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NUMBER 101

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

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January, 1977

In an article in Economica, August 1967, Professor H.G.Johnson concluded that "the larger the number of goods relative to the number of factors, the more likely is free trade to lead to factor price equalisation". It was to be understood that "more likely" meant that the introduction of additional commodities, by widening the range of feasible factor prices in one or both countries, would sometimes result in the emergence of an overlap in the ranges of the two countries, so that this necessary condition for factor price equalisation would be fulfilled where it would not otherwise have been fulfilled. Since the introduction of additional commodities could never narrow the range of feasible factor prices, additional commodities could only help and never hinder the fulfillment of the necessary condition for factor-price equalisation. Whether equalisation actually occurred would depend partly on demand, since this would influence which goods were actually produced. As Professor Johnson later made more explicit [1/], his conclusion embodied an implicit assumption that demand conditions would not prevent equalisation, so that in effect the necessary condition (an overlap in the ranges of feasible factor prices) became necessary and sufficient.

Professor Johnson restricts his conclusion to situation in which the strong Samuelson factor intensity assumption holds. The purpose of this note is to show that this restriction is redundant, and that a considerably weaker restriction is appropriate. It will be shown in fact

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1/ H.G.Johnson: "On Factor price equalisation when Commodities outnumber factors : a comment". Economica 1970.

that once the number of goods exceeds the number of factors, the existence of factor intensity reversals increases rather than reduces the likelihood, in Professor Johnson's sense, of factor price equalisation. In addition a diagrammatic proof will be given of the theorem (due to Meade and Samuelson) 2/ that if, in trading equilibrium, the number of goods produced in both countries and traded between them exceeds the number of factors, factor-price equalisation must have occurred, regardless of any other considerations relating to production functions or factor endowments,

