A SPECTRAL AND CROSS-SPECTRAL ANALYSIS OF THE LONG SWING HYPOTHESIS

Matthew J. Lambrinides

NUMBER 35

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

A SPECTRAL AND CROSS-SPECTRAL ANALYSIS OF THE LONG

SWING HYPOTHESIS

Matthew J. Lambrinides

NUMBER 35

University of Warwick October, 1973.

This paper is circulated for discussion purposes only and its contents should be considered preliminary.

TABLE OF CONTENTS

I. PURPOSE AND PLAN OF THE PAPER

The purpose of this paper is to subject to spectral and cross-spectral analysis four major economic and demographic U.S. time series with the purpose of investigating evidences of long swings (or Kuznets cycles, or Kuznets waves, as the long swings are alternatively called in the literature) and testing the plausibility of an elementary model of the generation of the 15 to 25 years wave.

Part II presents a survey of the non-spectral analyses of the long swing hypothesis and discusses the statistical techniques which these analyses employed.

Part III presents the basic concepts underlying the spectral methods of analysis.

The most common estimation problems to which the investigator must address himself when designing a spectral analysis are presented in Part IV.

Part V is concerned with the statistical tests of significance of spectral estimates.

Part VI supplies the empirical findings.

Some overall conclusions are presented in Part VII.

II. NON-SPECTRAL ANALYSES OF THE LONG SWING

A. Hypotheses of the Long-Swing

The hypothesis of fluctuations of an average duration of 15 to 25 years in the rate of growth of numerous economic variables started with the work of Wardwell, Kuznets, and Burns in the late 1920's and the 1930's.

Kuznets's study of data from the U.S., the U.K., Belgium, France, and Germany indicated the existence of long swings in the rate of growth of the production of many individual commodities as well as of prices.[1]

Carrying further the analysis of U.S. data, Burns demonstrated that not only are the Kuznets cycles diffused in many sectors of the U.S. economy, but they tend to be general in the sense that swings in different series exhibit systematic relationships with one another.[2]

Later analysis of U.S. data by Kuznets revealed that 15-25 years long waves were also present in capital formation

---

[1] Simon S. Kuznets, Secular Movements in Production and Prices (Boston: Houghton Mifflin, 1930).

[2] Arthur F. Burns, Production Trends in the United States Since 1870 (New York: National Bureau of Economic Research, 1934.

(both aggregate and its components), population growth, and immigration.[3]

Abramowitz[4] finally showed that

> in the United States, Kuznets' cycles in output growth have arisen from swings in almost all the elements into which output growth can be resolved. Waves in the rate of change of output have been accompanied -- with certain characteristic differences in timing - not only by swings in additions to the labor force, but also by fluctuations in additions to the capital stock, in the rate of increase of output per unit of resources employed, and in indicators of the intensity of resource utilization.[5]

---

[3] Simon S. Kuznets, "Long-Term Changes in National Income of the United States Since 1870," in S.S. Kuznets (ed.), Income and Wealth, Series II (Cambridge, 1952), and

Simon S. Kuznets, "Long Swings in the Growth of Population and in Related Economic Variables," in Proceedings of the American Philosophical Society, Vol. CII, No. 1 (Feb. 1958), pp. 25-52.

[4] Moses Abramowitz, Statement in United States Congress, Joint Economic Committee, Employment, Growth, and Price Levels, Hearings (86th Congress, 1st Session), Part II (Washington, 1959), pp. 411-66.

[5] Moses Abramowitz, "The Nature and Significance of Kuznets Cycles," in American Economic Association, Readings in Business Cycles, Vol. X (Homewood: Richard D. Irwin, 1965), pp. 523-524.

Studies of the Kuznets wave have been made for economies other than the U.S.

Cairncross[6] and especially Brinley Thomas[7] have suggested that the long swings in American and British capital construction and more general economic activity have been inversely related to each other and that this relationship is to be accounted for by British emigration and capital exports to the U.S.

Maurice Wilkinson[8] identified Kuznets cycles in numerous Swedish economic and demographic time series covering the periods of both before and after World War I. The study of the lead-lag relationships of the several variables led him to the formulation of the following model of the long swing mechanism.

There occurs first an investment boom in the manufacturing and mining sectors, possibly derived from an export boom.

The investment boom results in an increase in the demand for labor and per capita income, a decrease in emigration, an increase in marriages and births, and a wave of internal migration from urban to rural areas.

---

[6] A.K. Cairncross, Home and Foreign Investment, 1870-1914 (Cambridge: At the University Press, 1953).

[7] Brinley Thomas, Migration and Economic Growth: A Study of Great Britain and the Atlantic Economy (Cambridge: At the University Press, 1964).

[8] Maurice Wilkinson, Swedish Economic Growth (unpublished Ph.D. dissertation, Harvard University, 1965), Ch.III.

The increase in population growth and movement generates a long swing in residential and other capital construction.

Manufacturing sectors derive demand for consumer and capital goods.

Ohkawa and Rosovsky have identified long swings in the growth of output, investment, and productivity of the Japanese economy and have concluded that investment spurts are the primary causes of the Kuznets cycles of the Japanese economy.[9]

O'Leary and Lewis have, finally, demonstrated that the Kuznets cycle is widespread not only in industrialized but also in agricultural economies. The capital exports of industrial countries constitute the means by which the swing in industrial economies is transmitted to the periphery.[10]

---

[9] K. Ohkawa and H. Rosovsky, "Economic Fluctuations in Prewar Japan: A Preliminary Analysis in Cycles and Long Swings," in Hitotsubashi Journal of Economics, Vol. III, No.1 (October, 1962), and,

K. Ohkawa and H. Rosovsky, Postwar Japanese Growth in Historical Perspective: A Second Look (mimeo.)

[10] P.J. O'Leary and W. Arthur Lewis, "Secular Swings in Production and Trade," in The Manchester School, vol. XXIII (May, 1955).

B. Statistical Technique

The studies referred to in the preceding section employed several statistical techniques for the identification of the long swing. The techniques used can be conveniently grouped into three categories.

A) Kuznets (in the Secular Trends), Burns, and Thomas first removed the 'primary trend' by fitting a logistic curve or a second degree parabola, then took five- or nine-years moving averages of the residuals in order to remove the influence of the Kitchin and the Juglar cycles, and finally examined the smoothed residuals for evidences of Kuznets cycles.

O'Leary and Lewis employed a variation of the above method. They fitted an exponential curve and looked at the unsmoothed residuals for evidences of long waves.

B) The National Bureau of Economic Research (NBER) method employed by Hickman and sometimes by Abramowitz[11] consists of:

---

[11]Bert G. Hickman, "The Postwar Retardation: Another Long Swing in the Rate of Growth?" American Economic Review: Papers and Proceedings, vol. LIII, No. 2 (May, 1963).

Moses Abramowitz, Evidences of Long Swings in Aggregate Construction Since the Civil War (New York: NBER, 1964), chapter 8.

1. removing the influence of the 'specific' cycle from the primary series of GNP, investment, etc. by taking peak-to-peak and trough-to-trough averages and locating them at the midpoints of the specific cycles,

2. calculating the average growth rates of the smoothed series on a peak-to-peak and trough-to-trough basis again and locating these growth rates at the midpoints of their specific bands of years, and,

3. looking at this smoothed series of growth rates for evidences of long swings.

Wilkinson used a variation of the NBER method in which he did not smooth the original series but rather applied the specific cycle analysis directly on the series of unsmoothed growth rates.

C) In his study of Capital in the American Economy: Its Formation and Financing (New York: NBER, 1964), Kuznets first smoothed the primary series by taking five-years moving averages, then estimated average growth rates for overlapping decades, and finally investigated the resulting smoothed series of growth rates.

The objection has been raised to the above techniques that the Kuznets cycles might merely be an 'artifact' created by the smoothing procedures with which the different analysts operated on their series.[12] The Yule-Slutsky theorem concerning the effect of moving averages on a series of random numbers has been invoked to give theoretical basis to the possibility that the Kuznets cycles are spurious.[13] We will consider the effects of smoothing procedures in section IV-B. It is one of the advantages of spectral analysis that it provides a way of determining the effects of smoothing and other transformations on the final results of the analysis.

III. SPECTRAL METHODS OF ANALYSIS: BASIC THEORY

A. Methods of Analysis of Stationary Stochastic Processes

The decomposition of time-dependent economic phenomena into a set of independent components has held a prominent position in pure economic theory as well as in the statistical analysis of economic time series. The 'momentary', 'short-', 'medium', and 'long-run' are the key economic-theoretic concepts describing the decomposition over the time axis of the effects of economic phenomena. The statistical techniques for decomposing a time series into a trend; a seasonal, one or more cyclic, and a random component constitute the equivalent approach in econometrics.

---

[12] See, for example, Irma Adelman, "Kuznets Cycles. Fact or Artifact?", American Economic Review, vol. LV, No. 4, Dec. 1965.

[13] See, for example, Hickman, ibid., p. 492, and Moses Abramowitz, "Resources and Output Trends in the U.S. Since 1870," American Economic Review: Papers and Proceedings, Vol.XLVI, No. 2 (May, 1956), p. 21.

Spectral methods of analysis generalize the concept of decomposition of a time series into a set of statistically independent components, each component corresponding to a different frequency of occurrence.

Spectral analysis can be applied only on covariance stationary stochastic processes. The time series being subject to spectral analysis is thus viewed as a realization $[x(t);\ t = 1, 2, \ldots T]$ of a stationary stochastic process. Stationarity implies that the mean, variance, and covariance of the series do not vary with time,

i.e. $E\,[x(t)] = m$, and

$$E\,[(x(t) - m)(x\,(t+\ell) - m)] = C(\ell),$$

$$\ell = 0, 1, \ldots, M;\ \text{all } t.$$

There are two fundamental statistical approaches to the analysis of the intertemporal dependence of stationary stochastic processes:

1. Analysis of the process in the time domain, and,
2. Analysis of the process in the frequency domain.

The remaining sections of this part of the paper will be devoted to stating the main techniques employed in each of the two approaches, and to pointing out their similarities and differences. Several advantages of the spectral methods will be emphasized in the course of the discussion.

B. Description of a Time Series in the Time Domain

Consider the time series $[x(t);\ t = 1, \ldots T]$ which has been normalized to mean zero. The autocovariance of $x(t)$ is defined as

$$C(t, \ell) = E[x(t)\ x\ (t+\ell)] \qquad \ell = 0, 1, \ldots, M.$$

ℓ is the lag for which the autocovariance is computed.

Since we are concerned with covariance stationary series, C is independent of t, and is a function of the lag ℓ only; that is:

$$C(\ell) = E[x(t)\ x\ (t+\ell)]$$

for $\ell = 0, 1, \ldots, M$, and for all t.

