CONTINGENT PRICES FOR THE PROPERTY RIGHTS

OF NATURAL RESOURCES

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This paper is circulated for discussion purposes only and its contents
should be considered preliminary.

There has recently been increased interest in the economics of the development of mineral and other natural resources. This has been stimulated partly by large changes in the traded prices of minerals, other raw materials and land, and partly by the wish to find a 'correct' inter-temporal pricing policy for those commodities which are exhaustible.[1] As well as the obvious international redistribution of income which has arisen from the formation of cartels to promote higher prices, the profit potential from both developed and undeveloped natural resources has increased implying different incomes, actual or potential, to either or both of the original owner of the property rights to the resource and the resource developer who has or will acquire these rights. The original owner and the resource developer will often be different economic agents. To develop natural resources, whether they are known resources or simply a possibility awaiting exploration, generally will require the services of a specialist developer: rarely can it be achieved by the original owner of the rights to the resource acting alone.

The distribution of any profit between the two agents may be contingent upon the outcome of the development according to a contract. Thus the contract would distribute risk in addition to expectations of incomes. It is useful to distinguish between pure ex ante contracts and contracts which involve ex post adjustments of terms. The former can be defined as one where the terms specify the distribution of profit under all contingencies and are binding on both parties. This can be contrasted with a contract where the terms can be adjusted as a result of the outcome of the development and thus ex post adjustments are involved. A contract consisting only of a profit sharing agreement with share α is a pure ex ante contract if α is binding on both parties to the contract. If one party, however, can and does force an ex post adjustment by changing

α in response to profitability and thus controls the other's profit ex

post, then the contract is not purely ex ante. Examples of ex post

adjustment include nationalisation and the 'windfall tax' currently being

proposed in the U.S.A. Garnaut and Clunies Ross ({1} p.274) state one

of the difficulties of using ex post adjustments, even if one of the

parties to the contract has the legal power to do so, as:

> "... investors assess political risk and the stability
> of negotiated tax systems, by reference to ex post
> treatment of established investments ... (this) causes
> ex post adjustment in one project to affect ex ante
> expectations of the after-tax profitability of future
> projects. Ex post adjustments that are based on
> clearly articulated principles might, in principle,
> cause new investors to expect consistent treatment,
> but in reality the fact of ex post adjustments would
> probably contribute to some uncertainty about the
> stability of all systems ... and, given investors'
> aversion to risk, would probably raise the supply
> price of investment ..."

Garnaut and Clunies Ross go on to propose a Resource Rent Tax

(RRT) as an appropriate contingent price in the context of a pure ex

ante contract. This paper is also confined to considering contingent

prices compatible with pure ex ante contracts. A 'price' will be considered

as an amalgam of an immediate transfer of a given sum of money from one

party to the other followed by a continuous stream of payments from the

developer to the owner until time L when the property rights of the

resource that remains revert to the owner. The two extreme cases are

first when the developer just pays a lump sum to the owner, and second

when the developer is paid a fixed fee and simply acts as the owner's

agent, transferring all profits (positive or negative) to the owner. In

the first case the owner's income is non-stochastic and in the second the

developer's income is non-stochastic. A problem with the second case

is that the developer has no incentive to adopt a policy which is Pareto

efficient in respect of the owner's and developer's utilities. A price

that does provide this incentive when the uncertainties have been resolved

is an _ideal_ price and can be shown to have welfare advantages over other

policies.[2]   Also the price should be such as to produce policy decisions

taken in anticipation of stochastic events which are Pareto efficient in

terms of expected utilities.

We will see in the next section that an 'optimal' price should

have four properties:

P1   The price should be _ideal_.

P2   The price should produce decisions, which

          have to be taken by the developer in anticipation

          of stochastic events, that are Pareto efficient

          in terms of expected utilities.

P3   The price should approximately optimally

          distribute risk.

P4   A price satisfying P1 → P3 can be constructed

          for any particular distribution (i.e. ratio)

          of expected utilities.

These properties also relate to such diverse economic phenomena

as sharecropping (see Reid {7} and Stiglitz {9}), agent-principal relationships

(Ross {8}), penalty systems (Ireland {4}) and incentives (Mirrlees {6}).

An added dimension in the present case is that the contingent element of

the price is a stream of contingent payments rather than a single payment.

It is the time profile of such payments that is of particular interest.