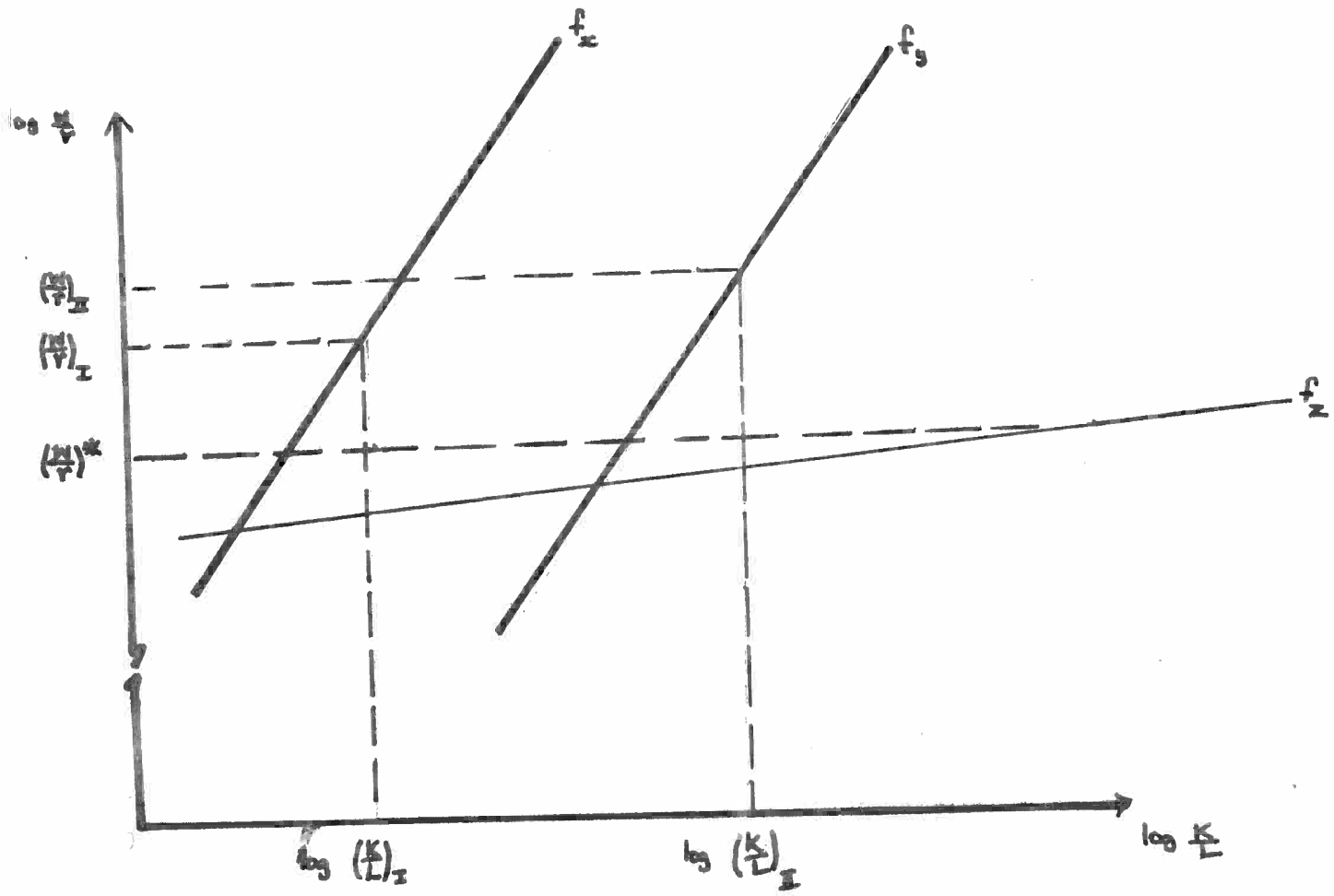


Figure 1

2/ J.E.Meade: "The Equalisation of Factor Prices : The Two-Country Two-Factor Three Product Case", Metroeconomica, December 1950. P.A.Samuelson: "Price of Factors & Goods in General Equilibrium, Rev.Econ.Studies 1953-4. See also J.R.Melvin: "Production and Trade with Two Factors and Three Goods", American Economic Review, December 1968.

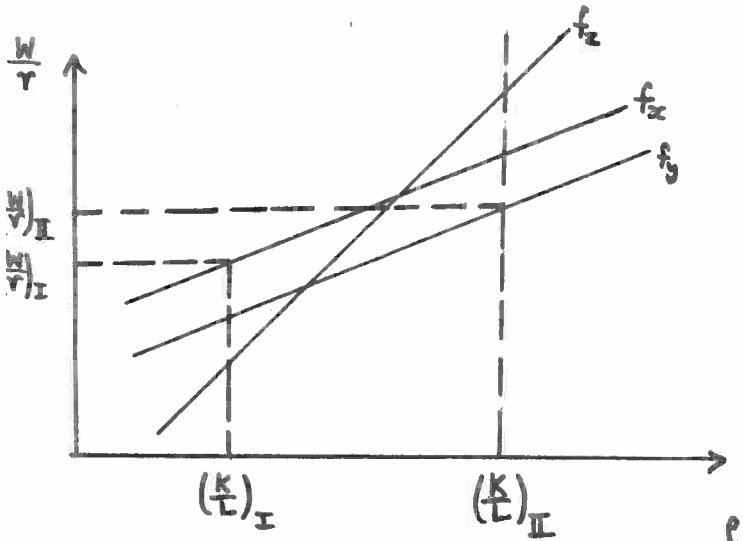
To prove the first point, consider an initial situation in which there are two factors K and L, two commodities x and y, and two countries I and II. The factor demand curves f_x and f_y for x and y are shown in Figure 1. We have chosen logarithmic scales on the axes since this has the convenient consequence that the gradient of the factor demand curve (or, more precisely, the reciprocal of the gradient) measures the elasticity of substitution. Now, if the factor endowments of I and II are $\left(\frac{K}{L}\right)_I$ and $\left(\frac{K}{L}\right)_{II}$ it is obvious that factor price equalisation cannot occur, since there is no overlap between the range of feasible factor prices in I and the corresponding range in II. This is because the maximum value of log $\frac{w}{r}$ in I $k\left(\frac{w}{r}\right)_I$ and the mimimum in II is $\left(\frac{w}{r}\right)_{II}$.