If we normalize the autocovariance function by dividing by C(0) - the variance of the series - , we obtain

$$R(\ell) = \frac{C\ (\ell)}{C\ (0)}, \quad \ell = 1, \ldots, M,$$

the autocorrelation function.

The plot of the autocorrelation function against ℓ is called the correlogram of the series. The corresponding plot of the autocovariance function is called the covariogram. The correlogram and the covariogram are the most widely employed statistical tools for analyzing a stationary stochastic process in the time domain.

C. Description of a Time Series in the Frequency Domain

It can be shown[14] that any covariance stationary stochastic process x(t) can be represented in the form (eq.III-1)

$x(t) = \int_0^\infty [\cos wt\, dU(w) + \sin wt dV(w)]$, where U and V are stochastic processes, and w is frequency of occurrence.

dU(w) and dV(w) have the following properties:

1. $E\ [dU(w)] = E\ [dV(w)] = 0.$

2. dU and dV are orthogonal for any pair of frequencies, that is:

$E\ [dU(w)\ dU(w')] = E\ [dV(w)\ dV(w')] = 0$, for all $w \neq w'$.

3. dU and dV are mutually orthogonal for all frequencies;

$E\ [dU\ (w)\ dV(w')] = 0$, all w and w'.

4. dU and dV have the same variance at all frequencies;

$E\ [dU(w)^2] = E\ [dV(w)^2] = dF(w)$

dF(w) is the power spectrum of x(t).

$f(w) = \frac{dF(w)}{C\ (o)}$ is the spectral density of x(t).

It can be shown that the properties of the random variables dU(w) and dV(w) imply that

$C(\ell) = \int_0^\infty \cos w\ell\ dF(w)$. For $\ell = 0$, we have:

$C(0) = \int_0^\infty (\cos 0)\ dF(w) = \int_0^\infty dF(w).$

---

[14] Yaglom, A.M. An Introduction to the Theory of Stationary Random Functions (Prentice-Hall, 1965).

This last equality implies that the spectrum describes a time series in the frequency domain by decomposing its variance C(0) in terms of the contributions of a set of components indexed on the entire frequency domain.

D. Fundamental Theorem of Time Series Analysis

Recalling that

$$C(\ell) = \int_0^\infty \cos w\ell \, dF(w), \text{ we obtain}$$

$$C(\ell) = \int_0^\infty \frac{e^{iw\ell} + e^{-iw\ell}}{2} \, dF(w),$$

or $$C(\ell) = \int_{-\infty}^{\infty} e^{iw\ell} \frac{dF(w)}{2} \qquad \text{(eq.III-2)}.$$

where by definition $dF(-w) = dF(w)$.

The autocovariance function is thus the Fourier transform of the power spectrum.

Dividing (eq.III-2) through by C(0) we obtain

$$\frac{C(\ell)}{C(0)} = R(\ell) = \tfrac{1}{2} \int_{-\infty}^{\infty} e^{iw\ell} \frac{dF(w)}{C\ (0)} .$$

$\frac{dF(w)}{C(0)}$ is the spectral density.function which is thus the Fourier transform of the autocorrelation coefficient.

We thus arrive to the Fundamental Theorem of Time Series Analysis which states that there is a one-to-one correspondence between the power spectral and the autocovariance function (and, similarly, between the spectral density and the autocorrelation function) of stochastic process, since each pair of functions forms a Fourier transform pair.

E. Comparison of Autocovariance Function and Power Spectrum

According to the Fundamental Theorem of Time Series Analysis, knowledge of either one of the autocovariance and power spectral functions entails knowledge of the other. The following are the main reasons that make it preferable to study intertemporal dependence in the frequency domain using the spectrum (or spectral density) rather than in the time domain using the covariogram (or the correlogram).[15]

1. The spectral function describes a covariance stationary process in terms of additive contributions to the variance of the process of a set of uncorrelated frequency components. In contrast, the value at ℓ_o of the autocovariance function is a weighted sum of its values at ℓ_1, ℓ_2, ... where $\ell_1, \ell_2, \ldots < \ell_o$ and ℓ_o is a linear combination of $\ell_1, \ell_2, \ldots$.

The power spectral function thus gives us information as to the relative importance of the contributions of the different cyclical movements which have generated the observed time series, while the autocovariance function does not.

2. Whereas consistent estimates of the power spectral function can be made, the sampling distribution of the autocovariance function is unstable.

[15] See, for example, C.W.J. Granger and M. Hatanaka, Spectral Analysis of Economic Time Series (Princeton, 1964), Ch. 1 and 2.

F. Cross-Spectral Analysis

The ideas of spectral analysis of a single time series are extended to the cross-spectral analysis of two covariance-stationary time series. This extension is based on a theorem demonstrating that not only is each frequency component of one time series independent of all the other frequency components of the series, but it is also independent of all such components of another stationary series except for the latter's component that corresponds to the same given frequency.[16]

The following measures of the association of the frequency components of two processes are central in cross-spectral analysis:[17]

(1) Coherence, C(w). It can be shown that $0 \le C(w) \le 1$. C(w) is analogous to the square of the correlation coefficient between two time series. The greater C(w) is the closer is the association between the two series at frequency w.

(2) Phase difference, Y(w). The phase difference between the frequency components of the two series measures the extent to which at each particular frequency the cyclical movement of one series leads, coincides with, or lags the corresponding cyclic movement of the second series.

---

[16] See Granger C.W.J. and Hatanaka M, Spectral Analysis of Economic Time Series (Princeton, 1964), Ch. 6.

[17] Ibid.

IV. SPECTRAL METHODS OF ANALYSIS: ESTIMATION PROBLEMS

A. Spectral Averages: Lag Windows and Spectral Windows

The discrete analogue of (eq.III-1) is

$$I(w_i) = \frac{1}{\pi} [C(0) + 2 \sum_{\ell=1}^{M} \cos 2\pi w_i \, C(\ell)] ,$$

where $w_i = \frac{i}{2M}$, $i = 0, 1, \ldots, M$,

M is the maximum lag for which $C(\ell)$ is estimated, and

$I(w_i)$ is the periodogram ordinate for w_i measuring the variance contributed by the oscillation of frequency w_i.

The periodogram estimate of the power spectrum would suggest itself as a natural estimate of the power spectral function. We are, however, interested in economic time series which exhibit oscillations without a strict period. It can be shown that for this class of time series the periodogram ordinates

1) are asymptotically unbiased estimators of the power spectrum, but,

2) they are not consistent estimators.

In order to obtain estimates of the power spectrum that are statistically consistent we must estimate average power over a band of frequencies rather than power at specific frequencies.

A weighting scheme of power in the frequency domain is called a spectral window. The Fourier transform of a spectral window, called a lag window is the corresponding weighting scheme of autocovariances in the time domain.

An ideal window would be one that weighs equally the power of all frequencies $\pm\frac{1}{2}$ the distance between two chosen frequencies and gives zero weight to powers of frequencies outside this band. The perfect window being a mathematical impossibility, the choice of an appropriate window becomes a crucial issue in spectral analysis.

B. Filters- Prewhitening and Recoloring

The window through which we look at specific frequencies of the spectrum concentrates its weight on an arbitrarily small band around a designated frequency, but there is always some leakage through the edges of the window. Some weight, in other words, is given to all frequencies outside the band of interest.

High power at certain frequencies will thus tend to distort the spectral estimates of other frequency components however distant the latter frequencies are from the one with the exceptionally high power.

Most economic processes are not stationary but rather exhibit a trend in their mean. In any finite realization of such a process the trend will be indistinguishable from low frequency components. In order to obtain undistorted spectral estimates we must thus 'filter out' some of the power at the very low frequencies by appropriately transforming our data. Such transformations are known as prewhitening. Taking moving averages, fitting a polynomial in time, or differencing the series are the most commonly employed pre-whitening procedures.

We can now recall the various 'traditional' techniques of analyzing the long swing. The smoothing techniques there employed are in effect pre-whitening techniques. But the spectrum of the pre-whitened series is not the same as the spectrum of the original series. One advantage of spectral over other methods of analyzing time series is that the spectrum of the pre-whitened series can be recolored, i.e. we can move back to an estimate of the spectrum of the original series in which however leakage is not as pronounced as it was in the original series before pre-whitening was performed. 'Traditional' echniques in contrast do not recolor their estimates and thus, in effect, draw inferences about the spectrum of the original series from the spectrum of the pre-whitened series, a procedure which is unwarranted and which may lead to erroneous inferences since the pre-whitened series may exhibit cycles which do not exist in the primary series.

C. The Choice of Maximum Laq - The Uncertainty Principle of Time Series Analysis.

In order that the estimated spectrum averages are representative of the true spectrum, the spectral window must concentrate its main lobe around the particular frequency of interest. The ability of a window to concentrate its focusing power on particular frequencies is measured by the band-width. The band width is defined as the width of the rectangle whose height and area correspond to those of the spectral window at the frequencies of interest.

For the Tukey-Hanning window, the averaging procedure employed in this paper,

$$\text{Bandwidth} = B(M) = \frac{2\pi}{M}.$$

For a normal process the variance of a spectral estimate is

$$V(M) = 0.75 \frac{M}{T} f^2 (w).$$ [18]

The above two equations demonstrate the uncertainty principle of time series analysis. As we increase M we increase the resolution capacity of our window but we do so at the expense of an estimate of increased variance. Hence the choice of the maximum lay M becomes an important issue in the design of spectral analysis.

---

[18] Fishman, G.S., Spectral Methods in Econometrics, (RAND, R-453-PR, 1968), p. 137.

D. Aliasing and Nyquist Frequency.

A recorded time series is the result of sampling an economic phenomenon at equally spaced points in time, each successive two points being Dt apart. We can, therefore, estimate the spectrum only for the frequencies 0 to π/Dt. π/Dt, the highest frequency about which we can obtain direct information is called the Nyquist frequency.

But whereas no direct information can be obtained about frequencies higher than the Nyquist one, these frequencies will tend to be confounded with the frequencies for which the spectrum is measured. Such a confounding of frequencies is known as aliasing. If aliasing of frequencies is serious, then the observed spectrum differs from the true one.

V. TESTS OF SIGNIFICANCE OF SPECTRAL ESTIMATES.

In his study "A Spectral Analysis of the Long-Swing Hypothesis," Howrey performs the following test for the existence of a long swing in an economic time series.

