THE BALANCE OF TRADE IN A MODEL OF TEMPORARY EQUILIBRIUM WITH RATIONING*

Avinash Dixit

NUMBER 98

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

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NUMBER 98

November, 1976.

* I thank Alan Kirman, Richard Portes and John Williamson for comments on an earlier draft.

I. Introduction

The importance of accepting the failure of prices to adjust instantaneously to their Walrasian equilibrium levels is now widely recognised. When economic activity is attempted at such sticky prices, the quantities must adjust to attain a short run or temporary equilibrium. The first effect is that in any market where the price fails to adjust, the 'short' side of the market determines the actual amount transacted, and the 'long' side is rationed. But even more important are the repercussions that arise as transactors who have failed to satisfy their initial demands or supplies in one market recalculate their actions in other markets. This is Clower's dual decision approach, which leads to a distinction between such recalculated or effective demands and the original or notional ones. The nature of the initial market imbalance depends on the values assumed by the sticky prices, and the resulting temporary equilibria with rationing are of different kinds.

Recent years have produced some models of closed economies based on this approach. In particular, Barro and Grossman (1976) and Malinvaud (1976) have completed such classification of equilibria in a model with labour, aggregate output and fiat money. They calculate equilibria starting with assumed sticky values for the money wage and the price level, and divide the wage-price space into three regions corresponding to three kinds of equilibria. In the first region the real wage is too high, and consumers face rationing both in buying commodities and in selling labour; this is labelled the case of classical unemployment. The second is a Keynesian region of excess supply in both commodity and labour markets, and the third, repressed inflation, has general excess demand. In these two cases, the real balance effect on demand is an important force.

These developments have not so far touched international trade theory. The conventional approach embodied in the diagrams of Swan (1963) and Mundell (1968) is a simple income-expenditure one. It largely neglects wages and prices, and assumes output to be demand-determined. This is a poor choice-theoretic framework, and inappropriate for equilibria other than the Keynesian kind. The work of Meade (1951), from which this approach can be said to derive, pays more attention to some prices, but is still short of giving a full account of the dual-decision choices. The monetary approach which has recently come into prominence following Frenkel and Johnson (1976), while paying more attention to prices and choices, often goes to the other extreme by assuming instantaneous Walrasian equilibrium in commodity and labour markets. An exception is Rodriguez's paper in that volume, but he simply replaces a Walrasian component by an income-expenditure one with its attendant defects.

This paper is a first attempt at giving a more satisfactory model of the balance of trade, based on the dual decision hypothesis. To focus on the new points that such an approach highlights, I use a very simple model that ignores several complications. The model is very similar to Malinvaud's, with one aggregate commodity, one type of labour, and fiat money, but no other assets. The basic setting is that of a small country in short-run equilibrium, with a given money supply $\bar{m}$, wage rate w and price level p. We can take $p = p^f r$, where p^f is the foreign price level assumed fixed, and r is the exchange rate either fixed by the government with sufficient reserves, or sticky in a market. It is not my purpose to argue for any lasting rigidity in any of these prices. But as soon as instantaneous equilibration is denied, quantity adjustments occur and such models must be considered seriously. Further, results of flexible exchange rates emerge as special cases of the analysis below.

The short-run equilibrium in question is attained by quantity adjustment. However, since commodities are freely tradeable, there is no need for rationing on that market. Any discrepancy between domestic output and demand simply shows up as the balance-of-trade surplus. In course of time, this will affect the money supply and therefore shift the equilibrium as happens in the monetary approach. However, my main concern here will be the characterisation of one short-run equilibrium, and the dynamic effects will be mentioned only in passing.

Rationing is then confined to the labour market, and the (w,p) space can be divided into two regions depending on which side of the market is rationed. One region has unemployment where the consumers are rationed as to the amount of labour they can sell, and the other region has an excess demand for labour, with the firms rationed as to the amount of labour they can buy. On this picture I superimpose a division into regions of surplus and deficit in the balance of payments, thus obtaining four regions each of which corresponds to one type of short-run equilibrium. This procedure of division into four zones is of course familiar from the Swan diagram and from Mundell's work, but the space in question is different. Where Swan considers fiscal policy and the exchange rate, and Mundell studies monetary and fiscal policies, my focus is on the wage rate and the price level. Exchange rate policy corresponds to a choice of p, but the explicit introduction of the wage rate is a new feature. Monetary and fiscal policies are studied by the comparative static method of shifting these regions, and by some rudimentary dynamic considerations.

II. The Model

Having outlined the general structure of the model in the introduction, the next step is to study the behaviour of its component parts, and finally to link them into an overall equilibrium. I shall work with a representative consumer and a representative firm. I shall compute the unconstrained behaviour for each, as well as the recalculation (dual decision) that is necessary given rationing on the labour market.

The consumer's utility function will be assumed to be of the form

$$u = U(x, \ell, m)$$

where x is the amount of commodities consumed, ℓ is the amount of labour supplied, and m the amount of money carried forward. This is a derived utility function, where the amounts of future consumption and labour have been optimised. Therefore parameters of this optimisation, such as expected future prices, wages, employment prospects and profit distribution should be included as arguments in U. The usual method would be to use the argument m/p', where p' is the expected future price level. Since the expected future prices, wages etc. depend on the current ones, all demand elasticities must be understood to include the effects arising through expectations. This is done in Dixit (1976), and I shall simplify matters here by assuming this done and making no explicit mention of expectations. This makes no qualitative difference as far as the characterisation of equilibrium is concerned once its existence is ensured.

