VOLUNTARY PUBLIC GOODS

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Abstract

A voluntary public good is a non-rivalrously-consumed commodity which, nonetheless, individuals can only consume if they make some enabling expenditure. E.g., with visual broadcasting a television is required and, in much of Europe, a licence. First, we characterise the population of voluntary consumers at arbitrary levels of the licence fee. Next, we consider the optimal licence fee chosen by a utilitarian government which recognises it does not have a captive population of public good consumers. We compare this outcome with that arising when the public good is financed from general taxation via a uniform poll tax and thus everyone consumes it.

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0. Introduction

By a voluntary public good we mean a commodity which, although non-rivalrous in consumption is such that, to be able to consume it, an individual or household has to make some enabling expenditure. For example, to enjoy visual broadcasting a household has to purchase a TV set and (at least in the UK and other European countries) a licence. Non-purchasers of sets or non-payers of the licence are effectively excluded. Among others, parks, museums and public performances (each up to a congestion limit) also have this feature if they are fenced and an entrance fee is levied.[1/] One presumes that, when determining the licence, the government or licensing authority takes account of the fact that some individuals might vote against the public good by keeping their cheques in their pockets. It will not behave as if it has a captive population.

Recognition of this feature raises many interesting and important issues. First and most obvious, we need to characterise the optimum with a variable population of users of the public good. In particular, how is the usual Samuelsonian condition for optimum provision of a public good (Samuelson, 1954, Atkinson and Stiglitz, 1980) modified? Second, given freedom to choose whether to participate in using the good, who will be voluntary consumers? (Another way of viewing this is, will the licence fee be set so that all the population uses the good, or will it be set so that some, e.g. the relatively poor, are excluded?) Related to this, will the licence fee with a captive population exceed that when non-using individuals need not pay, thus there are only voluntary consumers? Again, will the overall level of provision of the good to the captive population exceed that arising when consumption of the good is voluntary? We will use a very simple model to address these and related issues.[2/]

We will consider only uniform or poll tax-like licence fees. A licence fee is often unavoidably of the uniform or poll tax variety. For example, the licensing authority might be unable to monitor a user's income, tastes, or level of consumption. However, such a poll tax is likely to be highly inegalitarian. Intuition then suggests that distributional considerations would militate against a utilitarian government charging the poor in a captive population case a licence fee as high as that levied on the relatively rich who will be the likely purchasers with voluntary consumption. Our results indicate this intuition is unlikely to be universally valid.

We structure the paper as follows. In Section I we detail the economic environment. Our population is a continuum of heterogeneous individuals differing in endowments but not tastes. There is a composite private consumption commodity and one voluntary public good. The level of provision of the latter is given by the aggregated licence fees of those choosing to consume it. (Think of the BBC's income and value of output being given by the sum of the licence fees.) An individual spends his endowment on the private good and, possibly, the licence for the voluntary public good. He will consume the latter if the utility then resulting from consuming the rest of his endowment on the private good and from the level of public good available to him (incorporating his own contribution) is at least as great as that to be derived from spending his endowment solely on the private good. The utilitarian welfare maximising government takes account of this behaviour when setting the licence fee.

Section II examines in some detail the behaviour of the marginal man (or woman). That is, the one who at any given level of licence fee is indifferent between consuming only the private good and consuming both the goods. Of particular interest are how the identity of the marginal man changes with a variation in the licence fee and how the marginal man is characterised when the licence fee is zero. These are crucial for identification of the population of consumers of the public good and for treatment of the possibility of zero provision.

Section III "solves" the government's problem of finding the optimal licence fee. We relate the solution to that of an equivalent problem with an identical population but in which all voluntarily or involuntarily pay the licence fee and consume the public good. We then conclude with an evaluation of our results and their relationship to the public goods literature.