> The long-swing hypothesis can be interpreted as stating that the variance-contribution of [the long swing] band of frequencies is significantly greater than that of neighboring bands. This intuitive statement of the hypothesis suggests that its rejection be based on the absence of a local peak in the spectrum near this long-swing frequency. For the use of the estimated spectrum as a descriptive statistic, this statement of the hypothesis seems to be adequate. However, a more precise formulation of the hypothesis in terms

> of conventional tests of significance is possible. The (100 - 2α) percent confidence band for normally distributed independent random variables, referred to as white noise, can be determined from
>
> $$\Pr\{\chi^2_{1-\alpha}(d) \leq \frac{d\,\hat{f}(w)}{f(w)} \leq \chi^2_{\alpha}(d)\} = 1 - 2\alpha$$
>
> where d [stands] for the equivalent degrees of freedom .... <u>These confidence limits provide a method for testing the hypothesis that the underlying process is random. Specifically an estimate which lies outside the (100 - 2α) percent confidence limits is said to be significantly different from white noise at that level</u>. (emphasis added) [19]

Howrey's test is erroneous on two grounds; first, in terms of its underlying statistical logic, and second, in terms of its interpretation of the Long-Swing Hypothesis. We will deal with both issues at some length.

A) Howrey's method of testing spectral density estimates against the hypothesis of equality with the corresponding white noise density seems to be a common test of

---

[19] Howrey, Philip E., <u>A Spectral Analysis of the Long Swing Hypothesis</u> (Princeton, Econometric Research Program, Research Memo No. 78, 1965), p. 26.

significance in spectral analysis.[20] The underlined quote from Howrey provides the basis for the writer's contention that the test is logically erroneous.

The concept of randomness, when applied to the spectrum of a time series, is a concept that refers to the entire configuration of frequency bands and their spectral densities rather than to the spectral density of one frequency band alone. The estimate of one specific spectral density cannot be (or fail to be) significantly different from white noise. It is the entire time series which either is or is not significantly different from Gaussian white noise.

It is, therefore, perfectly correct to construct frequency bands for the spectral density function of the Gaussian white noise and compare the entire estimated spectral density function with the confidence limits. But once one spectral density is observed to lie outside the confidence intervals of the white noise spectrum, then the hypothesis that the underlying process is random must be rejected. It is logically absurd to then proceed and test whether particular densities could have arisen from white noise. How could they have arisen from a process the existence of which has already been rejected?

[20] See, for example, C.W.J. Granger and H.J.B. Rees, "Spectral Analysis of the Term Structure of Interest Rates," in The Review of Economic Studies, vol. XXXV(1), January 1968.

B) Howrey's procedure of examining only relative peaks of the spectrum is wrong because, in our analysis for example, the frequency band centered around 0.025 years per cycle (which corresponds to the Kondratieff cycle of 40 to 60 years period) neighbors the long swing band of 0.050 cycles per year. There would not be any reason for refusing to examine the contribution of the long-swing band to the variance of the series simply because the Kondratieff band contributed to this variation more than the long-swing band.

In this paper we will employ the following procedure for examining the importance of the long swing:

1. Test the entire spectrum against the null hypothesis Ho that it is not significantly different from the spectrum of white noise.

2. If Ho is rejected, we shall examine the importance of the long-swing frequency bands in terms of their contribution to the variance of the series relative to the contribution of bands corresponding to other important cyclical movements.

We shall regard the bands centered around 0.050 and 0.075 cycles per year as the long swing bands.[21]

---

[21] The corresponding periods are 20 and 13.3 years per cycle.

VI. EMPIRICAL FINDINGS

A. The Series Analyzed

In this part of the paper we will present and analyze the estimated spectra and cross-spectra of the annual growth rates from 1889 to 1967 of the following U.S. series:

1. Real GNP.
2. Real gross capital formation.
3. Real per capita consumption.
4. Population.[22]

The choice of the series was made with a model of the long swing in mind similar to that of Wilkinson.[23]

B. Estimation Considerations

1. Stationarity.

All four of the analyzed series of growth rates seem to satisfy the stationarity requirements. None of the traces of the series exhibits trend in the mean (see figures 1, 2, 3, and 4). The size of the sample precludes any test of heteroscedasticity and changing covariance.

2. Aliasing.

The period between two observations of all time series is one year. The Nyquist frequency is, therefore, 0.5 cycles

---

[22] Appendix 1 contains the primary data as well as a discussion of the operations performed on them.

[23] See Section A of Part II of this paper.