Should payments be made only after a reasonable return on capital has been

earned by the developer as embodied in the RRT?   Or should they be a

constant or varying proportion of (positive or negative) profit flow?

We are not concerned primarily with the distribution of expected

income between the owner and the developer although the nature of the

contingent prices available may effect the supply and demand for developers'

services. P4 above, however, states that the price should be capable

of satisfying any feasible distribution of expected utilities.

In Section II a model is constructed on the basis of a risk

premium approach and Proposition I states a price satisfying P1 → P4 on

the assumption that both the owner and the developer have the same discount

rate. Section III extends the analysis to differentiated discount rates.

The final section discusses the RRT in the light of the analysis and draws

conclusions concerning the relative merits in a normative context of

different policy prescriptions.

II

The model is constructed upon the following assumptions.

A1   The owner wishes to maximise his expected

       utility where his utility is U_G (Y_G) and

       the developer wishes to maximise his expected

       utility where his utility is U_D (Y_D)

       U_G, U_D are concave monotonic increasing

       functions and Y_G, Y_D are the present

       values of the net receipts, discounted at

       rate ρ, of the owner and developer

       respectively from the transactions in the

contract and the development of the

resource.

A2 Y_G, Y_D can be written:

$$Y_G = X_G + P \qquad\qquad (1)$$

$$Y_D = \Pi - P \qquad\qquad (2)$$

where $$P = \int_0^L p(t)\ e^{-\rho t}\ dt + h_o \qquad\qquad (3)$$

$$\Pi = \int_0^L \pi(t)\ e^{-\rho t}\ dt \qquad\qquad (4)$$

$$X_G = \int_0^L x(t)\ e^{-\rho t}\ dt \qquad\qquad (5)$$

P is the discounted stream of payments to
the owner where $p(t) = p(t, u, \theta)$ plus an initial
transfer h_o. Π is the discounted stream of
net receipts from the project where $\pi(t) = \pi(t, u, \theta)$.
X_G is the present value of money-equivalent receipts
other than P (positive or negative) of the owner
from the development.   Examples may be tax receipts
if the owner is the government and changes in the
value of neighbouring land under the same ownership
in the case of land development.   Again $x(t) = x(t, u, \theta)$.
u is a vector of decision variables under the control
of the developer, and is partitioned into $u = [u_1; u_2]$.
θ is a vector of random variables with $E(\theta) = 0$ and

$E(\theta\ \theta') = [c_{ij}]$. The decisions involving choice of u_1 are taken to maximise $E[U_D]$ in anticipation of the random effects θ and with knowledge of $E(\theta)$ and $E(\theta\ \theta')$. The decisions involving choice of u_2 are taken to maximise U_D given θ.

A3   Risk premiums R_G, R_D can be found such that

$$E[U_G\ (Y_G)] \simeq U_G\ (Y_G^{\ o} - R_G) \tag{6}$$

$$E[U_D\ (Y_D)] \simeq U_D\ (Y_D^{\ o} - R_D) \tag{7}$$

where superscript ^{o} denotes evaluation at $\theta = \overline{\theta}$, where $\overline{\theta}$ is a vector of fixed values of θ. R_G, R_D are found by expanding $U_G\ (Y_G)$ and $U_D\ (Y_D)$ around $\theta = \overline{\theta}$ and taking expectations. The expansion of $U_G\ (Y_G)$ yields:

$$U_G \simeq U_G^{\ o} + \left(\frac{dU_G}{dY_G}\right)^o \sum_i \left\{ \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o (\theta_i - \overline{\theta}_i) \right\}$$

$$+ \frac{1}{2} \left(\frac{d^2U_G}{dY_G^2}\right)^o \sum_i \sum_j \left\{ \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o (\theta_i - \overline{\theta}_i)(\theta_j - \overline{\theta}_j)\left(\frac{\partial Y_G}{\partial \theta_j}\right)^o \right\}$$

$$+ \frac{1}{2} \left(\frac{dU_G}{dY_G}\right)^o \sum_i \sum_j \left\{ \left(\frac{\partial^2 Y_G}{\partial \theta_i\ \partial \theta_j}\right)^o (\theta_i - \overline{\theta}_i)(\theta_j - \overline{\theta}_j) \right\}$$

$$\tag{8}$$

Taking expectations:

$$E[U_G] \simeq U_G^o - \left(\frac{dU_G}{dY_G}\right)^o \sum_i \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o \bar{\theta}_i$$