The budget constraint is

$$p\,x + m = w\,\ell + \bar{m} \tag{1}$$

where $\bar{m}$ is the money endowment, p the price level and w the wage rate. Maximisation yields the demand functions for commodities and money and the supply function for labour:

$$\left.\begin{aligned} x &= X(p,w,\bar{m}) \\ M &= M(p,w,\bar{m}) \\ \text{and} \quad \ell &= L(p,w,\bar{m}) \;. \end{aligned}\right\} \tag{2}$$

The producer's unconstrained behaviour is simply described by the profit function $\pi(p,w)$. This is increasing in p, decreasing in w, and convex and homogeneous of degree one in the two arguments together. This implies $\pi_{pp} > 0$, $\pi_{ww} > 0$ and $\pi_{pw} < 0$. Further, the commodity supply and labour demand functions are respectively

$$y = \pi_p(p,w)$$

and $$e = -\pi_w(p,w). \tag{3}$$

Note that it is assumed that the profits of one period do not accrue to the consumer as income in the same period. While this seems reasonable, changing the assumption will only cause mathematical complexities without significantly altering the results.

Now we compare the demand and supply in the labour market. If $\ell = e$, we have a conventional equilibrium. If $\ell > e$, the consumer is rationed, i.e. must accept employment at the level e determined by the short side of the market. Then he recalculates his demands for commodities and money, maximising $U(x,e,m)$ subject to the budget constraint

$$p\,x + m = w\,e + \bar{m}\,. \tag{4}$$

The outcomes are the effective demands

$$x = \hat{X}(p,w,e,\bar{m})\,,$$
$$m = \hat{M}(p,w,e,\bar{m})\,. \tag{5}$$

In the opposite case where $\ell < e$, the producer is rationed and can hire only ℓ units of labour. Then he produces $F(\ell)$ units of output where F is the production function. Clearly, in such a situation we must have $F'(\ell) > w/p$.

I shall assume that commodities and leisure are always normal, i.e. $X_{\bar{m}} > 0$, $\hat{X}_{\bar{m}} > 0$ and $L_{\bar{m}} < 0$. Then, as usual, $X_p < 0$ and

$\hat{X}_p < 0$. Further, I shall assume the substitution effect in labour supply to dominate the income effect, so $L_w > 0$. Next consider $\hat{X}_e$. The effect of a change in employment on effective demand is twofold. First we have the Keynesian income effect which acts by increasing the right hand side of the budget constraint (4), and secondly we have the general equilibrium effect arising from any complementarity or substitution between labour and commodities. Assuming the forer to dominate, we have $\hat{X}_e > 0$.

Further inferences can be drawn as follows :
$\hat{X}_w = e\,\hat{X}_{\bar{m}}$, since the only effect of w is to change the budget constraint (4). Thus $\hat{X}_w > 0$. Next we compare the notional and effective demands. The two must coincide if the rationed amount of employment happens to equal the desired labour supply, i.e.

$$X(p,w,\bar{m}) = \hat{X}(p,w, L(p,w,\bar{m}),\bar{m}) . \qquad (6)$$

Differentiating this with respect to w, we have

$$X_w = \hat{X}_w + \hat{X}_e L_w \qquad (7)$$

Then $X_w > \hat{X}_w > 0$. Differentiating (6) with respect to p, we have

$$X_p = \hat{X}_p + \hat{X}_e L_p \qquad (8)$$

The Le Chatelier-Samuelson principle suggests that the demand for one commodity is less price-elastic when some other commodity is rationed, i.e. $-X_p > -\hat{X}_p$. Then $L_p < 0$.

All this helps in dividing the (w,p) space into regions corresponding to the two cases of rationing. Consider the boundary curve defined by $\ell = e$, i.e.

$$L(p,w,\bar{m}) + \pi_w(p,w) = 0. \tag{9}$$

Its slope is found from the implicit function differentiation formula to be

$$\frac{dw}{dp} = -\frac{L_p + \pi_{wp}}{L_w + \pi_{ww}} . \tag{10}$$

Given the properties and assumptions stated earlier, L_p and π_{wp} are both negative and L_w and π_{ww} both positive, therefore the boundary is a positively sloped curve. The region above the curve has unemployment and the region below it has excess demand for labour.

The next task is to divide the space into regions of surplus and deficit in the balance of trade. In this model, we can simply work with the surplus defined in terms of the aggregated commodity, i.e. the difference between domestic production and demand

$$s = y - x. \tag{11}$$

However, care is necessary in determining this as a function of (w,p). This is because in each of the regions of unemployment and inflation, one of the relevant quantities is obtained from a recalculation given some rationing. In the region of unemployment, demand is constrained and we have

$$s = \pi_p(p,w) - \hat{X}(p,w, - \pi_w(p,w),\bar{m}), \tag{12}$$

while in the region of excess labour demand supply is constrained and

$$s = F(L(p,w,\bar{m})) - X(p,w,\bar{m}). \tag{13}$$

We should therefore expect the curve showing zero surplus on the balance of payments to change its slope as it crosses from one region into the other. But there is an even more serious problem, for it is difficult to determine the signs of $\partial s/\partial p$ and $\partial s/\partial w$ within each region. For example, in (13),

$$\partial s/\partial w = F_\ell L_w - X_w ,$$

and both terms are positive. A higher wage means a higher output demand, but it also means a higher labour supply and then, by relaxing the rationing on firms, a higher output supply. If we assume that a devaluation will increase the surplus, we have $\partial s/\partial p > 0$. But $\partial s/\partial w$ can change sign, and the balance of trade equilibrium curve can bend back.