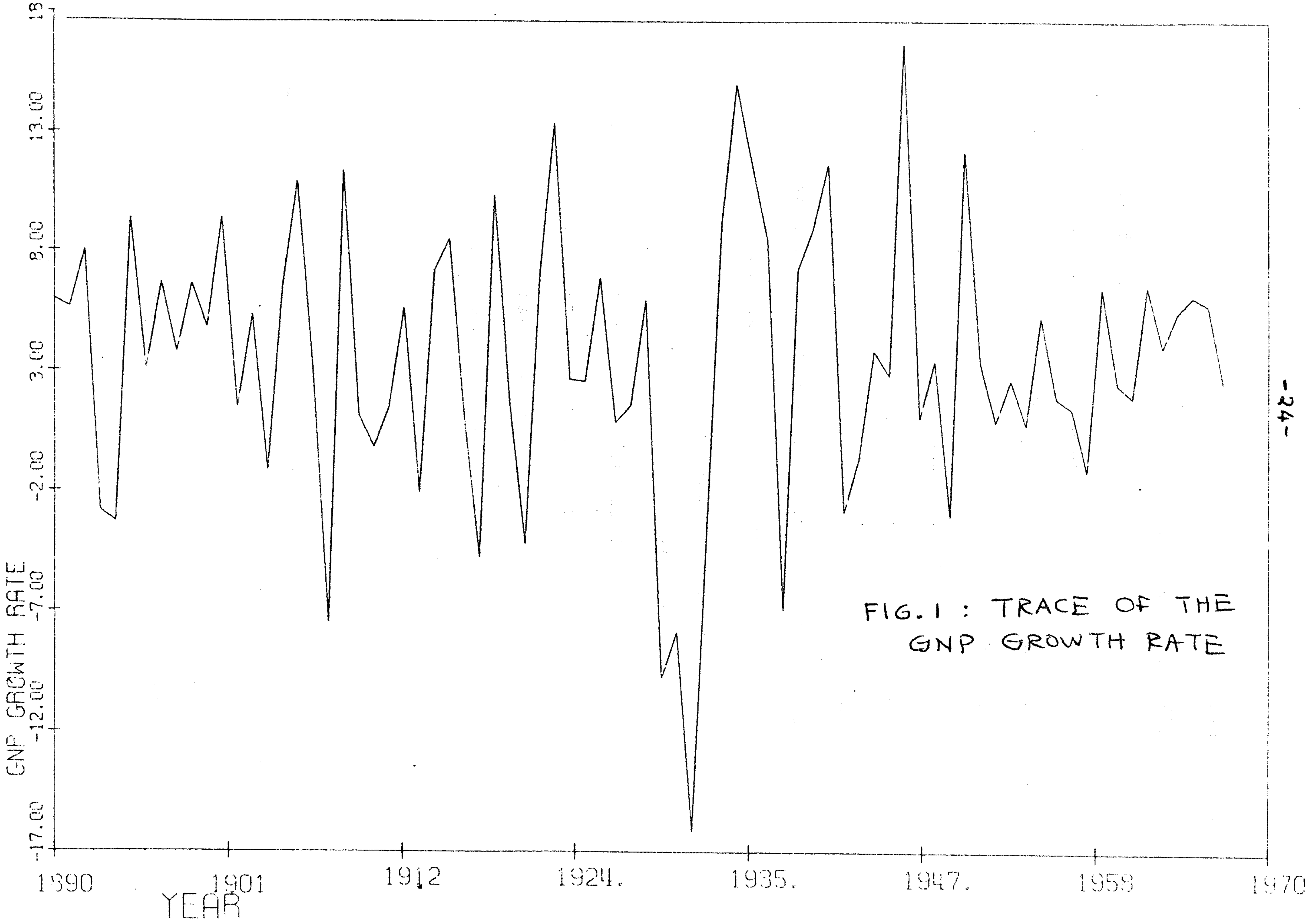

FIG. 1 : TRACE OF THE GNP GROWTH RATE

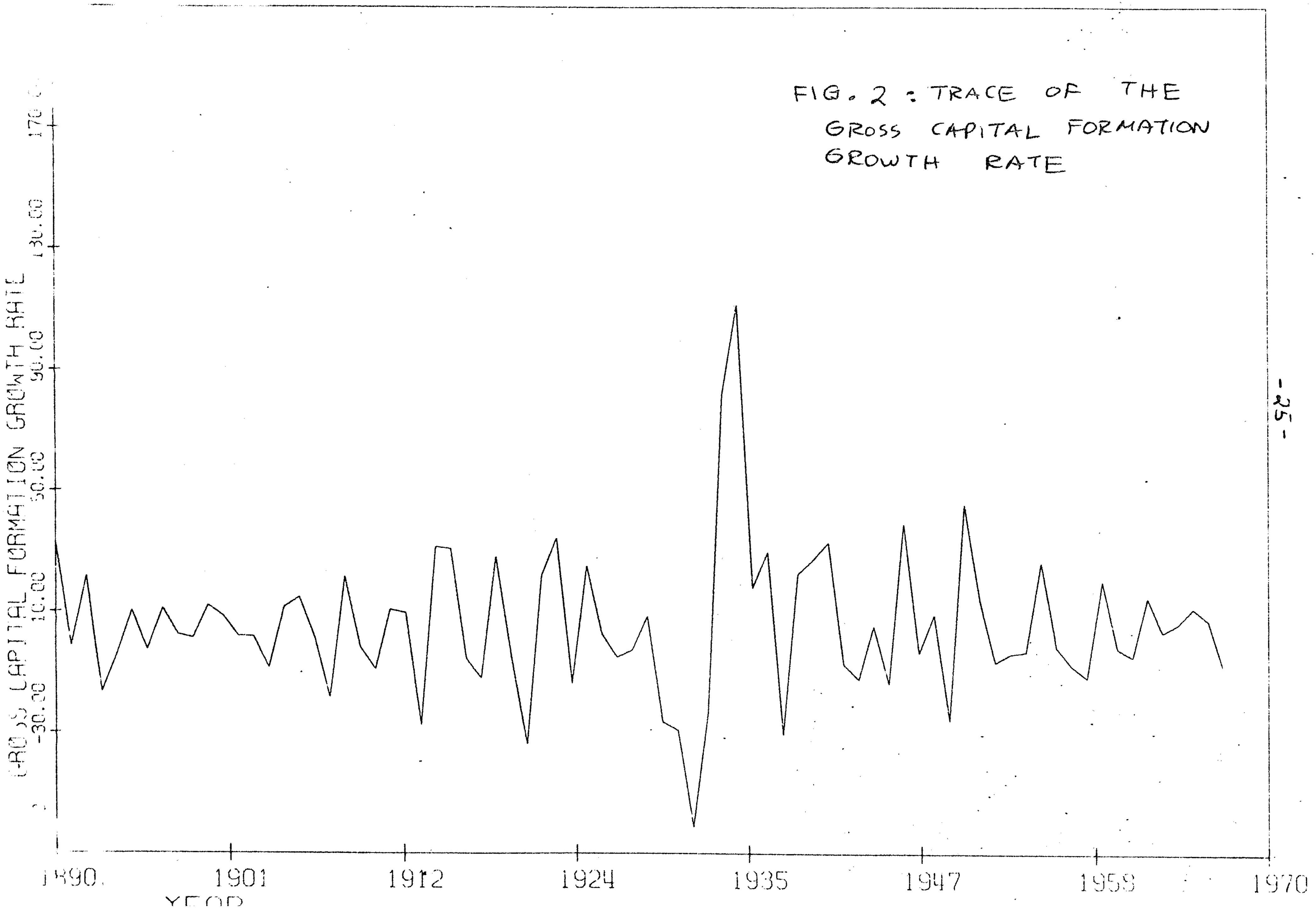

FIG. 2 : TRACE OF THE GROSS CAPITAL FORMATION GROWTH RATE

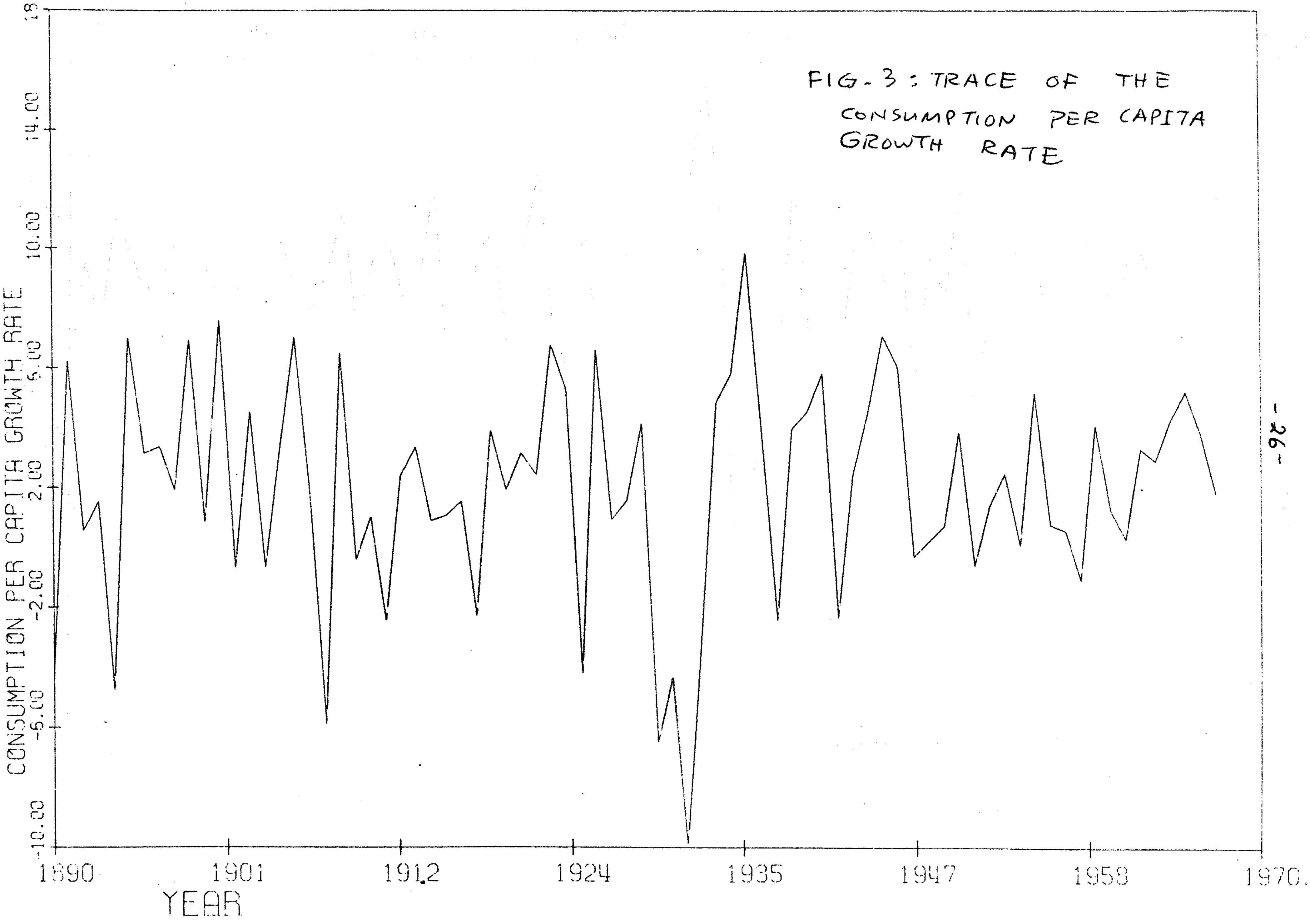

FIG-3: TRACE OF THE CONSUMPTION PER CAPITA GROWTH RATE

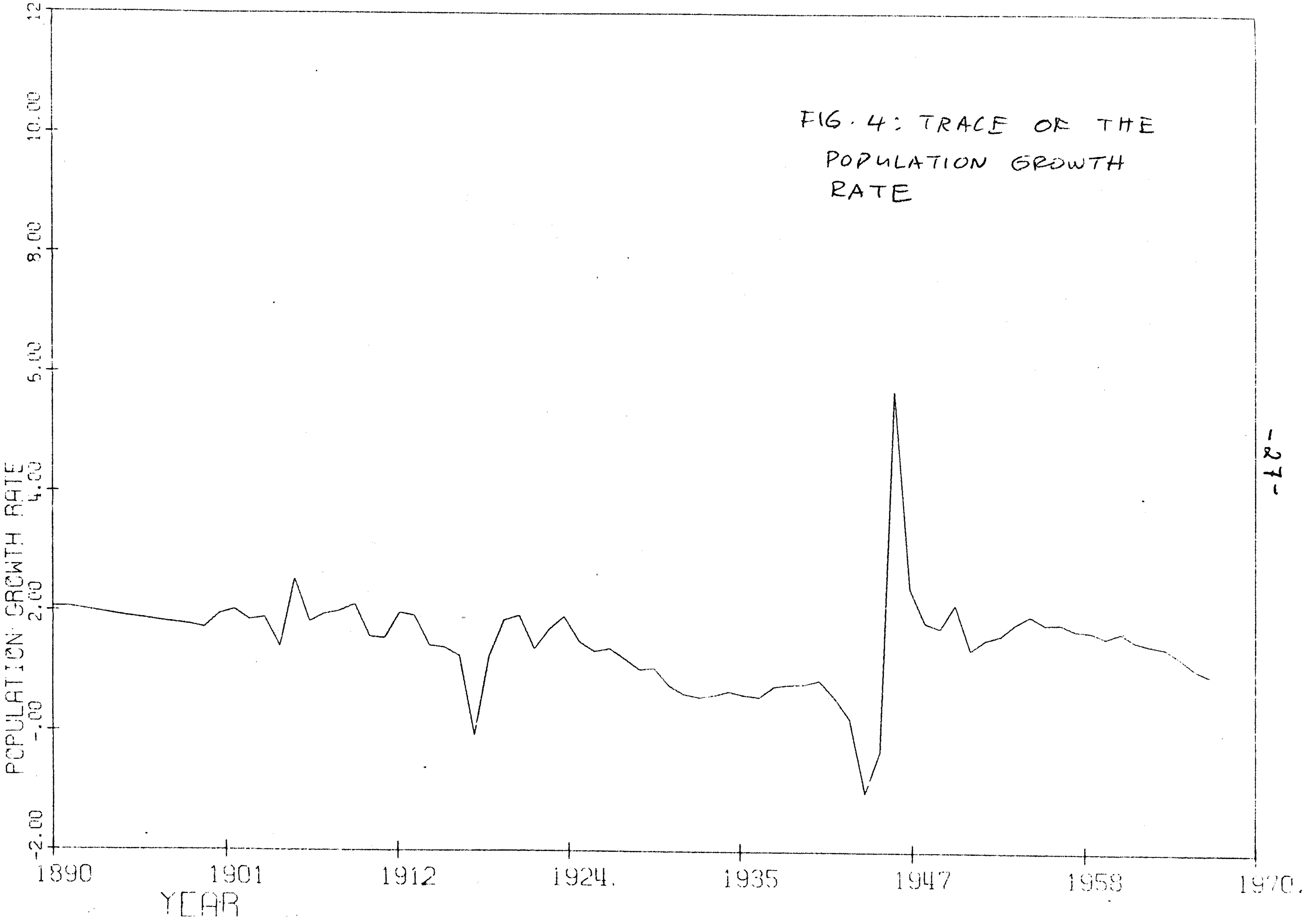

FIG. 4: TRACE OF THE POPULATION GROWTH RATE

per year. Given that the data are seasonally adjusted and that economic and demographic theory supply no evidence of other important cycles in the frequency band 0 to 0.5 cycles per year, it follows that our results are not obscured by aliasing of frequencies.

3. Choice of Window

The Tukey window was used in this paper. The characteristic measures of the window are given on Table 1.

4. Choice of Maximum Lag

In section C of Part IV, we stated that the choice of the maximum lag is an important part of the design of spectral analysis because of the conflict between resolvability and low variance of estimate.

Priestley[24] has proposed a rigorous method for choosing the maximum lag by fixing an acceptable level of bias of estimate and then choosing the lag that minimizes the variance; or, alternatively, by choosing M so as to minimize the mean squared error of the estimated ordinates.

The limited length of the four time series, unfortunately, made any rigorous choice of M impossible. Our choice of M was thus made very crudely. The analysis should satisfy the following two requirements:

---

[24] Priestley, M.B., "The Role of Band-Width in Spectral Analysis," in Applied Statistics, 1965.

TABLE 1

Characteristic Measures of the Tukey Window

Lag-Window	$\frac{1}{2}[1 + \cos(\pi\ell/M)]$ for $\ell < M$
	0 for $\ell \geq M$
Spectral Window	$\frac{1}{2\pi}\left\{\frac{\sin(M+\frac{1}{2})w}{\sin w/2} + \frac{1}{2}\left[\frac{\sin(m+\frac{1}{2})(w+\frac{\pi}{M})}{\sin\frac{1}{2}(w-\pi/M)} + \frac{\sin(M+\frac{1}{2})(w-\frac{\pi}{M})}{\sin\frac{1}{2}(w-\pi/M)}\right]\right\}$
Bandwidth	$2\pi/M$
Variance	$0.75\ (M/T)\ f^2(w)$
Equivalent Degrees of freedom	$2.7\ (T/M)$

Source: Fishman

1) The maximum lag should not be greater than approximately one fourth of the length of the time series.[25]

2) The maximum lag should not be too small compared to the length of each one of the periods corresponding to the long swing.