$$+ \frac{1}{2} \left(\frac{d^2 U_G}{dY_G^2}\right)^o \sum_i \sum_j \left\{ \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j}\right)^o \right\}$$

$$+ \frac{1}{2} \left(\frac{dU_G}{dY_G}\right)^o \sum_i \sum_j \left\{ \left(\frac{\partial^2 Y_G}{\partial \theta_i \partial \theta_j}\right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\}$$

$$(9)$$

Also by expanding $U_G (Y_G^o - R_G)$ around $R_G = 0$ we have

$$U_G (Y_G^o - R_G) \simeq U_G (Y_G^o) - \left(\frac{dU_G}{dY_G}\right)^o R_G \qquad (10)$$

Substituting (9) and (10) into (6) we have:

$$R_G \simeq \sum_i \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o \bar{\theta}_i + \frac{1}{2} A_G \sum_i \sum_j$$

$$\left\{ \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j}\right)^o \right\}$$

$$- \frac{1}{2} \sum_i \sum_j \left\{ \left(\frac{\partial^2 Y_G}{\partial \theta_i \partial \theta_j}\right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\}$$

$$(11)$$

where $A_G = -\left(\frac{d^2 U_G}{dY_G^2}\right)^o \Big/ \left(\frac{dU_G}{dY_G}\right)^o$ is the (assumed constant)

coefficient

of absolute risk aversion of the owner evaluated at
$\theta = \bar{\theta}$. Thus

$$Y_G^o - R_G \simeq E\left[Y_G\right] - \frac{1}{2} A_G \sum_i \sum_j \left\{ \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o \right. \left. (c_{ij} + \bar{\theta}_i \ \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j}\right)^o \right\}$$

$$(12)$$

Similarly

$$Y_D^o - R_D \simeq E\left[Y_D\right] - \frac{1}{2} A_D \sum_i \sum_j \left\{ \left(\frac{\partial Y_D}{\partial \theta_i}\right)^o \right. \left. (c_{ij} + \bar{\theta}_i \ \bar{\theta}_j) \left(\frac{\partial Y_D}{\partial \theta_j}\right)^o \right\}$$

$$(13)$$

where $A_D = - \left(\frac{d^2 U_D}{dY_D^2}\right)^o \bigg/ \left(\frac{dU_D}{dY_D}\right)^o$ and again is assumed constant.

Thus maximising (12) is (nearly) equivalent to maximising $E\left[U_G\right]$ and maximising (13) is (nearly) equivalent to maximising $E\left[U_D\right]$. The substance of A3 is that (12) and (13) are sufficiently near equivalents for the following analysis to hold. We proceed by proving Proposition 1 and then by considering the policy implications of various forms of X_G.

Proposition 1

Provided the owner is risk averse $(A_G > 0)$, a price consisting of an initial payment of h_o and a subsequent stream of payments of the form $p^*(t) = \lambda \pi(t) - (1 - \lambda) x(t)$ where λ is constant and equal to $A_D/(A_D + A_G)$ has properties $P1 \rightarrow P3$ and a value of h_o can be found to satisfy $P4$.

Proof of Proposition 1

First note that if changing $p(t)$ increases $Y_G^o - R_G$ by less than it decreases $Y_D^o - R_D$ then such a change is (Pareto) inferior to changing h_o. Thus an optimal payments stream maximises Z where Z is (12) plus (13).

$$Z \simeq E\left[\pi + X_G\right] - \frac{1}{2} \sum_i \sum_j \left\{ \left\{ A_G \left(\frac{\partial Y_G}{\partial \theta_i} \frac{\partial Y_G}{\partial \theta_j}\right)^o + A_D \left(\frac{\partial Y_D}{\partial \theta_i} \frac{\partial Y_D}{\partial \theta_j}\right)^o \right\} (c_{ij} + \overline{\theta}_i \overline{\theta}_j) \right\} \quad (14)$$

Write $p(t) = p^*(t) + \epsilon h(\theta, t)$ all t, θ (15)

where $h(\theta, t)$ is any arbitrary variation. If (14) is maximised by $p^*(t)$ then the derivative of (14) with respect to ϵ should be zero for any $h(\theta, t)$ at $\epsilon = 0$. Let $H = \int_0^L h(\theta, t) e^{-\rho t} dt$.