Some more information can be had by looking at the 'notional' trade balance, ignoring the dual decision constraints. We find

$$s = \pi_p(p,w) - X(p,w,\bar{m}). \tag{14}$$

and the slope of the curve $s = 0$ is

$$\frac{dw}{dp} = \frac{\pi_{pp} - X_p}{- \pi_{pw} + X_w} \tag{15}$$

This is positive, and the question arises whether the notional trade balance curve is steeper than the labour market equilibrium curve. I shall assume this to be the case, and argue justifications below. The outcome is shown in Figure 1.(*) The regions are labelled U for unemployment, E for an excess demand for labour, S for trade surplus and D for deficit. The curves are FE for full employment and BT for balanced trade, their intersection is W, the Walrasian equilibrium.

It is seen that the region to the north-east of W has unemployment and a trade surplus, i.e. general excess supply, and the region to the south-west has general excess demand. This accords with the idea that low wages and prices have a favourable real balance effect on demand. If the curve FE were steeper, the regions would be reversed. This supports the assumption that BT is steeper, and can be verified by rather tedious calculations.

An alternative explanation some may prefer works by considering a tatonnement process for w and p in response to notional excess demands or supplies in the respective markets. It is easily seen that if BT is the steeper curve, W is stable, while if FE is steeper, it is a saddle-point. In order to isolate the new features that quantity-adjusting equilibrium brings, it makes sense to assume Walrasian stability.

All this pertains to notional imbalances, but we can get some idea of how the classification of equilibria considering effective demands and supplies relates to the above notional one. When there is unemployment, effective commodity demand is less than the notional demand,

(*) See p. 21.

while supply is unaffected. Thus the effective surplus exceeds the notional, and the region US is enlarged when dual decisions are taken into account. Similarly the region ED is enlarged, and the other two regions must contract. The notional classification above is thus useful in providing bounds for the effective one.

As an alternative approach, I shall work through an example using specific utility and production functions in the next section. This allows a direct computation of the effective classification, and some comparative static and dynamic analyses can be performed. It will also be intuitively clear that some of the results generalise beyond the particular example.

III. An Example

I shall develop the case of Cobb-Douglas utility and production functions. A similar special case is used by Malinvaud (1976) for a closed economy. The utility function is

$$u = C \; x^{\alpha} \, (\bar{\ell} - \ell)^{\beta} \quad m^{\gamma} \tag{16}$$

where C, $\bar{\ell},\alpha,\beta$ and γ are constants, and $\alpha + \beta + \gamma = 1$. Expected future prices can be subsumed in C. The demand functions subject to the budget constraint alone are easily found to be

$$\begin{aligned} x &= \alpha(\bar{m} + w\,\bar{\ell})/p \\ \ell &= \left[(1-\beta)w\,\bar{\ell} - \beta\,\bar{m}\right] / w \\ m &= \gamma(\bar{m} + w\,\bar{\ell}) \end{aligned} \tag{17}$$

If employment is constrained at the level e, the dual decision yields effective demands

$$\left.\begin{aligned} x &= \frac{\alpha}{\alpha + \gamma} \; \frac{\bar{m} + w\,e}{p} \\ m &= \frac{\gamma}{\alpha + \gamma} \; (\bar{m} + w\,e) \end{aligned}\right\} \qquad (18)$$

The production function is

$$y = (\theta\, k\, e)^{1/\theta} \qquad (19)$$

where $\theta > 1$, and k is a constant which is used in examining comparative statics resulting from productivity shifts. Unconstrained profit-maximising choices are

$$\left.\begin{aligned} y &= (k\, p/w)^{1/(\theta - 1)} \\ e &= \theta^{-1}\, k^{1/(\theta - 1)}\, (p/w)^{\theta/(\theta - 1)} \end{aligned}\right\} \qquad (20)$$

Putting (17) and (20) together, it is easy to see that the full employment curve is given by

$$p^{\theta/(\theta - 1)} = \theta \left(\frac{w}{k}\right)^{\frac{1}{\theta-1}} \left[(1 - \beta)\; w\,\bar{\ell} - \beta\,\bar{m} \right] \qquad (21)$$

There is unemployment above this curve and excess demand for labour below it.

In the region of unemployment, we use the y and e from (20), and substitute this value of e in the constrained demand for commodities in (18). The resulting expression for the trade surplus yields the following locus of trade balance:

$$w = k \left\{ \frac{1 - \beta - \alpha/\theta}{\alpha \bar{m}} \right\}^{\theta - 1} p^{\theta} \quad . \tag{22}$$

There is trade surplus to the right of this and deficit to the left.

