Industry-Specific Income Taxes

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Abstract: From 1966 until 1973 the U.K. government levied the Selective Employment Tax which took the form of a weekly tax upon employees in service industries. By moving beyond competitive, constant returns models, the paper reconsiders the case for such industry-specific employment taxes. It is shown that the taxes are not justified by decreasing returns to scale even when profit taxation is non-optimal. In contrast, when imperfect competition is considered industry-specific income taxes become an integral part of an optimal tax system. Optimal tax rules are derived for an imperfectly competitive economy and the determinants of relative tax rates are investigated.

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1. Introduction

From 1966 until 1973 the U. K. government levied the Selective Employment Tax which took the form of a weekly tax upon employees in service industries. Existing studies of optimal taxation can provide no formal justification for an industry-specific tax of this form, in competitive models with constant returns this is simply a consequence of the Diamond-Mirrlees (1971) efficiency lemma. By moving beyond this special case, the following analysis reconsiders the case for industry-specific employment taxes.[1] It is shown that decreasing returns to scale do not provide a foundation for such taxes even when profit taxation is non-optimal. In contrast, when imperfect competition is considered, industry-specific income taxes become an integral part of an optimal tax system.

This analysis can be viewed in two ways. Firstly, from a formal perspective, the paper completes the analysis of optimal commodity taxation with imperfect competition that was begun in Myles (1989a). That paper assumed the tax upon labour was zero in order to focus upon the taxation of consumption goods, although it was recognised that this was more than just a normalisation. Commodity taxation and labour taxation are complementary instruments in an optimal tax system for an imperfectly competitive economy and the combined results of this and the previous paper characterise such a tax system.

Secondly, the practical interpretation of the paper is that it provides a new perspective from which to justify selective employment taxes. The stated motivation of the U.K. Selective Employment Tax was to transfer labour from service industries to manufacturing. Some may view the arguments put forward in support of it as convincing in that different forms of employment vary in their effects upon the aggregate economy and the market may not reach the correct allocation. As already

[1] As the notional incidence of the tax, ie. whether it is levied on firms (an "employment" tax) or on consumers (an "income"tax), does not affect the real equilibrium (Proposition 1) either term can be used to describe the class of taxes under consideration.

noted, formal support for this viewpoint cannot be found in the existing literature on optimal taxation and Reddaway's (1970) comprehensive study of the tax considered only its effects not its rationale. The contribution of this paper is to demonstrate that selective employment taxes should form part of an optimal tax system for an imperfectly competitive economy. It is worth noting that throughout the paper the industry is chosen as the basic unit for taxation, a choice motivated by the fact that the placement of firms into industries is a accepted procedure in the compilation of Standard Industrial Classifications. This should be contrasted to the practice of the Selective Employment Tax which was based on a division between service and manufacturing industry. Such a division proved too blurred to be practical and this was a contributing factor in the eventual elimination of the tax (Kay and King 1980).

Section 2 demonstrates the efficiency of zero labour taxes in competitive, decreasing returns models. In section 3, three simple examples of imperfectly competitive economies with additive taxes are analysed. These examples illustrate the determinants of relative tax rates, in particular the importance of returns to scale and industrial conduct are emphasised. An optimal tax rule is derived in section 4 that relates the reduction in supply from each industry due to the tax system to the returns to scale of the industry, the effect of the tax upon price and the interaction of the industry with the economy. Affine tax schedules are analysed in section 5 and it is shown that these have no advantage over additive taxes, more precisely one of the parameters describing the tax is redundant. Conclusions are given in section 6.

2. Decreasing returns

The literature on production efficiency, notably Dasgupta and Stiglitz (1972) and Mirrlees (1972), has demonstrated that optimal labour taxes should not be differentiated across industries in competitive models with decreasing returns to scale provided that appropriate profit taxes are levied. However, the requirement that profit taxes should be optimal does leave open the question of what should be done if they are not. The analysis of this section establishes the proposition that non-differentiated labour taxes are in fact optimal without the requirement to invoke optimality of profit taxation. The result is therefore a slight strengthening, in a specific direction, of existing productive efficiency results. More importantly, it also demonstrates that to find support for industry-specific income taxes it is necessary to move beyond the competitive framework.

The model employed in this section is based on Munk (1978) and with regard to the incidence of taxation, the model has two valid representations. The first is to fix the wage rate received by consumers at a level w. A firm j in industry i with output x_i^j facing a tax on its labour input of τ_i per unit then has cost function

$$C^i\left(x_i^j,\ w+\tau_i\right), \tag{1}$$

where $w + \tau_i$ is the post tax cost per unit of labour. The alternative is to assume the consumer pays the tax and maximises with respect to a wage rate w_i-τ_i, with a firm j in industry i having costs

$$C^i\left(x_i^j,\ w_i\right). \tag{2}$$

It is probably evident that these representations actually describe identical models. That this is indeed the case is stated formally as:

Proposition 1. The real equilibrium is independent of the notional incidence of the tax.