Now u_1^*, u_2^* the developer's optimal decisions are functions of ϵ for any given arbitrary $h(\theta, t)$: $u_1^* = u_1^*(\epsilon)$ and $u_2^* = u_2^*(\epsilon | \theta)$. Thus

$$\frac{dZ}{d\varepsilon} = \frac{\partial Z}{\partial u_1}\frac{du_1^*}{d\varepsilon} + \frac{\partial Z}{\partial u_2}\frac{du_2^*}{d\varepsilon} + \frac{\partial Z}{\partial \varepsilon} \tag{16}$$

Consider $\dfrac{\partial Z}{\partial \varepsilon}$:

$$\frac{\partial Z}{\partial \varepsilon} \approx \sum_i \left\{ \frac{\partial Z}{\partial\left(\frac{\partial Y_G}{\partial \theta_i}\right)}^o \left(\frac{\partial H}{\partial \theta_i}\right)^o - \frac{\partial Z}{\partial\left(\frac{\partial Y_D}{\partial \theta_i}\right)}^o \left(\frac{\partial H}{\partial \theta_i}\right)^o \right\}$$

$$\approx - \sum_i \sum_j \left\{ \left(A_G \left(\frac{\partial Y_G}{\partial \theta_j}\right)^o - A_D \left(\frac{\partial Y_D}{\partial \theta_j}\right)^o \right)\left(\frac{\partial H}{\partial \theta_i}\right)^o \right.$$
$$\left. (c_{ij} + \bar{\theta}_i\,\bar{\theta}_j) \right\} \tag{17}$$

(17) is zero for any $h(\theta, t)$ if

$$A_G\left(\frac{\partial Y_G}{\partial \theta_j}\right)^o - A_D\left(\frac{\partial Y_D}{\partial \theta_j}\right)^o = 0 \qquad \text{all } j \tag{18}$$

i.e.

$$A_G\left(\frac{\partial X_G}{\partial \theta_j}\right)^o + A_G\left(\frac{\partial P^*}{\partial \theta_j}\right)^o - A_D\left(\frac{\partial \Pi}{\partial \theta_j}\right)^o + A_D\left(\frac{\partial P^*}{\partial \theta_j}\right)^o = 0$$
$$\text{all } j \tag{19}$$

integrating with respect to θ_j :

$$P^{*o} = (A_D\,\Pi^o - A_G\,X_G^o)/(A_D + A_G) + h_o$$

$$= \lambda\,\Pi^o - (1 - \lambda)\,X_G^o + h_o \tag{20}$$

(20) is necessary only at the locality of $\theta = \bar{\theta}$. However if it holds for any θ, then $\dfrac{\partial Z}{\partial u_1} = \dfrac{\partial Z}{\partial u_2} = 0$ providing $A_G > 0$, by the following argument.

$$\text{If} \quad P = P^* \quad \text{where} \quad P^* = \lambda \Pi - (1 - \lambda) X_G + h_o \qquad (21)$$

then u_2^* maximises $\Pi - P^* = (1 - \lambda)(\Pi + X_G) - h_o$. i.e. u_2^* coincidently maximises $\Pi + X_G$. But Z is only a function of u_2^* via $\Pi + X_G$, as, given $P = P^*$, we can write:

$$Z \simeq E\left[\Pi + X_G\right] - \frac{1}{2}\left(A_G \lambda^2 + A_D (1 - \lambda)^2\right) S \qquad (22)$$

where

$$S = \sum_i \sum_j \left\{ \left(\frac{\partial \Pi + X_G}{\partial \theta_i}\right)^o \left(\frac{\partial(\Pi + X_G)}{\partial \theta_j}\right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\}$$

Thus $\dfrac{\partial \Pi - P^*}{\partial u_2} = 0$ implies $\dfrac{\partial Z}{\partial u_2} = 0$

Now u_1^* is chosen such that $\dfrac{\partial(Y_D^o - R_D)}{\partial u_1} = 0$

but

$$Y_D^o - R_D \simeq (1 - \lambda) E(\Pi + X_G) - \frac{1}{2} A_D (1 - \lambda)^2 S - h_o \qquad (23)$$

$$\simeq (1 - \lambda) Z - h_o \qquad (24)$$

Thus maximising $Y_D^o - R_D$ with respect to u_1 coincidently maximises Z and so $\dfrac{\partial Z}{\partial u_1} = 0$.

Thus if $P = P^*$, $\dfrac{\partial Z}{\partial u_1} = \dfrac{\partial Z}{\partial u_2} = \dfrac{\partial Z}{\partial \varepsilon} = 0$ and $\dfrac{dZ}{d\varepsilon} = 0$.