In the region of excess labour demand we use x and ℓ from (17) and constrain firms to use this labour supply and find effective output supply from (19). The resulting trade balance locus is

$$p = \frac{\alpha^{\theta}}{\theta k} \frac{w(\bar{m} + w \bar{\ell})}{(1-\beta) w \bar{\ell} - \beta \bar{m}} \tag{23}$$

Again we have surplus to its right and deficit to its left.

Both branches of the trade balance curve meet the full employment curve at the Walrasian equilibrium; its co-ordinates are

$$p^{*} = \frac{(\theta \alpha)^{(\theta-1)/\theta} (\alpha + \theta \beta)^{1/\theta}}{\theta (1 - \beta) - \alpha} \frac{\bar{m}}{(k \bar{\ell})^{1/\theta}} \tag{24}$$

$$w^{*} = \frac{\alpha + \theta \beta}{\theta (1 - \beta) - \alpha} \frac{\bar{m}}{\bar{\ell}}$$

Note that w^{*} and p^{*} are proportional to $\bar{m}$.

All this is depicted in Figure 2,(*) and the four regions corresponding to the four kinds of equilibria are marked in the notation explained earlier. All effective demands and supplies resulting from dual

(*) See p. 22.

decisions are now properly taken care of.

Having characterised the possible temporary equilibria, I shall examine the effects of three parameter changes on such an economy. The main focus will be on comparative statics. A parameter change will shift the curves, and I will examine where the initial position of the economy lies relative to the new curves. This tells us what will happen if w and p remain sticky at the original levels. I shall also consider some dynamics. This must be done in a very unsatisfactory manner, since several complexities make a rigorous treatment too difficult. Expectations and profit distribution both change in ways that are difficult to keep track of. But important aspects of monetary dynamics can be handled.

The first change is an increase in the money supply $\bar{m}$, and this is shown in Figure 3.(*) The old Walrasian equilibrium W is at the point of intersection of FE and BT, while the new one, W', is the meeting point of the corresponding new curves, FE' and BT'. The new picture is in fact a radial enlargement of the old one. Now suppose $\bar{m}$ increases suddenly, while w and p are sticky and initially remain at their original values. The quantity-adjusted temporary equilibrium that results is seen to have excess labour demand and trade deficit, since W lies in the region ID relative to W'. This is also as expected.

We can also consider the dynamics following the increase in $\bar{m}$. If no further active monetary policy is pursued i.e. no sterilisation of the trade deficit occurs, there will be a gradual reduction in $\bar{m}$ below its initially increased value. The Walrasian equilibrium will drift back south-west towards W. In the meantime, the pressure of demand in the labour

(*) See p. 23.

market will raise w. Consider first the case where p remains fixed, e.g. when foreign prices and the exchange rate are both fixed. Then the actual position of the economy will move vertically up from W. In course of time, it will cross into the region of unemployment and deficit relative to the then current Walrasian equilibrium. Cycles will occur, with ultimate convergence back to W, and a loss of foreign exchange reserves in the meanwhile. Next suppose the exchange rate can adjust slowly in response to the deficit. Then the actual position moves in a roughly north-easterly direction while the Walrasian equilibrium drifts to the south-west, and the two will meet between W and W', with some loss of reserves. Finally, if the exchange rate can adjust instantaneously, the actual position will move to the right from W at once to attain the new curve BT'. Then $\bar{m}$ will remain at its new value, and the economy will slowly move up along BT' to reach W'.

Conversely, a decrease in the money supply will produce an initial temporary equilibrium with unemployment and surplus, and the subsequent developments can be described similarly. It is evident that the qualitative features here should not depend on the restriction to Cobb-Douglas functions.

Turning to fiscal policy, consider the case where the government acts by purchasing an amount g of commodities. The trade surplus is now $s = y - x - g$, but nothing else is affected. Accordingly, the curve BT shifts to the right, and the curve FE is unaltered, as shown in Figure 4.(*) If this occurs while w and p stay sticky at any initial point, the outcome is a rise of g in the trade deficit,

(*) See p. 24.

and no change in the state of the labour market - a result made famous by the New Cambridge School, and recently discussed in an income-expenditure model by Cripps and Godley (1976).

Most economists seem to regard this result as an extreme case, and it would be worth spelling out the precise conditions that are responsible for it in my model. First we have the assumption of sticky prices. If the exchange rate is highly flexible, the economy will move quickly to the right from W to lie on the new curve BT' in Figure 4, and the effect of government expenditure will appear in the form of an excess demand for labour. Secondly, working with an aggregate commodity neglects the role of non-tradeables. Government expenditure on such commodities, or directly on labour, will obviously affect the labour market. Finally, taxes will affect wages and prices and therefore shift the FE curve, and it is not appropriate to think in terms of the budget deficit directly. It should be stressed that the result is not dependent on the Cobb-Douglas assumption.

Within its range of validity, however, the Cambridge result has more strength than is often realised. When we consider the monetary dynamics, we see that the monetisation of the budget deficit adds to the money supply exactly what is taken away as a result of the trade deficit. There is no net effect on the money supply and no further cause for shifts of the curves, so the result on the relation between government expenditure and the trade deficit extends to a longer run. In essence what happens is that the government spends reserves to buy commodities from abroad, as if bypassing the domestic economy.