Proof. Competition on the labour market must result in the returns across firms being equalised, hence

$$w_i - \tau_i = w,$$

or

$$w_i = w + \tau_i.$$

From this equality it is evident that the two methods of modelling are equivalent and determine identical real equilibria. Δ

Following proposition 1, it can be seen that it is possible to view the tax as both an income tax and a tax on labour input; the former interpretation is adopted below. In addition, I will model the tax as being paid by the firms and normalise the wage received by consumers at the fixed value w. It can also be shown that the real equilibrium is also unaffected by the choice of value for w.[2]

Consider a K industry competitive model in which each industry has decreasing returns to scale, labour is the only input into production and the wage rate is numeraire. The restriction that labour is the only input is employed to simplify notation; it is relaxed below without affecting the conclusion. For outputs $X_1,...,X_K$, aggregate profit π, and inverse demand functions $q_k(X_1,...,X_K, \pi)$, $k = 1,...,K$, marginal cost pricing implies that the equilibrium quantities must solve

$$q_k(X_1,...,X_K, \pi) = C_0^k(X_k, w+\tau_k), \; k = 1,...,K, \tag{3}$$

and

$$\pi = \sum_{k=1}^{K} \pi^k = \sum_{k=1}^{K} \left[q_k(X_1,...,X_K,\pi)X_k - C^k(X_k, w+\tau_k)\right], \tag{4}$$

where π^k is the profit level of industry k. If (4) is solved to express π as a function of $X_1,...,X_K$ and $\tau_1,...,\tau_K$, this solution can be substituted into (3) to find:

$$X_k = X_k(\tau_1,...,\tau_K), \; k = 1,...,K. \tag{5}$$

Now substitute (5) into the solution of (4) then both into the definition of each firm's profit, this gives

[2] Munk (1978) proves this for the competitive model. For the model of imperfect competition used in sections 3-5, the analogous result is proved in Cripps and Myles (1989).

$$\pi^k = \pi^k(\tau_1,\ldots,\tau_K),\ k = 1,\ldots,K. \tag{6}$$

Finally, (5) and (6) substituted into the inverse demand functions yield

$$q_k = q_k(\tau_1,\ldots,\tau_K),\ k = 1,\ldots,K. \tag{7}$$

Given the exogenous tax rates, (5), (6) and (7) determine the equilibrium values of the endogenous variables.

The proof of the proposition below requires the calculation of the effect of changes in taxes on profit levels. To derive these, consider industry k for which

$$\pi^k = q_k X_k - C^k(X_k,\ w+\tau_k). \tag{8}$$

Hence

$$\frac{\partial \pi^k}{\partial \tau_k} = X_k \frac{\partial q_k}{\partial \tau_k} - C_1^k + \left[q_k - C_0^k\right]\frac{\partial X_k}{\partial \tau_k}, \tag{9}$$

but, from optimality of X_k,

$$\frac{\partial \pi^k}{\partial \tau_k} = X_k \frac{\partial q_k}{\partial \tau_k} - C_1^k. \tag{10}$$

Similarly,

$$\frac{\partial \pi^k}{\partial \tau_i} = X_k \frac{\partial q_k}{\partial \tau_i}. \tag{11}$$

To concentrate only upon the efficiency of the tax system, assume zero revenue is to be collected. The optimal tax system then solves:

$$\max_{\{t_1,\ldots,t_K\}} V(q_1,\ldots,q_K,\pi) \text{ subject to } \sum_{k=1}^{K} \tau_k C_1^k(X_k, w+\tau_k) = 0. \tag{12}$$

I now prove

Proposition 2. $\tau_k = 0$, $k = 1,\ldots,K$ *satisfies the first-order necessary conditions for (12).*

Proof. The first-order conditions of the maximisation are:

$$\sum_{i=1}^{K} \frac{\partial V}{\partial q_i}\cdot\frac{\partial q_i}{\partial \tau_k} + \sum_{i=1}^{K} \frac{\partial V}{\partial \pi}\cdot\frac{\partial \pi^i}{\partial \tau_k} + \lambda\left[C_1^k + \sum_{i=1}^{K} \tau_i C_1^i \frac{\partial X_i}{\partial \tau_k}\right] = 0,\ k = 1,\ldots,K. \tag{13}$$

Using Roy's identity,

$$-\alpha\sum_{i=1}^{K} X_i\frac{\partial q_i}{\partial \tau_k} + \alpha\sum_{i=1}^{K} \frac{\partial \pi^i}{\partial \tau_k} + \lambda\left[C_1^k + \sum_{i=1}^{K} \tau_i C_1^i \frac{\partial X_i}{\partial \tau_k}\right] = 0,\ k = 1,\ldots,K. \tag{14}$$

Employing (10) and (11), this becomes

$$-\alpha C_1^k + \lambda\left[C_1^k + \sum_{i=1}^{K} \tau_i C_1^i \frac{\partial X_i}{\partial \tau_k}\right] = 0,\ k = 1,\ldots,K. \tag{15}$$

Multiplying each equation by the respective value of τ_k and summing,

$$-\left(\frac{\alpha-\lambda}{\lambda}\right)\sum_{k=1}^{K} \tau_k C_1^k = 0 = \sum_{k=1}^{K}\sum_{i=1}^{K} \tau_k \tau_i C_1^i \frac{\partial X_i}{\partial \tau_k}, \tag{16}$$

where the first term is zero by the definition of the revenue constraint. It is obvious from (16) that $\tau_k = 0$, $k = 1,\ldots,K$ is a solution to this equation as was to be proved. Δ

This result can easily be extended to more general production technologies. The equivalent equations, where it is now simpler to work with X as a function of q and π, would be

$$q_k = C_0^k(X_k(q_1,\ldots,q_K,\pi), q_1,\ldots,q_K,\ w+\tau_k),\ k = 1,\ldots,K, \tag{3'}$$

and

$$\pi = \sum_{k=1}^{K} \pi^k = \sum_{k=1}^{K} \left[q_k X_k(q_1,...,q_K,\pi) - C^k(X_k(q_1,...,q_K,\pi),q_1,...,q_K,\ w+\tau_k)\right] \qquad (4')$$

These equations can be solved to determine the dependence of the equilibrium upon the tax rates and the following proposition proved:

Proposition 2'. $\tau_k = 0$, *k = 1,...,K satisfies the first-order necessary conditions for the model described by (3') and (4').*

Proof. By the method of calculation used for proposition 2. Δ

These propositions demonstrate the efficiency of non-differentiated income taxes in competitive models with decreasing returns. In conjunction with standard results on production efficiency, they imply that a justification for industry-specific income taxes will only be found outside the competitive framework. From this perspective, the following sections now consider the consequences of imperfect competition.