Second order conditions follow from maximising behaviour by the developer

and by:

$$\frac{\partial^2 Z}{\partial \varepsilon^2} \simeq - (A_G + A_D) \sum_i \sum_j \left\{ \left(\frac{\partial H}{\partial \theta_i}\right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \left(\frac{\partial H}{\partial \theta_j}\right)^o \right\} \quad (25)$$

(25) is negative as $\begin{bmatrix} c_{ij} + \bar{\theta}_i \ \bar{\theta}_j \end{bmatrix}$ is a positive definite matrix.[3]

Finally note that h_o is an arbitrary constant of integration and

can thus be used to satisfy P4 and also that a special form of (21) is

an initial payment h_o followed by a sequence of payments $p(t)$ where

$$p(t) = \lambda \ \pi(t) - (1 - \lambda) \ x(t) \qquad (26)$$

Q.E.D.

Equation (26) is the special case where the payments are made

according to current flows and no 'indebtedness' exists. This formulation

is preferred as such indebtedness may change the distribution of risk in

a way not included in the model.

The assumption of the constancy of A_D and A_G simplified the

proof of Proposition 1, but is not the most general assumption that can

be made. For instance if $A_D = R/(Y_D + W_D)$ and $A_G = R/(Y_G + W_G)$

Proposition 1 still holds and $\lambda = \dfrac{W_G}{W_D + W_G}$; W_G, W_D could be

interpreted in 'other wealth' or 'other income' terms. In general, however

non-constancy of A_D and A_G will imply that (26) is a less close approximation to an optimal payments sequence.

Despite this the simple policy implications of Proposition 1 are appealing, partly because the probability distributions of the random variables do not need to be known for the construction of P^*, although they are required for the developers control decisions u_1^*, and also for choosing amongst various prices offered by a number of potential owners/developers.

P^* is composed of an initial payment h_o (positive or negative) followed by a share in the <u>aggregate</u> net revenues $\pi(t) + x(t)$. If, for instance $X_G \equiv 0$, then apart from h_o which is the subject of bargaining, this is exactly the case of the owner giving up his property rights for a λ share of the profit (or loss). If $A_D \simeq A_G$, then an optimal policy for the owner might be to ask all potential developers if they would be interested in forming a joint company with half the stock held by the owner and half by the developer, but under the direction of the developer. Thus the developer would contribute entrepreneurship and expertise and the owner the property rights. In a competitive context, the owner would form the joint company with that developer who was willing to make the highest (positive or negative) side payment (h_o) to the owner. If owners and developers have different coefficients of absolute risk aversion but are equally efficient in providing development services, then the opportunity set from which the owner can choose the pair (λ, h_o) can be found by asking each developer to bid a value of h_o for each of a number of values of λ. The expected utility from each (λ, h_o) can be calculated and the pair which yields the highest expected utility is chosen. The owner will then choose $(\lambda^G, h_o^{\,G})$ to maximise $h_o + \lambda\, Z$,

while the developer will choose (λ^D, h_o^D) to maximise $- h_o + (1 - \lambda) Z$

from the pairs (λ, h_o) associated with owners with identical resource

prospects. Adding additional elements M relating to the quality of

entrepreneurial abilities and N relating to the quality of the resource

implies that the owner will accept that contract defined by (λ^G, h_o^G, M^G)

and the developer that contract defined by (λ^D, h_o^D, N^D). Thus an

equilibrium schedule of contracts can be found as the set $\{\lambda, h_o, M, N\}$

such that for each element in the set there is some developer who weakly

prefers that to any other and also some owner who weakly prefers that

element to any other. It is obvious that for given M, N, a relatively

risk averse developer will contract with a relatively non risk-averse

owner etc.

Once the contract has been settled, both the owner and developer

wish to maximise a monotonic increasing function of Z. This commonality

of objective function is related to the Principle of Similarity (see Ross {8}).

It implies that policy decisions would be agreed between the two parties

to the contract.