The new Walrasian temporary equilibrium can be attained by raising both w and p, but the former by a smaller proportion, i.e. by devaluation accompanied by some decline in the real wage.

It is seen that the proper dual decision analysis leads to qualitative differences between the effects of monetary and fiscal policies, even without bringing in interest rates and capital movements.

The third parameter change I examine is an increase in k, i.e. an upward shift in productivity. This is shown in Figure 5. We see that the new Walrasian equilibrium W' is to the left of the old one W. Therefore, if w and p are sticky at their old values, the initial impact of such a shift is to produce excess demand for labour and a trade surplus. This is in striking contrast to Malinvaud's result that in a closed economy, the initial impact of a productivity increase is to produce Keynesian unemployment. That rather Ricardian result does not appear in a small trading economy since it does not face the same problem of generating enough commodity demand. A commodity price higher than its Walrasian equilibrium value simply makes expanded production profitable while squeezing domestic demand, leaving export prospects untouched. A tight labour market and a trade surplus are the obvious consequences. I think this is a more realistic picture than Malinvaud's for most countries, and Germany is an obvious example. Conversely, a downward shift in productivity, which in practice must be interpreted as a smaller upward shift than that occuring in the rest of the world, will lead to unemployment and deficit. The case of Britain seems to illustrate this. Also, as is done by Malinvaud, we can stretch the model a little and interpret the production function (19) as a net revenue function, when a downward shift in k is formally like an increase in the price of an imported raw material that is an input to production. Again, the predicted outcome of unemployment and deficit seems amply borne out by evidence.

If the production function with labour and the imported primary input is Cobb-Douglas, the net revenue function will also be Cobb-Douglas and the above analysis will apply exactly. In this case, as W' lies due west from W, the simplest way to restore Walrasian equilibrium is to raise p, i.e. to devalue. No change in money wages is necessary. If the elasticity of substitution between labour and the primary input is less than one, it seems plausible that a reduction in the money wage will also be necessary.

If the economy is left to itself after an increase in k, then the surplus will lead to an increase in the money supply. The Walrasian equilibrium will drift to the north-east from W', while the wage rate will increase and perhaps the price level will fall, moving the actual position to the north or the north-west from W. Qualitatively, stability is indicated, but a more complicated analysis is needed before we can say whether there will be overshooting and cycles.

IV. Concluding Remarks

I would argue that the approach presented here has some advantages in discussing problems of the balance of payments. It is capable of encompassing the essential features of both the Keynes-Meade and the monetary approaches. It does this allowing some disequilibrium, and using the choice theoretic framework of the dual decision hypothesis which is becoming widely accepted as being appropriate to such disequilibrium. It allows us to classify different kinds of market imbalances and temporary equilibria, and enriches the analysis of effects of various policy changes or exogenous shocks.

It is obvious that the model should be extended to allow greater disaggregation of commodities, and to allow bonds, interest rates and international capital movement. These are directions for future research.

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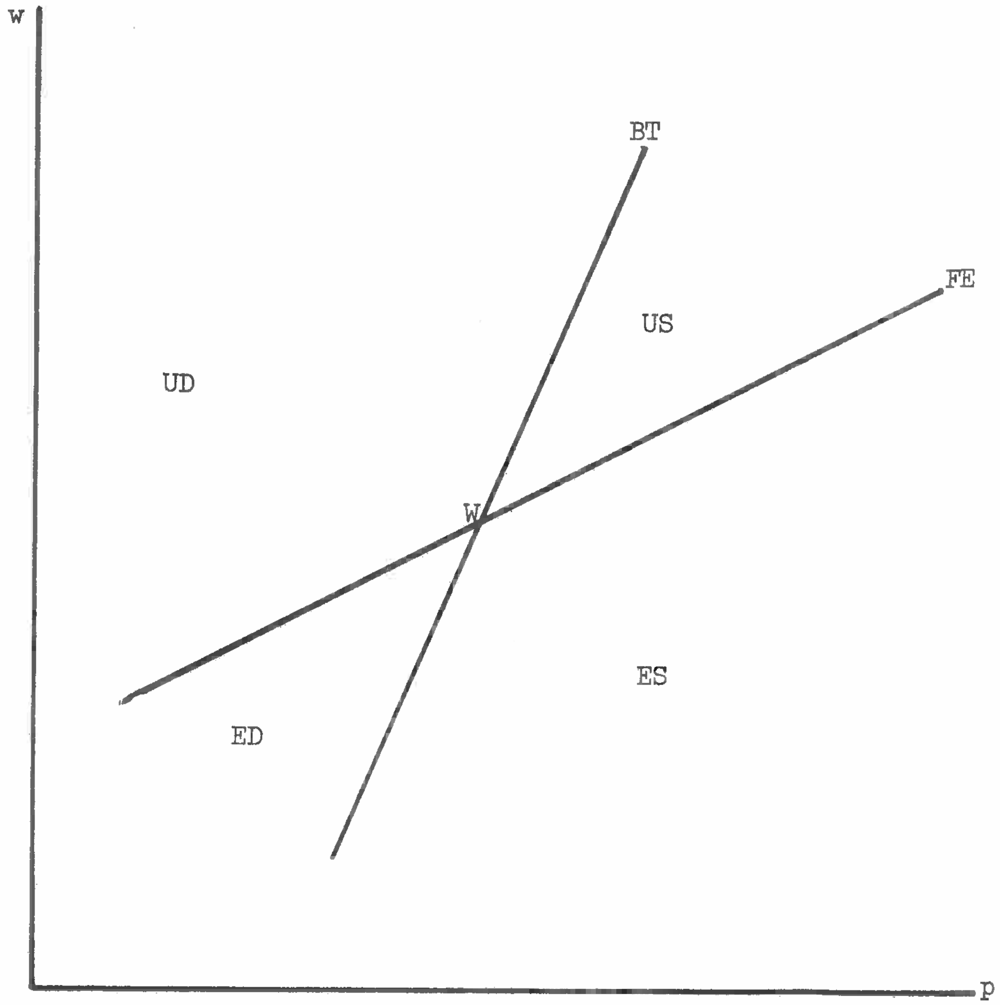


