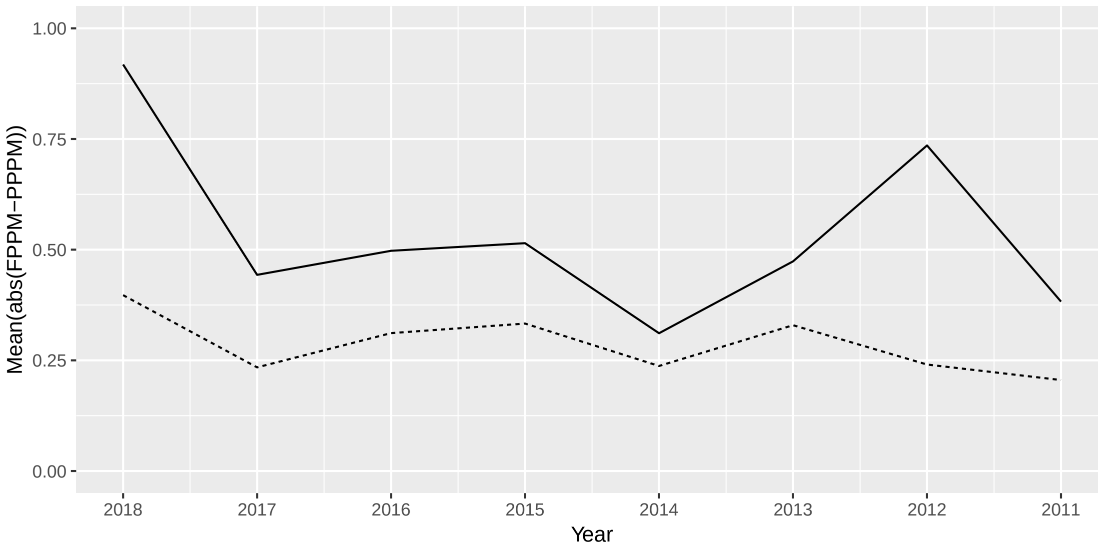

Figure 1: Mean of absolute value of the difference between FPPM and PPPM for English Premiership Round 5 2010/11 - 2017/18. Model 1 - solid line; Model 2 - dashed line

3.9.2 Predictive ability

Here we compare the projected points per match, PPPM, as estimated for each of the models under consideration, to the final points per match, FPPM, the points per match achieved at the end of the season, for each team. As before we will look at Round 5 results, and look at seasons 2010/11 to 2017/18.

Figure 1 shows a clear and consistent outperformance in the ability of Model 2 to achieve a projected points per match closer to the final points per match. Model 2 had a mean projection error of 0.29 points per match over the eight seasons. This is perhaps best understood as the equivalent of 6.3 League Points over the course of a twenty-two match season, with a maximum of five points available to a team in a single match. The mean difference between the two models was of a similar order, equating to 5.5 League Points over the season.

3.9.3 Summary

Neither model showed any notable sensitivity to the value of the structural parameters. However Model 2 showed a marked advantage compared to Model 1 when assessing based on its predictive ability. This is of an order that would seem to outweigh the attractiveness of the greater parsimony of Model 1.

4 Daily Mail Trophy

4.1 Data summary

The results for the Daily Mail Trophy have been supplied by SOCS, the organisation that administers the competition. The results are entered by the schools themselves, with, in almost all instances, a single entry for each match. The score is entered, and this is used to suggest a number of tries for each team which can then be amended. These inputs are not subject to any formal verification. This might suggest that data quality, especially as it relates to number of tries, may not be reliable. However the league tables are looked at keenly by players, coaches and parents, and corrections made where errors are found, and so data quality, especially at the top end of the table, is thought to be good. This analysis uses results from the three most recent seasons 2015/16 to 2017/18. There were 24 examples of inconsistencies or incompleteness that required assumptions to be made. All assumptions were checked with SOCS. Full details of these are given in the Appendix.

We begin by looking at the distribution of the result outcomes for the Daily Mail Trophy. In order to provide a comparison, we plot them above those for the Premiership in Figure 2. We see that in comparison to the Premiership result outcomes, there is a reduced home advantage and a reduced prevalence of narrow results, though the overall pattern of a higher proportion of wide than narrow results, and a low prevalence of draws is maintained. With respect to the try bonus outcomes, the notable difference is the higher prevalence of both teams gaining a bonus in the Premiership results with respect to all result outcomes.

4.2 Model selection

Noting our preference for Model 2 established in section 3.9, for now we continue looking at both models in the context of the Daily Mail Trophy results. We begin by looking at the parameter stability across the three seasons, and will move on to look at including a non-negligible prior and assess its impact.

In considering the stability of the structural parameters we take the same approach as we did with the English Premiership, looking at them over the available seasons' results. Tables 5 and 6 show no great variability in the structural parameters over the three seasons. Given the insensitivity to these parameters that we saw with respect to English Premiership results, we consider there to be no grounds here to have concerns with respect to these models. It can also be noted that the parameters are somewhat, though not dramatically, different to those seen in the English Premiership, with the home advantage and try bonus prevalence parameters slightly lower, as we might have expected from Figure 2.

As previously discussed there is limited scope with the Daily Mail Trophy data to compare the models based

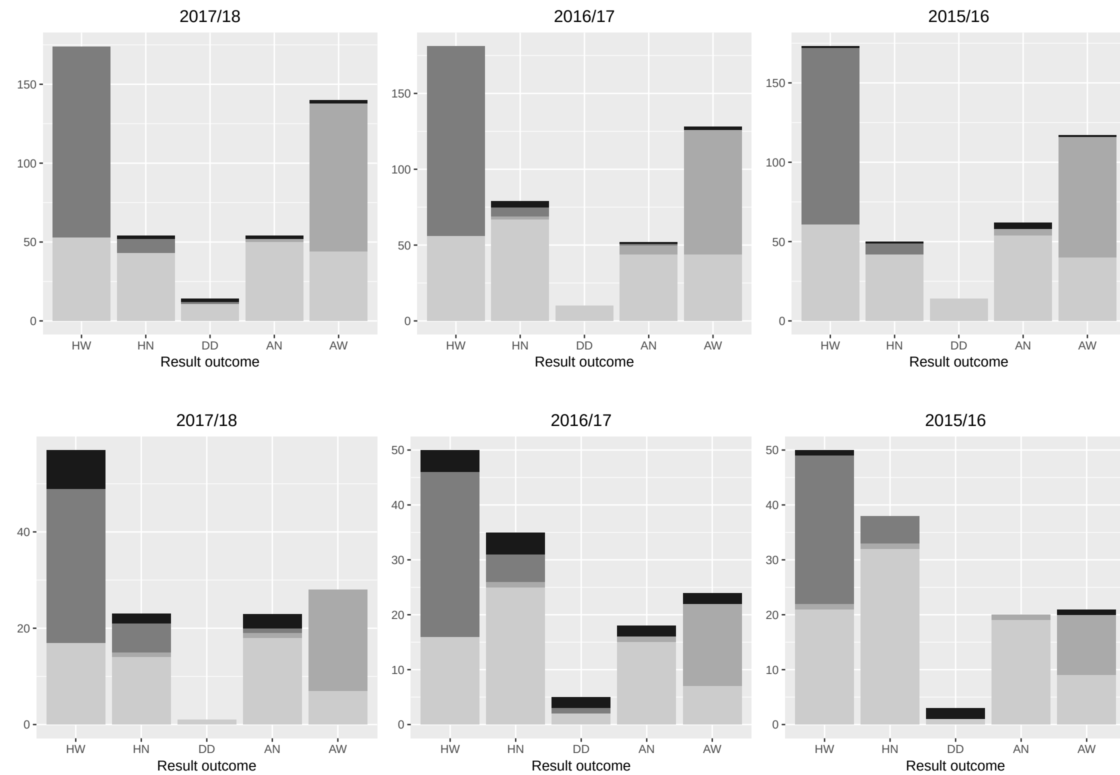

Figure 2: Distribution of outcomes for Daily Mail Trophy and English Premiership 2015/16 - 2017/18. Result outcome labels: HW - wide home win, HN - narrow home win, DD - Draw, AN - narrow away win, AW - wide away win; Shades indicate try bonus outcome, becoming lighter along scale: both bonus, home bonus, away bonus, zero bonus

	2017/18	2016/17	2015/16
κ	0.401	0.522	0.491
ν	0.207	0.182	0.242
ψ	0.274	0.234	0.267
τ	1.049	1.192	1.100

	Interpretation	2017/18	2016/17	2015/16
τ	Home advantage	1.049	1.192	1.100
λ	P(narrow result)	0.266	0.324	0.305
ρ	P(draw)	0.069	0.056	0.075
ξ	P(team gains bonus)	0.215	0.189	0.211

Table 5: Model 1 structural parameter values for Daily Mail Trophy 2015/16 - 2017/18

	2017/18	2016/17	2015/16
κ	0.399	0.520	0.490
ν	0.210	0.184	0.243
θ	0.043	0.036	0.052
ϕ	2.397	2.946	2.682
τ	1.049	1.190	1.100

	Interpretation	2017/18	2016/17	2015/16
τ	Home advantage	1.049	1.190	1.100
λ	P(narrow result)	0.265	0.323	0.304
ρ	P(draw)	0.070	0.057	0.075
ζ	P(single team bonus)	0.225	0.201	0.211
η	P(both gain bonus)	0.010	0.007	0.011

Table 6: Model 2 structural parameter values for Daily Mail Trophy 2015/16 - 2017/18

on their predictive capabilities, since it is not a round robin format. One could look at an earlier state in the tournament and compare to a later state where more information has become available, but such an approach is limited both by the number of matches that teams play (many play only five in total), by only having three season's worth of data on which to base it, and crucially by the fact that even in the later more informed state we have only an estimation of the ability as defined by League Points. In practice also, if we wished to maintain a temporal veracity to the analysis by choosing the first n games in the tournament to provide our projected points per match under our models, the lack of dates for the individual matches makes such an analysis impossible. We therefore move on to investigating the use of a prior without doing further analysis of this nature.

As discussed in section 3.7 the main aim of the use of a non-negligible prior is to reasonably account for the greater certainty we can have on a team's ability with the greater number of matches played. In this context therefore we provide details of the ranking produced using various priors and observe that against the relevant team's record (played, won, drawn, lost). Having found nothing in the stability in the structural parameters to deter us from doing so, we lean on our preference for Model 2 established in section 3.9 to focus just on that model from now on. We fix the structural parameters at the mean of the three season's values, namely $\kappa = 0.469$, $\nu = 0.212$, $\theta = 0.044$, $\phi = 2.675$, $\tau = 1.113$.

Looking at Figure 3 and comparing to the information in Table 7 we can see as we would expect that as we increase the prior weight that, in general, teams who have played fewer matches move lower, most notably Kingswood, and those who have played more move higher, most notably Sedbergh. This is not uniformly true with, for example, St Peter's moving higher despite having played relatively few matches and having a lower League Points per match score than either Kingswood or Northampton, who they overtake when prior weight is set to 8. Of course while the general pattern is clear and expected, the question of interest is

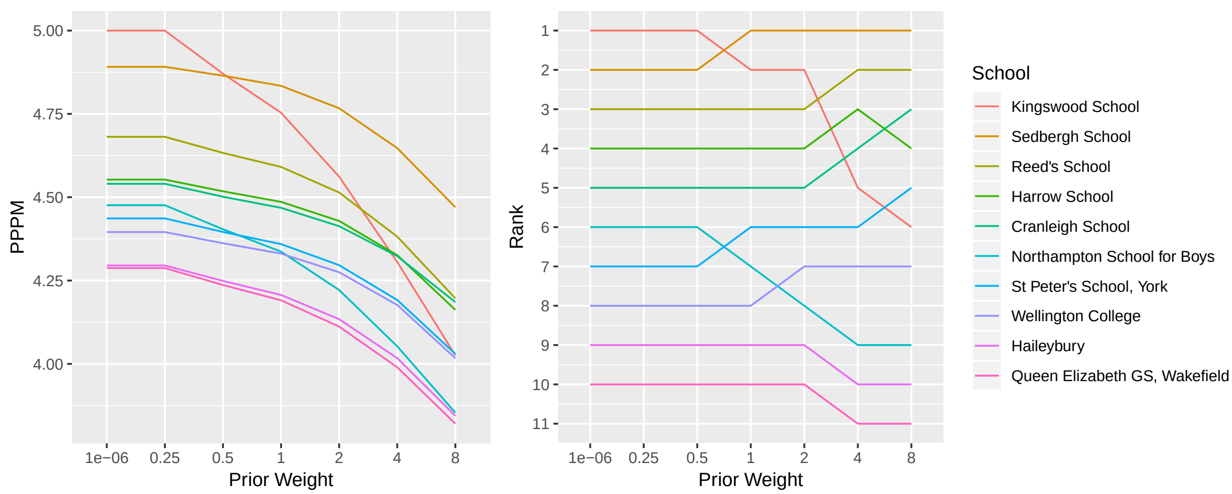

Figure 3: Top10 PPPM and Rank variation with prior weight for Daily Mail Trophy 2017/18

School	P	W	D	L	LPPM
Kingswood	4	4	0	0	5.00
Sedbergh	11	11	0	0	4.91
Reed's	10	10	0	0	4.80
Harrow	8	8	0	0	4.50
Cranleigh	8	8	0	0	4.63
Northampton	7	7	0	0	4.71
St Peter's, York	7	7	0	0	4.43
Wellington College	12	10	0	2	4.08
Haileybury	7	6	0	1	4.29
Queen Elizabeth Grammar	7	6	0	1	4.14

Table 7: Playing record for Top10 as ranked by Model 2, prior weight = 0.000001, for Daily Mail Trophy 2017/18. LPPM - League Points per match - the total number of League Points gained, including bonuses, divided by number of matches; P - Played, W - Win, D - draw, L - loss

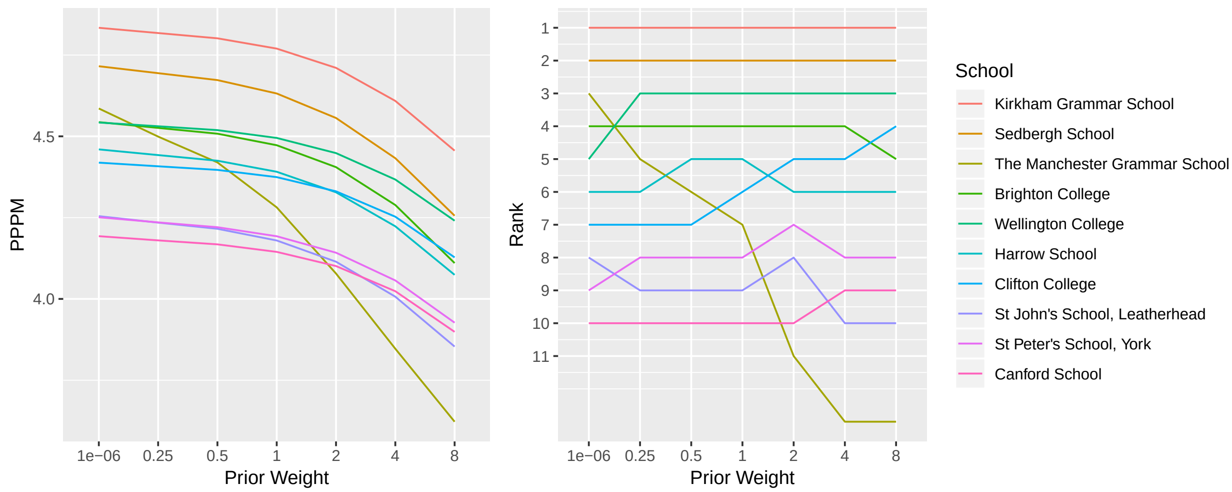

Figure 4: Top10 PPPM and Rank variation with prior weight for Daily Mail Trophy 2016/17

School	P	W	D	L	LPPM
Kirkham Grammar	12	12	0	0	4.75
Sedbergh	10	9	0	1	4.5
The Manchester Grammar	4	4	0	0	4.75
Brighton College	8	8	0	0	4.5
Wellington College	12	11	0	1	4.42
Harrow	9	8	0	1	4.44
Clifton College	10	9	0	1	4.5
St John's School, Leatherhead	9	7	0	2	3.89
St Peter's, York	9	9	0	0	4.33
Canford	10	9	0	1	4.2

Table 8: Playing record for Top10 as ranked by Model 2, prior weight = 0.000001, for Daily Mail Trophy 2016/17. LPPM - League Points per match - the total number of League Points gained, including bonuses, divided by number of matches; P - Played, W - Win, D - draw, L - loss

what absolute size for the prior should we choose. It seems somewhat reasonable to state that a team with a 100% winning record from four matches should not generally be ranked higher than a team with a 100% wining record from eight matches, assuming their schedule strength is not notably different. It certainly seems unlikely that all of the six other teams with 100% winning records below Kingswood should be ranked lower than them, which would imply a prior weight of at least one and more likely 4 or higher. To investigate further we will consider the other two seasons in the same way.