X_G can in principle be any function. It can relate to extern-

alities of the project suffered or gained by the owner: appreciation or

depreciation of neighbouring sites in land development or the present value

of capital stock at time L are examples. If the owner is the government

representing community welfare then employment created (in present value

money equivalent terms), taxes raised or pollution caused may be incorporated

as X_G. Also X_G may relate to the amount of an exhaustible resource

used up or extracted and sold during the lifetime of the project. The

cases we will consider in detail here are chosen partly because they

exemplify the role of the X_G function and partly because they are of

particular relevance to the development of exhaustible mineral resources where the property rights are owned by the government. The first case is that of an owner-collected profits tax/loss subsidy.[4] Suppose this is of rate τ of before tax profits Π^b. Thus from (26)

$$p^*(t) = \lambda(1 - \tau) \; \pi^b(t) - (1 - \lambda) \tau \; \pi^b(t)$$

$$= (\lambda - \tau) \; \pi^b(t) \tag{27}$$

$$\text{i.e.} \quad P^* = (\lambda - \tau) \; \Pi^b + h_o \tag{28}$$

(27) and (28) imply that the share of the enterprise taken by the owner should be the share that would be taken in the absence of taxation minus the tax rate.

The second case upon which we will focus is when the owner benefits according to the size of the stock of the resource remaining at L. Let the initial stock be K and total use be $Q = \int_0^L q(t) \, dt$. Then the stock at L is K - Q. Let the owner value this stock at present value price ϕ. Then $X_G = - Q \phi$ and is the loss in the value of terminal stocks due to the development operation and $\phi = \phi(\theta)$. Thus from (26):

$$p^*(t) = \lambda \pi(t) + (1 - \lambda) \; q(t) \; \phi \; e^{\rho t} \tag{29}$$

$$\text{and} \quad P^* = \lambda \Pi + (1 - \lambda) Q \phi . + h_o \tag{30}$$

(29) and (30) imply that an extraction tax should be levied by the owner and that the (specific) rate should increase by the rate of discount. The

initial rate should be $(1 - \lambda)$ times the present (time L) value ϕ,

which might be related to the world price of the resource at time L and

the costs specific to the extension of the project. The incorporation

of such a tax may be necessary to prevent too fast exploitation due to

the finite horizon of the property rights lease. Of course, the nature

of the ϕ function will be known when determining h_o, and so the

distribution of expected utility is unaffected by it. ϕ should in fact

be set equal to the uncertain present value shadow price of having more

resource at time L if Z were being maximised over an infinite horizon.

III

The application of Proposition 1, although general, is limited

to situations where both parties have the same rate of discount. A

contrary situation can occur if the two parties are involved in separate

capital markets or involve different risks to creditors. In particular,

differential discount rates are not compatible with capital transactions

between the two parties; otherwise the one with the higher discount rate

would wish to borrow an unbounded amount from the other. To abstract

out of this difficulty constraints have to be placed upon the class of price

profiles permissable in order to prove Proposition 2. An alternative and

preferable approach would be to extend the model so that the discount rate

is endogenous or to change to a discounted-stream-of-utility approach

implying limits, on borrowing or lending, which are themselves partly

endogenous. It is hoped that such an approach will be the subject of a

later paper.

Proposition 2

A contingent payments stream $p^*(t) = \lambda(t) \ \pi(t) - (1 - \lambda(t)) \ x(t)$

where $\lambda(t) = A_D e^{-\rho t} / (A_D e^{-\rho t} + A_G e^{-rt})$ and where ρ, r are

the discount rates of the developer and the owner respectively is Pareto

superior in the set of all payments streams

$$p(t) = p^*(t) + \Theta h(\theta, t) \quad \text{for any } \Theta \text{ and arbitrary } h(\theta, t)$$

where $\displaystyle\int_0^L \left(\frac{\partial h(\theta, t)}{\partial \theta_i}\right)^o (e^{-\rho t} - e^{-rt}) \, dt = 0 \qquad \text{all } i \qquad (30)$

and $\displaystyle E\left[\int_0^L h(\theta, t) (e^{-\rho t} - e^{-rt}) \, dt\right] = 0 \qquad (31)$

Note that the constraints eliminate the possibility of arbitrage

in risks and expected incomes respectively.