I. The Economic Environment

The population of size H comprises a continuum of households or individuals with a given member indexed by $h \varepsilon [0,H]$. An h-man has endowment $M^h \varepsilon [\underline{M},\bar{M}]$ with $\underline{M}^h$ distributed according to the continuous distribution function $F(M)$ and density $f(M)$. Thus $f(M^h)\ H$ is the number of men with income M^h. The government knows $F(\cdot)$ but not the identity of any given household for the purposes of levying the licence fee. Everyone is assumed to have identical strictly concave utility function, $U(X,G)$, defined over consumption of the private good, denoted X, and the voluntary public good, G. Thus, an h-man's utility is $U(X^h,G^h)$ where, if he chooses to not consume the public good, $G^h = 0$. If he purchases a licence and consumes the public good, G^h equals the level available to all other licensed individuals. The unit price and average and marginal cost of the private good is p while the public good is taken to be the numeraire. Constant returns in production of both goods are assumed, thus p^{-1} gives the marginal rate of transformation (MRT) between private and public goods.

Let $T \geqq 0$ denote the licence fee set by the utilitarian social welfare-maximising government. In general, if purchase of a licence, hence consumption of the public good, is voluntary, the proportion of the population which consumes it, denoted q, is a function of T. $q(T)$ expresses this functional dependence. Then, if h consumes G, $U[X^h,G^h] = U[(M^h-T)/p,q(T)HT]$. If he doesn's consume G, $U[X^h,G^h] = U[M^h/p,0]$. Note we are assuming the only enabling expenditure an individual need make in order to be entitled to consume G is on the licence. This means that, e.g. in the case of broadcasting, the annualised capital cost of a TV set is being ignored. We consider the additional complications created by fixed costs in a theory of voluntary public goods briefly in our concluding comments and then in a subsequent paper.

Now, at a given T, an h-man will consume G if

$$U[(M^h-T)/p,q(T)HT] \geqq U[M^h/p,0] \tag{1}$$

where $q(T)H$ includes the h-man. I.e., he will consume it if the utility from spreading his endowment over the private good and the licence (enabling consumption $q(T)HT$ of the public good), exceeds that derivable from spending all on the private good. The latter can be regarded as his reservation or status quo utility. He will be indifferent between consuming and not consuming G if (1) holds with equality. (1) can be termed the "participation constraint". The behaviour of the indifferent or marginal man is crucial in our analysis.[3/]

II. The Marginal Man (Or Woman)

At a given $T \geqq 0$, a marginal man has endowment $M^*(T)$ satisfying

$$U[(M^*(T)-T)/p,q(T)HT] = U[M^*(T)/p,0] \tag{2}$$

(q and M^*'s functional dependence upon T will often be suppressed in notation.) (2) collapses to $U[M^*/p,0] = U[M^*/p,0]$ when $T = 0$, suggesting casually that everyone is marginal at $T = 0$ and, in particular, everyone would be indifferent between having the presence of the public good and otherwise. This might produce a discontinuous jump in $q(T)$ for a move to even an infinitessimal $T > 0$. We therefore need to treat this corner possibly with some care.

What we do is to argue that those who would wish to purchase the licence to the public good at $T = 0$ are those whose utility would be increased by an infinitessimal increase in the licence fee and consequently public good supply from zero. (Of course, the infinitessimal supply then available to each aggregates the fee contribution of all others like them.) I.e., those wishing to purchase are those whose marginal utility from an infinitessimal level of G consumption is no less than the marginal disutility of the infinitessimal amount of private good foregone in purchasing a licence. The marginal man is then the one for whom $T = 0$ is optimal.

In detail, we argue that at $T = 0$ an individual wishing to purchase G is one for whom, for all "small" $\Delta \geq 0$, $U[(M^h-\Delta)/p, q(\Delta)H\Delta] \geqq U[M^h/p, 0]$. If this is to be satisfied, we must have

$$\frac{d}{d\Delta} U[(M^h-\Delta)/p, q(\Delta)H\Delta] \Big|_{\Delta = 0} \geqq 0,$$

i.e.,

$$\{-U_1[(M^h-\Delta)/p, q(\Delta)H\Delta]\ p^{-1} + U_2[(M^h-\Delta)/p, q(\Delta)H\Delta](qH+H\Delta dq/d\Delta)\} \Big|_{\Delta = 0} \geqq 0$$

or

$$-U_1[M^h/p, 0]p^{-1} + U_2[M^h/p, 0]Hq(0) \geqq 0 \qquad (3)$$

It is now convenient to establish what might already seem obvious: if a man with endowment $M^*(T)$ is marginal at a given $T \geqq 0$, then all men with $M^k > M^*(T)$ will wish to purchase a licence and consume the corresponding G at that T.