Looking at Figures 4 and 5 and comparing them to their respective leagues in Tables 8 and 9, we see, as we did before, that we need a prior on the larger end of our scale before teams playing a smaller number of matches are sufficiently penalised. Looking at the 2017/18 and 2015/16 seasons and the ranking of Kingswood School and Stockport Grammar School respectively, in particular, might suggest that of our tested priors, 4 or 8 would be most appropriate.

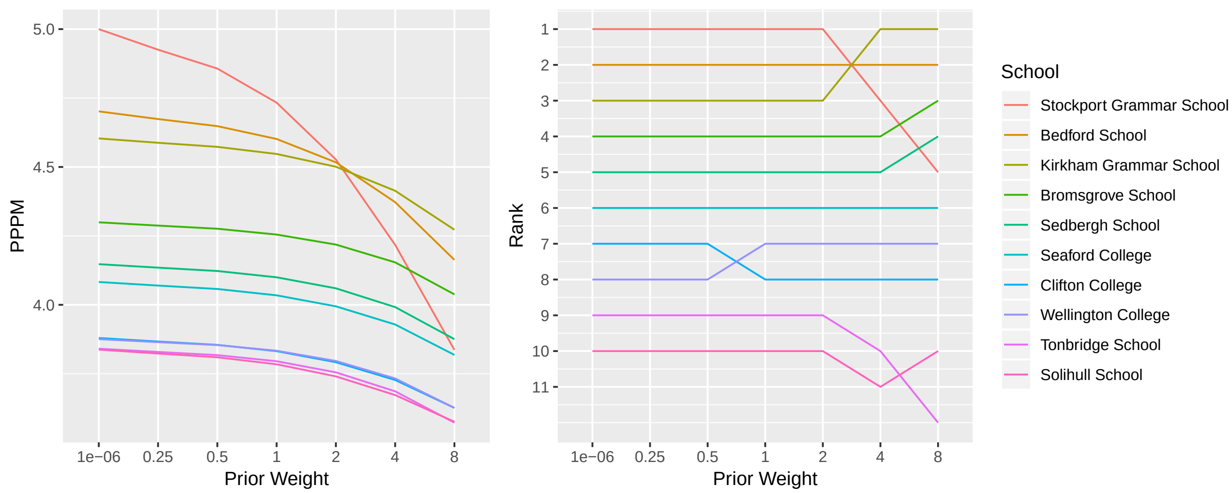

Figure 5: Top10 PPPM and Rank variation with prior weight for Daily Mail Trophy 2015/16

School	P	W	D	L	LPPM
Stockport Grammar	4	4	0	0	5
Bedford	8	8	0	0	4.75
Kirkham Grammar	11	11	0	0	4.64
Bromsgrove	9	8	1	0	4.11
Sedbergh	10	7	1	2	3.70
Seaford College	7	6	0	1	4.14
Clifton College	9	7	2	0	4.11
Wellington College	13	9	0	4	3.46
Tonbridge	9	7	0	2	3.44
Solihull	10	9	0	1	4.10

Table 9: Playing record for Top10 as ranked by Model 2, prior weight = 0.000001, for Daily Mail Trophy 2015/16. LPPM - League Points per match - the total number of League Points gained, including bonuses, divided by number of matches; P - Played, W - Win, D - draw, L - loss

An argument against this assertion might be that under current Daily Mail Trophy rules, teams playing fewer than five matches are excluded from the league table. In these two seasons Kingswood School and Stockport Grammar School therefore did not appear in the final Daily Mail Trophy league table. This rule could continue to be used to deal with cases of teams playing low numbers of matches rather than relying on the prior to do the job entirely.

On the other hand one can credibly argue that a robust ranking model should be able to deal with all result outcomes without an arbitrary inclusion cut off. It is also reasonable to assert that there is still useful information from these teams for the calibration of the model, whether they are included or not in the final table. With this in mind it seems sensible to select a prior of 4 or 8 from the values presented here. In deciding which of these to choose, we should consider the implications. If we look at effects other than those due to the re-ranking of Kingswood School and Stockport Grammar School from a change of prior from 4 to 8, then we see only that Harrow and Cranleigh swap in 2017/18, Clifton and Brighton in 2016/17 and Solihull and Tonbridge in 2015/16. It is not possible to say that either of these alternative rankings is definitively right in any of these three cases. In all these cases the projected points per match of the two teams remain very similar, and both alternatives would pass the criterion that we expressed at the outset that a ranking methodology should be such that all other relative rankings should not be perceivable as unreasonable by a large proportion of the tournament stakeholders. Likewise using either a prior weight of 4 or of 8 would meet our other important criterion that the ranking methodology should be such that the top-ranked team should not be obviously wrong in the opinion of a large proportion of stakeholders in the tournament.

It is therefore a very marginal decision given this evidence as to which one to use. It is worth briefly noting the intuition of what these prior weight values mean in the present context. Given that we award one point for a win against 'team 0' and that a team earns four points for a win under the League Points scoring rule, then a weighting of 4 effectively means that each team has won and lost a full match against our hypothetical mean ability team, with no losing or try bonuses for either side. For the purposes of further analysis we will use a prior weight of 4, leaning on the results of the 2015/16 tournament, and looking at the relative ranking of Stockport Grammar as set against Bromsgrove and Sedbergh. We consider that a record of four wins all with try bonuses should be considered superior to one of eight wins and a draw with only three try bonuses, or a record of seven wins, a draw and two losses, though of course such a statement takes no account of the relative opposition.

4.3 Comparison of scoring rule

We will therefore compare the current Daily Mail Trophy methodology with our Model 2 using a prior weight of 4. We look at this in two ways. We consider the effective adjustment to League Points per match provided

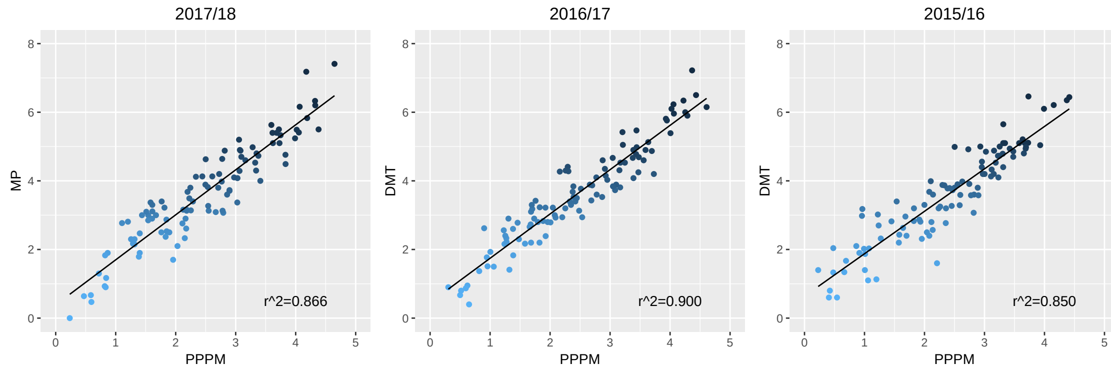

Figure 6: Scatterplot of Model 2 (prior weight = 4) ranking measure, PPPM, on x-axis, against Daily Mail Trophy (DMT) ranking measure, MP, on y-axis. Darker colours represent higher rank in Daily Mail Trophy. Top three teams as ranked by Daily Mail Trophy labeled.

by the Daily Mail Trophy method and by our model for the general population of all qualified teams, and second, noting the emphasis on the greater importance of ranking at the top of the table, we will inspect the top ten comparatively directly. We will look at each of these over the three seasons of available data. In the first part, in line with the Daily Mail Trophy itself, we will consider the matches with non-ranked teams in the creation of our Model 2 measure, but will not include them in the ranking.

4.3.1 General population

We might first consider the current Daily Mail Trophy ranking measure, Merit Points, against that from our model, the projected points per match, PPPM. As we see in Figure 6 there is at least broad agreement between the two measures. However this is not a particularly helpful way to look at the quality of the Daily Mail Trophy methodology, as a very large proportion of this can be put down to the base scoring rule of League Points per match, LPPM, which they both essentially have in common. A better way of making the comparison as to the effectiveness of the Daily Mail Trophy methodology in accounting for schedule strength, is to look at their implied adjustment to League Points per match as we do in Figure 7.

Here we see clear differences and a low correlation between the measures. Perhaps not surprisingly, some of the teams who perform well in the Daily Mail Trophy rankings seem to be those that are benefiting most from these differences, with Wellington College in particular, winner in two of the last three seasons, being a serial outlier in this regard. While this is concerning in its own right, the requirements on the measure are related almost solely to the ranking that they produce. And so we look at that ranking itself in Figure 8. Here we do indeed see considerable differences between the rankings produced by the two different methodologies. In order to focus more clearly on that difference we may look at this difference plotted against the Daily Mail

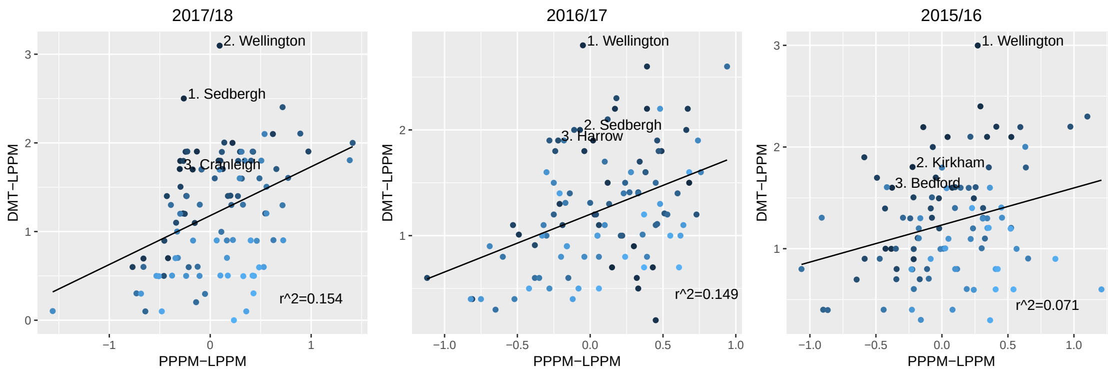

Figure 7: Scatterplot of points adjustment to League Points per match. Adjustment due to Model 2 (prior weight = 4), equal to PPPM-LPPM, on x-axis. Adjustment due to Daily Mail Trophy (DMT) methodology, MP-LPPM, on y-axis. Darker colours represent higher rank in Daily Mail Trophy ranking. Top three teams as ranked by Daily Mail Trophy labeled.

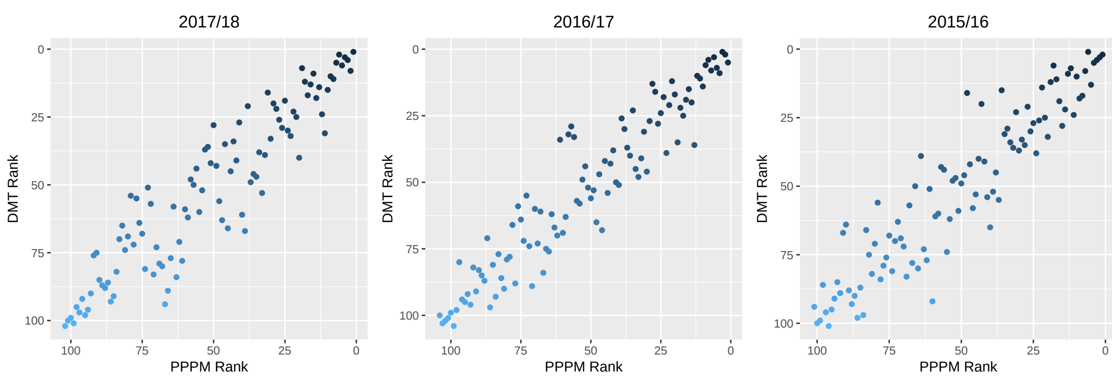

Figure 8: Scatterplot of Model 2 (prior weight = 4) rank on x-axis, against Daily Mail Trophy (DMT) rank on y-axis. Darker colours represent higher rank in Daily Mail Trophy.

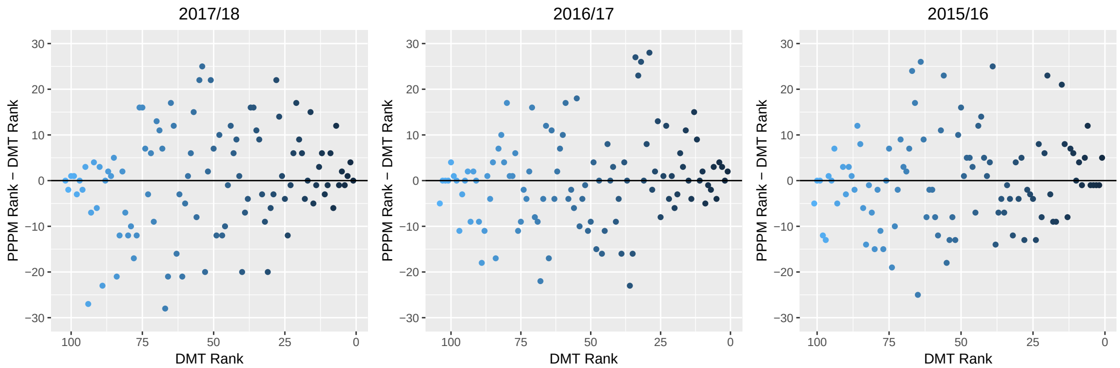

Figure 9: Scatterplot of Daily Mail Trophy rank on x-axis, against the gain in rank from Daily Mail Trophy methodology vs Model2 (prior weight = 4). Darker colours represent higher rank in Daily Mail Trophy.

Trophy rank. As we might expect, looking at Figure 9, if we consider the top (and bottom) quintile of the Daily Mail Trophy ranking we see a disproportionately positive (and negative) impact from the Additional Points adjustment of the Daily mail Trophy methodology. What is perhaps more notable is the size of some of these rank differences, up to 28 places in a tournament of approximately one hundred teams. Looked at across the population the mean absolute difference in rank is approximately eight. If we look at the typical difference in points between two teams eight places apart then we see this to be worth approximately 0.4 points per match, with some variation over year and methodology.

It seems reasonable therefore to say that over the general population of teams there is empirical cause for concern for the reasonableness of the Daily Mail Trophy methodology in its approach to adjusting for schedule strength.

4.3.2 Top teams

Here we simply look at the top ten teams for each season ranked as per the Daily Mail Trophy methodology and by Model 2 with a prior weight of 4.