Proof of Proposition 2

$$\text{Let} \quad P^\rho = \int_0^L p(t) e^{-\rho t} \, dt + h_o$$

$$P^r = \int_0^L p(t) e^{-rt} \, dt + h_o$$

$$H^\rho = \int_0^L h(\theta, t) e^{-\rho t} \, dt$$

$$H^r = \int_0^L h(\theta, t) e^{-rt} \, dt$$

$$\Pi = \int_0^L \pi(t) e^{-\rho t} \, dt$$

$$X_G = \int_0^L x(t) e^{-rt} \, dt$$

Then (12) + (13) becomes Z' where

$$Z' \simeq E\left[P^r - P^P + \Pi + X_G\right] - \frac{1}{2} \sum_i \sum_j \left\{ \left\{ A_G \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o \left(\frac{\partial Y_G}{\partial \theta_j}\right)^o \right.\right.$$

$$\left. + A_D \left(\frac{\partial Y_D}{\partial \theta_i}\right)^o \left(\frac{\partial Y_D}{\partial \theta_j}\right)^o \right\}$$

$$\left. \left\{ c_{ij} + \bar{\theta}_i \bar{\theta}_j \right\} \right\} \qquad (32)$$

Again construct $p(t) = p^*(t) + \epsilon h(\theta, t)$ all θ, t where $h(\theta, t)$ is any arbitrary variation satisfying the constraints (30) and (31).

Now

$$\frac{dZ'}{d\epsilon} = \frac{\partial Z'}{\partial u_1} \frac{du_1^*}{d\epsilon} + \frac{\partial Z'}{\partial u_2} \frac{du_2^*}{d\epsilon} + \frac{\partial Z'}{\partial \epsilon}$$

Again we show that $\dfrac{\partial Z'}{\partial u_1} = \dfrac{\partial Z'}{\partial u_2} = \dfrac{\partial Z'}{\partial \epsilon} = 0$ at $\epsilon = 0$

Consider $\dfrac{\partial Z'}{\partial \epsilon}$: $\dfrac{\partial E\left[P^r - P^\rho + \Pi + X_G\right]}{\partial \epsilon} = 0$ due to the constraints

on $h(\theta, t)$. Thus:

$$\frac{\partial Z'}{\partial \epsilon} \simeq -\frac{1}{2} \sum_i \sum_j \left\{ \left\{ A_G \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o \left(\frac{\partial H^r}{\partial \theta_j}\right)^o - A_D \left(\frac{\partial Y_D}{\partial \theta_i}\right)^o \left(\frac{\partial H^\rho}{\partial \theta_j}\right)^o \right.\right.$$

$$\left. + A_G \left(\frac{\partial Y_G}{\partial \theta_j}\right)^o \left(\frac{\partial H^r}{\partial \theta_i}\right)^o - A_D \left(\frac{\partial Y_D}{\partial \theta_j}\right)^o \left(\frac{\partial H^\rho}{\partial \theta_i}\right)^o \right\}$$

$$\left. \left\{ c_{ij} + \bar{\theta}_i \bar{\theta}_j \right\} \right\} \qquad (33)$$

For $\dfrac{\partial Z'}{\partial \epsilon} = 0$ for any permissible $h(\theta, t)$ we require

$$A_G \left(\frac{\partial Y_G}{\partial \theta_i}\right)^o - A_D \left(\frac{\partial Y_D}{\partial \theta_i}\right)^o = 0 \quad \text{all } i \quad (34)$$

integrating with respect to θ_i :

$$p^*(t)^o = (A_D e^{-\rho t} \pi(t)^o - A_G e^{-rt} x(t))/ A_D e^{-\rho t} + A_G e^{-rt}) \quad (35)$$

$$\frac{\partial Z}{\partial u_1} = \frac{\partial Z}{\partial u_2} = 0 \text{ if } p^*(t)^o \text{ holds over all } \theta \text{ can be shown}$$

in the same way as in the proof of Proposition 1. The second-order conditions
also follow similarly.

Thus $\quad p^*(t) = \lambda(t) \ \pi(t) - (1 - \lambda(t)) \ x(t)$

Q.E.D.

Proposition 2 states that if $\rho \neq r$, λ will change with t
as the party with the higher rate of discount will discount risk faster
and it will be optimal to allocate him an increasing proportion of the risk.
Note that

$$\frac{d\lambda(t)}{dt} \gtreqless 0 \quad \text{as} \quad r \gtreqless \rho .$$

Any difference in expected utility distribution due to the time
profile of $\lambda(t)$ would be countered by a changed initial payment.

