

RSS Euro 2020 Prediction Competition

Bradley-Terry Go Home

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Motivating ideas

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- 1 A significant proportion of the competition outcome would be luck.
- 2 We don't have much time!
- 3 Markets are good at evaluation.
- 4 Markets get things wrong in (somewhat) predictable ways.

The data - match probabilities

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Tomorrow, 11 Jun

Time	Match	1	X	2	ETS
20:00	Turkey - Italy	8.14	3.90	1.53	14
12 Jun 2021					
14:00	Wales - Switzerland	3.97	3.06	2.21	14
17:00	Denmark - Finland	1.46	4.20	9.08	14
20:00	Belgium - Russia	1.74	3.70	5.37	14
13 Jun 2021					
14:00	England - Croatia	1.68	3.78	5.78	14
17:00	Austria - North Macedonia	1.68	3.58	6.25	14
20:00	Netherlands - Ukraine	1.67	3.68	6.10	14
14 Jun 2021					
14:00	Scotland - Czech Republic	2.98	2.98	2.77	14
17:00	Poland - Slovakia	1.94	3.29	5.34	14
20:00	Spain - Sweden	1.47	4.33	7.96	13
15 Jun 2021					
17:00	Hungary - Portugal	7.49	4.27	1.50	14
20:00	France - Germany	2.69	3.17	2.87	14
16 Jun 2021					
14:00	Finland - Russia	5.31	3.74	1.71	13
17:00	Turkey - Wales	2.30	3.12	3.51	13
20:00	Italy - Switzerland	1.74	3.56	5.34	13
17 Jun 2021					

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The model - Bradley-Terry

Probability that i beats j

$$p_{ij} = \frac{\pi_i}{\pi_i + \pi_j},$$

where π_i is the 'strength' of i .

As a generalised linear model

$$\text{logit}(p_{ij}) = \lambda_i - \lambda_j,$$

where $\lambda_i = \log(\pi_i)$

Zermelo (1929); Bradley and Terry (1952)

Bradley-Terry - typical use

Bradley-Terry model applied to a set of results, for the purpose of prediction or ranking e.g. alt-3.uk

Parameters estimated by maximum likelihood estimation

$$L(\lambda) = \prod_{i < j} \binom{m_{ij}}{c_{ij}} p_{ij}^{c_{ij}} (1 - p_{ij})^{m_{ij} - c_{ij}},$$

where c_{ij} is the number of time i beats j and $m_{ij} = c_{ij} + c_{ji}$ is the number of matches between i and j .

Bradley-Terry - issues

But:

- 1 not enough recent useful results to estimate strengths reliably
- 2 market prices are likely to be more informative
- 3 draws in the group stages

Bradley-Terry - dealing with draws

Extension to draws (alt-3.uk, Davidson (1970))

$$\mathbb{P}(i \text{ beats } j) = \frac{\pi_i}{\pi_i + \pi_j + \nu(\pi_i\pi_j)^{\frac{1}{3}}}$$
$$\mathbb{P}(i \text{ draws with } j) = \frac{\nu(\pi_i\pi_j)^{\frac{1}{3}}}{\pi_i + \pi_j + \nu(\pi_i\pi_j)^{\frac{1}{3}}}$$

Note even with draws:

$$\frac{p_{ij}}{p_{ji}} = \frac{\pi_i}{\pi_j} \quad \text{or} \quad \text{logit}(p_{ij}) = \lambda_i - \lambda_j$$

Intra-group strength estimation

Can estimate the intra-group log-strengths $r_i = \log s_i$ by linear regression:

$$\log \left(\frac{p_{ij}}{p_{ji}} \right) = r_i - r_j,$$

since p_{ij} are known from market odds.

But how do we compare strengths between groups?

Overall strength estimation

Assumptions:

- 1 Team i 's overall strength π_i is a scaling of its intra-group strength s_i by a factor dependent on its group $\gamma_{G(i)}$

$$\pi_i = \gamma_{G(i)} s_i \quad \text{or equivalently} \quad \lambda_i = \log \gamma_{G(i)} + r_i$$

- 2 The strength of every team's unknown final opponent is the same

$$p_{i_o} = \mathbb{P}(i \text{ winning tournament} \mid i \text{ reaches final}) = \frac{\pi_i}{\pi_i + \pi_o},$$

where π_o is the strength of the unknown final opponent.

Overall strength estimation

We can calculate p_{io} from market odds since

$$p_{io} = \frac{\mathbb{P}(i \text{ winning tournament})}{\mathbb{P}(i \text{ reaches final})}$$

Then we have that

$$\log \left(\frac{p_{io}}{p_{oi}} \right) = \lambda_i - \lambda_o = \log \gamma_{G(i)} + r_i - \lambda_o,$$

and we can estimate $\log \gamma_{G(i)}$ and λ_o through linear regression.

Knock-out prediction

Now we can calculate the strengths of each team

$$\pi_i = \gamma_{G(i)} s_i,$$

and apply these through the Bradley-Terry model to predict the KO match results

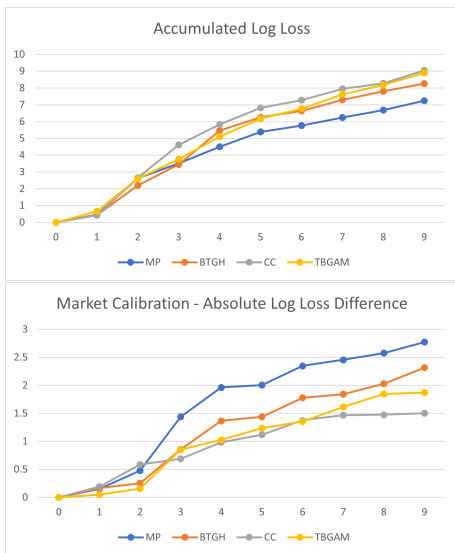
$$p_{ij} = \frac{\pi_i}{\pi_i + \pi_j}.$$

Miscellaneous notes

- 1 Parsimonious model - 120 data points (72 group stage match probabilities + 24 reach final + 24 tournament win); two linear regressions; two days
- 2 What happened to market being wrong in predictable ways?
- 3 Did we do well just because of taking market odds for the group stage?

How much was luck?

Performance graphs for KO stages alone based on the nine competition updates



Bibliography

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