Some things become immediately apparent. On the positive side, ignoring non-ranked teams, the top five always appear within the top ten of the other ranking methodology, and the top ten always within the top twenty. On the other hand they only agree on the first placed team in one of the three seasons, and in the two seasons where they differ, looking at Figures 4 and 5, there seems little evidence to support the team selected by the Daily Mail Trophy Merit Points methodology. In particular, in 2015/16 Wellington College, who were the winners of the tournament, are ranked seventh under our selected model and were a full 0.68 points behind the leader.

School	DMT Rank	DMT	PPPM Rank	PPPM
Sedbergh	1	7.41	1	4.65
Wellington College	2	7.18	7	4.18
Cranleigh	3	6.33	4	4.32
Harrow	4	6.20	3	4.33
Cheltenham College	5	6.16	8	4.07
St Peter's, York	6	5.83	6	4.19
Brighton College	7	5.63	20	3.59
Reed's	8	5.50	2	4.38
Clifton College	8	5.50	16	3.72
Haileybury	10	5.49	10	4.02

School	PPPM Rank	PPPM	DMT Rank	DMT
Sedbergh	1	4.65	1	7.41
Reed's	2	4.38	8	5.50
Harrow	3	4.33	4	6.20
Cranleigh	4	4.32	3	6.33
Kingswood	5	4.31	NR	NR
St Peter's, York	6	4.19	6	5.83
Wellington College	7	4.18	2	7.18
Cheltenham College	8	4.07	5	6.16
Northampton	9	4.05	11	5.41
Haileybury	10	4.02	10	5.49

Table 10: 2017/18: Top 10 by Daily Mail Trophy Merit Points and Model 2 projected points per match. NR - not ranked

School	DMT Rank	DMT	PPPM Rank	PPPM
Wellington College	1	7.22	3	4.37
Sedbergh	2	6.50	2	4.43
Harrow	3	6.34	6	4.22
St Peter's, York	4	6.23	8	4.06
Kirkham	5	6.15	1	4.61
Canford	6	6.10	9	4.02
Clifton College	7	6.00	5	4.25
Rugby	8	5.96	7	4.06
Brighton College	9	5.90	4	4.29
Woodhouse Grove	10	5.81	12	3.93

School	PPPM Rank	PPPM	DMT Rank	DMT
Kirkham Grammar	1	4.61	5	6.15
Sedbergh	2	4.43	2	6.50
Wellington College	3	4.37	1	7.22
Brighton College	4	4.29	9	5.90
Clifton College	5	4.25	7	6.00
Harrow	6	4.22	3	6.34
Rugby	7	4.06	8	5.96
St Peter's, York	8	4.06	4	6.23
Canford	9	4.02	6	6.10
St John's, Leatherhead	10	4.01	14	5.39

Table 11: 2016/17: Top 10 by Daily Mail Trophy Merit Points and Model 2 projected points per match

School	DMT Rank	DMT	PPPM Rank	PPPM
Wellington College	1	6.46	7	3.73
Kirkham	2	6.44	1	4.41
Bedford	3	6.35	2	4.37
Bromsgrove	4	6.21	4	4.15
Sedbergh	5	6.10	5	3.99
Woodhouse Grove	6	5.65	19	3.31
Millfield	7	5.21	13	3.64
Clifton College	8	5.11	8	3.73
Solihull	9	5.10	11	3.67
St Paul's	9	5.10	14	3.58

School	PPPM Rank	PPPM	DMT Rank	DMT
Kirkham Grammar	1	4.41	2	6.44
Bedford	2	4.37	3	6.35
Stockport Grammar	3	4.22	NR	NR
Bromsgrove	4	4.15	4	6.21
Sedbergh	5	3.99	5	6.10
Seaford College	6	3.93	13	5.04
Wellington College	7	3.73	1	6.46
Clifton College	8	3.73	8	5.11
Queen Elizabeth Grammar	9	3.69	17	4.98
Tonbridge	10	3.69	18	4.94

Table 12: 2015/16: Top 10 by Daily Mail Trophy Merit Points and Model 2 projected points per match. NR - not ranked

4.4 Summary

The agreement at the top end of the table, as seen by the consistency with which teams appear at the top end of each other's ranking, is perhaps more reassuring than might have been feared based on some of the more hypothetical possibilities of the Daily Mail Trophy scoring rule, as described in the introduction. However given the elaborated criteria for a scoring rule for this tournament, the poor levels of agreement between our model and the Daily Mail Trophy methodology on the adjustment to the League Points, and the inability of our model to justify the winner of the tournament in two of the three years, there is strong evidence for believing that a better scoring rule could be developed.

5 Concluding Remarks

We have shown in this report how the standard win, draw, loss elaboration of the Bradley-Terry model may be extended to account for the five result outcomes of the standard rugby union scoring rules used in the UK. This has been extended further to take account of the try bonus outcome, giving us a set of retrodictive models where the League Points gained represent a sufficient statistic for the ability of the teams. We have investigated two of these models further and made conclusions about their relative merit. We have seen that in the context of the English Premiership there are sufficient grounds for the maintenance of an additional parameter and so have preferred the model with a try bonus parametrisation dependent on the strength of opposition. We have gone on to investigate the use of a prior and have preferred a model using a prior weight of 4. We have then used that to evaluate the current scoring rule used in the Daily Mail Trophy. We have found strong grounds to believe the Daily Mail Trophy Merit Points scoring model can be substantially improved.

5.1 Methodology

In a sense the preferred model has met the brief exactly, providing a ranking measure where League Points are a sufficient statistic. On a broader view with respect to the Daily Mail Trophy there may be grounds for more caution. If our base point is not the standard league scoring rule but the rather more amorphous generalised assessment of tournament stakeholders, e.g. organisers, coaches, players, parents, as to the relative merit of different teams given their results then it is less clear. Partly this is because this is necessarily a poorly defined target, with individual stakeholders likely to hold differing views, and our ability to discern those views limited. However there are also some technical limitations in this regard. A classic critique of the Bradley-Terry model in the win, loss scenario is that it requires the assumption that if the odds that Team

A beats Team B are α and the odds that Team B beats Team C are β then the odds that Team A beats Team C are $\alpha\beta$, and that this is not necessarily realistic. There is not such a complete expression of this transitive property with the five result outcomes of rugby union. The property does hold for a team winning by a wide margin conditional on the result being wide. Or alternatively one could argue that there is an approximate transitivity if one is prepared to ignore the possibility of draws, which are in any case rare, and to note that the consequent odds of team i beating team j are $\frac{\pi_i^4(1+\pi_j)}{\pi_j^4(1+\pi_i)} \approx \frac{\pi_i^4}{\pi_j^4}$. In any case there is an imposed transitive-like structure that reasonable observers could consider was misrepresentative. With respect to the try bonus point, our preferred model had the try bonus probabilities dependent on the ability of the opposition but not the result. This is perhaps not hard to criticise from the perspective of real world intuition, where it seems reasonable to expect that the scoring of a try bonus is likely to be meaningfully dependent on the result outcome, indeed Figure 2 suggests so. Alternatively it could be criticised from the other direction. We have used two parameters in our try bonus model, the same as our result outcome, despite the try bonus being responsible for accruing only approximately a quarter of the points. Additionally were the model to be used as the scoring measure itself then it lacks transparency for a general audience.

Setting aside issues of transparency, the selection of the prior would provide an interesting area for further investigation. Here we have demonstrated the sensitivity to the selection of the prior for a range of 0.000001 to 8 under our model, as applied to the top ten. As well as investigating a more granular level of priors in the 4 to 10 range, there is more that can be done here. What we would like to determine is what level of prior would reasonably balance out the potential advantage from a low prior to a team with few fixtures with the advantage from a high prior to a team with a high number of fixtures, expressed in section 3.7. A sensible place to start investigating this would be to use two teams of the same or very similar underlying ability but playing different numbers of matches, and find a prior that ranks them equivalently. One possible methodology for doing this would be to simulate a tournament based on assigned abilities and according to the probabilities defined in the model, and have some teams of the same ability play fewer matches, and then seek a prior that would reasonably match their rankings. Alternatively we could try to do something similar with the existing results set, finding pairs of teams for which one has a fixture list that is largely a subset of the other's. If their performances against these common teams are similar then we may assume that they are of similar ability and we can look at how they are ranked differently based on the extra matches that one team plays. If enough examples of this can be found, or reasonably concocted, perhaps by simply ignoring some of the matches played by one team, then we could possibly draw more meaningful conclusions about the prior. Speculatively it seems likely that such a 'balancing' prior would vary both with the differences in the number of fixtures and with the gap of the ability of the group of teams being considered from the mean ability. In the context of the Daily Mail Trophy, we would likely need to lean on the greater weight being placed on the ranking to be good at the top of the table, rather than the bottom or middle, in our selection of an appropriate prior.

This concept of finding a balancing prior is strongly related to that of Stein's paradox as set out by Efron and Morris (1977) but it would be interesting to investigate whether there are any theoretical subtleties when applied in a retrodictive context and with respect to this scoring rule.

5.2 Daily Mail Trophy

There are limitations to the analysis provided here. Some of this is related to the available data. We currently have only three seasons of reliable results, and there is no tournament of a comparable number of competing teams in a full round robin format against which we might compare the models. We do not have the dates of the matches and so we have been unable to inspect the point during the season at which the participating teams become a connected set.

Other limitations are related to the scope of this particular investigation. Further investigations with respect to the Daily Mail Trophy might include an assessment of the five and eight parameter try bonus models of section 3.2.3, the self-reinforcing nature of the reliance on previous season's results to better critique the current scoring rule, and the potential existence of highly influential matches within our model. It might also be interesting to look in greater detail at the home advantage parametrisation. Since spectator attendance at schools rugby is not of the order of professional leagues, one might speculate that the predominant factor in any home advantage is distance travelled, in which case the home advantage parameter could be modelled as a function of this distance, potentially even with a single parameter, and so without dramatically expanding the complexity of the model. Ultimately we might seek to propose an alternative scoring rule that showed better agreement with the Bradley-Terry based model, but was of a more intuitive nature for the generalist audience. A potentially persuasive way to determine this might be to create a carefully chosen set of hypothetical results and ask stakeholders how they think teams should be ranked based on those, with different rankings implying different measures or model calibrations. We may also wish to repeat analyses as more data become available with time.

5.3 Other tournaments

With respect to rugby scoring rules more generally, it would be of interest to look at modeling other tournaments. Some of these have different bonus structures. Several have a losing bonus awarded to teams within five rather than seven points, which will not radically change the model but will give a slightly different interpretation to the structural parameters. The French Top14 has a try bonus which is only awarded to the winning team and then only if they score three more tries than their opposition. The format of the Pro14, which is the premier competition for clubs from Ireland, Italy, Scotland, and Wales, and also includes two teams from South Africa, makes it a particularly interesting subject for an investigation of this nature. It is formatted into two conferences with matches played across those, and uneven numbers of matches between teams to allow for more games against local rivals. There are also expansive European tournaments in rugby union, including the majority of teams in the the English Premiership, the French Top14, and the Pro14. This may make the creation of a Europe wide ranking an interesting and plausible project.

Outside of rugby union, the general framework for defining a model to meet the condition of a sufficient statistic in line with a league scoring rule, and the integration of bonus points, may be applied to other sports. One obvious candidate might be County Championship cricket, where there is a more complex bonus point structure, with up to five batting and three bowling bonus points available to each team, and a four level result outcome that includes draws and ties as separate outcomes. As well as giving a more informed in-season ranking, such a model may also provide an alternative treatment for matches rained off completely due to the vagaries of the English summer.

6 Appendix

6.1 Further details of other try bonus models

Throughout we use here the notation introduced in section 3.4, where the subscript ij defines the home and away teams.

6.1.1 Independent of result and opposition

Define $\psi = \frac{\xi}{(1-\xi)}$ then the non-normalised probabilities become

$$
\begin{aligned}
P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) &\propto \psi^2 \pi_i \pi_j \\
P(\text{only team } i \text{ gains try bonus point}) &\propto \psi \tau \pi_i \\
P(\text{only team } j \text{ gains try bonus point}) &\propto \frac{\psi \pi_j}{\tau} \\
P(\text{neither team gains try bonus point}) &\propto 1
\end{aligned}
$$

and we have likelihood

$$L(\boldsymbol{\pi},\kappa,\nu,\theta,\phi,\tau;W,N,D,BB,B,Z)=\prod_{i=1}^{m}\prod_{j=1}^{m}\frac{(\tau^4\pi_i^4)^{hw_{ij}}(\kappa\tau^3\pi_i^4\pi_j)^{hn_{ij}}(\nu\pi_i^2\pi_j^2)^{d_{ij}}(\frac{\kappa\pi_i\pi_j^4}{\tau^3})^{an_{ij}}(\frac{\pi_j^4}{\tau^4})^{aw_{ij}}}{(\tau^4\pi^4+\kappa\tau^3\pi^4\pi_j+\nu\pi_i^2\pi_j^2+\frac{\kappa\pi_i\pi_j^4}{\tau^3}+\frac{\pi_j^4}{\tau^4})^{r_{ij}}}\frac{(\psi^2\pi_i\pi_j)^{bb_{ij}}(\psi\tau\pi_i)^{hb_{ij}}(\frac{\psi\pi_j}{\tau})^{ab_{ij}}}{(\psi^2\pi_i\pi_j+\psi\tau\pi_i+\frac{\psi\pi_j}{\tau}+1)^{r_{ij}}}$$

Then defining total number of try bonus points $t=\sum_i\sum_j(2bb_{ij}+hb_{ij}+ab_{ji})$ we have

$$L(\boldsymbol{\pi},\kappa,\nu,\psi,\tau;W,N,D,BB,B)=\frac{\kappa^n\nu^d\psi^t\tau^h\prod_{k=1}^{m}\pi_k^{s_k}}{\prod_{i=1}^{m}\prod_{j=1}^{m}(\tau^4\pi_i^4+\kappa\tau^3\pi_i^4\pi_j+\nu\pi_i^2\pi_j^2+\frac{\kappa\pi_i\pi_j^4}{\tau^3}+\frac{\pi_j^4}{\tau^4})^{r_{ij}}(\psi^2\pi_i\pi_j+\psi\tau\pi_i+\frac{\psi\pi_j}{\tau}+1)^{r_{ij}}}$$

and the statistic $(\boldsymbol{s},n,d,t,h)$ is a sufficient statistic for $(\boldsymbol{\pi},\kappa,\nu,\psi,\tau)$.

For the loglinear representation, $\alpha_{ijk\cdot}$ is defined as before and

$$\alpha_{ij\cdot l}=\begin{cases}\delta_i+\delta_j+2\gamma_{tb} & \text{if both home and away try bonuses}\\ \delta_i+\gamma_{tb}+\sigma & \text{if home try bonus only}\\ \delta_j+\gamma_{tb}-\sigma & \text{if away try bonus only}\\ 0 & \text{if no try bonus for either side}\end{cases}$$

where $\psi=\exp(\gamma_{tb})$.

6.1.2 Dependent on result and opposition - Eight parameters

Let us consider first the model with eight parameters i.e. where try bonus probabilities are considered to be dependent on wide result, narrow result or draw. In addition to the notation defined already we define

$$\varphi_{wb}=\frac{\eta_w}{1-\eta_w-\zeta_{ww}-\zeta_{wl}}\quad\varphi_{ww}=\frac{\zeta_{ww}}{1-\eta_w-\zeta_{ww}-\zeta_{wl}}\quad\varphi_{wl}=\frac{\zeta_{wl}}{1-\eta_w-\zeta_{ww}-\zeta_{wl}}$$

$$\varphi_{nb}=\frac{\eta_n}{1-\eta_n-\zeta_{nw}-\zeta_{nl}}\quad\varphi_{nw}=\frac{\zeta_{nw}}{1-\eta_n-\zeta_{nw}-\zeta_{nl}}\quad\varphi_{nl}=\frac{\zeta_{nl}}{1-\eta_n-\zeta_{nw}-\zeta_{nl}}$$

$$\varphi_{db} = \frac{\eta_d}{\zeta_d} \quad \varphi_{dz} = \frac{1 - \eta_d - 2\zeta_d}{\zeta_d}$$

where the first letter of the subscript indicates the result outcome (wide, narrow, draw) and the second the try bonus outcome (both, winner, loser, zero).