The choice of a lag equal to twenty seemed to achieve the best balance between the above two requirements.

C. The Estimated Spectral Densities

The estimated spectral densities are presented on Tables 2, 3, 4, and 5, and on Figures 5, 6, 7, and 8. The 90 and 95 percent confidence limits for the spectral densities of Gaussian white noise for the length of series and maximum lag of our analysis are given on Table 6.[26]

The spectral density functions of all four series are significantly different from the white noise spectrum at the 95 percent confidence level.

The contribution to the overall variance of each series of the long-swing frequency bands is high for all series.

---

[25] This is a rule of thumb followed by all students of spectral analysis. See, for example, Granger and Hatanaka, or Fishman.

[26] See Appendix B for the derivation of the confidence limits.

TABLE 2.

U.S.: Spectral Estimates of the Growth Rate of GNP

Frequency (cycles/year)	Spectral Density
0	0.46
0.025	1.08
0.050	2.13
0.075	3.04
0.100	2.76
0.125	1.59
0.150	2.41
0.175	4.10
0.200	3.62
0.225	1.63
0.250	0.49
0.275	2.06
0.300	3.40
0.325	2.10
0.350	1.09
0.375	0.56
0.400	0.64
0.425	1.15
0.450	1.58
0.475	1.86
0.500	2.25

TABLE 3

U.S.: Spectral Estimates of the Growth Rate of Gross Capital Formation

Frequency (cycles/year)	Spectral Density
0	0.37
0.025	0.72
0.050	1.68
0.075	2.75
0.100	2.88
0.125	2.31
0.150	3.39
0.175	5.02
0.200	3.92
0.225	1.44
0.250	1.21
0.275	1.62
0.300	2.29
0.325	2.61
0.350	1.16
0.375	0.36
0.400	0.47
0.425	1.26
0.450	1.86
0.475	1.49
0.500	1.19

TABLE 4.

U.S.: Spectral Estimates of the Growth Rate of Per Capita Consumption

Frequency (Cycles/Year)	Spectral Density
0	0.34
0.025	1.47
0.050	2.89
0.075	2.53
0.100	2.50
0.125	2.25
0.150	1.83
0.175	2.03
0.200	2.57
0.225	1.97
0.250	0.91
0.275	1.60
0.300	1.79
0.325	0.76
0.350	0.68
0.375	1.54
0.400	2.71
0.425	1.80
0.450	0.82
0.475	2.49
0.500	0.45

TABLE 5.

U.S.: Spectral Estimates of the Population Growth Rate

Frequency (cycles/year)	Spectral Density
0	6.89
0.025	5.94
0.050	2.83
0.075	1.70
0.100	2.08
0.125	2.26
0.150	1.98
0.175	1.70
0.200	1.60
0.225	1.70
0.250	1.88
0.275	1.98
0.300	1.51
0.325	1.13
0.350	0.76
0.375	0.76
0.400	0.57
0.425	0.57
0.450	0.76
0.475	0.76
0.500	0.66

TABLE 6.

Confidence Limits of Spectral Density of Gaussian White Noise for T = 78 and L = 20

CONFIDENCE LEVEL	LOWER LIMIT	UPPER LIMIT
90 percent	0.83	3.58
95 percent	0.69	3.98