But if we now introduce a 3rd commodity z with factor demand curve f_z as shown in the figure, the range of possible factor prices in II is widened and an overlap with I appears, making equalisation possible. For example, the factor price ratio $\left(\frac{w}{r}\right)^*$ is feasible in both countries, with I producing x and either y or z or both, and II producing z and either x or y or both.

From the diagram it will be seen that introducing z widens the range of feasible factor prices in at least one country, provided f_z does not lie entirely between f_x and f_y over the interval $\left(\frac{K}{L}\right)_I$ to $\left(\frac{K}{L}\right)_{II}$. But if f_z were to lie entirely between f_x and f_y, it would follow that, at any factor price ratio, the factor intensity of z could be expressed as a linear combination, with non-negative weights, of the factor intensities of x and y. In this sense, its production function would not be independent of the production functions of the other two goods, and

the existence of the 3rd would contribute nothing to the likelihood of factor price equalisation. Provided z had a production function which is independent, in the sense just described, of the production functions of x and y, the introduction of z as a 3rd commodity may widen and could never narrow the range of feasible factor prices, and will therefore increase the likelihood of factor price equalisation in Professor Johnson's sense. In the figure we have given z a different elasticity of substitution from x and y, so that a factor intensity reversal between y and z occurs in the interval $\left(\frac{K}{L}\right)_I$ to $\left(\frac{K}{L}\right)_{II}$. This reversal, far from precluding equalisation, increases its likelihood since it guarantees that z has an independent production function in the sense desc ribed above.

Figure 2



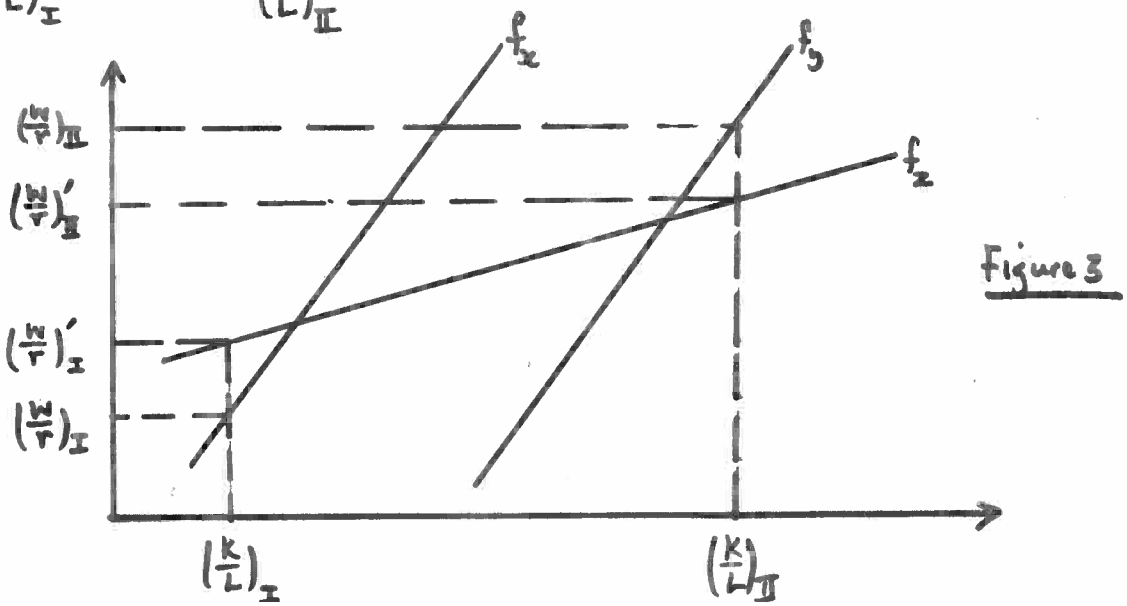


Figure 3

Of course, it is not necessary that the third commodity should have a differing elasticity of substitution in order for it to widen the range of feasible factor prices. If f_z lies parallel with, and above, f_x or below f_y the range of feasible factor prices is certainly widened. This simple conclusion does not require diagrammatic illustration.