With these we may define the twenty individual (normalised) outcome probabilities

$$P(\text{home wide win, both bonus}) = \frac{\tau^4\pi_i^4}{A}\frac{\varphi_{wb}\pi_i\pi_j}{B}$$
$$P(\text{home wide win, home bonus}) = \frac{\tau^4\pi_i^4}{A}\frac{\varphi_{ww}\tau\pi_i}{B}$$
$$P(\text{home wide win, away bonus}) = \frac{\tau^4\pi_i^4}{A}\frac{\varphi_{wl}\pi_j/\tau}{B}$$
$$P(\text{home wide win, zero bonus}) = \frac{\tau^4\pi_i^4}{A}\frac{1}{B}$$

$$P(\text{home narrow win, both bonus}) = \frac{\kappa\tau^3\pi_i^4\pi_j}{A}\frac{\varphi_{nb}\pi_i\pi_j}{C}$$
$$P(\text{home narrow win, home bonus}) = \frac{\kappa\tau^3\pi_i^4\pi_j}{A}\frac{\varphi_{nw}\tau\pi_i}{C}$$
$$P(\text{home narrow win, away bonus}) = \frac{\kappa\tau^3\pi_i^4\pi_j}{A}\frac{\varphi_{nl}\pi_j/\tau}{C}$$
$$P(\text{home narrow win, zero bonus}) = \frac{\kappa\tau^3\pi_i^4\pi_j}{A}\frac{1}{C}$$

$$P(\text{draw, both bonus}) = \frac{\nu\pi_i^2\pi_j^2}{A}\frac{\varphi_{db}\pi_i\pi_j}{D}$$
$$P(\text{draw, home bonus}) = \frac{\nu\pi_i^2\pi_j^2}{A}\frac{\tau\pi_i}{D}$$
$$P(\text{draw, away bonus}) = \frac{\nu\pi_i^2\pi_j^2}{A}\frac{\pi_j/\tau}{D}$$
$$P(\text{draw, zero bonus}) = \frac{\nu\pi_i^2\pi_j^2}{A}\frac{\varphi_{dz}}{D}$$

$$P(\text{away narrow win, both bonus}) = \frac{\kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\varphi_{nb}\pi_i\pi_j}{E}$$
$$P(\text{away narrow win, home bonus}) = \frac{\kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\varphi_{nl}\tau\pi_i}{E}$$
$$P(\text{away narrow win, away bonus}) = \frac{\kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\varphi_{nw}\pi_j/\tau}{E}$$
$$P(\text{away narrow win, zero bonus}) = \frac{\kappa\pi_i\pi_j^4/\tau^3}{A}\frac{1}{E}$$

$$P(\text{away wide win, both bonus}) = \frac{\pi_j^4/\tau^4}{A}\frac{\varphi_{wb}\pi_i\pi_j}{F}$$
$$P(\text{away wide win, home bonus}) = \frac{\pi_j^4/\tau^4}{A}\frac{\varphi_{wl}\pi_j/\tau}{F}$$
$$P(\text{away wide win, away bonus}) = \frac{\pi_j^4/\tau^4}{A}\frac{\varphi_{ww}\tau\pi_i}{F}$$
$$P(\text{away wide win, zero bonus}) = \frac{\pi_j^4/\tau^4}{A}\frac{1}{F}$$

where

$$A = \tau^4\pi_i^4 + \kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \frac{\kappa\pi_i\pi_j^4}{\tau^3} + \frac{\pi_j^4}{\tau^4}$$

is the standard normalisation for the result outcome, and

$$B = \varphi_{wb}\pi_i\pi_j + \varphi_{ww}\tau\pi_i + \varphi_{wl}\pi_j/\tau + 1$$
$$C = \varphi_{nb}\pi_i\pi_j + \varphi_{nw}\tau\pi_i + \varphi_{nl}\pi_j/\tau + 1$$
$$D = \varphi_{db}\pi_i\pi_j + \tau\pi_i + \pi_j/\tau + \varphi_{dz}$$
$$E = \varphi_{nb}\pi_i\pi_j + \varphi_{nl}\tau\pi_i + \varphi_{nw}\pi_j/\tau + 1$$
$$F = \varphi_{wb}\pi_i\pi_j + \varphi_{wl}\tau\pi_i + \varphi_{ww}\pi_j/\tau + 1$$

are the normalisations for the conditional probabilities relating to the five result outcomes

Consistent with the notation used in the definitions in section 3.4, allow $x.y_{ij}$ to be the composite outcome of result outcome x_{ij} and try bonus outcome y_{ij} from a match where team i is the home team and team j is the away team e.g. $hw.bb_{ij}$ would represent the number of home wins where both teams gain a try bonus point, then we define

$$r_{wb} = \sum_i\sum_j(hw.bb_{ij} + aw.bb_{ij}) \quad r_{ww} = \sum_i\sum_j(hw.hb_{ij} + aw.ab_{ij}) \quad r_{wl} = \sum_i\sum_j(hw.ab_{ij} + aw.hb_{ij})$$

$$r_{nb} = \sum_i\sum_j(hn.bb_{ij} + an.bb_{ij}) \quad r_{nw} = \sum_i\sum_j(hn.hb_{ij} + an.ab_{ij}) \quad r_{nl} = \sum_i\sum_j(hn.ab_{ij} + an.hb_{ij})$$

$$r_{db} = \sum_i\sum_j(d.bb_{ij}) \quad r_{dz} = \sum_i\sum_j(d.zb_{ij})$$

Also define R to be the set of all possible composite outcomes and $X = \{wb, ww, wl, nb, nw, nl, db, dz\}$ then we have

$$L(\boldsymbol{\pi}, \kappa, \nu, \tau, \boldsymbol{\varphi}; R) = \frac{\kappa^n \nu^d \tau^h \prod_{k=1}^{m} \pi_k^{s_k} \prod_{x \in X} \varphi_x^{r_x}}{\prod_{i=1}^{m}\prod_{j=1}^{m}(\tau^4\pi_i^4 + \kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \frac{\kappa\pi_i\pi_j^4}{\tau^3} + \frac{\pi_j^4}{\tau^4})^{r_{ij}} B^{hw_{ij}} C^{hn_{ij}} D^{d_{ij}} E^{an_{ij}} F^{aw_{ij}}}$$

which is suggestive of the statistic $(\boldsymbol{s}, n, d, h, \boldsymbol{r})$ being a sufficient statistic for $(\boldsymbol{\pi}, \kappa, \nu, \tau, \boldsymbol{\varphi})$, where $\boldsymbol{r} = \{r_{wb}, r_{ww}, r_{wl}, r_{nb}, r_{nw}, r_{nl}, r_{db}, r_{dz}\}$ and $\boldsymbol{\varphi} = \{\varphi_{wb}, \varphi_{ww}, \varphi_{wl}, \varphi_{nb}, \varphi_{nw}, \varphi_{nl}, \varphi_{db}, \varphi_{dz}\}$.

Proceeding along these lines we have a loglinear representation

$$log\, m_{ijkl} = \alpha_{ij} + \alpha_{ijk\cdot} + \alpha'_{ijkl}$$

where α_{ij} is a normalisation parameter, and $\alpha_{ijk\cdot}$ and α'_{ijkl} represent the part due solely to the result outcome and that due to the try outcome, but conditional on the result outcome, respectively. That is

$$\alpha_{ijk\cdot} = \begin{cases} 4\delta_i + 4\sigma & \text{if home win by wide margin} \\ 4\delta_i + \delta_j + \beta_n + 3\sigma & \text{if home win by narrow margin} \\ 2\delta_i + 2\delta_j + \beta_d & \text{if draw} \\ \delta_i + 4\delta_j + \beta_n - 3\sigma & \text{if away win by narrow margin} \\ 4\delta_j - 4\sigma & \text{if away win by wide margin} \end{cases}$$

and we must consider the five result outcomes separately in defining α'_{ijkl}.

In the case of a wide home win

$$\alpha'_{ijkl} = \begin{cases} \delta_i + \delta_j + \chi_{wb} & \text{if both home and away try bonuses} \\ \delta_i + \chi_{ww} + \sigma & \text{if home try bonus only} \\ \delta_j + \chi_{wl} - \sigma & \text{if away try bonus only} \\ 1 & \text{if no try bonus for either side} \end{cases}$$

in the case of a narrow home win

$$\alpha'_{ijkl} = \begin{cases} \delta_i + \delta_j + \chi_{nb} & \text{if both home and away try bonuses} \\ \delta_i + \chi_{nw} + \sigma & \text{if home try bonus only} \\ \delta_j + \chi_{nl} - \sigma & \text{if away try bonus only} \\ 1 & \text{if no try bonus for either side} \end{cases}$$

in the case of a draw

$$\alpha'_{ijkl} = \begin{cases} \delta_i + \delta_j + \chi_{db} & \text{if both home and away try bonuses} \\ \delta_i + \sigma & \text{if home try bonus only} \\ \delta_j - \sigma & \text{if away try bonus only} \\ \chi_{dz} & \text{if no try bonus for either side} \end{cases}$$

in the case of a narrow away win

$$\alpha'_{ijkl} = \begin{cases} \delta_i + \delta_j + \chi_{nb} & \text{if both home and away try bonuses} \\ \delta_i + \chi_{nl} + \sigma & \text{if home try bonus only} \\ \delta_j + \chi_{nw} - \sigma & \text{if away try bonus only} \\ 1 & \text{if no try bonus for either side} \end{cases}$$

and in the case of a wide away win

$$\alpha'_{ijkl} = \begin{cases} \delta_i + \delta_j + \chi_{wb} & \text{if both home and away try bonuses} \\ \delta_i + \chi_{wl} + \sigma & \text{if home try bonus only} \\ \delta_j + \chi_{ww} - \sigma & \text{if away try bonus only} \\ 1 & \text{if no try bonus for either side} \end{cases}$$

with the previously defined parameters taking the same definitions and $\varphi_x = \exp(\chi_x)$

6.1.3 Dependent on result and opposition - Five parameters

The five parameter try bonus model follows that of the eight parameter model closely, with the difference that the try bonus probabilities become conditional on an aggregated close result, encompassing narrow win, draw and narrow loss in a single parametrisation. That is the probabilities for the wide home and away wins are maintained but the non-normalised probability for the close result outcome are given by

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \eta_c \pi_i \pi_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \zeta_c \pi_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \zeta_c \pi_j$$
$$P(\text{neither team gains try bonus point}) \propto 1 - \eta_c - 2\zeta_c$$

where in the case of a close result in a match between two teams of mean ability, η_c represents the probability that both gain a try bonus, and ζ_c that team i alone (or equivalently team j alone) gains a try bonus.

This gives us normalised probabilities of

$$\begin{aligned}
P(\text{home wide win, both bonus}) &= \frac{\tau^4\pi_i^4}{A}\frac{\varphi_{wb}\pi_i\pi_j}{B} \\
P(\text{home wide win, home bonus}) &= \frac{\tau^4\pi_i^4}{A}\frac{\varphi_{ww}\tau\pi_i}{B} \\
P(\text{home wide win, away bonus}) &= \frac{\tau^4\pi_i^4}{A}\frac{\varphi_{wl}\pi_j/\tau}{B} \\
P(\text{home wide win, zero bonus}) &= \frac{\tau^4\pi_i^4}{A}\frac{1}{B}
\end{aligned}$$

$$\begin{aligned}
P(\text{close result, both bonus}) &= \frac{\kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\varphi_{cb}\pi_i\pi_j}{G} \\
P(\text{close result, home bonus}) &= \frac{\kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\tau\pi_i}{G} \\
P(\text{close result, away bonus}) &= \frac{\kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\pi_j/\tau}{G} \\
P(\text{close result, zero bonus}) &= \frac{\kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \kappa\pi_i\pi_j^4/\tau^3}{A}\frac{\varphi_{cz}}{G}
\end{aligned}$$

$$\begin{aligned}
P(\text{away wide win, both bonus}) &= \frac{\pi_j^4/\tau^4}{A}\frac{\varphi_{wb}\pi_i\pi_j}{F} \\
P(\text{away wide win, home bonus}) &= \frac{\pi_j^4/\tau^4}{A}\frac{\varphi_{wl}\pi_j/\tau}{F} \\
P(\text{away wide win, away bonus}) &= \frac{\pi_j^4/\tau^4}{A}\frac{\varphi_{ww}\tau\pi_i}{F} \\
P(\text{away wide win, zero bonus}) &= \frac{\pi_j^4/\tau^4}{A}\frac{1}{F}
\end{aligned}$$

where definitions are maintained and in addition we have

$$\varphi_{cb} = \frac{\eta_c}{\zeta_c}$$
$$\varphi_{cz} = \frac{1 - \eta_c - 2\zeta_c}{\zeta_c}$$

and

$$G = \varphi_{cb}\pi_i\pi_j + \tau\pi_i + \pi_j/\tau + \varphi_{cz}$$

and employing analogous notation to previously we have a likelihood of

$$L(\boldsymbol{\pi}, \kappa, \nu, \tau, \boldsymbol{\varphi}; R) = \frac{\kappa^n \nu^d \tau^h \prod_{k=1}^{m} \pi_k^{s_k} \prod_{x \in X} \varphi_x^{r_x}}{\prod_{i=1}^{m}\prod_{j=1}^{m}(\tau^4\pi_i^4 + \kappa\tau^3\pi_i^4\pi_j + \nu\pi_i^2\pi_j^2 + \frac{\kappa\pi_i\pi_j^4}{\tau^3} + \frac{\pi_j^4}{\tau^4})^{r_{ij}} B^{hw_{ij}} G^{c_{ij}} F^{aw_{ij}}}$$

with X redefined as $X = \{wb, ww, wl, cb, cz\}$ for the five parameter model. This is suggestive of the statistic $(\boldsymbol{s}, n, d, h, \boldsymbol{r})$ being a sufficient statistic for $(\boldsymbol{\pi}, \kappa, \nu, \tau, \boldsymbol{\varphi})$, where $\boldsymbol{r}$ and $\boldsymbol{\varphi}$ are similarly redefined to be the sets $\boldsymbol{r} = \{r_{wb}, r_{ww}, r_{wl}, r_{cb}, r_{cz}\}$ and $\boldsymbol{\varphi} = \{\varphi_{wb}, \varphi_{ww}, \varphi_{wl}, \varphi_{cb}, \varphi_{cz}\}$.