<u>Lemma 1</u>. If $U_{12} \geqq 0$, $q(T)=1-F(M^*(T))$.

<u>Proof</u>. Consider $T=0$ first. Suppose a $q(0)$ satisfying (3) with equality exists. This defines the marginal man at $T=0$ with $M=M^*(0)$. Consider $M^k > M^*(0)$. By concavity, $U_1[M^*/p,0] > U_1[M^k/p,0](>0)$. Thus $(0>)-U_1[M^k/p,0]p^{-1} > -U_1[M^*/p,0]p^{-1}$. Also, $U_2[M^*/p,0]\{\gtrless\}U_2[M^k/p,0]$ as $U_{21}\{\lesseqgtr\}0$. Thus, if $U_{12} \geqq 0$, $U_2[M^k/p,0] \geqq U_2[M^*/p,0]$. Then
$-U_1[M^k/p,0]p^{-1}+U_2[M^k/p,0]q(0)H =$
$-[U_1(M^k/p,0)-U_1(M^*/p,0)]p^{-1}+q(0)H[U_2(M^k/p,0)-U_2(M^*/p,0)] > 0$. I.e., the individual with $M^k > M^*(0)$ would wish for an fnfinitessimal T and infinitessimal G. It remains to show in this case that a q satisfying (3) with equality exists. This is straightforward. For arbitrary M^*, $U_1(M^*/p,0)p^{-1}$ and $U_2(M^*/p,0)$ are fixed and finite. Thus, clearly, for any given $q > 0$, an H can be found to ensure (3) holds with equality. (However, note that the converse - that for any H a $q \geqq 0$ to ensure (3) holds with equality exists - need not be true. This is because q is restricted to [0,1].)

Next, consider $T > 0$. The marginal man has M^* satisfying $U[(M^*(T)-T)/p,q(T)HT] = U[M^*(T)/p,0]$. To establish our result we need an increase in M from M^* to increase the LHS of this equality more than it does the RHS. Now, by concavity, $U_1[(M^*-T)/p,0] > U_1[M^*/p,0]$. Thus, if $U_{12} \geqq 0$, $U_1[M^*-T)/p,q(T)HT]p^{-1} > U_1[M^*/p,0]p^{-1}$ and $U[(M^k-T)/p,q(T)HT] > U[M^k/p,0]$. By a symmetric argument we can show that all men with endowments $< M^*$ will not wish to purchase the licence to consume G . Finally, note that if all men with $M > M^*$ wish to purchase the licence and all with $M < M^*$ do not, then $q(T) = 1 - F(M^*(T))$. Q.E.D.

Perhaps the most interesting possibility raised by Lemma 1 is this. If $U_{12} < 0$, at a given T a marginal man might occur at more than one income level. That is, it might not be the relatively rich above a certain endowment level who feel able to afford G while all the relatively poor below that level are excluded from consumption. Rather, some of the relatively poor will consume G while some will not, and similarly for the relatively rich.[4/]

Does the number of purchasers of the licence increase with the fee? Equivalently, how does the identity of the marginal man change with T?[5/] An increase in T increases the provision of G, other things equal. This enhances the desirability of having the licence. This effect could, one imagines, overcome the income and substitution effects of the T increase (both of which would tend to reduce the demand for G). Hence $dM^*/dT < 0$ is a possibility intuitively. The following proposition (P) establishes some conditions under which this intuition is confirmed and when it is overturned.

<u>Proposition (P)1</u>. (i) For $T > 0$, if U exhibits relative risk aversion w.r.t. public good consumption of at least unity (i.e., if $-(1-F(M^*(T)))HT\, U_{22}(\cdot)/U_2(\cdot) \geq 1$) and $U_{21} \geq 0$, then $dM^*/dT > 0$. (ii) If F is uniform and U is log-quasilinear of the form $U(X,G) = \log X + G$, then at the optimum T, $\hat{T}$, $dM^*/dT \{\gtrless\} 0$ as $2M^* \{\lessgtr\} \bar{M}$. (iii) If $T = 0$, $dM^*/dT = 0$.