3. Linear income taxes: some examples

This section establishes that differential income taxes can increase welfare in a model with imperfect competition and considers the basic determinants of relative taxes by analysing three examples of approximately increasing generality. It is assumed throughout that the revenue requirement is zero, so that only efficiency aspects of taxation are involved, and that utility is additive in labour supply so that income effects can be ignored. The details of the derivations will mostly be omitted; they are of a similar form to those in Myles (1989b).

For the first example, let industry 1 be competitive, employing 1 unit of labour to produce each unit of output. Hence

$$q_1 = w + \tau_1. \tag{17}$$

Industry 2 is composed of a profit maximising monopolist who faces demand

$$X_2 = X_2(q_2), \tag{18}$$

and has costs

$$C^2 = C^2(X_2,\ w+\tau_2). \tag{19}$$

From (18), it can be seen that all income effects fall on labour supply.

From the monopolist's first-order condition for profit maximisation it can be calculated that

$$\frac{dq_2}{d\tau_2} = \frac{C^2_{01}\dfrac{\partial X_2}{\partial q_2}}{2\dfrac{\partial X_2}{\partial q_2} + \dfrac{\partial^2 X_2}{\partial {q_2}^2}\left[q_2 - C^2_0\right] - C^2_{00}\left(\dfrac{\partial X_2}{\partial q_2}\right)^2} > 0\ , \tag{20}$$

and

$$\frac{d\pi_2}{d\tau_2} = -C^2_1 < 0\ , \tag{21}$$

where $C^2_0 \equiv \dfrac{\partial C^2}{\partial X_2}$, $C^2_1 \equiv \dfrac{\partial C^2}{\partial [w+\tau_2]}$.

Writing L^i for the labour demand of industry i, the government budget constraint is

$$\tau_1 L^1 + \tau_2 L^2 = 0. \tag{22}$$

Finally, social welfare is determined by the indirect utility function

$$V(q_1,\ q_2) + \pi_2\ . \tag{23}$$

Maximising (23) with respect to τ_1 and τ_2 subject to (22) and solving the necessary conditions, the optimal value of τ_2 can be characterised by

$$\tau_2 = \frac{L^1\left[\frac{\partial V}{\partial q_2}\cdot\frac{\partial q_2}{\partial \tau_2} + \frac{\partial \pi_2}{\partial \tau_2}\right] - L^2\frac{\partial V}{\partial q_1}\cdot\frac{\partial q_1}{\partial \tau_1}}{\frac{\partial L^2}{\partial \tau_2}\cdot\frac{\partial V}{\partial q_1}\cdot\frac{\partial q_1}{\partial \tau_1} + \frac{L^2}{L^1}\cdot\frac{\partial L^1}{\partial \tau_1}\left[\frac{\partial V}{\partial q_2}\cdot\frac{\partial q_2}{\partial \tau_2} + \frac{\partial \pi_2}{\partial \tau_2}\right]}. \tag{24}$$

The denominator in (24) is positive under standard assumptions[3], hence the sign of τ_2 is that of the numerator. Using Roy's identity and noting that $L^1 = X_1$, $L^2 = C_1^2$, the numerator can be written

$$-X_1X_2\frac{\partial q_2}{\partial \tau_2} < 0, \tag{25}$$

by using (20). Therefore, in this example, the labour input to the monopolist should be subsidised and that to the competitive firm should be taxed.

This result is in accordance with the standard partial equilibrium conclusion that the output from the monopolist should be increased. However, it must be noted that (25) was derived in a general equilibrium context and that the subsidy to the monopolist is financed by a tax on the labour input of a competitive industry. That the monopolist should always be subsidised is, perhaps, surprising.

The second example is designed to gain insight into the determinants of relative rates of tax on labour employed by two distinct imperfectly competitive industries. Hence let both industries 1 and 2 be monopolistic[4] with demands

$$X_1 = X_1(q_1),\ X_2 = X_2(q_2). \tag{26}$$

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[3] $\frac{\partial L^i}{\partial \tau_i} < 0,\ \frac{\partial q_i}{\partial \tau_i} > 0,\ \frac{\partial \pi_i}{\partial \tau_i} < 0.$

[4] The use of "monopolist" is justified here due to the separable demands in (26).

