Predicting the Past

A Retrodictive Model for Modern Rugby Union

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A: Daily Mail Trophy!

Motivation

Q: Wouldn't it be nice (for me, at least) if there was a sport with which I was familiar, where the points system was just a bit more complicated, where there was a system of matches that do not make up a full round robin, and there was an actual tournament based on the results of these matches, and the methodology they currently use could do with some serious improvement?

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A: Full house!

Bradley Terry

In the context of tournaments, the probability that team i beats team j is given by

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j}$$

where π_i is positive-valued, and can be thought of as a parameter reflecting the strength of team i.

Zermelo (1929), Bradley & Terry (1952)

Extension to include ties

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j + \nu\sqrt{\pi_i \pi_j}}$$

$$P(i \approx j) = \frac{\nu\sqrt{\pi_i \pi_j}}{\pi_i + \pi_j + \nu\sqrt{\pi_i \pi_j}}$$

Davidson (1970)

Extension to account for home advantage (order effects)

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \gamma\pi_j + \nu\sqrt{\pi_i\pi_j}}$$

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Davidson & Beaver (1977)

Applying to 3 for a win, 1 for a draw

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j + \nu(\pi_i \pi_j)^{\frac{1}{3}}}$$

$$P(i \approx j) = \frac{\nu(\pi_i \pi_j)^{\frac{1}{3}}}{\pi_i + \pi_j + \nu(\pi_i \pi_j)^{\frac{1}{3}}}$$

See: alt-3.uk

Firth (2017)

Rugby union scoring rule

League Points:

- 4 points for a win
- 2 points for a draw
- 0 points for a loss
- 1 bonus point for losing by less than seven points
- 1 bonus point for scoring four or more tries

Summary

Model	B-T	Davidson	Firth	Rugby
Points - win	1	2	3	4
Points - draw	NA	1	1	2
Points - other	NA	NA	NA	1 (try,losing)
Model - i win	π_i	π_i	π_i	???
Model - draw	NA	$(\pi_i\pi_j)^{1/2}$	$(\pi_i\pi_j)^{1/3}$	???
Model - other	NA	NA	NA	???

RASR (pronounced 'razor') - Ranking Algorithm for Schools Rugby

Part one: result outcome

$$P(\text{team } i \text{ beats team } j \text{ by wide margin}) \propto \tau^4 \pi_i^4$$

$$P(\text{team } i \text{ beats team } j \text{ by narrow margin}) \propto \kappa \tau^3 \pi_i^4 \pi_j$$

$$P(\text{team } i \text{ draws with team } j) \propto \nu \pi_i^2 \pi_j^2$$

$$P(\text{team } j \text{ beats team } i \text{ by narrow margin}) \propto \frac{\kappa \pi_i \pi_j^4}{\tau^3}$$

$$P(\text{team } j \text{ beats team } i \text{ by wide margin}) \propto \frac{\pi_j^4}{\tau^4}$$

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Part two: try bonus outcome

$$P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \theta\pi_i\pi_j$$
$$P(\text{only team } i \text{ gains try bonus point}) \propto \tau\pi_i$$
$$P(\text{only team } j \text{ gains try bonus point}) \propto \frac{\pi_j}{\tau}$$
$$P(\text{neither team gains try bonus point}) \propto \phi$$

A principle-based approach

Maximise entropy

$$S(p) = -\sum_{i,j}\sum_{a,b} p^{ij}_{a,b} \log p^{ij}_{a,b} \quad ,$$

subject to conditions,

$$\sum_{a,b} p^{ij}_{a,b} = 1 \quad , \tag{1}$$

and

$$\sum_{j}\sum_{a,b} a p^{ij}_{a,b} = \sum_{j}\sum_{a,b} a m^{ij}_{a,b} \quad , \tag{2}$$

where $p^{ij}_{a,b}$ is the probability that i gains a points and j gains b points, and $m^{ij}_{a,b}$ is the number of matches that have resulted with i gaining a points and j gaining b points.

A principle-based approach

Taking the Lagrangian and differentiating wrt $p^{ij}_{a,b}$ we have

$$\log p^{ij}_{a,b} = -\lambda_{ij} - a\lambda_i - b\lambda_j - 1 \quad , \tag{3}$$

which gives us that

$$p^{ij}_{a,b} \propto \pi^a_i \pi^b_j \quad , \tag{4}$$

where the $\pi_i = \exp(-\lambda_i)$, may be used to rank the teams, and $\exp(-\lambda_{ij} - 1)$ is the constant of proportionality.

Potential models

Examples:

- Try bonus dependent on result outcome and opposition
- Try bonus independent of result outcome but dependent on opposition
- Try bonus independent of result outcome and opposition
- Offensive-defensive strengths
- Home-away strengths

To prior or not to prior?

Introduce a dummy $team_0$ against whom each other team wins one and loses one, then decide how much weight to give these matches.

Pros:

- Ensures connectedness therefore rating from start of season
- Explicitly controls fairness in situations of varying fixture numbers
- Allows for estimation of structural parameters even with existence of 100% record

Cons:

- Might not match intuition / round robin outcomes

Intuitive Measure

Projected Points per Match

$$\text{PPPM}_i = \frac{1}{n-1} \sum_j \sum_{a,b} ap_{a,b}^{ij}$$

Intuitive measure that converges to the rating in round robin

Daily Mail Trophy

League Points per Match + Additional Points

Additional Points in the Daily Mail Trophy are awarded based on the ranking of the current season's opponents in the previous season's tournament:

Rank 1 to 25:	0.3
Rank 26 to 50:	0.2
Rank 51 to 75:	0.1
Otherwise:	0

Results 2015/16

School	DMT Rank	DMT	PPPM Rank	PPPM
Wellington College	1	6.46	7	3.73
Kirkham	2	6.44	1	4.41
Bedford	3	6.35	2	4.37
Bromsgrove	4	6.21	4	4.15
Sedbergh	5	6.10	5	3.99
Woodhouse Grove	6	5.65	19	3.31
Millfield	7	5.21	13	3.64
Clifton College	8	5.11	8	3.73
Solihull	9	5.10	11	3.67
St Paul's	9	5.10	14	3.58

Results 2016/17

School	DMT Rank	DMT	PPPM Rank	PPPM
Wellington College	1	7.22	3	4.37
Sedbergh	2	6.50	2	4.43
Harrow	3	6.34	6	4.22
St Peter's, York	4	6.23	8	4.06
Kirkham	5	6.15	1	4.61
Canford	6	6.10	9	4.02
Clifton College	7	6.00	5	4.25
Rugby	8	5.96	7	4.06
Brighton College	9	5.90	4	4.29
Woodhouse Grove	10	5.81	12	3.93

Results 2017/18

School	DMT Rank	DMT	PPPM Rank	PPPM
Sedbergh	1	7.41	1	4.65
Wellington College	2	7.18	7	4.18
Cranleigh	3	6.33	4	4.32
Harrow	4	6.20	3	4.33
Cheltenham College	5	6.16	8	4.07
St Peter's, York	6	5.83	6	4.19
Brighton College	7	5.63	20	3.59
Reed's	8	5.50	2	4.38
Clifton College	8	5.50	16	3.72
Haileybury	10	5.49	10	4.02