<u>Proof</u>. (i) Let $X_T^h \equiv (X^h-T)/p$, $X_N^h \equiv M^h/p$. For $T > 0$, differentiating the condition defining the marginal man ((2)),

$$p^{-1}U_1(X_T^*,.)(dM^*/dT-1)+U_2(X_T^*,.)[(1-F(M^*))H-HTf(M^*)dM^*/dT]=p^{-1}U_1(X_N,0)dM^*/dT$$

Rearranging,

$$\partial M^*/\partial T=[p^{-1}U_1(X_T^*,.)-U_2(X_T^*,.)(1-F(M^*))H][p^{-1}\{U_1(X_T^*,.)-U_1(X_N^*,.)\}-U_2(X_T^*,.)HTf(M^*)]^{-1}$$

$\equiv N_1D_1^{-1}$. To sign N_1 and D_1 consider Figure 1.

FIGURE 1 HERE

Figure 1 graphs against T the utility of a man with endowment M^* and another with $M^k > M^*$. At fixed $(1-F(M^h))$ the utility schedules are clearly concave in T and (provided $\lim_{X_T \to 0} U_1 = \infty$, or similar) have the shape shown. Wherever the M^* schedule crosses the T axis $(T > 0)$ is the T at which that man is marginal. At that T, his utility must be decreasing in T. I.e., $\partial U[(M^*-T)/p,(1-F(M^*))TH]/\partial T$ $= -p^{-1}U_1(X_T^*,.) + U_2(X_T^*,.)(1-F(M^*))H = -N_1 < 0$. Thus $N_1 > 0$.

D_1 equals the derivative w.r.t. M of the condition describing the marginal man, but holding T constant. Its sign depends on whether, at the same T at which a slightly poorer man was marginal, someone would wish to purchase a licence at the slightly lower level of provision arising from the displacement of the slightly poorer man. Evidently, from the figure, the key is whether or not an increase in M induces a parallel shift in the utility schedule, a shift increasing in T or one decreasing in T . Thus we require

$\text{sgn}(\partial^2 U/\partial M\partial T)$. Now,

$$\partial^2 U/\partial M\partial T = -p^{-2}U_{11}(X_T,.) + p^{-1}U_{12}(X_T,.)HTf(M)$$

$$+ (1-F(M))H[p^{-1}U_{12}(X_T,.) - U_{22}(X_T,.)HTf(M)] - f(M)HU_2(X_T,.)$$

The presence of the last term makes this ambiguously-signed in general. However, combining the last two terms, these equal $f(M)H[-U_{22}(1-F(M))HT - U_2]\{\gtrless\}0$ as $-\frac{U_{22}}{U_2}[(M-T)/p,(1-F(M))HT](1-F(M))HT\{\gtrless\}1$, Hence, $U_{12} \geqq 0$ and $-\frac{U_{22}}{U_2}[\cdot,G]G \geqq 1$ are sufficient for $\partial^2 U/\partial M\partial T > 0$, hence for the two schedules in Figure 1 to diverge as T increases, hence for $D_1 > 0$. (ii) See Appendix A. (iii) At T = 0, differentiating the condition defining the marginal man ((3) with equality),
$-p^{-2}U_{11}[M^*(0))/p,0]dM^*/dT + p^{-1}U_{21}[M^*(0)/p,0][1-F(M^*(0))]HdM^*/dT$
$-Hf(M^*(0))U_2[M^*(0)/p,0]dM^*/dT = 0$. The only solutions to this equation are either $dM^*/dT = 0$ or dM^*/dT undefined (if $-p^{-2}U_{11}(\cdot) + p^{-1}U_{21}(\cdot)$ $-Hf(M^*(0))U_2(\cdot) = 0$). But the marginal man is optimising at T = 0. Thus, a first-order change in T from T = 0 will have no effect on this man's behaviour or welfare and, by continuity, only a similar order effect on the men adjacent. Hence it leaves unchanged the population wishing to purchase G at T = 0 . Q.E.D.