Repeating the maximisation of welfare, with indirect utility now equal to $V(q_1, q_2) + \pi_1 + \pi_2$, the solution for τ_2 can be written implicitly as

$$\tau_2 = \frac{L^1\left[\frac{\partial V}{\partial q_2}.\frac{\partial q_2}{\partial \tau_2} + \frac{\partial \pi_2}{\partial \tau_2}\right] - L^2\left[\frac{\partial V}{\partial q_1}.\frac{\partial q_1}{\partial \tau_1} + \frac{\partial \pi_1}{\partial \tau_1}\right]}{\frac{\partial L^2}{\partial \tau_2}.\left[\frac{\partial V}{\partial q_1}.\frac{\partial q_1}{\partial \tau_1} + \frac{\partial \pi_1}{\partial \tau_1}\right] + \frac{L^2}{L^1}.\frac{\partial L^1}{\partial \tau_1}\left[\frac{\partial V}{\partial q_2}.\frac{\partial q_2}{\partial \tau_2} + \frac{\partial \pi_2}{\partial \tau_2}\right]}. \tag{27}$$

The denominator of (27) is positive so the sign of τ_2 is again determined by that of the numerator. Substituting from Roy's identity and from the relevant extension of (21),[5] the numerator can be written

$$X_1 C_1^2 \frac{\partial q_1}{\partial \tau_1} - X_2 C_1^1 \frac{\partial q_2}{\partial \tau_2}. \tag{28}$$

Using the definitions of the price derivatives[6], and dividing across by the first derivatives of the cost functions, the sign of (28) is given by the sign of:

$$X_1\frac{C_{01}^1}{C_1^1}\left[\frac{\frac{\partial X_1}{\partial q_1}}{2\frac{\partial X_1}{\partial q_1} + \frac{\partial^2 X_1}{\partial q_2{}^1}\left[q_1 - C_0^1\right] - C_{00}^1\left(\frac{\partial X_1}{\partial q_1}\right)^2}\right] - X_2\frac{C_{01}^2}{C_1^2}\left[\frac{\frac{\partial X_2}{\partial q_2}}{2\frac{\partial X_2}{\partial q_2} + \frac{\partial^2 X_2}{\partial q_2{}^2}\left[q_2 - C_0^2\right] - C_{00}^2\left(\frac{\partial X_2}{\partial q_2}\right)^2}\right] \tag{29}$$

To interpret (29), note that $X_i\frac{C_{01}^i}{C_1^i}$ is a measure of returns to scale: $X_i\frac{C_{01}^i}{C_1^i} > 1$ represents decreasing returns to scale, a value of 1 constant returns and < 1 increasing returns. Now assume that the two bracketed terms are equal. It then follows that if

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5 $\frac{\partial \pi_i}{\partial \tau_i} = - C_1^i$, i = 1, 2.

6 $\frac{dq_i}{dt_i} = \frac{C_{01}^i\frac{\partial X_i}{\partial q_i}}{2\frac{\partial X_i}{\partial q_i} + \frac{\partial^2 X_i}{\partial q_i{}^2}\left[q_i - C_0^i\right] - C_{00}^i\left(\frac{\partial X_i}{\partial q_i}\right)^2} > 0$, i = 1, 2.

$$X_1\frac{C^1_{01}}{C^1_1} > X_2\frac{C^2_{01}}{C^2_1}, \tag{30}$$

then $\tau_2 > 0$. That is, the firm with lower returns to scale should have its labour input subsidised when the market conditions are identical. In contrast, if the returns to scale are the same for both firms, the bracketed terms describe the rate at which the firms would forward-shift any commodity tax. Hence the firm that would forward-shift most should have the labour input subsidised. In this case, if $C^i_{00} = 0$, then $\tau_2 > 0$ if the elasticity of the slope of the inverse demand function, Seade's "E" (Seade 1985), facing firm 1 is greater than the elasticity of that facing firm 2.

From the analysis above, it can be seen how relative taxes are determined by a composition of market demand and returns to scale properties. The role of these can be understood by viewing the aim of the taxes as the achievement of a more balanced exploitation of returns to scale but the cost of this process is the consequent change in market prices, with the rate of change of prices being dependent upon the demand elasticity facing the firm. It is the resolution of these two effects that is captured in (29).

The final example considers two oligopolistic industries and allows for differences in both the size, in terms of the number of firms, and the conduct of the industries. The firms comprising each industry are assumed to compete by choosing quantities of a homogeneous product. To introduce differences of conduct, it is assumed that each firm conjectures how aggregate output will change in response to a change in their output. As the industries are taken to be symmetric, all firms in each industry have the same conjecture. The conjecture is denoted by λ_i, $i = 1, 2$. A value of $\lambda_i = 0$ represents "Bertrand" competition and $\lambda_i = 1$ "Cournot" competition.

With n_i firms in industry i and x^j_i denoting the output of firm j in industry i, the inverse demand functions are defined as

$$q_i = q_i(X_i),\ \ X_i = \sum_{j=1}^{n_i} x_i^j,\ i = 1, 2. \tag{31}$$

From the conditions for profit maximisation, it follows that

$$\frac{dq_i}{d\tau_i} = \frac{n_i C^i_{01}\frac{\partial q_i}{\partial X_i}}{[n_i+\lambda_i]\frac{\partial q_i}{\partial X_i} + x_i n_i \lambda_i \frac{\partial^2 q_i}{\partial X_i^2} - C^i_{00}} > 0, i = 1, 2, \tag{32}$$

and

$$\frac{d\pi_i^j}{d\tau_i} = \frac{C^i_{01}\left[q_i + x_i n_i \frac{\partial q_i}{\partial X_i} - C^i_0\right]}{[n_i+\lambda_i]\frac{\partial q_i}{\partial X_i} + x_i n_i \lambda_i \frac{\partial^2 q_i}{\partial X_i^2} - C^i_{00}} - C^i_0,\ j = 1,...,n_i,\ i = 1, 2, \tag{33}$$

where x_i is the output of each firm at the symmetric equilibrium and $C^i(\cdot)$ is the cost function common to all firms in industry i.