Figure 1

Notional equilibrium classification

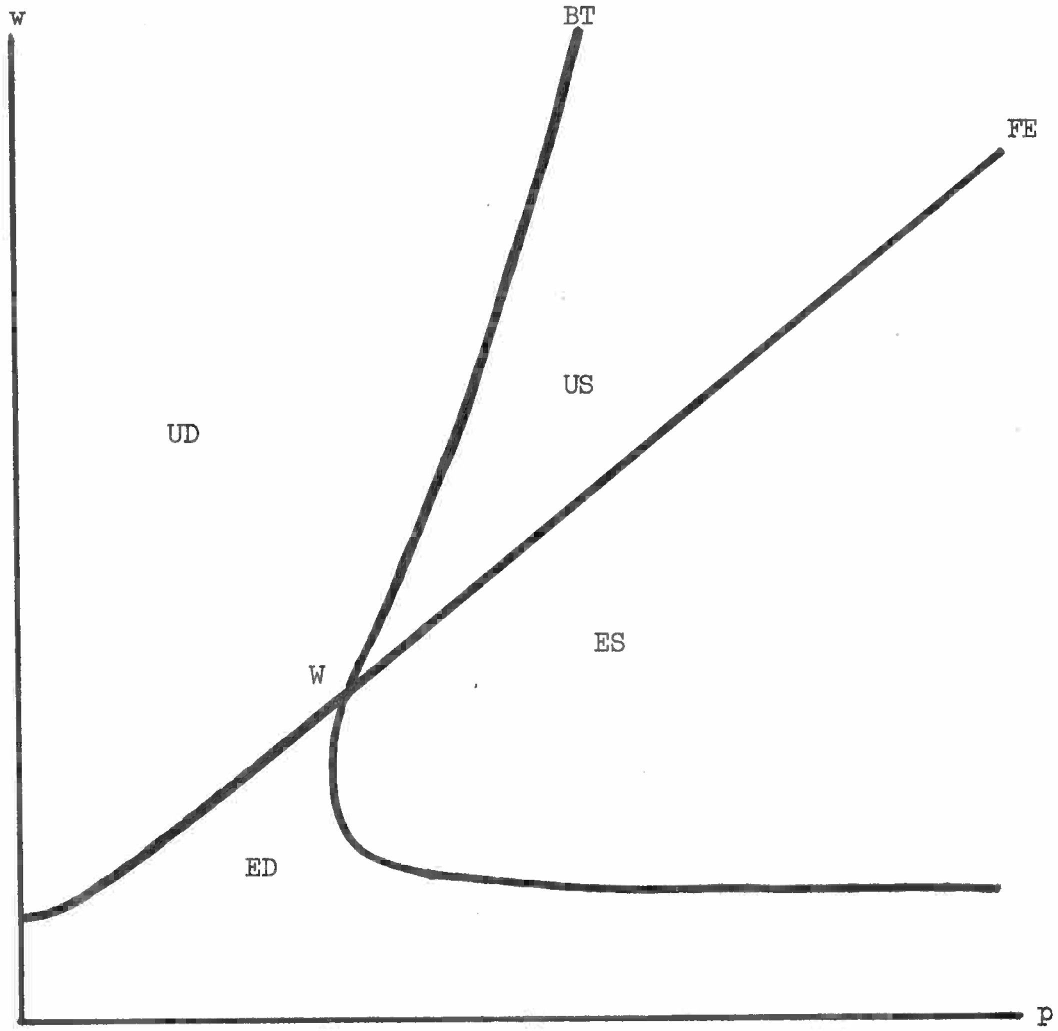


Figure 2

Effective equilibrium