Retrodictive Modeling of Modern Rugby Union

An extension of Bradley-Terry to multiple outcomes

Hint: some questions you might want to ask me....

- Hi, how are you?
- Is this your first poster?
- The π_is are not a very intuitive measure - is there something more interpretable we can convert them to?
- Don't you need a constraint on the π_is?
- Why don't you use a predictive machine learning based algorithm (similar to how they use NET in NCAA basketball)?
- Rugby is kind of like football, right?
- Would I ever want to calculate the parameters without using a prior?
- Isn't there some sort of big rugby tournament on at the moment?
- How did you actually calculate the parameter estimates?
- I don't suppose you have any business cards?

Ranking vs Rugby

How do we produce a coherent ranking, respecting the scoring rules of a sport, when teams play different numbers of matches, against different strength of opposition, with different proportions at home or away?

In modern rugby union a team earns:

4 for a win
2 for a draw
0 for a loss
1 bonus for losing by less than seven
1 bonus for scoring four or more tries

This gives a system of five result outcomes: wide win, narrow win, draw, narrow loss, wide loss, plus a try bonus point.

The Model

Given a set of rugby match results with team i the home team and team j the away team, the model estimates a vector of ratings $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_n)$ such that

$$P(i \text{ beats } j \text{ wide}) \propto \tau^4\pi_i^4$$
$$P(i \text{ beats } j \text{ narrow}) \propto \kappa\tau^3\pi_i^4\pi_j$$
$$P(i \text{ draws with } j) \propto \nu\pi_i^2\pi_j^2$$
$$P(j \text{ beats } i \text{ narrow}) \propto \kappa\pi_i\pi_j^4/\tau^3$$
$$P(j \text{ beats } i \text{ wide}) \propto \pi_j^4/\tau^4$$

and:

$$P(i \text{ and } j \text{ both try bonus}) \propto \theta\pi_i\pi_j$$
$$P(\text{only } i \text{ try bonus}) \propto \tau\pi_i$$
$$P(\text{only } j \text{ try bonus}) \propto \pi_j/\tau$$
$$P(\text{neither team try bonus}) \propto \phi$$

with τ, κ, ν, θ, and ϕ structural parameters.

Ranking in practice

The Daily Mail Trophy is an annual competition between around a hundred of the best school teams in England. Schools play anywhere between five and thirteen matches, with the strength of opposition varying.

The tournament organisers have expressed a desire for a methodology that is more transparent to stakeholders than they believe this model to be. However the model provides insights on the design and calibration of an alternative.

This led us to propose the 'Dapper' (Damped and Adjusted Points Per match) family of models, which improves their current methodology by incorporating a per match adjustment and a parameter that allows for record preference to be explicitly controlled (see Calibrating Fairness).

Calibrating Fairness

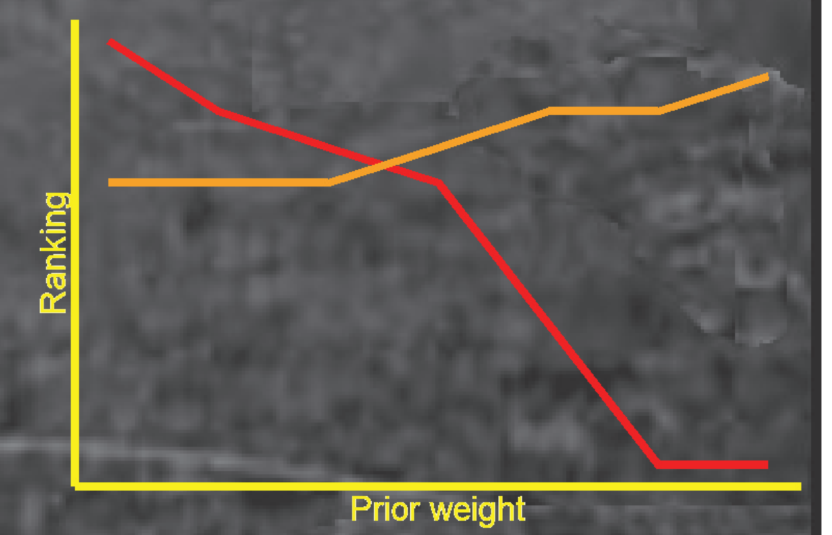
Which of these schools in the 2016/17 Daily Mail Trophy deserved to be ranked higher?

School	P	W	D	L	Bonus	Points per Match
Manchester Grammar	4	4	0	0	3	4.75
Clifton College	10	9	0	1	9	4.5

This is really a question for the tournament stakeholders rather than the statistician, but we can provide means by which a preference may be calibrated.

Adjusting the weight of the prior allows us to do exactly this.

The line chart shows the effect on the rankings of these schools in the 2016/17 tournament from adjusting the weight of the prior. Manchester Grammar is shown in red, Clifton College in orange.



The Evolution of a Sports Ranking Model

1928

The probability that i beats j is defined as

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j}$$

where π_i is a positive-valued rating of i.

Originally proposed by Zermelo (1929) in the context of chess tournaments, before being rediscovered by Bradley and Terry (1952).

1970

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j + \nu\sqrt{\pi_i\pi_j}}$$
$$P(i \prec j) = \frac{\pi_j}{\pi_i + \pi_j + \nu\sqrt{\pi_i\pi_j}}$$
$$P(i \approx j) = \frac{\nu\sqrt{\pi_i\pi_j}}{\pi_i + \pi_j + \nu\sqrt{\pi_i\pi_j}}$$

... extended by Davidson (1970) to allow for ties.

1977

$$P(i \succ j) = \frac{\gamma\pi_i}{\gamma\pi_i + \pi_j + \nu\sqrt{\pi_i\pi_j}}$$
$$P(i \prec j) = \frac{\pi_j}{\gamma\pi_i + \pi_j + \nu\sqrt{\pi_i\pi_j}}$$
$$P(i \approx j) = \frac{\nu\sqrt{\pi_i\pi_j}}{\gamma\pi_i + \pi_j + \nu\sqrt{\pi_i\pi_j}}$$

... further extended by Davidson and Beaver (1977) to, in the context of sports ranking, account for home advantage.

2017

$$P(i \succ j) = \frac{\gamma\pi_i}{\gamma\pi_i + \pi_j + \nu(\pi_i\pi_j)^{\frac{1}{3}}}$$
$$P(i \prec j) = \frac{\pi_j}{\gamma\pi_i + \pi_j + \nu(\pi_i\pi_j)^{\frac{1}{3}}}$$
$$P(i \approx j) = \frac{\nu(\pi_i\pi_j)^{\frac{1}{3}}}{\gamma\pi_i + \pi_j + \nu(\pi_i\pi_j)^{\frac{1}{3}}}$$

... and updated by Firth (2017) to account for modern soccer, where teams are awarded three for a win and one for a draw.

For more details please visit: warwick.ac.uk/IanHamilton