The optimal value of τ_2 will again be characterised by (27). Assuming the denominator to be positive, substitution from (32) and (33) allows the numerator to be written as

$$n_1 C^1_1\left[\frac{n_2 C^2_{01}[q_2 - C^2_0]}{[n_2+\lambda_2]\frac{\partial q_2}{\partial X_2} + x_2 n_2 \lambda_2 \frac{\partial^2 q_2}{\partial X_2^2} - C^2_{00}}\right] - n_2 C^2_1\left[\frac{n_1 C^1_{01}[q_1 - C^1_0]}{[n_1+\lambda_1]\frac{\partial q_1}{\partial X_1} + x_1 n_1 \lambda_1 \frac{\partial^2 q_1}{\partial X_1^2} - C^1_{00}}\right] \tag{34}$$

From (34) it is possible to describe the result in a number of limiting cases. Firstly, it illustrates that with marginal cost pricing in both industries the income taxes will both be zero, which is an alternative view of proposition 2, and that if one industry practices marginal cost pricing, the labour input for the other, assuming price is above cost, should be subsidised. Secondly, provided the derivatives are bounded, as the number of firms in both industries increases the taxes will both tend to zero.

Using the first-order condition for profit maximisation of the individual firms, (34) can be usefully re-written as

$$n_1C_1^1\left[\frac{-n_2C_{01}^2x_2\lambda_2\dfrac{\partial q_2}{\partial X_2}}{\left[n_2+\lambda_2\right]\dfrac{\partial q_2}{\partial X_2}+x_2n_2\lambda_2\dfrac{\partial^2 q_2}{\partial X_2^2}-C_{00}^2}\right]-n_2C_1^2\left[\frac{-n_1C_{01}^1x_1\lambda_1\dfrac{\partial q_1}{\partial X_1}}{\left[n_1+\lambda_1\right]\dfrac{\partial q_1}{\partial X_1}+x_1n_1\lambda_1\dfrac{\partial^2 q_1}{\partial X_1^2}-C_{00}^1}\right]. \quad (35)$$

Hence the taxes will both be zero if competition is Bertrand ($\lambda_i = 0$, $i = 1,2$), in other cases, with demands and costs equal, the industry with the larger value of λ_i will be subsidised. Finally, as n_1 tends to infinity, the labour input for industry 2 should be subsidised.

This completes the consideration of examples. The fact that differential income taxes are efficient in an imperfectly competitive economy has been established and several factors have emerged as relevant to the determination of relative rates of tax. Amongst these are the returns to scale of the industries' production processes, the demand function facing the industry as captured by Seade's E, the conduct of each industry and the size of the industry in terms of the number of firms. This latter factor is representative of a notion of the competitiveness of the industry.

4. Linear taxes: a general model

A general model, based on that of Myles (1989a), is now presented of an imperfectly competitive economy and rules are derived that describe optimal industry-specific income taxes. The economy has N industries, of which K are competitive with index k = 1,...,K and N-K are imperfectly competitive with index r = K+1,...,N. Each firm produces a single output using labour as the only input into production and the wage rate is normalised as above at w. Competitive firms produce with constant returns to scale and all the firms in each industry employ the same technology. It is also assumed that all imperfectly competitive firms choose quantities[7] and that equilibria are symmetric. These assumptions are not necessary in order to perform the analysis but do considerably simplify the presentation. As the wage rate is taken as numeraire, it is suppressed as an argument of the functions that follow.

If the production of good k, k= 1,..,K, requires c^k units of labour and industry k faces a labour tax of τ_k, it follows that the equilibrium price at marginal cost is

$$q_k = c^k.[w+\tau_k]\ ,\ k = 1,\dots,K. \tag{36}$$

For imperfectly competitive firms, the outcome of the maximisation facing a firm j in industry r, which is composed of n_r firms, is a choice of output

$$x_r^{j*} = \operatorname{argmax}\left\{\pi_r^j = x_r^j q_r - C^r\left(x_r^j,\ w+\tau_r\right)\right\},\ j = 1,\dots,n_r\ ,\ r = K+1,\dots,n. \tag{37}$$

Under appropriate convexity assumptions[8] the maximising choice can be written

$$x_r^{j*} = \sigma^{rj}\left(q_1,\dots,q_{r-1},\tau_r,q_{r+1},\dots,q_N,\pi_{-jr}\right),\ j = 1,\dots,n_r\ ,\ r = K+1,\dots,n, \tag{38}$$

where

$$\pi_{-jr} = \sum_{\substack{i=K+1 \\ i\neq r}}^{N} \sum_{f=1}^{n_i} \pi_i^f + \sum_{\substack{g=1 \\ g\neq j}}^{n_r} \pi_r^g\ . \tag{39}$$

Aggregate supply of good r is the summation of the outputs from the firms in industry r

$$X_r^* = \sum_{j=1}^{n_r} \sigma^{rj} = X_r\left(q_1,\dots,q_{r-1},\tau_r\ ,q_{r+1}\ ,\dots,q_N,\pi_{-1r}\ ,\dots,\pi_{-n_r r}\right). \tag{40}$$

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[7] This is an inconsequential restriction as it was shown in Myles (1989a) how the analysis could be applied to price-setting firms and industries with free-entry. The qualitative conclusions are not affected by the specific form of imperfect competition.