Proceeding along these lines we have a loglinear representation

$$log\, m_{ijkl} = \alpha_{ij} + \alpha_{ijk\cdot} + \alpha'_{ijkl}$$

where m_{ijkl} and α_{ij} are as previously seen, and $\alpha_{ijk\cdot}$ and α'_{ijkl} represent the part due solely to the result outcome, and that due to the try outcome but conditional on the result outcome respectively. That is $\alpha_{ijk\cdot}$ is as before and we must consider the three result outcomes separately in defining α'_{ijkl}, with wide home win and wide away wins being as in the eight parameter case and in the case of a close result

$$\alpha'_{ijkl} = \begin{cases} \delta_i + \delta_j + \chi_{cb} & \text{if both home and away try bonuses} \\ \delta_i + \sigma & \text{if home try bonus only} \\ \delta_j - \sigma & \text{if away try bonus only} \\ \chi_{cz} & \text{if no try bonus for either side} \end{cases}$$

6.1.4 Independent offensive and defensive ability

Suppose we consider a team's ability to be the product of its offensive and defensive ability

$$\pi_i = o_i.d_i$$

Then it seems reasonable to consider a team's propensity to score a try bonus as a function of its own offensive ability and the opposition's defensive ability. So define the non-normalised probability of team i's try bonus outcome in a match between teams i and j as

$$P(\text{team } i \text{ gains try bonus point}) \propto o_i$$
$$P(\text{team } i \text{ does not gain try bonus point}) \propto d_j$$

Clearly given π_i we need only define one further parameter per team. Since we have used d to refer to draws earlier in the document and have in general used greek letters for our parameters, for the avoidance of confusion, let us use ω_i instead for our defensive ability then we have for our non-normalised probabilities

$$P(\text{team } i \text{ gains try bonus point}) \propto \frac{\pi_i}{\omega_i}$$
$$P(\text{team } i \text{ does not gain try bonus point}) \propto \omega_j$$

this gives the non-normalised try bonus outcome probabilities

$$P(\text{both home and away try bonuses}) \propto \frac{\tau\pi_i}{\omega_i}\frac{\pi_j}{\tau\omega_j} = \frac{\pi_i\pi_j}{\omega_i\omega_j}$$
$$P(\text{home try bonus only}) \propto \frac{\tau\pi_i}{\omega_i}\omega_i = \tau\pi_i$$
$$P(\text{away try bonus only}) \propto \frac{\pi_j}{\tau\omega_j}\omega_j = \frac{\pi_j}{\tau}$$
$$P(\text{no try bonus for either side}) \propto \omega_i\omega_j$$

Letting $\boldsymbol{\omega} = (\omega_1 \ldots \omega_m)$ our likelihood becomes

$$L(\boldsymbol{\pi}, \boldsymbol{\omega}, \kappa, \nu, \tau; R) = \prod_{i=1}^{m}\prod_{j=1}^{m} \frac{(\tau^4\pi_i^4)^{hw_{ij}}(\kappa\tau^3\pi_i^4\pi_j)^{hn_{ij}}(\nu\pi_i^2\pi_j^2)^{d_{ij}}(\frac{\kappa\pi_i\pi_j^4}{\tau^3})^{an_{ij}}(\frac{\pi_j^4}{\tau^4})^{aw_{ij}}}{(\tau^4\pi^4 + \kappa\tau^3\pi^4\pi_j + \nu\pi_i^2\pi_j^2 + \frac{\kappa\pi_i\pi_j^4}{\tau^3} + \frac{\pi_j^4}{\tau^4})^{r_{ij}}} \frac{(\frac{\pi_i\pi_j}{\omega_i\omega_j})^{bb_{ij}}(\tau\pi_i)^{hb_{ij}}(\frac{\pi_j}{\tau})^{ab_{ij}}(\omega_i\omega_j)^{zb_{ij}}}{(\frac{\pi_i\pi_j}{\omega_i\omega_j} + \tau\pi_i + \frac{\pi_j}{\tau} + \omega_i\omega_j)^{r_{ij}}}$$

Defining $t_i = \sum_j zb_{ij} + zb_{ji} - bb_{ij} - bb_{ji}$, which may be thought of as the difference between the number of matches involving team i where defenses performed better and those where offenses performed better then

$$L(\boldsymbol{\pi}, \boldsymbol{\omega}, \kappa, \nu, \tau; R) = \frac{\kappa^n \nu^d \tau^h \prod_{k=1}^{m} \pi_k^{s_k} \prod_{l=1}^{m} \omega_l^{t_l}}{\prod_{i=1}^{m} \prod_{j=1}^{m} (\tau^4 \pi_i^4 + \kappa\tau^3 \pi_i^4 \pi_j + \nu \pi_i^2 \pi_j^2 + \frac{\kappa \pi_i \pi_j^4}{\tau^3} + \frac{\pi_j^4}{\tau^4})^{r_{ij}} (\frac{\pi_i \pi_j}{\omega_i \omega_j} + \tau \pi_i + \frac{\pi_j}{\tau} + \omega_i \omega_j)^{r_{ij}}}$$

and the statistic $(\boldsymbol{s}, n, d, h, \boldsymbol{t})$ is a sufficient statistic for $(\boldsymbol{\pi}, \kappa, \nu, \tau, \boldsymbol{\omega})$.

For the loglinear representation, $\alpha_{ijk\cdot}$ is defined as before and

$$\alpha_{ij\cdot l} = \begin{cases} \delta_i + \delta_j - \varphi_i - \varphi_j & \text{if both home and away try bonuses} \\ \delta_i + \sigma & \text{if home try bonus only} \\ \delta_j - \sigma & \text{if away try bonus only} \\ \varphi_i + \varphi_j & \text{if no try bonus for either side} \end{cases}$$

with $\omega_i = exp(\varphi_i)$ and the other parameters as before.

6.2 Data assumptions

While there were no means to validate the data independently, there were 24 occasions of identifiable self-inconsistencies or incompleteness in the data across the three seasons of interest, 15 of which impacted the result or try outcomes for at least one of the teams involved. The treatment of all of these is described below. These were checked for reasonableness with SOCS, the administrator for the tournament.

1. Where the score could not have produced the try outcome. Since a try is worth five points in rugby union, then the score of any team may not be more than five times their number of tries. If swapping the number of tries recorded for home and away teams produced consistency then this was done. If this did not resolve the issue then the number of tries was adjusted down to the maximum number of tries possible given the score (The number of incidences of this were: two in 2017/18, five in 2016/17, four in 2015/16. Of those that meant a team's try bonus status changed there were just one in 2017/18 and two in 2016/17)
2. Where Venue had been entered as "tbc", the Venue was set to Neutral (five in 2016/17, two in 2015/16)

3. Where matches were entered as a win for one side but score and tries were both given as 0-0. On speaking to SOCS, their speculation was that these may have related to matches where there had been some sort of 'gentleman's agreement' e.g. one team had lost very heavily and they had agreed not to make the result public, or the teams had agreed to deselect certain players (in particular those with representative honours), and so it was used as a means of recognising that a fixture had taken place, but not giving it full status. In our analysis the winning team is awarded four points for a win, the losing team one for a narrow loss, and no try bonus is awarded to either side (two in 2017/18, two in 2016/17)
4. Where the try count was not given for one of the two teams, the number of tries was taken to be the maximum number of tries possible given the score (one in 2016/17)
5. Where the result outcome (Won, Draw, Loss) did not agree with the score but did agree with the try outcome, but was consistent if the score was reversed, then the score was reversed (one in 2017/18. This did not impact the analysis).

6.3 Further results

6.3.1 Structural parameters

	2018	2017	2016	2015	2014	2013	2012	2011	Min	Max	Mean
κ	0.655	0.948	1.107	0.910	1.345	1.277	0.700	1.007	0.655	1.345	0.994
ν	0.032	0.232	0.147	0.220	0.165	0.196	0.196	0.184	0.032	0.232	0.172
ψ	0.522	0.447	0.260	0.286	0.233	0.211	0.116	0.174	0.116	0.522	0.281
τ	1.114	1.203	1.221	1.214	1.089	1.200	1.200	1.202	1.089	1.221	1.180
λ	0.392	0.459	0.508	0.451	0.554	0.538	0.389	0.480	0.389	0.554	0.471
ρ	0.010	0.056	0.034	0.054	0.034	0.041	0.055	0.044	0.010	0.056	0.041
ξ	0.343	0.309	0.207	0.222	0.189	0.174	0.104	0.148	0.104	0.343	0.212

Table 13: Model 1 structural parameters for English Premiership 2010/11 - 2017/18

	2018	2017	2016	2015	2014	2013	2012	2011	Min	Max	Mean
κ	0.617	0.904	0.411	0.702	1.263	1.186	1.339	0.914	0.411	1.339	0.917
ν	0.032	0.233	0.233	0.147	0.165	0.196	0.184	0.183	0.032	0.233	0.172
θ	0.418	0.525	0.239	0.116	0.301	0.096	0.063	0.054	0.054	0.525	0.227
ϕ	1.880	2.586	5.682	5.598	4.978	4.584	6.687	5.665	1.880	6.687	4.707
τ	1.114	1.204	1.204	1.220	1.090	1.199	1.104	1.201	1.090	1.220	1.167
λ	0.378	0.447	0.269	0.395	0.538	0.519	0.551	0.456	0.269	0.551	0.444
ρ	0.010	0.058	0.076	0.041	0.035	0.043	0.038	0.046	0.010	0.076	0.043
ζ	0.233	0.196	0.126	0.130	0.137	0.150	0.114	0.130	0.114	0.233	0.152
η	0.097	0.103	0.030	0.015	0.041	0.014	0.007	0.007	0.007	0.103	0.039

Table 14: Model 2 structural parameters for English Premiership 2010/11 - 2017/18

6.3.2 Full rankings

School	DMT Rank	DMT	PPPM Rank	PPPM
Sedbergh School	1	7.41	1	4.65
Wellington College	2	7.18	7	4.18
Cranleigh School	3	6.33	4	4.32
Harrow School	4	6.20	3	4.33
Cheltenham College	5	6.16	8	4.07
St Peter's School, York	6	5.83	6	4.19
Brighton College	7	5.63	20	3.59
Reed's School	8	5.50	2	4.38
Clifton College	8	5.50	16	3.72
Haileybury	10	5.49	10	4.02
Northampton School for Boys	11	5.41	9	4.05
St Edward's School, Oxford	12	5.40	17	3.68
Canford School	12	5.40	19	3.61
Blundell's School	14	5.33	14	3.75
Queen Elizabeth Grammar, Wakefield	15	5.24	11	3.99
Oakham School	16	5.20	33	3.06
Campion School, Essex	17	5.10	15	3.73
Woodhouse Grove School	17	5.10	18	3.62
Bedford School	19	4.98	27	3.28
Seaford College	20	4.90	31	3.07
Bromsgrove School	21	4.88	30	3.08
St Paul's School	21	4.88	40	2.82
Stamford	23	4.80	24	3.35
Whitgift School	24	4.76	13	3.83
Dean Close School	25	4.73	23	3.38
Monmouth School	26	4.70	29	3.09
Eton College	27	4.64	43	2.78
Rugby School	28	4.63	52	2.50
The Grammar School at Leeds	29	4.60	28	3.16
Denstone College	30	4.53	26	3.33
RGS, Newcastle	31	4.49	12	3.83
Colston's School	32	4.30	25	3.34
Bradford Grammar School	33	4.29	32	3.06
Sherborne School	34	4.20	45	2.73
Hampton School	35	4.13	54	2.44
Berkhamsted School	35	4.13	48	2.61
Solihull School	37	4.12	55	2.34
Hurstpierpoint College	38	4.10	36	2.97
Epsom College	39	4.08	34	3.03
Warwick School	40	4.00	22	3.41
Dulwich College	41	3.98	44	2.77
St John's School, Leatherhead	42	3.89	53	2.50
Sir Thomas Rich's School	43	3.82	51	2.54
Tonbridge School	44	3.80	46	2.71
RGS, Guildford	44	3.80	58	2.24
Durham School	46	3.73	38	2.90
Merchant Taylors' School, Northwood	47	3.71	37	2.90
RGS, High Wycombe	48	3.68	60	2.20
St George's College, Weybridge	49	3.60	39	2.86
Caterham School	50	3.49	59	2.23
Bristol Grammar School	51	3.40	56	2.29
The King's School, Worcester	51	3.40	79	1.77
Bloxham School	53	3.37	35	3.03
Loughborough Grammar School	53	3.37	85	1.58
Old Swinford Hospital	55	3.30	83	1.61
Magdalen College School	56	3.27	50	2.54
King's College School (KCS), Wimbledon	57	3.22	78	1.81
Reading Blue Coat School	58	3.16	66	2.13
The King's School, Macclesfield	58	3.16	62	2.18
Trinity School, Croydon	60	3.14	57	2.25

School	PPPM Rank	PPPM	DMT Rank	DMT
Sedbergh School	1	4.65	1	7.41
Reed's School	2	4.38	8	5.50
Harrow School	3	4.33	4	6.20
Cranleigh School	4	4.32	3	6.33
Kingswood School	5	4.31	NR	NR
St Peter's School, York	6	4.19	6	5.83
Wellington College	7	4.18	2	7.18
Cheltenham College	8	4.07	5	6.16
Northampton School for Boys	9	4.05	11	5.41
Haileybury	10	4.02	10	5.49
Queen Elizabeth Grammar, Wakefield	11	3.99	15	5.24
RGS, Newcastle	12	3.83	31	4.49
Whitgift School	13	3.83	24	4.76
Blundell's School	14	3.75	14	5.33
Campion School, Essex	15	3.73	17	5.10
Clifton College	16	3.72	8	5.50
St Edward's School, Oxford	17	3.68	12	5.40
Woodhouse Grove School	18	3.62	17	5.10
Canford School	19	3.61	12	5.40
Brighton College	20	3.59	7	5.63
Wirral Grammar School for Boys	21	3.59	NR	NR
Warwick School	22	3.41	40	4.00
Dean Close School	23	3.38	25	4.73
Stamford	24	3.35	23	4.80
Colston's School	25	3.34	32	4.30
Denstone College	26	3.33	30	4.53
Bedford School	27	3.28	19	4.98
The Grammar School at Leeds	28	3.16	29	4.60
Monmouth School	29	3.09	26	4.70
Bromsgrove School	30	3.08	21	4.88
Seaford College	31	3.07	20	4.90
Bradford Grammar School	32	3.06	33	4.29
Oakham School	33	3.06	16	5.20
Epsom College	34	3.03	39	4.08
Bloxham School	35	3.03	53	3.37
Hurstpierpoint College	36	2.97	38	4.10
Merchant Taylors' School, Northwood	37	2.90	47	3.71
Durham School	38	2.90	46	3.73
St George's College, Weybridge	39	2.86	49	3.60
St Paul's School	40	2.82	21	4.88
Bishop Wordsworth's School	41	2.79	67	3.07
Wilmslow High School	42	2.78	61	3.13
Eton College	43	2.78	27	4.64
Dulwich College	44	2.77	41	3.98
Sherborne School	45	2.73	34	4.20
Tonbridge School	46	2.71	44	3.80
Millfield School	47	2.67	66	3.09
Berkhamsted School	48	2.61	35	4.13
Bryanston School	49	2.55	61	3.13
Magdalen College School	50	2.54	56	3.27
Sir Thomas Rich's School	51	2.54	43	3.82
Rugby School	52	2.50	28	4.63
St John's School, Leatherhead	53	2.50	42	3.89
Hampton School	54	2.44	35	4.13
Solihull School	55	2.34	37	4.12
Bristol Grammar School	56	2.29	51	3.40
Trinity School, Croydon	57	2.25	60	3.14
RGS, Guildford	58	2.24	44	3.80
Caterham School	59	2.23	50	3.49
RGS, High Wycombe	60	2.20	48	3.68