Intuition for the role of the elasticity of marginal utility w.r.t. public good consumption ($-GU_{22}/U_2$) in P1(i) is clear. If this marginal utility falls sufficiently fast, the incentive for the initial marginal man to continue purchasing G disappears, given its increased "cost", even if total supply increases with the increase in T . P1 (ii)'s sharpness naturally owes much to the strong assumptions made for it. Put simply, it states that the impact of a T increase at the optimum depends on whether the marginal man and his neighbours are able to bear it. Thus, e.g., if the marginal man is of relatively high income (measured against $\bar{M}$, not $\underline{M}$), hence there are relatively few voluntary purchasers of G ($\bar{M} - 2M^* < 0$), an increase in its fee will increase the number of purchasers ($dM^*/dT < 0$).[6/] The converse holds if the marginal man is relatively poor.

We are now placed to examine the optimal T.

III. The Optimal Licence Fee

We assume the utilitarian government chooses T to maximise social welfare (W). That is, it solves

$$\underset{T}{\text{Max.}}\ W = \int_{M^*}^{\bar{M}} U[(M^h-T)/p,(1-F(M^*))HT]Hf(M^h)dM + \int_0^{M^*} U[M^h/p,0]Hf(M^h)dM \tag{4}$$

where M^* satisfies (2), for $T > 0$, and (3) with equality for $T = 0$. The first-order necessary condition (FONC) is (in notation used before)

$$H^{-1}\partial W/\partial T = -p^{-1}\int_{M^*}^{\bar{M}} U_1(X_T^h,.)f(M^h)dM + [(1-F(M^*))H - HTf(M^*)\frac{dM^*}{dT}]\int_{M^*}^{\bar{M}} U_2(X_T^h,.)f(M^h)dM^h$$

$$-U[(M^*-T)/p,(1-F(M^*))HT]f(M^*)\frac{dM^*}{dT} + U[M^*/p,0]f(M^*)\frac{dM^*}{dT} \leqq 0 \tag{5}$$

But, because of the definition of the marginal man, the last two terms of (5) cancel and it collapses to

$$H^{-1}\partial W/\partial T = -p^{-1} \int_{M^*}^{\bar{M}} U_1(X_T^h,.)f(M^h)dM + [(1-F(M^*))H - HTf(M^*)\frac{dM^*}{dT}] \int_{M^*}^{\bar{M}} U_2(X_T^h,.)f(M^h)dM$$

$$\leqq 0 \qquad (6)$$

In general, W can lose concavity in T because of the dependence of M* on T and the complex way this enters the maximand. We will therefore assume the second order conditions are satisfied at T = 0 or any T satisfying (6) with equality. Our next proposition (P2) then rules out a corner solution:

Proposition 2. Let $\hat{T}$ denote the optimal licence fee. $\hat{T} > 0$ unambiguously if $U_{12} \geqq 0$.

Proof. Suppose $\hat{T} = 0$. Then, from P1(iii), $dM^*(0)/dT = 0$. Substituting this into (6) yields

$$- p^{-1} \int_{M^*}^{\bar{M}} U_1[M^h/p,0]f(M^h)dM + (1-F(M^*))H \int_{M^*}^{\bar{M}} U_2[M^h/p,0]f(M^h)dM \leqq 0 \qquad (7)$$

But from (3) and Lemma 1 we know that under P2's conditions $-p^{-1}U_1[M^h/p,0] + U_2[M^h/p,0][1-F(M^*(0))]H \geqq 0$, $M^h \geqq M^*(0)$, with strict inequality for $M^h > M^*(0)$. Thus

$$- p^{-1} \int_{M^*}^{\bar{M}} U_1[M^h/p,0]f(M^h)dM + (1-F(M^*))H \int_{M^*}^{\bar{M}} U_2[M^h/p,0]f(M^h)dM > 0 \qquad (8)$$

(8) contradicts (7), thus social welfare is strictly increasing in T at $T = 0$, hence $\hat{T} > 0$.