[8] If there is an upper bound on labour supply and all functions are continuous, the profit/output possibility set is compact for each firm and thus a maximum exists. Strict convexity of this set implies the continuity of the maximising element with respect to its conditioning variables.

After substituting (40) into the inverse demand function for good r and into the definition of profits for firm j, the solution of the resulting simultaneous equations characterises the price and profit level as

$$q_r = f^r(q_1,\ldots,q_{r-1},\tau_r,q_{r+1},\ldots,q_N,\pi_{-r}), \tag{41}$$

and

$$\pi_r^j = g^{jr}(q_1,\ldots,q_{r-1},\tau_r,q_{r+1},\ldots,q_N,\pi_{-r}),\ j = 1,\ldots,n_r, \tag{42}$$

with

$$\pi_{-r} = \sum_{\substack{i=K+1 \\ i\neq r}}^{N} \sum_{f=1}^{n_i} \pi_i^f . \tag{43}$$

This is the argument for a single industry. Collecting the equations from each industry and using (36), the equilibrium of the model will then be the solution of the resultant equation system. Assuming a solution exists and that the implicit function theorem can be applied, the general equilibrium of the model can be written

$$q_k = c^k.[w+\tau_k],\quad k = 1,\ldots,K, \tag{44}$$

$$q_r = \Phi^r(\tau_1,\ldots,\tau_r,\ldots,\tau_N),\quad r = K+1,\ldots,N, \tag{45}$$

and

$$\pi_r^j = \Omega^{j,r}(\tau_1,\ldots,\tau_r,\ldots,\tau_N),\quad j = 1,\ldots,n_r,\ r = K+1,\ldots,N. \tag{46}$$

The terms

$$\Phi_i^r,\ \Omega_i^{j,r} \tag{47}$$

will represent the derivatives of these functions with respect to their i'th argument.

To simplify the notation in the analysis of policy, I assume that there is a single consumer whose preferences can be represented by the indirect utility function $V(q_1,...,q_n,\pi)$,[9] where

$$\pi = \sum_{i=K+1}^{N} \sum_{f=1}^{n_f} \pi_i^f . \tag{48}$$

In order to construct the government budget constraint it is necessary to derive the labour demand, L^i, from each industry i = 1,...,N. For competitive industry k,

$$\begin{aligned} L^k &= c^k X_k, \\ &= c^k X_k(q_1,...,q_N,\pi), \\ &= c^k X_k\left(c^1.[w+\tau_1],...,c^N.[w+\tau_N], \Phi_{K+1},...,\Phi_N, \sum_{i=K+1}^{N} \sum_{f=1}^{n_f} \Omega^{f,i} \right), \\ &= c^k X_k(\tau_1,...,\tau_N), \\ &= L^k(\tau_1,...,\tau_N). \end{aligned} \tag{49}$$

The labour demand from imperfectly competitive industry r can be derived as:

$$\begin{aligned} L^r &= \sum_{j=1}^{n_r} \frac{\partial C^r(x_r^j, w+\tau_r)}{\partial (w+\tau_r)}, \\ &= \sum_{j=1}^{n_r} C_1^r(x_r^j, w+\tau_r), \\ &= \sum_{j=1}^{n_r} C_1^r(\sigma^{rj}(q_1,...,\tau_r,...,q_N,\pi_{-jr}), w+\tau_r), \\ &= \sum_{j=1}^{n_r} C_1^r\left(\sigma^{rj}\left(c^1.[w+\tau_1],...,\tau_r,...,\Phi_N, \sum_{\substack{i=K+1 \\ i\neq r}}^{N} \sum_{f=1}^{n_i} \Omega^{f,i} + \sum_{\substack{g=1 \\ g\neq j}}^{n_r} \Omega^{g,r} \right), w+\tau_r \right), \end{aligned}$$

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[9] As the return to labour is equalised across firms, it is unnecessary to specify the firm from which any labour income is received or the allocation of consumers to firms.

$$= \sum_{j=1}^{n_r} C_1^r(\sigma^{rj}(\tau_1,\ldots,\tau_N),\, \tau_r),$$

$$= L^r(\tau_1,\ldots,\tau_N)\,. \tag{50}$$

Using these definitions, the income taxes are chosen to maximise

$$V(q_1,\ldots,q_N,\pi)\,, \tag{51}$$

subject to the budget constraint

$$\sum_{i=1}^{K} \tau_i L^i(\tau_1,\ldots,\tau_N) + \sum_{i=K+1}^{N} \tau_i L^i(\tau_1,\ldots,\tau_N) = R\,, \tag{52}$$

and equations (44) - (46). In (52) R is equal to wL^G, the government labour requirement valued at the normalised wage rate, as can be demonstrated by use of the labour market equilibrium condition and the consumer's and firms' budget constraints.