At the same time, it is not sufficient that the third commodity should have a different elasticity of substituion. If the elasticity of substitution of z differs sufficiently from that of x and y, as in Figure 2, f_z may intersect both f_x and f_y in the interval $\left(\frac{K}{L}\right)_I$ to $\left(\frac{K}{L}\right)_{II}$, in which case it will contribute nothing to the likelihood of factor price equalisation, despite having an independent production function. This is because the existence of z merely lowers the minimum feasible factor price ratio in I, and raises it in II.

It might be thought that, in the case just considered, z contributes nothing to the likelihood of equalisation because it exhibits a reversal both with x and y over the relevant range. In Figure 3 however the existence of z appears to contribute something towards the possibility of equalisation by lowering the minimum value of $\frac{w}{r}$ in II from $\left(\frac{w}{r}\right)_{II}$ to $\left(\frac{w}{r}\right)'_{II}$ and raising the maximum value in I from $\left(\frac{w}{r}\right)_I$ to $\left(\frac{w}{r}\right)'_I$. Though the (necessarily positive) slope of f_z precludes any overlap, the existence of z would appear to permit partial equalisation to proceed further than would otherwise have been possible.

However this case must be handled with caution. Factor prices in II can only go below $\left(\frac{w}{r}\right)_{II}$ if output of z is increasing at the expense of x or y. Similarly factor prices in I can only go above $\left(\frac{w}{r}\right)_I$ if output of z is increasing. If z is a traded good, trade will lead to the expansion of z production in one country and its

contraction in the other. Hence factor prices will move in the same direction in both countries. (If only x and y had existed, factor prices would have converged). Whether the absolute difference between factor prices in the two countries, in equilibrium, will be greater or less than it would have been had only x and y existed it is impossible, a priori, to say. It is possible but not in any sense likely, that the existence of commodity z may result in greater equality. If on the other hand z is a non-traded good, its output could increase in both countries as a result of trade. In this case the factor price ratio could approach* $\left(\frac{w}{r}\right)'_I$ in I and $\left(\frac{w}{r}\right)'_{II}$ in II, resulting in a greater movement towards equalisation than would have been possible had z not existed.

We now summarise on the likelihood of factor price equalisation when commodities outnumber factors. Full factor price equalisation requires an overlap in the range of feasible factor prices and this is only possible if there are two commodities which do not reverse their intensities at any capital/labour ratio between the endowment ratios of the two countries. If increasing the number of commodities increases the diversity of slope and intercept of the factor demand curves, the likelihood of finding two such commodities is increased. If a third commodity is added which reverses its factor intensity relative to one (but not both) of