IV

It is of interest to consider the RRT in the light of the
preceding analysis, although it should be stressed that the analysis is
normative and a different approach or formulation may alter its implication.
In essence the RRT consists of a tax on the return earned on capital above
a given reasonable rate.   As the tax is not symmetric with regard to all
policies that could be pursued by the developer, the higher the tax the
less relatively appealing will be policies the outcome of which is very
uncertain, as very high profits would be taxed away but very high losses
would not be subsidised.   If the owner is less risk averse than the
developer, this implies that too cautious a policy will be followed: for
instance investment would take place in established coal seams rather than
a search being undertaken for new seams.

Arguments have been advanced elsewhere (see Heal {2}, p.216)
concerning why the owner is likely to be risk averse even if the owner
is the government.   However, if the owner is much less risk averse than
the developer, then the optimal contract (by Proposition 1) is for the
developer to act almost as the owner's agent with a small shareholding
to act as an incentive,[5] thus most of the risk is borne by the owner.
The RRT cannot allocate risk in this way; it simply reduces the probability
of the developer making very high profits.   It does not reduce the
probability of the developer making very low or negative profits.

The RRT concentrates payments to the owner (government) in the
later years of the project.   This is stated to be advantageous if "the
supply price of loan capital to the government is below the supply price
of investment" ({1}, p.276), i.e. $r < \rho$.   But Proposition 2 states

that $\dfrac{d\lambda(t)}{dt} < 0$ if $r < \rho$. This is partly because Proposition 2 disallows the use of the contract to transfer finance – surely there is a case for treating this separately – and partly because <u>variations</u> in profit in later years are relatively unimportant to the developer if he has the higher rate of discount.

The above points are made in a purely normative spirit as is the analysis in this paper. Implementation costs may prohibit the adoption of the price given in (26), and in particular it may not be possible for the government to find the finance to provide its share of initial costs from either taxation or capital markets if a large number of development projects are to be undertaken. However the analysis in Sections II and III does incorporate the rate of discount and the degree of risk aversion of the government explicitly and it does solve for all the parameters in the system with the exception of the distribution of expected utility (i.e. h_o). The choice of the values and number of parameters in the RRT is left open due to the lack of a normative framework. Also it is stated ([1}, p.284) that "the advantage for the government is being able to get the benefits of high returns without actually purchasing equity in that it removes the risk of absolute loss that equity investment entails." This statement ignores the premium that the developer is willing to pay for a reduction in his risk due to equity investment by the owner specific to the one project. If this premium is greater than the government's premium for accepting this risk, the joint expected utility is increased. The advocacy of a consortium of developers ([1}, p.275 fn2) is in a similar risk-sharing spirit, but the promotion and organisation of such a consortium plus the need to arrive at optimal development policy decisions may prove problematic. Also competition for the development contract may be reduced and this may produce a reduction in the expected utility of the government.

Finally any pricing or taxation policy must be able to take account of outcomes of the project other than simply its profitability. Revenue paid directly to the owner is rarely the sum total of his concern.

In this paper, it has been argued that owner or government participation in the development of natural resources as proposed in Propositions 1 and 2 is optimal in three ways. First, it produces Pareto efficient decisions in terms of expected utility by the developer in anticipation of stochastic events. Second, it produces Pareto efficient decisions in terms of utility given the stochastic events. Third, it produces an (approximately) optimal distribution of risk. It is also argued that there is a contract of this nature for any feasible distribution of expected utility (choice of h_o). The argument is based on an assumption of the existence of approximate risk premiums and implementation costs are ignored. However, the conclusion remains that there exists considerable theoretical foundation for a policy of joint owner-developer participation in projects with uncertain outcomes. Such a policy can be seen to exist or be emerging in several cases of present resource development, not least the case of North Sea Oil.

Footnotes

(1) See for instance Heal {3}.

(2) For a discussion of ideal prices and an analysis of their welfare advantages see Ireland {5}.

(3) The term inside the summation in (25) can be written

$$E \left[\left(\sum_i \left\{ \theta_i \left(\frac{\partial H}{\partial \theta_i} \right)^o \right\} \right)^2 \right] + \left(\sum_i \bar{\theta}_i \left(\frac{\partial H}{\partial \theta_i} \right)^o \right)^2 \quad \text{which is}$$

positive for non-trivial $\left[\frac{\partial H}{\partial \theta_i} \right]$.

(4) It is assumed that the developer can use any loss incurred in this development to offset tax liabilities in others.

(5) Note that there does not exist an 'optimal' contract in the sense of Proposition 1 if $A_G = 0$. Z increases as λ approaches unity, but at unity the lack of incentive for (jointly) optimal policy prevents this being optimal.

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