Source: Appendix B

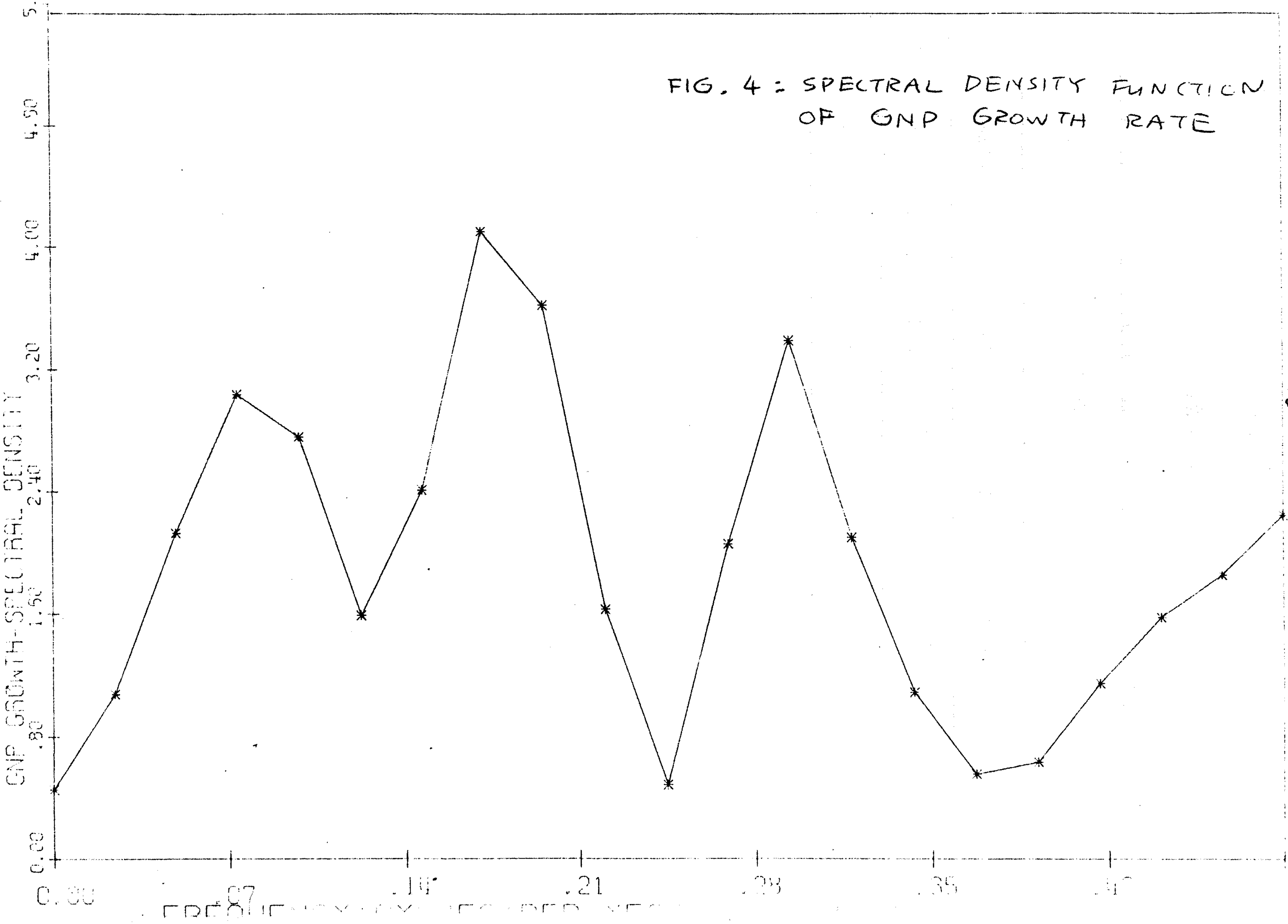

FIG. 4 : SPECTRAL DENSITY FUNCTION OF GNP GROWTH RATE

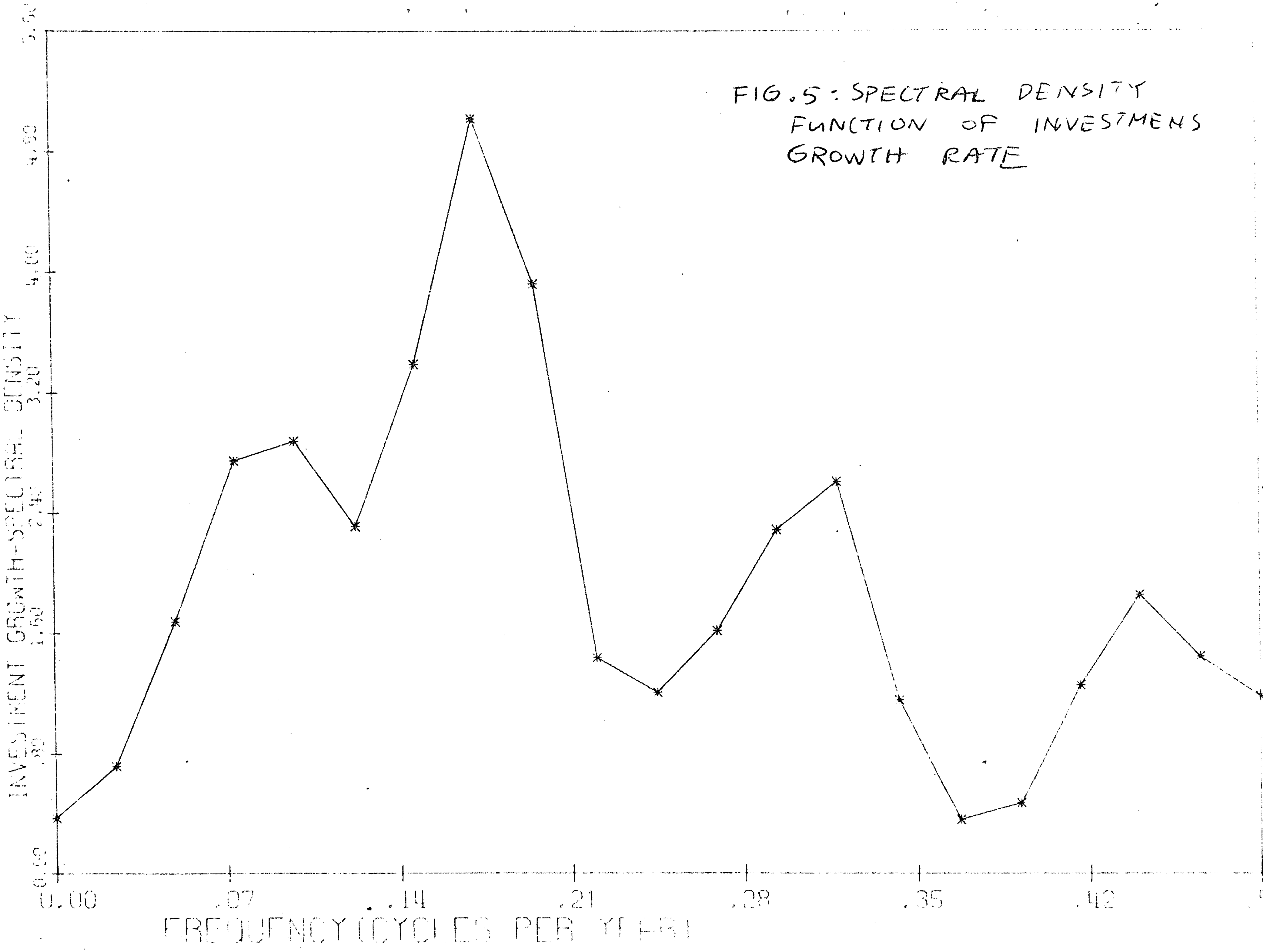

FIG. 5: SPECTRAL DENSITY FUNCTION OF INVESTMENS GROWTH RATE

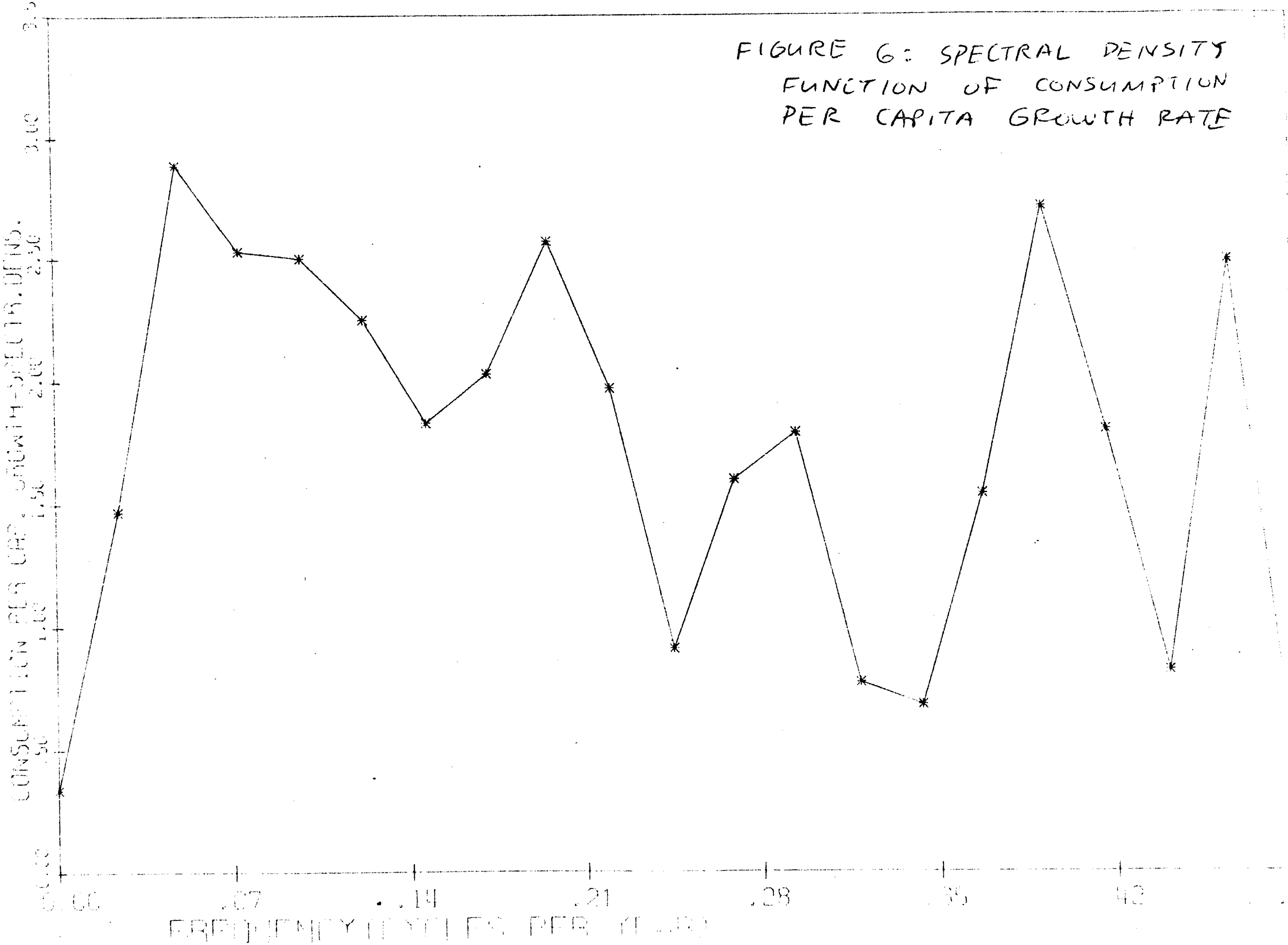

FIGURE 6: SPECTRAL DENSITY FUNCTION OF CONSUMPTION PER CAPITA GROWTH RATE

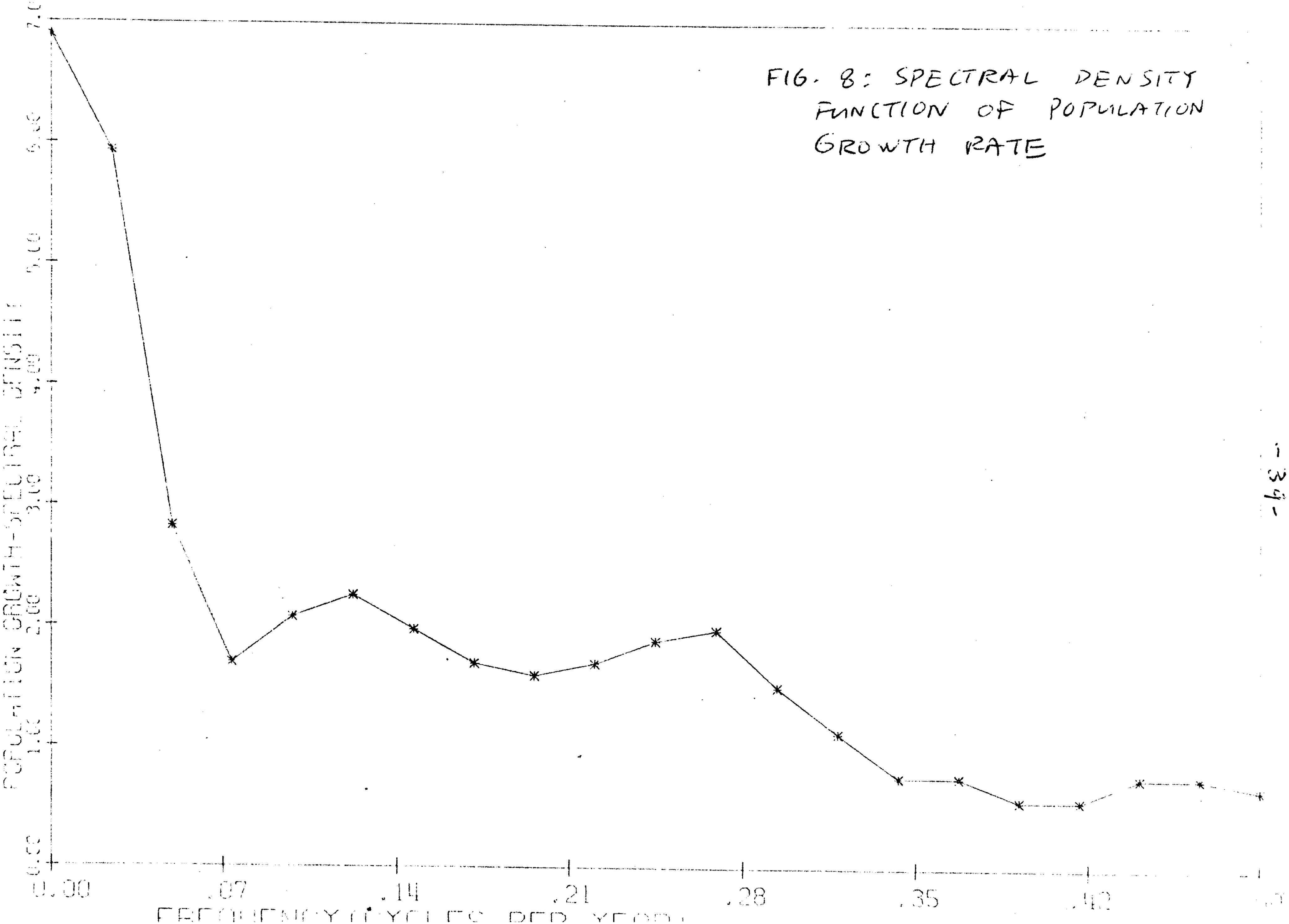

FIG. 8: SPECTRAL DENSITY FUNCTION OF POPULATION GROWTH RATE

Concerning the spectrum of the GNP growth rate, only the bands corresponding to the 3-, 5- and 6-years cycles contribute more to the variance of the series than does the 0.075 cycles per year band. Similarly, the contribution of the 0.05 cycles per year band stands above average on the ranking.

The importance of the long swing bands is less pronounced, though still apparent in the case of gross capital formation.

The 0.05 cycles per year band is the single most important band of the consumption per capita spectrum.

Finally, the 0.05 cycles per year band is third in importance in the population growth spectrum.

D. The Estimated Cross-Spectra

Table 8 exhibits the coherence and phase angle for GNP and investment. The phase angle, being an angle derived from arctan, can be altered by addition or subtraction of any multiple of 2π. We therefore present two estimates of the phase angle, the one differing from the other by 2π. Our discussion is in each case based on that one of the two angles that seems to be more plausible on the basis of theory and common sense. The fact that one of the two phase angles in most cases implied implausible leads or lags made our choice easy.

Table 7 presents the critical values for estimated coherence when the true coherence is zero. Comparing the estimate of Table 8 with these critical values, we see that the coherence between GNP and investment is significantly different from zero at the 95 percent level for both the long swing bands. The long swing in investment leads the long swing in GNP by approximately 4 to 5 years.

Per capita consumption and population exhibit very low coherences (see Table 9).

Per capita consumption and GNP exhibit significant coherences and are approximately coincident. (see Table 10).

Per capita consumption and investment exhibit coherences that are not significant at the 90 percent confidence level, but come closely to being so. Investment leads per capita consumption by 3 to 4 years (see Table 11).

The cross-spectra of investment and population on the one hand and GNP and population on the other do not exhibit coherences significantly different from zero at the 90 percent confidence level (see Tables 12 and 13).

The results of the cross-spectral analysis thus suggest that the long swing in investment sets off a corresponding wave in GNP (a proxy for productive activity) and GNP per capita (a proxy for the standard of living). No link was found between per capita consumption and population.

TABLE 7.

Critical Values for C (w) when C(w) = 0 for T/M = 4

Median	0.454
90%	0.732
95%	0.795

Source: Granger and Hatanaka, p. 79.

TABLE 8.

U.S.: Coherence and Phase Angle of the Growth Rates of GNP and Gross Capital Formation.