classification

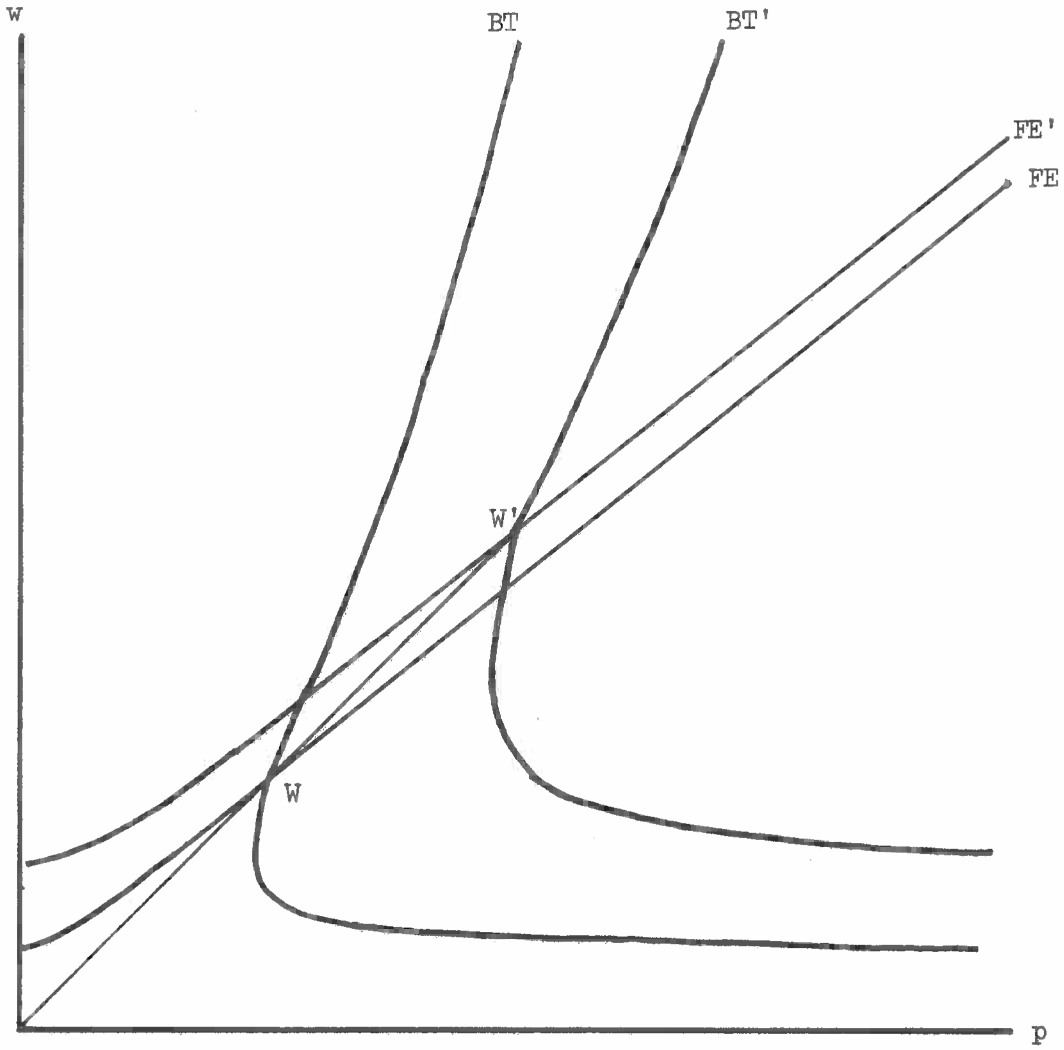


Figure 3

Monetary Policy

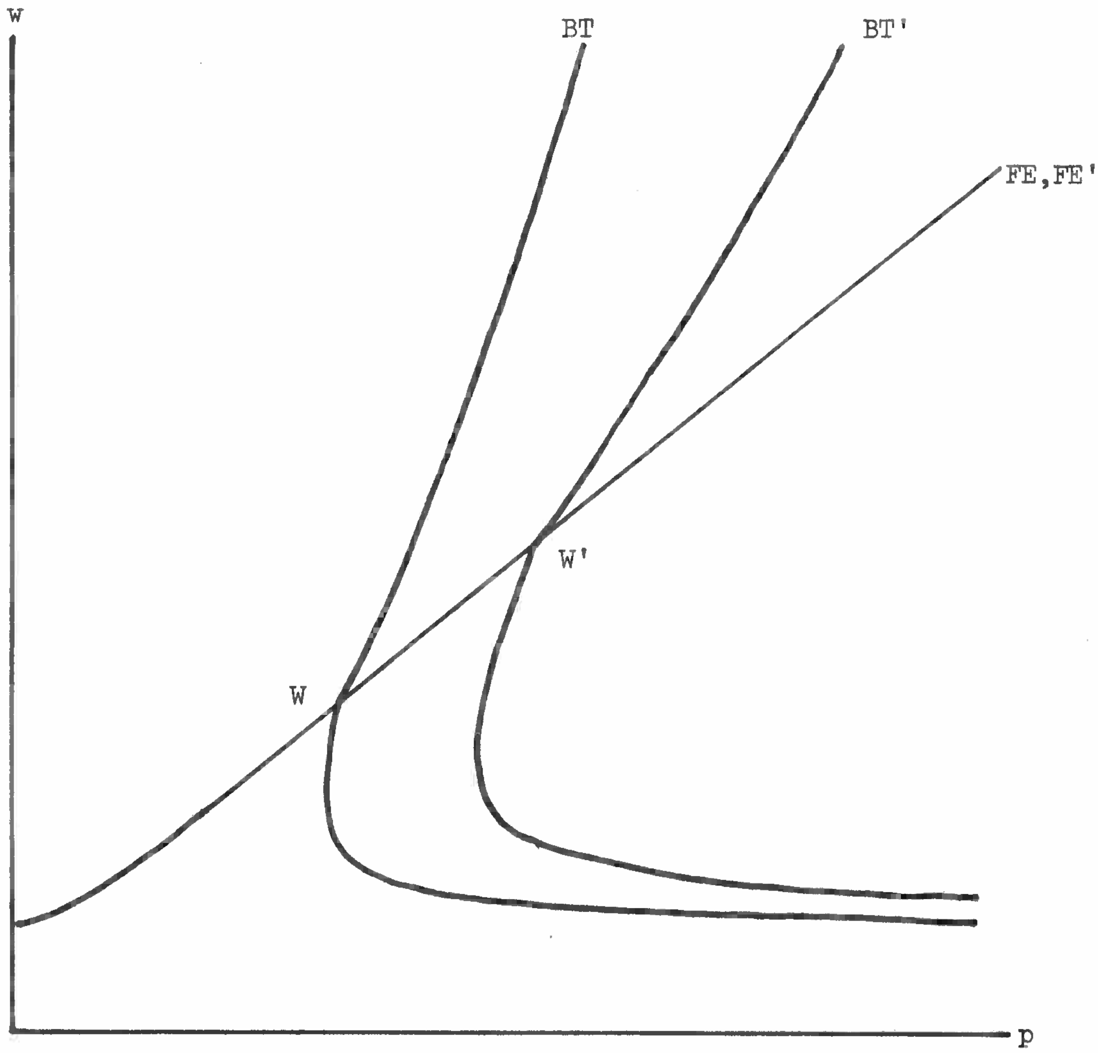