School	DMT Rank	DMT	PPPM Rank	PPPM
Wilmslow High School	61	3.13	42	2.78
Bryanston School	61	3.13	49	2.55
Barnard Castle School	61	3.13	61	2.18
Wimbledon College	64	3.11	82	1.61
Stowe School	65	3.10	90	1.51
Millfield School	66	3.09	47	2.67
Bishop Wordsworth's School	67	3.07	41	2.79
Radley College	68	3.00	87	1.54
Marlborough College	68	3.00	91	1.44
Pate's Grammar School	68	3.00	81	1.67
Ampleforth College	71	2.90	64	2.16
Abingdon School	71	2.90	84	1.61
Pocklington School	73	2.87	76	1.85
Uppingham School	74	2.85	88	1.54
Oundle School	75	2.81	99	1.20
Christ's Hospital	76	2.77	101	1.11
Eastbourne College	77	2.76	67	2.11
Hymers College	78	2.61	63	2.18
John Fisher School	79	2.53	75	1.85
RGS Worcester	80	2.50	73	1.89
Ellesmere College	80	2.50	80	1.76
King Edward's School, Birmingham	82	2.47	92	1.40
St Albans School	83	2.37	77	1.83
Malvern College	84	2.33	65	2.15
St Benedict's School	85	2.30	98	1.26
Worth School	85	2.30	95	1.31
Emanuel School	87	2.17	97	1.29
Skinners' School	88	2.15	96	1.31
The King's School, Canterbury	89	2.10	69	2.03
Trent College	90	1.90	93	1.40
Tiffin School	90	1.90	104	0.87
Ashville College	92	1.83	108	0.82
The Portsmouth Grammar School	93	1.79	94	1.39
St Ambrose College	94	1.70	71	1.96
Churcher's College	95	1.30	110	0.72
The Oratory School	96	1.17	105	0.84
St Peter's RC High School, Gloucester	97	0.93	109	0.82
Yarm School	98	0.90	107	0.83
Nottingham High School	99	0.67	113	0.59
London Oratory School	100	0.64	114	0.47
Aylesbury Grammar School	101	0.47	112	0.60
Reigate Grammar School	102	0.00	115	0.23
Kingswood School	NR	NR	5	4.31
Wirral Grammar School for Boys	NR	NR	21	3.59
Crossley Heath School	NR	NR	68	2.11
The Manchester Grammar School	NR	NR	74	1.89
Stockport Grammar School	NR	NR	70	2.03
Adams Grammar School	NR	NR	89	1.51
Lancaster Royal Grammar School	NR	NR	100	1.15
Bedford Modern School	NR	NR	86	1.57
Dame Allan's Schools	NR	NR	102	1.02
St Joseph's College, Ipswich	NR	NR	72	1.91
King's College, Taunton	NR	NR	103	0.94
Cokethorpe School	NR	NR	111	0.65
Lymm High School	NR	NR	106	0.84

School	PPPM Rank	PPPM	DMT Rank	DMT
Barnard Castle School	61	2.18	61	3.13
The King's School, Macclesfield	62	2.18	58	3.16
Hymers College	63	2.18	78	2.61
Ampleforth College	64	2.16	71	2.90
Malvern College	65	2.15	84	2.33
Reading Blue Coat School	66	2.13	58	3.16
Eastbourne College	67	2.11	77	2.76
Crossley Heath School	68	2.11	NR	NR
The King's School, Canterbury	69	2.03	89	2.10
Stockport Grammar School	70	2.03	NR	NR
St Ambrose College	71	1.96	94	1.70
St Joseph's College, Ipswich	72	1.91	NR	NR
RGS Worcester	73	1.89	80	2.50
The Manchester Grammar School	74	1.89	NR	NR
John Fisher School	75	1.85	79	2.53
Pocklington School	76	1.85	73	2.87
St Albans School	77	1.83	83	2.37
King's College School (KCS), Wimbledon	78	1.81	57	3.22
The King's School, Worcester	79	1.77	51	3.40
Ellesmere College	80	1.76	80	2.50
Pate's Grammar School	81	1.67	68	3.00
Wimbledon College	82	1.61	64	3.11
Old Swinford Hospital	83	1.61	55	3.30
Abingdon School	84	1.61	71	2.90
Loughborough Grammar School	85	1.58	53	3.37
Bedford Modern School	86	1.57	NR	NR
Radley College	87	1.54	68	3.00
Uppingham School	88	1.54	74	2.85
Adams Grammar School	89	1.51	NR	NR
Stowe School	90	1.51	65	3.10
Marlborough College	91	1.44	68	3.00
King Edward's School, Birmingham	92	1.40	82	2.47
Trent College	93	1.40	90	1.90
The Portsmouth Grammar School	94	1.39	93	1.79
Worth School	95	1.31	85	2.30
Skinners' School	96	1.31	88	2.15
Emanuel School	97	1.29	87	2.17
St Benedict's School	98	1.26	85	2.30
Oundle School	99	1.20	75	2.81
Lancaster Royal Grammar School	100	1.15	NR	NR
Christ's Hospital	101	1.11	76	2.77
Dame Allan's Schools	102	1.02	NR	NR
King's College, Taunton	103	0.94	NR	NR
Tiffin School	104	0.87	90	1.90
The Oratory School	105	0.84	96	1.17
Lymm High School	106	0.84	NR	NR
Yarm School	107	0.83	98	0.90
Ashville College	108	0.82	92	1.83
St Peter's RC High School, Gloucester	109	0.82	97	0.93
Churcher's College	110	0.72	95	1.30
Cokethorpe School	111	0.65	NR	NR
Aylesbury Grammar School	112	0.60	101	0.47
Nottingham High School	113	0.59	99	0.67
London Oratory School	114	0.47	100	0.64
Reigate Grammar School	115	0.23	102	0.00

Table 15: 2017/18: Full league table by Daily Mail Trophy Merit Points and Model 2 projected points per match. NR - Not ranked

School	DMT Rank	DMT	PPPM Rank	PPPM
Wellington College	1	7.22	3	4.37
Sedbergh School	2	6.50	2	4.43
Harrow School	3	6.34	6	4.22
St Peter's School, York	4	6.23	8	4.06
Kirkham Grammar School	5	6.15	1	4.61
Canford School	6	6.10	9	4.02
Clifton College	7	6.00	5	4.25
Rugby School	8	5.96	7	4.06
Brighton College	9	5.90	4	4.29
Woodhouse Grove School	10	5.81	12	3.93
Millfield School	11	5.76	11	3.95
Hampton School	12	5.47	22	3.44
Bromsgrove School	13	5.42	29	3.21
St John's School, Leatherhead	14	5.39	10	4.01
Bedford School	15	5.13	16	3.64
Cheltenham College	16	5.05	28	3.21
Warwick School	17	4.98	21	3.44
Epsom College	18	4.90	17	3.59
Bristol Grammar School	18	4.90	25	3.39
Sir Thomas Rich's School	20	4.87	15	3.70
Whitgift School	21	4.78	23	3.43
Bloxham School	22	4.69	19	3.48
Trent College	23	4.67	26	3.38
Nottingham High School	23	4.67	36	3.04
RGS, Newcastle	25	4.60	18	3.56
St George's College, Weybridge	25	4.60	40	2.88
St Peter's RC High School, Gloucester	27	4.53	27	3.25
Abingdon School	27	4.53	30	3.17
RGS, High Wycombe	29	4.41	59	2.29
Sherborne School	30	4.35	39	2.91
Haileybury	31	4.31	32	3.16
Hurstpierpoint College	32	4.30	60	2.26
St Paul's School	33	4.28	58	2.31
Eton College	34	4.27	63	2.16
The King's School, Worcester	35	4.25	20	3.47
Cranleigh School	36	4.20	14	3.73
Berkhamsted School	37	4.15	38	2.93
Barnard Castle School	38	4.10	44	2.77
Oakham School	39	4.08	24	3.39
Queen Elizabeth Grammar, Wakefield	40	4.03	37	2.95
Monmouth School	41	3.98	43	2.78
Denstone College	42	3.89	33	3.10
Tonbridge School	42	3.89	47	2.66
Pocklington School	44	3.87	45	2.70
Durham School	45	3.84	35	3.05
King's College School (KCS), Wimbledon	45	3.84	54	2.39
Bishop Wordsworth's School	47	3.81	31	3.17
Stamford	48	3.77	49	2.51
Blundell's School	49	3.73	34	3.08
Northampton School for Boys	50	3.68	55	2.38
Oundle School	51	3.53	41	2.87
Stowe School	51	3.53	53	2.39
The King's School, Macclesfield	53	3.50	51	2.45
The Portsmouth Grammar School	54	3.43	46	2.69
Dulwich College	55	3.42	77	1.76
St Edward's School, Oxford	56	3.40	52	2.42
RGS, Guildford	56	3.40	57	2.33
Ashville College	56	3.40	105	0.82
Bedford Modern School	59	3.30	56	2.35
John Fisher School	59	3.30	80	1.70
St Benedict's School	61	3.23	74	1.83
Bradford Grammar School	62	3.22	67	2.05
Eastbourne College	62	3.22	72	1.92
Reed's School	64	3.20	61	2.25
Hymers College	65	3.18	79	1.70
Malvern College	66	3.13	50	2.48
RGS Worcester	67	3.10	82	1.68
Kingswood School	68	3.00	65	2.08
Seaford College	69	2.94	48	2.54
Wilmslow High School	69	2.94	62	2.20

School	PPPM Rank	PPPM	DMT Rank	DMT
Kirkham Grammar School	1	4.61	5	6.15
Sedbergh School	2	4.43	2	6.50
Wellington College	3	4.37	1	7.22
Brighton College	4	4.29	9	5.90
Clifton College	5	4.25	7	6.00
Harrow School	6	4.22	3	6.34
Rugby School	7	4.06	8	5.96
St Peter's School, York	8	4.06	4	6.23
Canford School	9	4.02	6	6.10
St John's School, Leatherhead	10	4.01	14	5.39
Millfield School	11	3.95	11	5.76
Woodhouse Grove School	12	3.93	10	5.81
The Manchester Grammar School	13	3.85	NR	NR
Cranleigh School	14	3.73	36	4.20
Sir Thomas Rich's School	15	3.70	20	4.87
Bedford School	16	3.64	15	5.13
Epsom College	17	3.59	18	4.90
RGS, Newcastle	18	3.56	25	4.60
Bloxham School	19	3.48	22	4.69
The King's School, Worcester	20	3.47	35	4.25
Warwick School	21	3.44	17	4.98
Hampton School	22	3.44	12	5.47
Whitgift School	23	3.43	21	4.78
Oakham School	24	3.39	39	4.08
Bristol Grammar School	25	3.39	18	4.90
Trent College	26	3.38	23	4.67
St Peter's RC High School, Gloucester	27	3.25	27	4.53
Cheltenham College	28	3.21	16	5.05
Bromsgrove School	29	3.21	13	5.42
Abingdon School	30	3.17	27	4.53
Bishop Wordsworth's School	31	3.17	47	3.81
Haileybury	32	3.16	31	4.31
Denstone College	33	3.10	42	3.89
Blundell's School	34	3.08	49	3.73
Durham School	35	3.05	45	3.84
Nottingham High School	36	3.04	23	4.67
Queen Elizabeth Grammar, Wakefield	37	2.95	40	4.03
Berkhamsted School	38	2.93	37	4.15
Sherborne School	39	2.91	30	4.35
St George's College, Weybridge	40	2.88	25	4.60
Oundle School	41	2.87	51	3.53
Reigate Grammar School	42	2.80	NR	NR
Monmouth School	43	2.78	41	3.98
Barnard Castle School	44	2.77	38	4.10
Pocklington School	45	2.70	44	3.87
The Portsmouth Grammar School	46	2.69	54	3.43
Tonbridge School	47	2.66	42	3.89
Seaford College	48	2.54	69	2.94
Stamford	49	2.51	48	3.77
Malvern College	50	2.48	66	3.13
The King's School, Macclesfield	51	2.45	53	3.50
St Edward's School, Oxford	52	2.42	56	3.40
Stowe School	53	2.39	51	3.53
King's College School (KCS), Wimbledon	54	2.39	45	3.84
Northampton School for Boys	55	2.38	50	3.68
Bedford Modern School	56	2.35	59	3.30
RGS, Guildford	57	2.33	56	3.40
St Paul's School	58	2.31	33	4.28
RGS, High Wycombe	59	2.29	29	4.41
Hurstpierpoint College	60	2.26	32	4.30
Reed's School	61	2.25	64	3.20
Wilmslow High School	62	2.20	69	2.94
Eton College	63	2.16	34	4.27
Loughborough Grammar School	64	2.09	71	2.93
Kingswood School	65	2.08	68	3.00
Lymm High School	66	2.05	NR	NR
Bradford Grammar School	67	2.05	62	3.22
Magdalen College School	68	2.01	77	2.79
Crossley Heath School	69	1.98	NR	NR
The Oratory School	70	1.96	75	2.80

School	DMT Rank	DMT	PPPM Rank	PPPM
Loughborough Grammar School	71	2.93	64	2.09
Radley College	72	2.90	78	1.74
Trinity School, Croydon	72	2.90	91	1.31
Ampleforth College	74	2.83	73	1.88
The Oratory School	75	2.80	70	1.96
Solihull School	75	2.80	76	1.80
Magdalen College School	77	2.79	68	2.01
The Grammar School at Leeds	78	2.78	87	1.46
Yarm School	79	2.73	83	1.68
St Ambrose College	80	2.66	84	1.66
Marlborough College	81	2.62	104	0.90
King Edward's School, Birmingham	82	2.60	89	1.38
Caterham School	83	2.56	96	1.23
Christ's Hospital	84	2.40	94	1.25
Bryanston School	85	2.39	71	1.93
Tiffin School	86	2.33	93	1.27
Ellesmere College	87	2.30	86	1.48
Wimbledon College	88	2.22	92	1.27
Lancaster Royal Grammar School	89	2.20	75	1.82
Colston's School	89	2.20	81	1.68
Merchant Taylors' School, Northwood	91	2.17	85	1.58
Uppingham School	92	2.16	95	1.24
Campion School, Essex	93	1.93	100	1.00
Cokethorpe School	94	1.83	88	1.39
Skinners' School	95	1.77	102	0.95
St Albans School	96	1.51	101	0.96
The King's School, Canterbury	97	1.50	98	1.06
Dean Close School	98	1.41	90	1.32
Pate's Grammar School	99	0.95	109	0.62
London Oratory School	100	0.90	113	0.30
Old Swinford Hospital	101	0.87	110	0.60
Dame Allan's Schools	102	0.80	111	0.52
Worth School	103	0.67	112	0.50
Aylesbury Grammar School	104	0.40	107	0.65
The Manchester Grammar School	NR	NR	13	3.85
Reigate Grammar School	NR	NR	42	2.80
Crossley Heath School	NR	NR	69	1.98
Lymm High School	NR	NR	66	2.05
Stockport Grammar School	NR	NR	97	1.12
Wirral Grammar School for Boys	NR	NR	103	0.95
Adams Grammar School	NR	NR	108	0.63
King's College, Taunton	NR	NR	106	0.79
St Joseph's College, Ipswich	NR	NR	99	1.05

School	PPPM Rank	PPPM	DMT Rank	DMT
Bryanston School	71	1.93	85	2.39
Eastbourne College	72	1.92	62	3.22
Ampleforth College	73	1.88	74	2.83
St Benedict's School	74	1.83	61	3.23
Lancaster Royal Grammar School	75	1.82	89	2.20
Solihull School	76	1.80	75	2.80
Dulwich College	77	1.76	55	3.42
Radley College	78	1.74	72	2.90
Hymers College	79	1.70	65	3.18
John Fisher School	80	1.70	59	3.30
Colston's School	81	1.68	89	2.20
RGS Worcester	82	1.68	67	3.10
Yarm School	83	1.68	79	2.73
St Ambrose College	84	1.66	80	2.66
Merchant Taylors' School, Northwood	85	1.58	91	2.17
Ellesmere College	86	1.48	87	2.30
The Grammar School at Leeds	87	1.46	78	2.78
Cokethorpe School	88	1.39	94	1.83
King Edward's School, Birmingham	89	1.38	82	2.60
Dean Close School	90	1.32	98	1.41
Trinity School, Croydon	91	1.31	72	2.90
Wimbledon College	92	1.27	88	2.22
Tiffin School	93	1.27	86	2.33
Christ's Hospital	94	1.25	84	2.40
Uppingham School	95	1.24	92	2.16
Caterham School	96	1.23	83	2.56
Stockport Grammar School	97	1.12	NR	NR
The King's School, Canterbury	98	1.06	97	1.50
St Joseph's College, Ipswich	99	1.05	NR	NR
Campion School, Essex	100	1.00	93	1.93
St Albans School	101	0.96	96	1.51
Skinners' School	102	0.95	95	1.77
Wirral Grammar School for Boys	103	0.95	NR	NR
Marlborough College	104	0.90	81	2.62
Ashville College	105	0.82	56	3.40
King's College, Taunton	106	0.79	NR	NR
Aylesbury Grammar School	107	0.65	104	0.40
Adams Grammar School	108	0.63	NR	NR
Pate's Grammar School	109	0.62	99	0.95
Old Swinford Hospital	110	0.60	101	0.87
Dame Allan's Schools	111	0.52	102	0.80
Worth School	112	0.50	103	0.67
London Oratory School	113	0.30	100	0.90