For a typical competitive industry k, the first-order condition for the choice of τ_k can be written in the form

$$\frac{\tau^k}{X_k}.\frac{\partial X_k}{\partial \tau^k} = \left[\frac{\alpha}{\lambda} - 1\right] + \left[\frac{1}{c^k X_k}\right].\Psi^k,\; k = 1,\ldots,K, \tag{53}$$

where

$$\Psi^k = \left[\frac{\alpha}{\lambda}\right] \sum_{r=K+1}^{N} X_r \Phi_k^r - \sum_{\substack{i=1 \\ i\neq k}}^{K} c^i \tau_i \frac{\partial X_i}{\partial \tau_k} - \sum_{r=K+1}^{N} \tau_r \sum_{f=1}^{n_r} C_{01}^r \frac{\partial \sigma^{rf}}{\partial \tau_k} - \left[\frac{\alpha}{\lambda}\right] \sum_{r=K+1}^{N} \sum_{f=1}^{n_r} \Omega_k^{f,r}\,.$$

The interpretation of (53) is that the left-hand side of the equality approximates the proportional reduction in the equilibrium quantity of good k supplied due to the labour tax on that industry, evaluated with other taxes at optimal levels, and that this should be equal to a constant plus Ψ^k. Ψ^k captures the interaction of industry k with the rest of the economy. If the model were entirely competitive then Ψ^k would be zero and the optimal tax scheme would raise revenue by reaching point at which the reduction in supply due to the tax would be equal for each industry. When Ψ^k is not zero, if the tax on industry k tends to raise the prices of the imperfectly competitive industries, its reduction in supply should be less. The same applies when it reduces profits or the supplies from other industries

For a typical imperfectly competitive industry s, the tax rule is

$$\sum_{g=1}^{n_s} \frac{\tau_s}{\sigma^{sg}} \cdot \frac{\partial \sigma^{sg}}{\partial \tau_s} = \left[\frac{\alpha}{\lambda} \cdot \frac{\Phi_s^s}{C_{10}^s} - \sum_{g=1}^{n_s} \frac{C_1^s}{\sigma^{sg} C_{10}^s}\right] + \left[\frac{1}{C_{10}^s X_s}\right] \cdot [\Psi^s], \quad s = K+1,\ldots,N, \qquad (54)$$

with

$$\Psi^s = \left[\frac{1}{X_s C_{10}^s}\right] \left[\left[\frac{\alpha}{\lambda}\right] \sum_{\substack{r=K+1 \\ r \neq s}}^{N} X_r \Phi_s^r - \sum_{i=1}^{K} c^i \tau_i \frac{\partial X_i}{\partial \tau_s} - \sum_{\substack{r=K+1 \\ r \neq s}}^{N} \tau_r \sum_{f=1}^{n_r} C_{01}^r \frac{\partial \sigma^{rf}}{\partial \tau_s} - \left[\frac{\alpha}{\lambda}\right] \sum_{r=K+1}^{N} \sum_{f=1}^{n_r} \Omega_s^{f,r} \right].$$

As for (53), the left-hand term approximates the reduction in supply. Taking the terms on the right-hand side in turn, this should be smaller when the effect of the tax on price, Φ_s^s, is great. It was noted in section 3 that $\frac{C_1^s}{\sigma^{sg} C_{10}^s}$ is a measure of returns to scale, a low value representing decreasing returns to scale. The reduction in supply should thus be inversely related to the returns to scale, this is in accord with the interpretation following (29). Ψ^s captures the interaction of industry s with the rest of the economy and its terms can be interpreted as for Ψ^k .

5. Affine tax schedules

I now wish to extend the analysis to consider a more general class of "affine" income tax schedules. As it has been demonstrated above that there is a role for linear income taxes in an imperfectly competitive economy, it may be possible that a more general class will also be justified.

The model can be given two interpretations. Firstly, it can be taken as a direct extension of the above but with allowance for affine modifications of the wage rate. Alternatively, it can be interpreted as a model where it is the total income from employment that is taxed and that the level of pre-tax income is determined by a standard working period. Following the second interpretation, consider an individual with a gross of tax wage w_i from employment in industry i. Assuming that there is a standard working period of h units in all industries, income from this employment is

$$W_i = hw_i . \tag{55}$$

The tax system facing each industry is described by two parameters a_i and b_i, such that the net of tax income from employment in industry i is defined by

$$\widehat{W}_i = [1 - b_i]W_i - a_i , \tag{56}$$

where a_i is the lump-sum tax payable at zero income and b_i is the marginal rate of income tax. The net of tax income schedule is therefore affine from the consumer's viewpoint. Assuming that the labour market is competitive, the income from employment will be equalised across firms at a value W, hence $\widehat{W}_i = W$, all i.[10] From (55) the cost to a firm in industry i of an employee for the standard working period is

$$W_i = \frac{W + a_i}{1 - b_i} , \tag{57}$$

and its appropriately defined cost function will be written

$$C^i = C^i(X_i, W_i) = C^i\left(X_i, \frac{W + a_i}{1 - b_i}\right). \tag{58}$$

---

[10] Note that a_i is measured in units of income. The normalisation rule is to select a value W and determine each a_i relative to this.

The form of cost function in (58) makes it clear that the equilibrium is only affected by the value of W_i, not its composition in terms of a_i and b_i. As any desired value of W_i can be attained by the use of just one of these instruments, it is clear that there is a redundancy involved: either a_i or b_i can be set equal to zero. Stated formally

Proposition 3. From each pair of tax instruments $\{a_i, b_i\}$, one of the tax instruments is redundant.

Proof. Follows from the above text. A formal proof is given in the appendix for an extension of the second example. Δ

The conclusion of this proposition is that there is no gain in welfare by using affine taxes rather than simple additive taxes.

6. Conclusions

The paper has considered the role of industry-specific income taxes in models of optimal taxation. In a competitive model with constant returns, it follows from the Diamond-Mirrlees Production Efficiency lemma that such taxes are not efficient. With decreasing returns, the existing literature demonstrates only that they should not be used when optimal profit taxes are levied. The conclusion that non-differentiated labour taxes are optimal was shown above to also hold without the taxation of profits, extending the existing results. A justification for industry-specific income taxes can therefore only arise outside the standard competitive model.