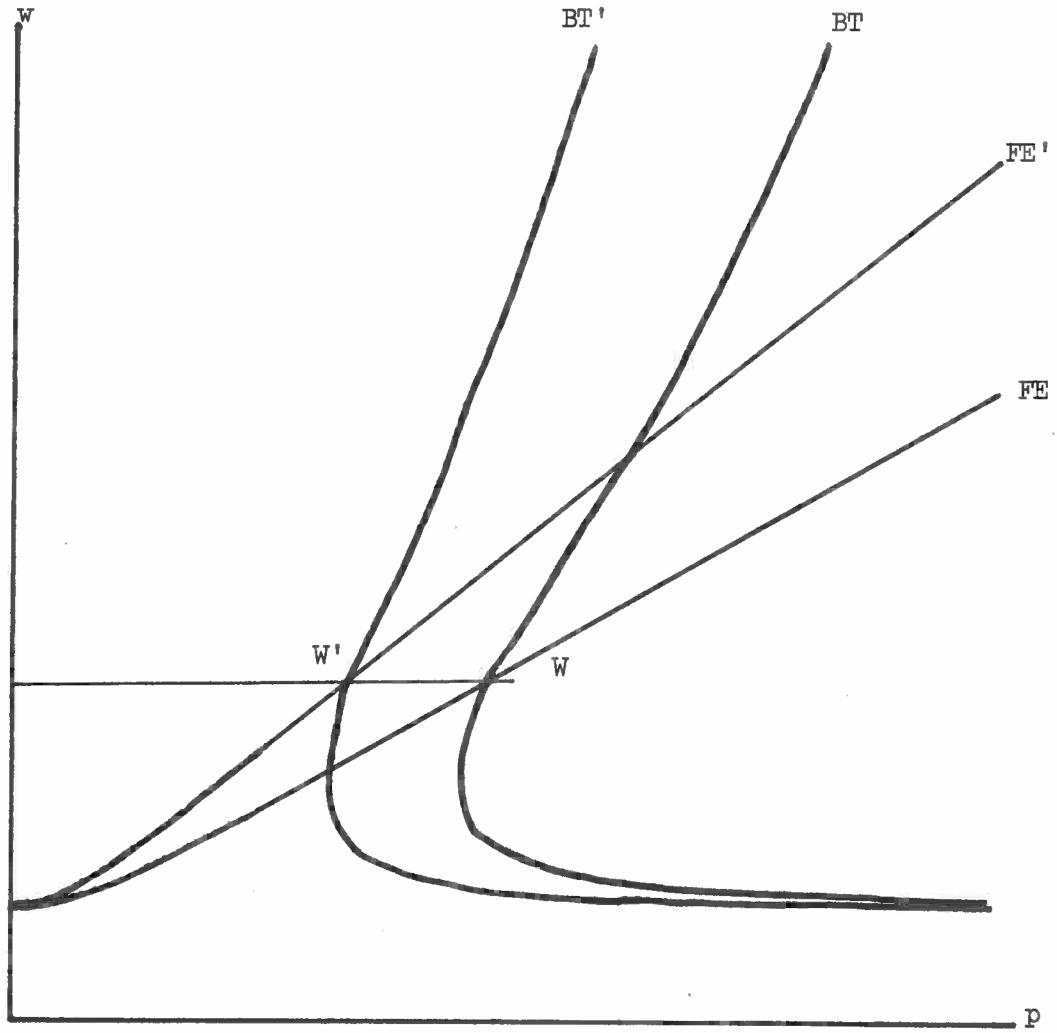


Figure 4

Fiscal policy



<u>Figure 5</u>

Productivity shift