Table 16: 2016/17: Full league table by Daily Mail Trophy Merit Points and Model 2 projected points per match. NR - Not ranked

School	DMT Rank	DMT	PPPM Rank	PPPM
Wellington College	1	6.46	7	3.73
Kirkham Grammar School	2	6.44	1	4.41
Bedford School	3	6.35	2	4.37
Bromsgrove School	4	6.21	4	4.15
Sedbergh School	5	6.10	5	3.99
Woodhouse Grove School	6	5.65	19	3.31
Millfield School	7	5.21	13	3.64
Clifton College	8	5.11	8	3.73
Solihull School	9	5.10	11	3.67
St Paul's School	9	5.10	14	3.58
Stowe School	9	5.10	18	3.34
Whitgift School	9	5.10	20	3.31
Seaford College	13	5.04	6	3.93
Hampton School	14	5.00	23	3.25
St Peter's School, York	14	5.00	38	2.93
Bradford Grammar School	16	4.99	50	2.50
Queen Elizabeth Grammar, Wakefield	17	4.98	9	3.69
Tonbridge School	18	4.94	10	3.69
Cranleigh School	18	4.94	17	3.42
Durham School	20	4.92	45	2.73
Sherborne School	21	4.88	29	3.16
Oundle School	22	4.86	15	3.48
King's College School (KCS), Wimbledon	23	4.85	33	3.02
Berkhamsted School	24	4.80	12	3.66
Reed's School	25	4.79	22	3.30
Barnard Castle School	26	4.73	26	3.23
Sir Thomas Rich's School	26	4.73	24	3.25
Harrow School	28	4.70	16	3.48
RGS, High Wycombe	29	4.56	36	2.96
Hereford Cathedral School	30	4.53	27	3.19
Bishop Wordsworth's School	31	4.40	21	3.31
Brighton College	31	4.40	37	2.96
Blundell's School	33	4.33	31	3.12
Eastbourne College	34	4.20	30	3.15
Warwick School	34	4.20	34	3.00
Dulwich College	34	4.20	35	2.97
Wilmslow High School	37	4.13	32	3.11
St Peter's RC High School, Gloucester	38	4.10	25	3.23
John Fisher School	39	3.99	66	2.10
Epsom College	40	3.91	44	2.75
The King's School, Worcester	41	3.90	49	2.55
RGS, Newcastle	42	3.88	59	2.30
Denstone College	43	3.87	58	2.33
Stamford	44	3.80	40	2.89
Bloxham School	44	3.80	51	2.50
St Edward's School, Oxford	46	3.79	54	2.42
St John's School, Leatherhead	47	3.78	55	2.38
Reigate Grammar School	48	3.73	52	2.47
St Benedict's School	49	3.68	68	2.08
St Ambrose College	50	3.60	41	2.83
Monmouth School	50	3.60	46	2.63
Bristol Grammar School	50	3.60	63	2.14
Abingdon School	53	3.59	47	2.60
Eton College	54	3.58	39	2.90
Pate's Grammar School	54	3.58	43	2.78
Canford School	56	3.30	71	2.00
The King's School, Macclesfield	57	3.29	48	2.59
Lymm High School	58	3.27	53	2.45
Trinity School, Croydon	59	3.25	60	2.26
Bryanston School	60	3.20	61	2.24
Magdalen College School	60	3.20	56	2.36
Colston's School	60	3.20	76	1.82
Nottingham High School	63	3.18	96	0.96
The Manchester Grammar School	64	3.07	42	2.82
Cheltenham College	65	3.02	89	1.22
Ampleforth College	66	2.98	97	0.96
Pocklington School	67	2.96	79	1.68
Oakham School	68	2.87	75	1.91
Trent College	69	2.83	77	1.82
Radley College	70	2.82	84	1.45

School	PPPM Rank	PPPM	DMT Rank	DMT
Kirkham Grammar School	1	4.41	2	6.44
Bedford School	2	4.37	3	6.35
Stockport Grammar School	3	4.22	NR	NR
Bromsgrove School	4	4.15	4	6.21
Sedbergh School	5	3.99	5	6.10
Seaford College	6	3.93	13	5.04
Wellington College	7	3.73	1	6.46
Clifton College	8	3.73	8	5.11
Queen Elizabeth Grammar, Wakefield	9	3.69	17	4.98
Tonbridge School	10	3.69	18	4.94
Solihull School	11	3.67	9	5.10
Berkhamsted School	12	3.66	24	4.80
Millfield School	13	3.64	7	5.21
St Paul's School	14	3.58	9	5.10
Oundle School	15	3.48	22	4.86
Harrow School	16	3.48	28	4.70
Cranleigh School	17	3.42	18	4.94
Stowe School	18	3.34	9	5.10
Woodhouse Grove School	19	3.31	6	5.65
Whitgift School	20	3.31	9	5.10
Bishop Wordsworth's School	21	3.31	31	4.40
Reed's School	22	3.30	25	4.79
Hampton School	23	3.25	14	5.00
Sir Thomas Rich's School	24	3.25	26	4.73
St Peter's RC High School, Gloucester	25	3.23	38	4.10
Barnard Castle School	26	3.23	26	4.73
Hereford Cathedral School	27	3.19	30	4.53
Lancaster Royal Grammar School	28	3.17	NR	NR
Sherborne School	29	3.16	21	4.88
Eastbourne College	30	3.15	34	4.20
Blundell's School	31	3.12	33	4.33
Wilmslow High School	32	3.11	37	4.13
King's College School (KCS), Wimbledon	33	3.02	23	4.85
Warwick School	34	3.00	34	4.20
Dulwich College	35	2.97	34	4.20
RGS, High Wycombe	36	2.96	29	4.56
Brighton College	37	2.96	31	4.40
St Peter's School, York	38	2.93	14	5.00
Eton College	39	2.90	54	3.58
Stamford	40	2.89	44	3.80
St Ambrose College	41	2.83	50	3.60
The Manchester Grammar School	42	2.82	64	3.07
Pate's Grammar School	43	2.78	54	3.58
Epsom College	44	2.75	40	3.91
Durham School	45	2.73	20	4.92
Monmouth School	46	2.63	50	3.60
Abingdon School	47	2.60	53	3.59
The King's School, Macclesfield	48	2.59	57	3.29
The King's School, Worcester	49	2.55	41	3.90
Bradford Grammar School	50	2.50	16	4.99
Bloxham School	51	2.50	44	3.80
Reigate Grammar School	52	2.47	48	3.73
Lymm High School	53	2.45	58	3.27
St Edward's School, Oxford	54	2.42	46	3.79
St John's School, Leatherhead	55	2.38	47	3.78
Magdalen College School	56	2.36	60	3.20
Ellesmere College	57	2.35	73	2.77
Denstone College	58	2.33	43	3.87
RGS, Newcastle	59	2.30	42	3.88
Trinity School, Croydon	60	2.26	59	3.25
Bryanston School	61	2.24	60	3.20
Wirral Grammar School for Boys	62	2.21	91	1.60
Bristol Grammar School	63	2.14	50	3.60
Skinners' School	64	2.14	76	2.57
Campion School, Essex	65	2.12	72	2.80
John Fisher School	66	2.10	39	3.99
Kingswood School	67	2.08	79	2.40
St Benedict's School	68	2.08	49	3.68
Crossley Heath School	69	2.07	NR	NR
Haileybury	70	2.04	77	2.50

School	DMT Rank	DMT	PPPM Rank	PPPM
The Oratory School	71	2.81	73	1.93
Campion School, Essex	72	2.80	65	2.12
Ellesmere College	73	2.77	57	2.35
Loughborough Grammar School	74	2.70	88	1.24
Hurstpierpoint College	75	2.63	80	1.64
Skinners' School	76	2.57	64	2.14
Haileybury	77	2.50	70	2.04
Marlborough College	78	2.43	81	1.58
Kingswood School	79	2.40	67	2.08
Yarm School	79	2.40	78	1.70
RGS Worcester	81	2.32	86	1.27
Malvern College	82	2.31	72	1.96
Bedford Modern School	83	2.20	82	1.57
Uppingham School	84	2.10	99	0.86
Hymers College	85	2.04	104	0.48
RGS, Guildford	86	2.03	91	1.08
Tiffin School	87	2.02	95	0.99
St George's College, Weybridge	88	1.90	98	0.91
Adams Grammar School	89	1.87	93	1.01
Wimbledon College	90	1.67	100	0.69
Wirral Grammar School for Boys	91	1.60	62	2.21
Dame Allan's Schools	92	1.40	94	1.01
The Grammar School at Leeds	92	1.40	107	0.23
Ashville College	94	1.37	83	1.54
King's College, Taunton	95	1.34	101	0.66
Old Swinford Hospital	96	1.33	103	0.48
Christ's Hospital	97	1.13	90	1.20
Sandbach School	98	1.10	92	1.06
King Edward's School, Birmingham	99	0.80	105	0.42
The Portsmouth Grammar School	100	0.60	106	0.41
St Albans School	100	0.60	102	0.54
Stockport Grammar School	NR	NR	3	4.22
Lancaster Royal Grammar School	NR	NR	28	3.17
Crossley Heath School	NR	NR	69	2.07
Caterham School	NR	NR	74	1.92
London Oratory School	NR	NR	85	1.44
Northampton School for Boys	NR	NR	87	1.27

School	PPPM Rank	PPPM	DMT Rank	DMT
Canford School	71	2.00	56	3.30
Malvern College	72	1.96	82	2.31
The Oratory School	73	1.93	71	2.81
Caterham School	74	1.92	NR	NR
Oakham School	75	1.91	68	2.87
Colston's School	76	1.82	60	3.20
Trent College	77	1.82	69	2.83
Yarm School	78	1.70	79	2.40
Pocklington School	79	1.68	67	2.96
Hurstpierpoint College	80	1.64	75	2.63
Marlborough College	81	1.58	78	2.43
Bedford Modern School	82	1.57	83	2.20
Ashville College	83	1.54	94	1.37
Radley College	84	1.45	70	2.82
London Oratory School	85	1.44	NR	NR
RGS Worcester	86	1.27	81	2.32
Northampton School for Boys	87	1.27	NR	NR
Loughborough Grammar School	88	1.24	74	2.70
Cheltenham College	89	1.22	65	3.02
Christ's Hospital	90	1.20	97	1.13
RGS, Guildford	91	1.08	86	2.03
Sandbach School	92	1.06	98	1.10
Adams Grammar School	93	1.01	89	1.87
Dame Allan's Schools	94	1.01	92	1.40
Tiffin School	95	0.99	87	2.02
Nottingham High School	96	0.96	63	3.18
Ampleforth College	97	0.96	66	2.98
St George's College, Weybridge	98	0.91	88	1.90
Uppingham School	99	0.86	84	2.10
Wimbledon College	100	0.69	90	1.67
King's College, Taunton	101	0.66	95	1.34
St Albans School	102	0.54	100	0.60
Old Swinford Hospital	103	0.48	96	1.33
Hymers College	104	0.48	85	2.04
King Edward's School, Birmingham	105	0.42	99	0.80
The Portsmouth Grammar School	106	0.41	100	0.60
The Grammar School at Leeds	107	0.23	92	1.40

Table 17: 2015/16: Full league table by Daily Mail Trophy Merit Points and Model 2 projected points per match. NR - Not ranked

6.4 R Code

6.4.1 Data preparation

```
## import and prepare data
ResultsPrepare < function(tournament, season, proportion) {
  season.data < read.csv("RugbyResults.csv", header = TRUE)
  # pull all data  held in consolidated file of both tournaments
  # with columns: Competition, Year, Date, Venue, HomeTeam, AwayTeam,
      HomeScore, AwayScore, HomeTries, AwayTries

  # filter for desired tournament
  if(tournament == "DMT")
  {season.data < subset(season.data, Competition == "Daily␣Mail␣Trophy")}
  else if (tournament == "Prem")
  {season.data < subset(season.data, Competition == "Premiership")}
  else {print("Tournament␣not␣recognised")}

  # filter for desired season
  season.data < subset(season.data, Year == season)

  # take proportion of season (N.B. DMT date not currently supplied)
  season.data < season.data[order(as.Date(season.data$Date, format="%d/%m/%Y"
      )),]
  season.data < season.data[1:round(proportion*nrow(season.data)),]

  # add result and try outcomes
  # result outcome
  season.data$FTR < 0
  season.data$FTR < ifelse (season.data $ HomeScore  season.data $ AwayScore
      > 7 , "HW",
                            ifelse (season.data $ HomeScore  season.data $
                              AwayScore > 0, "HN",
                                ifelse (season.data $ HomeScore  season.
                                  data $ AwayScore == 0, "DD",
```

```
                                          ifelse (season.data$HomeScore
                                             season.data$AwayScore >= 7, "
                                            AN" , "AW"))))

  # try bonus outcome
  season.data$FTB < 0
  season.data$FTB < ifelse (season.data $ HomeTries >= 4 ,
                          ifelse (season.data $ AwayTries >= 4, "BB", "HB") ,
                          ifelse (season.data $ AwayTries >=4, "AB" , "ZB"))

  teamNames < sort(union(season.data$HomeTeam, season.data$AwayTeam)) #
      create team names list

  return(season.data)
}


## Re express data as count data
ExpandResults < function(season.data, priorWeight, teams){

  # expand wrt result outcome
  expandedR < gnm::expandCategorical(season.data, "FTR", idvar = "match")
  expandedR$FTB < as.factor(0)

  # expand wrt try bonus outcome
  expandedB < gnm::expandCategorical(season.data, "FTB", idvar = "match")
  expandedB$FTR < as.factor(0)

  expandedR < expandedR[c(1:10,12,11,13:14)] # reverse (FTR, FTB) so columns
      match in expandedR and expandedB

  levels(expandedB $ match) < paste0(levels(expandedB $ match), "b") # allow
      for different normalisation
```

```r
  ## set up the "prior data" part of the dataframe
  priorData < data.frame(Competition = NA, Year = 0, Date = NA,
                         HomeTeam = teams, AwayTeam = "team0",
                         Venue = NA, HomeScore=0, AwayScore=0, HomeTries=0,
                             AwayTries=0,
                         FTR = 0, FTB = 0)
  priorData[1,11] < "loss"
  priorData[2:length(teams),11] < "win"

  # expand to express as count data
  expandedP < gnm::expandCategorical(priorData, "FTR",
                                     idvar = "match", group = FALSE)
  expandedP < expandedP[, c(1:10, 12, 11, 14, 13)] # columns match expandedR
      and expandedB
  levels(expandedP $ match) < paste0(levels(expandedP $ match), "p") #
      identify as different matches
  expandedP $ count < 1

  # add weights
  expandedP $ weight < priorWeight
  expandedR $ weight < 1
  expandedB $ weight < 1

  # combine
  expanded < rbind(expandedP, expandedR, expandedB)

  return(expanded)
}


## define design matrix wrt abilities
makeX < function(season.data,teams){
  X < matrix(0,
             nrow(season.data),
```

```
            length(teams)
)
colnames(X) < teamNames
for (team in colnames(X)) {

  # 5 possible Home team result outcomes
  X[season.data$HomeTeam == team & season.data$FTR == "HW", team] < 4
  X[season.data$HomeTeam == team & season.data$FTR == "HN", team] < 4
  X[season.data$HomeTeam == team & season.data$FTR == "DD", team] < 2
  X[season.data$HomeTeam == team & season.data$FTR == "AN", team] < 1
  X[season.data$HomeTeam == team & season.data$FTR == "AW", team] < 0

  # 5 possible Away team result outcomes
  X[season.data$AwayTeam == team & season.data$FTR == "HW", team] < 0
  X[season.data$AwayTeam == team & season.data$FTR == "HN", team] < 1
  X[season.data$AwayTeam == team & season.data$FTR == "DD", team] < 2
  X[season.data$AwayTeam == team & season.data$FTR == "AN", team] < 4
  X[season.data$AwayTeam == team & season.data$FTR == "AW", team] < 4

  # 4 possible Home Try bonus outcomes
  X[season.data$HomeTeam == team & season.data$FTB == "BB", team] < 1
  X[season.data$HomeTeam == team & season.data$FTB == "HB", team] < 1
  X[season.data$HomeTeam == team & season.data$FTB == "AB", team] < 0
  X[season.data$HomeTeam == team & season.data$FTB == "ZB", team] < 0

  # 4 possible Away Try bonus outcomes
  X[season.data$AwayTeam == team & season.data$FTB == "BB", team] < 1
  X[season.data$AwayTeam == team & season.data$FTB == "HB", team] < 0
  X[season.data$AwayTeam == team & season.data$FTB == "AB", team] < 1
  X[season.data$AwayTeam == team & season.data$FTB == "ZB", team] < 0

  # 2 possible "outcomes" in the prior data
  X[season.data$HomeTeam == team & season.data$FTR == "win", team] < 1
  X[season.data$HomeTeam == team & season.data$FTR == "loss", team] < 0
```

```
  }
  return(X)
}
```