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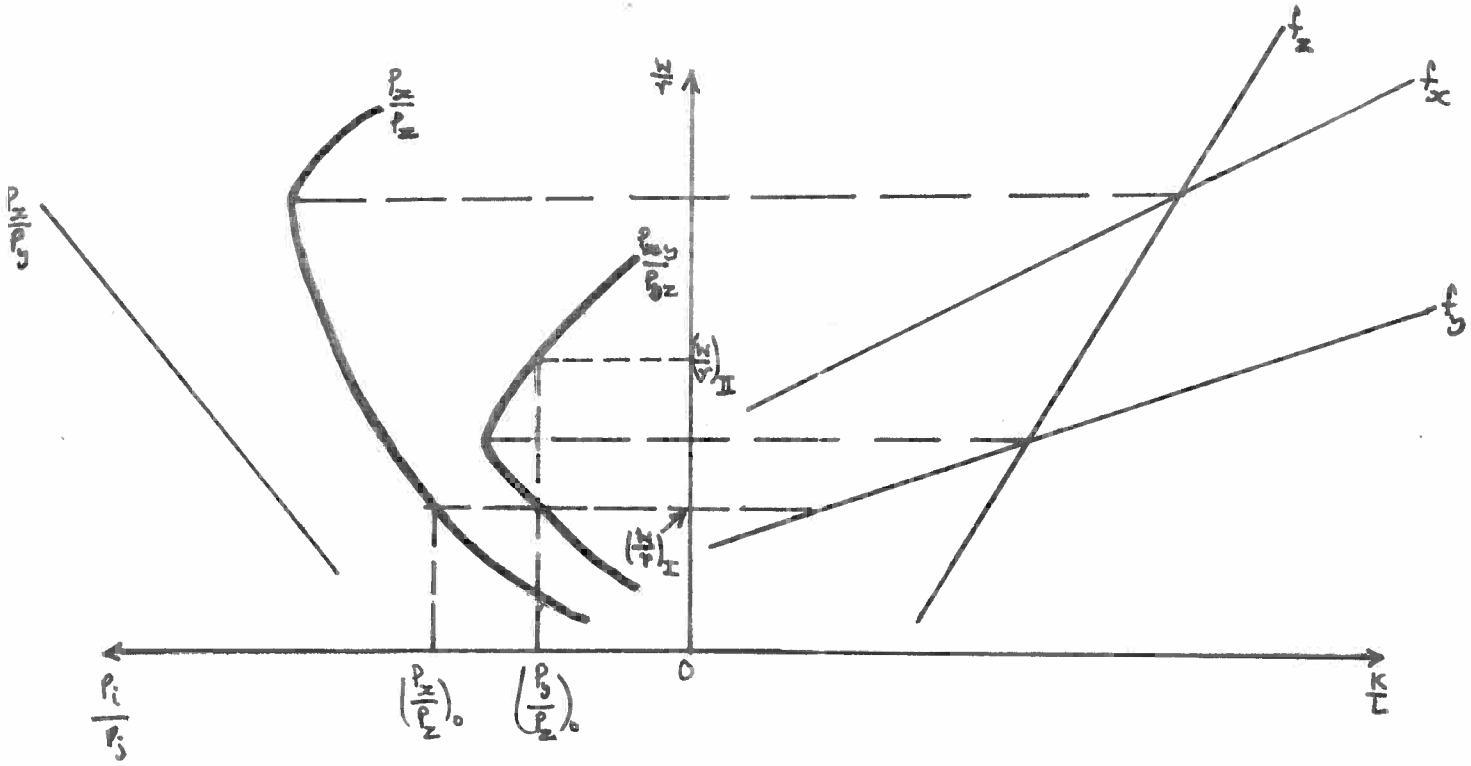
* The factor price ratio could not reach the limits of $\left(\frac{w}{r}\right)'_I$ and $\left(\frac{w}{r}\right)'_{II}$ in I and II without extinguishing trade; for at these points production of x and y cases in both countries.

the pre-existing goods in the relevant range of factor intensities, this increases the likelihood of factor price equalisation. If a third commodity is added which does not reverse its itensity relative to either of the pre-existing goods over the relevant range, this adds to the likelihood of factor price equalisation only if its factor intensity, at any factor price ratio, is not a weighted average of the intensities of the pre-existing goods. A key question is whether, when commodities outnumber factors, the existence of factor intensity reversals increases or reduces the likelihood of factor price equalisation. On reflection it is apparent that the existence of factor intensity reversals increases the likelihood of factor price equalisation. This is because, as the number of commodities increases, it must become less and likely that the next commodity added will exhibit a reversal of intensity, relative to all pre-existing goods, within the relevant range of factor intensities. If the next good added reverses its intensity with all other goods except one, we then have a pair of goods which do not reverse their relative intensities, and full factor price equalisation then becomes possible. To put the point another way, if factor intensity reversals are precluded, factor demand curves can differ only in their intercept on the $\frac{w}{r}$ axis. If factor intensity reversals are permitted, factor demand curves differ both in their slope and intercept. This extra degree of freedom increases the likelihood that an overlap in the range of feasible factor prices will occur, and therefore the likelihood of factor price equalisation. This conclusion is in interesting contrast to the two commodity case in which a reversal of factor intensity in the relevant range precludes equalisation.

Figure 4



The necessity of factor price equalisation when the number of produced and traded goods exceeds the number of factors.

The second point to be demonstrated is that if the number of commodities produced in both countries in trading equilibrium exceeds the number of factors, factor prices must be equalised. In Figure 4 three factor demand curves are shown in the right hand quadrant. In the left hand quadrant are shown the 3 commodity price/factor price relationships - one for each pair of commodities. In the usual way, the $\left(\frac{P_x}{P_z}\right)$ commodity price ratio has its turning point at the value of $\frac{w}{r}$ at which a factor intensity reversal between x and z occurs, and similarly the $\left(\frac{P_y}{P_z}\right)$

price ratio has its turning point at the value of $\frac{w}{r}$ at which y and z reverse their intensities. The $\left(P_x/P_y\right)$ price ratio is monotonic, reflecting the absence of a reversal between commodities x and y.