Frequency (Cycles/year)	Coherence	Phase Angle from GNP to Gross Capital Formation (fraction of a circle) A	B
0	0.70	0.73	-.27
0.025	0.82	0.81	-.19
0.050	0.78	0.88	-.12
0.075	0.90	0.95	-.05
0.100	0.95	0.96	-.04
0.125	0.86	0.95	-.05
0.150	0.93	0.00	-1.00
0.175	0.97	0.00	-1.00
0.200	0.87	0.98	-.02
0.225	0.42	0.97	-.03
0.250	0.23	0.97	-.03
0.275	0.60	0.96	-.04
0.300	0.72	0.96	-.04
0.325	0.79	0.97	-.03
0.350	0.87	0.00	-1.00
0.375	0.32	0.03	-.97
0.400	0.27	0.22	-.78
0.425	0.60	0.06	-.94
0.450	0.92	0.00	-1.00
0.475	0.78	0.97	-.03
0.500	0.67	0.97	-.03

TABLE 9.

U.S.: Coherence and Phase Angle of the Growth Rates of Per Capita Consumption and Population

Frequency (cycles/year)	Coherence	Phase Angle From Consumption to Population (fraction of a circle) A	B
0	0.55	0.14	-.86
0.025	0.31	0.24	-.76
0.050	0.27	0.30	0.70
0.075	0.06	0.04	-.96
0.100	0.22	0.19	-.81
0.125	0.24	0.32	-.68
0.150	0.07	0.89	-.11
0.175	0.27	0.11	-.89
0.200	0.71	0.23	-.77
0.225	0.45	0.28	-.72
0.250	0.14	0.44	-.56
0.275	0.00	0.01	-.99
0.300	0.28	0.04	-.96
0.325	0.43	0.97	-.03
0.350	0.01	0.77	-.23
0.375	0.01	0.71	-.29
0.400	0.13	0.04	-.96
0.425	0.39	0.12	-.88
0.450	0.20	0.06	-.94
0.475	0.02	0.46	-.54
0.500	0.17	0.48	-.52

TABLE 10.

U.S.: Coherence and Phase Angle of the Growth Rates of GNP and Per Capita Consumption

Frequency (cycles/year)	Coherence	Phase Angle from GNP to Per Capita Consumption (fraction of a circle)	
0	0.84	0.98	-.12
0.025	0.88	0.01	-.99
0.050	0.90	0.01	-.99
0.075	0.85	0.99	-.01
0.100	0.81	0.99	-.01
0.125	0.79	0.02	-.98
0.150	0.81	0.00	-1.00
0.175	0.95	0.97	-.03
0.200	0.93	0.96	-.04
0.225	0.89	0.95	-.05
0.250	0.33	0.02	-.98
0.275	0.69	0.09	-.91
0.300	0.66	0.06	-.94
0.325	0.41	0.04	-.96
0.350	0.58	0.03	-.97
0.375	0.55	0.05	-.95
0.400	0.50	0.95	-.05
0.425	0.48	0.91	-.09
0.450	0.37	0.98	-.02
0.475	0.43	0.04	-.96
0.500	0.55	0.02	-.98

TABLE 11.

U.S.: Coherence and Phase Angle of the Growth Rates of Per Capita Consumption and Gross Capital Formation

Frequency (cycles/year)	Coherence	Phase Angle from Consumption to Investment (fraction of a circle) A	
0	0.50	0.77	-0.23
0.025	0.65	0.80	-0.20
0.050	0.60	0.85	-0.15
0.075	0.66	0.94	-0.06
0.100	0.71	0.96	-0.04
0.125	0.67	0.90	-0.10
0.150	0.68	0.98	-0.02
0.175	0.98	0.03	-0.97
0.200	0.78	0.02	-0.98
0.225	0.23	0.00	-1.00
0.250	0.11	0.61	-0.39
0.275	0.31	0.80	-0.20
0.300	0.28	0.85	-0.15
0.325	0.10	0.89	-0.11
0.350	0.34	0.96	-0.04
0.375	0.00	0.10	-0.90
0.400	0.51	0.39	-0.61
0.425	0.18	0.27	-0.73
0.450	0.18	0.04	-0.96
0.475	0.08	0.86	-0.14
0.500	0.08	0.88	-0.12

TABLE 12.

U.S.: Coherence and Phase Angle of the Growth Rates of Gross Capital Formation and Population

Frequency	Coherence	Phase Angle from Investment to Population	
		A	B
0	0.89	0.40	-.60
0.025	0.77	0.40	-.60
0.050	0.19	0.45	-.55
0.075	0.22	0.97	-.03
0.100	0.03	0.15	-.85
0.125	0.18	0.55	-.45
0.150	0.33	0.84	-.16
0.175	0.32	0.08	-.92
0.200	0.64	0.20	-.80
0.225	0.06	0.08	-.92
0.250	0.25	0.89	-.11
0.275	0.03	0.22	-.78
0.300	0.03	0.35	-.65
0.325	0.05	0.83	-.17
0.350	0.02	0.04	-.96
0.375	0.41	0.38	-.62
0.400	0.05	0.59	-.41
0.425	0.45	0.99	-.01
0.450	0.72	0.02	-.98
0.475	0.29	0.06	-.94
0.500	0.07	0.15	-.85

TABLE 13.

U.S.: Coherence and Phase Angle of the Growth Rates of GNP and Population

Frequency (cycles/year)	Coherence	Phase Angle from GNP to Population (fraction of a circle) I	II
0	0.80	0.11	-0.89
0.025	0.47	0.22	-0.78
0.050	0.26	0.29	-0.71
0.075	0.21	0.96	-0.04
0.100	0.09	0.09	-0.91
0.125	0.07	0.40	-0.60
0.150	0.23	0.89	-0.89
0.175	0.39	0.08	-0.92
0.200	0.75	0.17	-0.83
0.225	0.44	0.19	-0.81
0.250	0.03	0.09	-0.91
0.275	0.02	0.08	-0.92
0.300	0.10	0.08	-0.92
0.325	0.11	0.95	-0.05
0.350	0.00	0.04	-0.96
0.375	0.00	0.36	-0.64
0.400	0.33	0.00	-1.00
0.425	0.81	0.04	-0.96
0.450	0.67	0.03	-0.97
0.475	0.14	0.08	-0.92
0.500	0.03	0.29	-0.71

VII. CONCLUSION

Being based on only four series, our spectral and cross-spectral analysis of the long swing hypothesis was necessarily elementary.

The results of our research differed in two ways with the conclusions of previous spectral analyses of the Kuznets wave.

First, our spectral estimates indicated that the long swing is an important cyclical movement in terms of its contribution to the variance of the four time series that we studied.

Second, the implications of the cross-spectral estimates were in agreement with the model of long swing outlined by Wilkinson, except for the fact that we could not establish any inter-relationship between population growth and the other three variables.

BIBLIOGRAPHY

I. BOOKS

1. Abramovitz, Moses, Evidences of Long Swings in Aggregate Construction Since the Civil War. New York: National Bureau of Economic Research, 1964.

2. American Economic Association, Readings in Business Cycles, vol. X. Homewood, Illinois: Richard D. Irwin, 1965.

3. Burns, Arthur F. Production Trends in the United States Since 1870. New York: National Bureau for Economic Research, 1934.

4. Fishman, G.S., Spectral Methods in Econometrics. RAND: R-453-PR, 1968.

5. Granger, C.W.J. and Hatanaka, M., Spectral Analysis of Economic Time Series, Princeton University Press, 1964.

6. Kendrick, John W., Productivity Trends in the United States, National Bureau for Economic Research, No. 71 General Series, 1961.

7. Kuznets, Simon. Capital in the American Economy: Its Formation and Financing. National Bureau for Economic Research, 1964.

8. Kuznets, Simon. Secular Movements in Production and Prices. Boston: Houghton and Mifflin, 1930.

II. DOCUMENTS AND PERIODICALS

1. American Economic Review, vol. XLVI and LIII.

2. Applied Statistics, vol.

3. Econometrica, vol. 32.

4. Hitotsubashi Journal of Economics, vol. III.

4. *The Manchester School*, vol. XXIII.

5. *Proceedings of the American Philosophical Society*, vol. CII.

6. *Review of Economic Studies*, vol. XXXV.

7. *Technometrics*, vol.

8. U.S. Department of Congress, *Historical Statistics of the U.S. - Colonial Times to 1957*. Washington, 1961.

III. UNPUBLISHED MATERIAL

1. Hatanaka, M., and Howrey, Philip E., *Another View of the Long Swing: Comments on Adelman's Study of Long Cycles*. Princeton University: Econometric Research Program, Research Memorandum No. 77, July 1965.

2. Howrey, Philip E., *A Spectral Analysis of the Long Swing Hypothesis*. Princeton University: Econometric Research Project, Research Memorandum No. 78, August 1965.

3. Ohkawa, K., and Rosovsky, H., *Postwar Japanese Growth in Perspective: A Second Look*, (mimeo.)

4. Wilkinson, Maurice, *Swedish Economic Growth: A Model of the Long Swing*. Unpublished Ph.D. dissertation, Harvard University, 1965.