After introducing an element of imperfect competition into the model, it was established, via examples, that differential income taxes could be justified by efficiency considerations. In addition, the examples illustrated some determinants of the relative rates of taxation. The labour input into an industry will generally be taxed at a lower rate when it has low returns to scale and equilibrium price increases markedly in response to the tax. The second of these factors can be broken down further to distinguish between the consequences of the shape of the demand curve facing the industry (summarised by Seade's E), the conduct of the industry in terms of conjectures and its competitiveness measured, approximately, by the number of firms in the industry.

Optimal rules for the choice of income taxes were also derived. These reflected the features noted above and also the importance of the interaction of the industry with the remaining economy. A general interpretation of these rules was given in terms of an approximation of the reduction in supply from each industry. A complete commodity tax system for an imperfectly competitive economy must consist of taxes on all final goods and labour; with the results of this paper suggesting that the labour taxes should be industry specific. Labour and commodity taxes are therefore complementary in the optimal tax system and a complete tax system can be described by a juxtaposition of the present results with those of Myles (1989a).

In summary, industry specific income taxes arise as a natural feature of an optimal commodity tax system for an imperfectly competitive economy. Furthermore, the levels of such taxes are easily characterised in terms of returns to scale for each industry, its conduct and the form of demand function it faces.

Appendix

This appendix extends the second example of section 3 to allow for income effects and for cross-price effects in demand and the resulting model used to present a formal proof of proposition 3.

The demand functions facing the two monopolists are defined by

$$X_i = X_i(q_1, q_2, \pi_1+\pi_2),\ i = 1, 2\,. \tag{A1}$$

Equilibrium of the model is the simultaneous solution to:

$$X_1 + \left[q_1 - C_0^1\right].\frac{\partial X_1}{\partial q_1} = 0, \tag{A2}$$

$$\pi_1 - q_1X_1 + C^1 = 0, \tag{A3}$$

$$X_2 + \left[q_2 - C_0^2\right].\frac{\partial X_2}{\partial q_2} = 0, \tag{A4}$$

and

$$\pi_2 - q_2X_2 + C^2 = 0. \tag{A5}$$

(A2) and (A4) are the first-order conditions for profit maximisation of firms 1 and 2 respectively and (A3) and (A5) are the definitions of profit. This system of four equations should determine the endogenous variables q_1, q_2, π_1, π_2.

Optimal values of a_1, b_1, a_2 and b_2 are chosen to maximise $V(q_1, q_2, \pi_1+\pi_2)$ subject to the revenue constraint

$$\left[\frac{b_1W+a_1}{1-b_1}\right].C_1^1 + \left[\frac{b_2W+a_2}{1-b_2}\right].C_1^2 = 0\,. \tag{A6}$$

I now establish proposition 3 for this model:

Proposition 3. From each pair of tax instruments $\{a_i, b_i\}$, one of the tax instruments is redundant.

Proof. Perturbing the equilibrium of (A2) - (A5), it is easily calculated that

$$\frac{\partial q_i}{\partial b_j} = \left[\frac{W+a_j}{1-b_j}\right].\frac{\partial q_i}{\partial a_j},\ \frac{\partial \pi_i}{\partial b_j} = \left[\frac{W+a_j}{1-b_j}\right].\frac{\partial \pi_i}{\partial a_j},\ i = 1, 2, j = 1, 2. \tag{A7}$$

The necessary conditions for the choice of a_1 and b_1 are

$$\sum_{i=1}^{2}\frac{\partial V}{\partial q_i}.\frac{\partial q_i}{\partial a_1} + \sum_{i=1}^{2}\frac{\partial V}{\partial \pi}.\frac{\partial \pi_i}{\partial a_1}$$

$$+ \lambda\left[\left[\frac{b_1W+a_1}{1-b_1}\right].\left[C_{10}^1\frac{\partial X_1}{\partial a_1} + C_{11}^1\left[\frac{1}{1-b_1}\right]\right] + C_1^1\left[\frac{1}{1-b_1}\right] + C_{10}^2\frac{\partial X_2}{\partial a_1}\left[\frac{b_2W+a_2}{1-b_2}\right]\right] = 0\,, \tag{A8}$$

and

$$\sum_{i=1}^{2} \frac{\partial V}{\partial q_i} \cdot \frac{\partial q_i}{\partial b_1} + \sum_{i=1}^{2} \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi_i}{\partial b_1}$$

$$+ \lambda \left[\left[\frac{b_1 W + a_1}{1-b_1} \right] \cdot \left[C^1_{10} \frac{\partial X_1}{\partial b_1} + C^1_{11} \left[\frac{W + a_1}{[1-b_1]^2} \right] \right] + C^1_1 \left[\frac{W + a_1}{[1-b_1]^2} \right] + C^2_{10} \frac{\partial X_2}{\partial b_1} \left[\frac{b_2 W + a_2}{1-b_2} \right] \right] = 0 \; . \quad \text{(A9)}$$

Using (A7), it can be seen that (A9) is equal to (A8) multiplied by $\left[\frac{W+a_1}{1-b_1}\right]$. As this can be eliminated from the equation, (A8) and (A9) describe identical conditions from which it can be concluded that one of the instruments is redundant. Δ

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