Because the quantity unit of each commodity is arbitrary, the scale on the price axis cannot be specified. But the location of the three price relationships is constrained by their mutual dependence. Because f_x lies entirely above f_y, we know that $\left(P_x/P_y\right)$ is a monotonic increasing function of $\left(\frac{w}{r}\right)$. At the same time, the values of $\left(P_x/P_z\right)$ and $\left(P_y/P_z\right)$, are constrained by the need to satisfy the identity :-

$$\frac{P_x}{P_y} \equiv \frac{P_x}{P_z} \Big/ \frac{P_y}{P_z}$$

at every value of $\frac{w}{r}$.

Thus the ratio of $\frac{P_x}{P_z}$ to $\frac{P_y}{P_z}$ must increase as $\frac{w}{r}$ increases and this dictates the relative slopes of these two curves. (It will be apparent also that one of these curves is redundant, being derivable from the other two. We shall treat the $\left(P_x/P_y\right)$ relation as redundant, and shall not refer to it again).

If we now assume that free trade leads to $\left(\frac{w}{r}\right)_I$ in country I, it is apparent that if all three goods are produced in I their relative

prices must be given by $\left(P_x/P_z\right)_0$ and $\left(P_y/P_z\right)_0$. Under free trade with no transport costs, these prices must also prevail in II. But from inspection of the diagram it is clear that no other value of $\frac{w}{r}$ could have generated these commodity prices, so that if all goods are produced in II, factor prices must be the same in II as in I. If factor prices in II were, say, $\left(\frac{w}{r}\right)_{II}$, only commodities y and z could be produced competitively in II. The reader may easily check for himself that the conclusion is quite independent of the shapes 3/ of the curves in Figure 4. It results solely from the fact that, with 3 commodity price/factor price relationships, each factor price ratio generates a unique set of commodity prices.

This result is somewhat surprising because it appears to permit us to state circumstances under which factor price equalisation will occur, without any direct reference to production functions or factor endowments. This appearance is deceptive however because of course whether or not the two countries are able to produce all 3 goods after trade commences will depend on production functions and factor endowments. The result tells us nothing ex ante about the likelihood of equalisation but only states a sufficient condition for equalisation to occur. Fortunately, however, for the purposes of empirical generalisation we are not interested in the ex ante likelihood of equalisation, but rather in the question whether the apparent absence of equalisation in the real world constitutes an

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3/ There is one exception, which has been described by Chipman as pathological. If a reversal of factor intensity between each of the three pairs of commodities occurs in the relevant range, all three factor-price/commodity price relationships will exhibit turning points. If these relationships are sufficiently symmetrical about their turning points, it may be possible to find two factor price ratios consistent with a given set of commodity prices. This is formally equivalent to finding two solutions, with the same x co-ordinate, to a pair of simultaneous equations in two variables x and y. In Figure 4 a necessary (but not sufficient) condition for this is that the 3 factor demand curves should intersect at a common point.

adequate refutation of the theory. Provided we are satisfied that the number of commodities produced in common and traded between a pair of countries exceeds the number of their (non-tradeable) factors of production, we can confidentially predict complete factor prices equalisation except to the extent that tariffs or transport costs prevent commodity prices being equalised. Though some writers 4/ have refused to accept that the number of commodities exceeds the number of factors, and the question cannot be settled until the terms "commodity" and "factor" are precisely defined, most of us would accept that the number of distinct traded commodities exceeds the number of distinct factors. At the same time, despite the considerable movement towards free trade in the last two decades, there appears no strong tendency towards factor price equalisation. This provides therefore indirect evidence for rejecting a key assumption of the Heckscher/Ohlin model; that of identity of production functions between countries.

4/ Pearce for example is not prepared to grant this. "All published results fail to contradict the hypothesis that more than two basic factors of production exist". (I.F.Pearce: "International Trade", Macmillan, 1970), Vol.II).