APPENDIX A:

PRIMARY DATA AND TRANSFORMS

TABLE A-1

PRIMARY DATA

	G.N.P. (billions 1929 $) (1)	Consumpt. (bill. 1929 $) (2)	Gross Inv. (billions 1929 $) (3)	Population (thousands) (4)	Per capita Cons. (2/4) (5)
1889	23.4	18.2	5.3	61775	294
1890	24.8	17.8	7.0	63056	282
1891	26.2	19.3	6.9	64361	299
1892	28.3	19.8	8.4	65666	301
1893	27.5	20.5	7.0	66970	306
1894	26.6	19.9	6.7	68275	291
1895	29.1	21.7	7.4	69580	311
1896	30.0	22.8	7.2	70885	321
1897	32.0	24.0	8.0	72189	332
1898	33.2	24.9	8.2	73494	338
1899	35.4	27.1	8.3	74799	362
1900	37.1	27.8	9.3	76094	365
1901	40.6	30.5	10.1	77585	393
1902	41.2	30.9	10.3	79160	390
1903	43.4	32.9	10.5	80632	408
1904	42.9	33.3	9.6	82165	405
1905	45.7	34.9	10.7	83320	418
1906	50.7	38.3	12.3	85437	448
1907	52.2	39.7	12.5	87000	456
1908	48.3	38.1	10.2	88709	429
1909	53.8	41.4	12.4	90492	457
1910	54.4	42.1	12.2	92407	455
1911	54.3	43.2	11.1	93868	460
1912	55.1	42.8	12.3	95331	448
1913	58.2	44.7	13.5	97227	459
1914	57.0	47.1	9.8	99118	475
1915	61.1	48.2	12.9	100549	479
1916	66.3	49.4	16.9	101966	484
1917	66.9	50.8	16.0	103266	491
1918	63.7	49.6	14.1	103203	480
1919	70.3	52.2	18.1	104512	499
1920	71.4	54.2	17.2	106466	509
1921	68.4	57.0	11.4	108541	525
1922	73.2	59.2	13.9	110055	537
1923	83.0	64.3	18.7	111950	574
1924	85.2	69.0	16.2	114113	604
1925	87.4	67.1	20.3	115832	579
1926	93.4	72.5	20.9	117399	617
1927	94.2	74.2	19.9	119038	623
1928	95.7	76.3	19.4	120501	633

TABLE A-1 (continued)

	(1)	(2)	(3)	(4)	(5)
1929	101.4	80.3	21.1	121770	659
1930	91.5	75.9	15.6	123077	616
1931	84.3	73.2	11.1	124040	590
1932	70.7	66.4	4.3	124840	531
1933	68.3	65.0	3.3	125579	517
1934	74.6	68.6	6.0	126374	542
1935	85.8	73.1	12.7	127250	574
1936	95.8	80.8	15.0	128053	630
1937	103.9	84.4	19.5	128825	655
1938	96.7	83.0	13.6	129825	639
1939	103.7	87.0	16.7	130880	664
1940	113.0	91.7	21.3	131954	694
1941	126.2	97.9	28.4	133121	735
1942	122.6	96.2	26.4	133920	718
1943	121.9	98.8	23.2	134245	735
1944	126.6	102.2	24.5	132885	769
1945	130.2	109.1	21.2	132481	823
1946	151.9	122.3	29.6	140054	873
1947	153.5	124.9	28.6	143446	870
1948	158.8	127.5	31.3	146093	872
1949	154.0	130.7	23.3	148665	879
1950	172.8	138.7	34.0	151868	913
1951	178.6	139.8	38.8	153982	907
1952	180.2	144.0	36.3	156393	920
1953	185.0	150.0	35.0	158956	943
1954	186.4	152.9	34.0	161884	944
1955	196.2	164.0	43.1	165069	993
1956	199.9	168.3	42.6	168088	1001
1957	202.8	172.4	39.4	171187	1007
1958	200.4	173.5	34.9	174149	996
1959	213.3	183.7	42.1	177135	1037
1960	218.6	189.0	41.4	179992	1050
1961	222.8	192.8	39.5	183057	1053
1962	237.4	202.3	45.5	185890	1088
1963	246.9	211.3	47.2	188658	1120
1964	260.4	223.5	50.2	191372	1167
1965	276.4	238.2	56.1	193815	1229
1966	292.4	250.0	60.4	195936	1275
1967	299.9	257.1	55.5	197863	1299

NOTES ON TABLE A-1

A) The GNP, Gross Capital Formation, and Consumption series are taken from the following sources:

1889-1918: S. Kuznets, Capital in the American Economy (N.B.E.R., 1961), version III.

1919-1952: John W. Kendrick, Productivity Trends in the U.S. (N.B.E.R., 1961), Appendix A.

1953-1967: Economic Report of the President, 1968, Statistical Appendix. The conversion of the data taken from this source to 1929 prices is based on the figures given on Table A-2.

B) The population series measures total resident population, excluding armed forces overseas, but including armed forces domestically (as measured on July 1).

The series is taken from the following sources:

1889-1949: U.S. Department of Commerce and Bureau of the Census, Historical Statistics of the U.S., Colonial Times to 1957 (U.S. Government Printing Office, Washington, D.C., 1960), Series A, 1-3, col. 2, p. 7.

1950-1967: "Estimates of the Population of the U.S.; January 1, 1950, to May 1, 1968," in Current Population Reports, Series P-25, no. 395 (U.S. Department of Commerce, Bureau of the Census, June 14, 1968).

TABLE A-2.

	GNP	CONSUMPTION	INVESTMENT
	(billions of dollars; 1958 prices)		
1953	412.8	250.8	61.2
1954	407.0	255.7	59.4
1955	438.0	274.2	75.4
1956	446.1	281.4	74.3
1957	452.5	288.2	68.8
1958	447.3	290.1	60.9
1959	475.9	307.3	73.6
1960	487.7	316.1	72.4
1961	497.2	322.5	69.0
1962	529.8	338.4	79.4
1963	551.0	353.3	82.5
1964	581.0	373.7	87.8
1965	617.7	398.4	98.0
1966	652.6	418.0	105.6
1967	669.2	429.9	96.9

IMPLICIT PRICE DEFLATORS FOR

	GNP	CONSUMPTION	INVESTMENT
1929	50.6	55.3	39.4
1958	100	100	100

TABLE A-3

ANNUAL GROWTH RATES

	G.N.P. (1)	Per cap. consump. (2)	Gross invest. (3)	Population (4)
1890	5.98	-4.18	32.08	2.07
1891	5.65	6.23	-1.43	2.07
1892	8.02	0.55	21.74	2.03
1893	-2.83	1.52	-16.67	1.99
1894	-3.27	-4.78	-4.29	1.95
1895	9.40	7.00	10.45	1.91
1896	3.09	3.13	-2.70	1.88
1897	6.67	3.36	11.11	1.84
1898	3.75	1.91	2.50	1.81
1899	6.63	6.94	1.22	1.78
1900	4.80	0.84	12.05	1.73
1901	9.43	7.60	8.60	1.96
1902	1.48	-0.70	1.98	2.03
1903	5.34	4.53	1.94	1.86
1904	-1.15	-0.67	-8.57	1.90
1905	6.53	3.35	11.46	1.41
1906	10.94	7.02	14.95	2.54
1907	2.96	1.79	1.63	1.83
1908	-7.47	-5.88	-18.40	1.96
1909	11.39	6.52	21.57	2.01
1910	1.12	-0.42	-1.61	2.12
1911	-0.18	1.02	-9.02	1.58
1912	1.47	-2.45	10.81	1.56
1913	5.63	2.40	9.76	1.99
1914	-2.06	3.36	-27.41	1.94
1915	7.19	0.88	31.63	1.44
1916	8.51	1.07	31.01	1.41
1917	0.90	1.54	-5.33	1.27
1918	-4.78	-2.30	-11.87	-0.06
1919	10.36	3.92	28.37	1.27
1920	1.56	1.93	-4.97	1.87
1921	-4.20	3.16	-33.72	1.95
1922	7.02	2.43	21.93	1.39
1923	13.39	6.78	34.53	1.72
1924	2.65	5.28	-13.37	1.93
1925	2.58	-4.20	25.31	1.51
1926	6.86	6.61	2.96	1.35
1927	0.86	0.94	-4.78	1.40
1928	1.59	1.58	-2.51	1.23

TABLE A*3(continued)

	(1)	(2)	(3)	(4)
1929	5.96	4.15	8.76	1.05
1930	-9.76	-6.48	-26.07	1.07
1931	-7.87	-4.31	-28.85	0.78
1932	-16.13	-9.87	-61.26	0.64
1933	-3.39	-2.68	-23.26	0.59
1934	9.22	4.87	81.82	0.63
1935	15.01	5.83	111.67	0.69
1936	11.66	9.84	18.11	0.63
1937	8.46	3.83	30.00	0.60
1938	-6.93	-2.42	-30.26	0.78
1939	7.24	3.97	22.79	0.81
1940	8.97	4.54	27.54	0.82
1941	11.68	5.83	33.33	0.88
1942	-2.85	-2.32	-7.04	0.60
1943	-0.57	2.45	-12.12	0.24
1944	3.86	4.50	5.60	-1.01
1945	2.84	7.08	-13.47	-0.30
1946	16.67	6.04	39.62	5.72
1947	1.05	-0.29	-3.38	2.42
1948	3.45	0.23	9.44	1.85
1949	-3.02	0.74	-25.56	1.76
1950	12.21	3.88	45.92	2.15
1951	3.36	-0.59	14.12	1.39
1952	0.90	1.42	-6.44	1.57
1953	2.66	2.49	-3.58	1.64
1954	0.76	0.09	-2.86	1.84
1955	5.26	5.19	26.76	1.97
1956	1.89	0.78	-1.16	1.83
1957	1.45	0.58	-7.51	1.84
1958	-1.18	-1.07	-11.42	1.73
1959	6.44	4.09	20.63	1.71
1960	2.48	1.25	-1.66	1.61
1961	1.92	0.30	-4.59	1.70
1962	6.55	3.33	15.19	1.55
1963	4.00	2.92	3.74	1.49
1964	5.47	4.27	6.36	1.44
1965	6.14	5.23	11.75	1.28
1966	5.79	3.82	7.66	1.09
1967	2.56	1.84	-8.11	0.98

Note: These are the data which were subject to spectral and cross-spectral analysis.

APPENDIX B:

CONSTRUCTION OF CONFIDENCE BANDS OF GAUSSIAN WHITE NOISE

The confidence limits for Gaussian white noise presented on Table 6 were derived as follows:

On the assumption that spectral density estimates are distributed as chi square, we have:

$$\Pr\left\{ x^2_{1-\alpha}(d) \leq \frac{d \cdot \hat{f}(\omega)}{f(\omega)} \leq x^2_{\alpha}(d) \right\} = 1 - 2\alpha,$$

where $f(\omega)$ = the true spectral density,

$\hat{f}(\omega)$ = the estimated spectral density, and,

d = the degrees of freedom.

For the Tukey window,

$d = 2.7(T/M)$.

For $T = 78$ and $M = 20$ (that is, for the length of our series and the maximum lag employed in our analysis), d is approximately equal to 11.

The spectral density of white noise, when its frequency is measured in cycles per year, is equal to 2 for all ω.

For $d = 11$ and $f(\omega) = 2$, we obtain:

(eq. B-1) $$\Pr\left\{ \frac{x^2_{1-\alpha}(11)}{5 \cdot 5} \leq \hat{f}(\omega) \leq \frac{x^2_{\alpha}(11)}{5 \cdot 5} \right\} = 1-2\alpha.$$

The values of $x^2_{\alpha}(11)$ and $x^2_{1-\alpha}(11)$ for $\alpha = .025$ and $\alpha = .05$ are given on Table B-1.

The confidence limits of Table 6 are obtained by inserting the values of chi square, as supplied on Table B-1, in equation B-1.

TABLE 3.[illegible].

Confidence Limits of Chi Square for 11 Degrees of Freedom

CONFIDENCE LEVEL	LOWER LIMIT	UPPER LIMIT
90 per cent	4.57	19.7
95 per cent	3.82	21.9

Source: A. M. Mood and F. A. Graybill, Introduction to the Theory of Statistics (New York: McGraw-Hill, 1963).