6.4.2 Model 1

```
## Function to fit model 1
fitRugby1 < function(season.data, teamNames, coefs=TRUE){
  season.data$X < makeX(season.data, teamNames)
  nteams < ncol(season.data$X)

  # insert columns for narrow, draw, both, zero
  season.data$narrow < as.numeric(season.data$FTR == "HN" | season.data$FTR
     == "AN")
  season.data$draw < as.numeric(season.data$FTR == "DD")

  # insert column for try
  season.data$try < 0
  season.data < within(season.data, try[FTB == "BB"] < 2)
  season.data < within(season.data, try[FTB == "HB"] < 1)
  season.data < within(season.data, try[FTB == "AB"] < 1)
  season.data < within(season.data, try[FTB == "ZB"] < 0)

  # insert column for home advantage
  season.data$home < 0

  # home based on result outcome
  season.data < within(season.data, home[FTR == "HW"] < 4)
  season.data < within(season.data, home[FTR == "HN"] < 3)
  season.data < within(season.data, home[FTR == "DD"] < 0)
  season.data < within(season.data, home[FTR == "AN"] <  3)
  season.data < within(season.data, home[FTR == "AW"] <  4)
```

```
# home based on try bonus outcome
season.data < within(season.data, home[FTB == "BB"] < 0)
season.data < within(season.data, home[FTB == "HB"] < 1)
season.data < within(season.data, home[FTB == "AB"] <  1)
season.data < within(season.data, home[FTB == "BB"] < 0)

season.data < within(season.data, home[Venue == "Neutral"] < 0)
#  Used for DMT where Neutral Venue == "N"

thefit < gnm(count ~ +  1 + X + narrow + draw + try + home,
             eliminate = match,
             family = poisson,
             weights = weight,
             data = season.data)

thecoefs < coef(thefit)
names(thecoefs)[1:nteams] < colnames(season.data$X)
narrow < thecoefs[nteams + 1]
draw < thecoefs[nteams + 2]
try < thecoefs[nteams + 3]
home < thecoefs[nteams + 4]

abilities < rev(sort(thecoefs[1:nteams]))
abilities < exp(abilities)
narrow < exp(narrow)
draw < exp(draw)
try < exp(try)
home < exp(home)

# Parameter names used for clarity. Dissertation text mapping:
# narrow   Kappa = exp(Beta_n)
# draw   Nu = exp(Beta_d)
# try   Psi = exp(Gamma_tb)
# home   Tau = exp(Sigma)
```

```
  lambda < 2*narrow / (2+2*narrow+draw)
  rho < draw / (2+2*narrow+draw)

  xi < try/(1+try)

  pppm < pppm1(abilities, narrow, draw, try, home)

  if (coefs)
    return(c(pppm, abilities, narrow, draw, try, home, lambda, rho, xi))    ##
        translated from log scale
  else return(thefit)
}


## function to fit model 1 with structural parameters constrained
fitRugby1constrain < function(season.data, teamNames, setParameters = c
    (0,0,0,0), coefs=TRUE){
  season.data$X < makeX(season.data, teamNames)
  nteams < ncol(season.data$X)

  # insert columns for narrow, draw, both, zero
  season.data$narrow < as.numeric(season.data$FTR == "HN" | season.data$FTR
     == "AN")
  season.data$draw < as.numeric(season.data$FTR == "DD")

  # insert column for try
  season.data$try < 0
  season.data < within(season.data, try[FTB == "BB"] < 2)
  season.data < within(season.data, try[FTB == "HB"] < 1)
  season.data < within(season.data, try[FTB == "AB"] < 1)
  season.data < within(season.data, try[FTB == "ZB"] < 0)

  # insert column for home advantage
```

```
season.data$home < 0

# home based on result outcome
season.data < within(season.data, home[FTR == "HW"] < 4)
season.data < within(season.data, home[FTR == "HN"] < 3)
season.data < within(season.data, home[FTR == "DD"] < 0)
season.data < within(season.data, home[FTR == "AN"] <  3)
season.data < within(season.data, home[FTR == "AW"] <  4)

# home based on try bonus outcome
season.data < within(season.data, home[FTB == "BB"] < 0)
season.data < within(season.data, home[FTB == "HB"] < 1)
season.data < within(season.data, home[FTB == "AB"] <  1)
season.data < within(season.data, home[FTB == "BB"] < 0)

season.data < within(season.data, home[Venue == "Neutral"] < 0)
#  Used for DMT where there are some matches played at a Neutral Venue

# require log(parameters) for offset
logParameters < as.numeric(lapply(setParameters,log))

season.data$narrowOffset < season.data$narrow * logParameters[1]
season.data$drawOffset < season.data$draw * logParameters[2]
season.data$tryOffset < season.data$try * logParameters[3]
season.data$homeOffset < season.data$home * logParameters[4]

thefit < gnm(count ~ +  1 + X ,
             eliminate = match,
             family = poisson,
             weights = weight,
             data = season.data,
             offset = narrowOffset + drawOffset + tryOffset + homeOffset)

thecoefs < coef(thefit)
```

```
  names(thecoefs)[1:nteams] < colnames(season.data$X)
  abilities < rev(sort(thecoefs[1:nteams]))
  abilities < exp(abilities)


  # Parameter names used for clarity. Dissertation text mapping:
  # narrow   Kappa = exp(Beta_n)
  # draw   Nu = exp(Beta_d)
  # both   Theta = exp(Gamma_bb)
  # zero   Phi = exp(Gamma_zb)
  # home   Tau = exp(Sigma)

  lambda < 2*setParameters[1] / (2+2*setParameters[1]+setParameters[2])
  rho < setParameters[2] / (2+2*setParameters[1]+setParameters[2])

  xi < setParameters[3]/(1+setParameters[3])

  pppm < pppm1(abilities, setParameters[1], setParameters[2], setParameters
      [3], setParameters[4])

  if (coefs)
    return(c(pppm, abilities, narrow, draw, both, zero, home, lambda, rho,
        zeta, eta))   ## translated from log scale
  else return(thefit)
}


## projected points per match based on model 1
pppm1 < function(pi, kappa, nu, psi, tau) {
  pts < outer(pi, pi, function(x, y) {

    #home result outcome expected points
    (4 * tau^4 * x^4 + 4 * kappa * tau^3 * x^4 * y + 2 * nu * x^2 * y^2 +
        kappa * x * y^4 / tau^3) /
```

```
    (tau^4 * x^4 + kappa * tau^3 * x^4 * y + nu * x^2 * y^2 + kappa * x * y
       ^4 / tau^3 + y^4 / tau^4) +

    #away result outcome expected points
    (kappa * tau^3 * y^4 * x + 2 * nu * y^2 * x^2 + 4 * kappa * y * x^4 /
       tau^3 + 4 * x^4 / tau^4) /
    (tau^4 * y^4 + kappa * tau^3 * y^4 * x + nu * y^2 * x^2 + kappa * y * x
       ^4 / tau^3 + x^4 / tau^4) +

    #home try bonus expected points
    (psi^2 * x * y + psi * tau * x) /
    (psi^2 * x * y + psi * tau * x + psi * y / tau + 1) +

    #away try bonus expected points
    (psi^2 * x * y + psi * x / tau) /
    (psi^2 * y * x + psi * tau * y + psi * x / tau + 1)

  })
  diag(pts) < 0
  rowSums(pts) / (2 * (length(pi)  1))
}
```

6.4.3 Model 2

```
## Function to fit model 2
fitRugby2 < function(season.data, teamNames, coefs=TRUE){
  season.data$X < makeX(season.data, teamNames)
  nteams < ncol(season.data$X)

  # insert columns for narrow, draw, both, zero
  season.data$narrow < as.numeric(season.data$FTR == "HN" | season.data$FTR
     == "AN")
  season.data$draw < as.numeric(season.data$FTR == "DD")
  season.data$both < as.numeric(season.data$FTB == "BB")
  season.data$zero < as.numeric(season.data$FTB == "ZB")
```

```
# insert column for home advantage
season.data$home < 0

# home based on result outcome
season.data < within(season.data, home[FTR == "HW"] < 4)
season.data < within(season.data, home[FTR == "HN"] < 3)
season.data < within(season.data, home[FTR == "DD"] < 0)
season.data < within(season.data, home[FTR == "AN"] <  3)
season.data < within(season.data, home[FTR == "AW"] <  4)

# home based on try bonus outcome
season.data < within(season.data, home[FTB == "BB"] < 0)
season.data < within(season.data, home[FTB == "HB"] < 1)
season.data < within(season.data, home[FTB == "AB"] <  1)
season.data < within(season.data, home[FTB == "BB"] < 0)

season.data < within(season.data, home[Venue == "Neutral"] < 0)
#  Used for DMT where Neutral Venue == "N"

thefit < gnm(count ~ +  1 + X + narrow + draw + both + zero + home,
             eliminate = match,
             family = poisson,
             weights = weight,
             data = season.data)

thecoefs < coef(thefit)
names(thecoefs)[1:nteams] < colnames(season.data$X)
narrow < thecoefs[nteams + 1]
draw < thecoefs[nteams + 2]
both < thecoefs[nteams + 3]
zero < thecoefs[nteams + 4]
home < thecoefs[nteams + 5]
abilities < rev(sort(thecoefs[1:nteams]))
```

```
  abilities < exp(abilities)
  narrow < exp(narrow)
  draw < exp(draw)
  both < exp(both)
  zero < exp(zero)
  home < exp(home)

  # Parameter names used for clarity. Dissertation text mapping:
  # narrow   Kappa = exp(Beta_n)
  # draw   Nu = exp(Beta_d)
  # both   Theta = exp(Gamma_bb)
  # zero   Phi = exp(Gamma_zb)
  # home   Tau = exp(Sigma)

  lambda < 2*narrow / (2+2*narrow+draw)
  rho < draw / (2+2*narrow+draw)

  zeta < 1 / (zero + both + 2)
  eta < both / (zero + both + 2)

  pppm < pppm2(abilities, narrow, draw, both, zero, home)

  if (coefs)
    return(c(pppm, abilities, narrow, draw, both, zero, home, lambda, rho,
        zeta, eta))   ## translated from log scale
  else return(thefit)
}

## Function to fit model 2 with structural parameters constrained
fitRugby2constrain < function(season.data, teamNames, setParameters = c
    (0,0,0,0,0), coefs=TRUE){
  season.data$X < makeX(season.data, teamNames)
  nteams < ncol(season.data$X)
```

```
# insert columns for narrow, draw, both, zero
season.data$narrow < as.numeric(season.data$FTR == "HN" | season.data$FTR
    == "AN")
season.data$draw < as.numeric(season.data$FTR == "DD")
season.data$both < as.numeric(season.data$FTB == "BB")
season.data$zero < as.numeric(season.data$FTB == "ZB")

# insert column for home advantage
season.data$home < 0

# home based on result outcome
season.data < within(season.data, home[FTR == "HW"] < 4)
season.data < within(season.data, home[FTR == "HN"] < 3)
season.data < within(season.data, home[FTR == "DD"] < 0)
season.data < within(season.data, home[FTR == "AN"] <  3)
season.data < within(season.data, home[FTR == "AW"] <  4)

# home based on try bonus outcome
season.data < within(season.data, home[FTB == "BB"] < 0)
season.data < within(season.data, home[FTB == "HB"] < 1)
season.data < within(season.data, home[FTB == "AB"] <  1)
season.data < within(season.data, home[FTB == "BB"] < 0)

season.data < within(season.data, home[Venue == "Neutral"] <  0)
#  Used for DMT where there are some matches played at a Neutral Venue

# require log(parameters) for offset
logParameters < as.numeric(lapply(setParameters,log))

season.data$narrowOffset < season.data$narrow * logParameters[1]
season.data$drawOffset < season.data$draw * logParameters[2]
season.data$bothOffset < season.data$both * logParameters[3]
season.data$zeroOffset < season.data$zero * logParameters[4]
season.data$homeOffset < season.data$home * logParameters[5]
```

```
thefit < gnm(count ~ + 1 + X ,
             eliminate = match,
             family = poisson ,
             weights = weight ,
             data = season.data,
             offset = narrowOffset + drawOffset + bothOffset + zeroOffset +
                 homeOffset)

thecoefs < coef(thefit)
names(thecoefs)[1:nteams] < colnames(season.data$X)
abilities < rev(sort(thecoefs[1:nteams]))
abilities < exp(abilities)


# Parameter names used for clarity. Dissertation text mapping:
# narrow    Kappa = exp(Beta_n)
# draw    Nu = exp(Beta_d)
# both    Theta = exp(Gamma_bb)
# zero    Phi = exp(Gamma_zb)
# home    Tau = exp(Sigma)

lambda < 2*setParameters[1] / (2+2*setParameters[1]+setParameters[2])
rho < setParameters[2] / (2+2*setParameters[1]+setParameters[2])

zeta < 1 / (setParameters[4] + setParameters[3] + 2)
eta < setParameters[3] / (setParameters[4] + setParameters[3] + 2)

pppm < pppm2(abilities, setParameters[1], setParameters[2], setParameters
    [3], setParameters[4], setParameters[5])

if (coefs)
    return(c(pppm, abilities, narrow, draw, both, zero, home, lambda, rho,
        zeta, eta))   ## translated from log scale
```

```
  else return(thefit)
}

## projected points per match based on model 2
pppm2 < function(pi, kappa, nu, theta, phi, tau) {
  pts < outer(pi, pi, function(x, y) {

    #home result outcome expected points
    (4 * tau^4 * x^4 + 4 * kappa * tau^3 * x^4 * y + 2 * nu * x^2 * y^2 +
       kappa * x * y^4 / tau^3) /
      (tau^4 * x^4 + kappa * tau^3 * x^4 * y + nu * x^2 * y^2 + kappa * x * y
         ^4 / tau^3 + y^4 / tau^4) +

      #away result outcome expected points
      (kappa * tau^3 * y^4 * x + 2 * nu * y^2 * x^2 + 4 * kappa * y * x^4 /
         tau^3 + 4 * x^4 / tau^4) /
      (tau^4 * y^4 + kappa * tau^3 * y^4 * x + nu * y^2 * x^2 + kappa * y * x
         ^4 / tau^3 + x^4 / tau^4) +

      #home try bonus expected points
      (theta * x * y + tau * x) /
      (theta * x * y + tau * x + y / tau + phi) +

      #away try bonus expected points
      (theta * y * x + x / tau) /
      (theta * y * x + tau * y + x / tau + phi)

  })
  diag(pts) < 0
  rowSums(pts) / (2 * (length(pi)    1))
}
```

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