P2 is not surprising. There is an apparent bias towards $\hat{T} > 0$ in our model where we have neglected any fixed costs of supplying or consuming the public goods (e.g., the cost of TV sets and transmitters). Thus, in principle, if even one man (the richest) wishes to consume some G at a positive T, however, small, it will be supplied at that T. This also highlights a hidden condition for P2 : it is only valid if at least one man desires the public good at a positive T .[7/]

P2 means we can concentrate on cases where the licence fee, hence G are positive. These satisfy (6) with equality. This immediately indicates that $d[(1 - F(M^*(\hat{T})))H\hat{T}]/dT = (1-F(M^*(\hat{T})))H-H\hat{T}f(M^*(\hat{T}))dM^*(\hat{T})/dT > 0$, as already used in P1(ii). I.e., <u>a first-order increase in T from $\hat{T}$, while having no impact on social welfare (because it is occurring at the welfare optimum), will produce a first-order increase in licence fee revenue, hence in the provision of G</u> . Clearly, this is true even if the fee increase resulted in a decline in the population of purchasers of G. Hence, another way of understanding this result is: the fee elasticity of the population of voluntary consumers of the public good is less than unity at the optimum - that is, $1 > - \dfrac{\hat{T}d[(1-F(M^*(\hat{T})))H]/dT}{[1-F(M^*(\hat{T})]H}$.

Comparisons with the Captive Population Case.

To effect this comparison, initially we need to rearrange the equality form of (6) to an expression akin to the variants of $\sum_h MRS^h = MRT$ in the conventional public goods literature. Thus, let $X^h_{\hat{T}} \equiv (M^h - \hat{T})/p$, $\hat{G} \equiv [1-F(M^*(\hat{T}))]H\hat{T}$, $\hat{MRS}^h \equiv U_2[X^h_{\hat{T}}, \hat{G}]/U_1[X^h_{\hat{T}}, \hat{G}]$, $\hat{\beta}^h \equiv U_1[X^h_{\hat{T}}, \hat{G}]$, $\hat{\bar{\beta}} \equiv [(1-F(M^*)H]^{-1} \int_{M^*}^{\bar{M}} U_1[X^h_{\hat{T}}, \hat{G}]f(M^h)dM$. Then, some routine manipulations transform (6) to

$$p^{-1} = \frac{(1-F(M^*))H}{[(1-F(M^*))H - H\hat{T}f(M^*)dM^*/dT]} \int_{M^*}^{\bar{M}} \hat{MRS}^h \frac{\hat{\beta}^h}{\hat{\bar{\beta}}} f(M^h)dM \qquad (9)$$

The corresponding modified Samuelsonian condition for the case with a captive population of consumers of G is (cf. Atkinson and Stiglitz, 495-496):[8/]

$$p^{-1} = \int_{\underline{M}}^{\bar{M}} \frac{\beta^h}{\bar{\beta}} MRS^h f(M^h)dM \qquad (10)$$

where, now, $\beta^h \equiv U_1[X^h_{\bar{T}}, \bar{G}]$, $\bar{\beta} \equiv H^{-1} \int_{\underline{M}}^{\bar{M}} U_1[X^h_{\bar{T}}, \bar{G}]f(M^h)dM$, $X^h_{\bar{T}} \equiv (M^h - \bar{T})/p$, $\bar{G} \equiv H\bar{T}$, $MRS^h \equiv U_2[X^h_{\bar{T}}, \bar{G}]/U_1[X^h_{\bar{T}}, \bar{G}]$ and $\bar{T} \equiv$ the optimal licence fee in the captive population case.

Comparing (10) with the outer equality in (9) we see that

$$\int_{\underline{M}}^{\bar{M}} \frac{\beta^h}{\bar{\beta}} MRS^h f(M^h)dM = \frac{(1-F(M^*))H}{(1-F(M^*))H - H\hat{T}f(M^*)dM^*/dT} \int_{M^*}^{\bar{M}} \frac{\hat{\beta}^h}{\hat{\bar{\beta}}} \hat{MRS}^h f(M^h)dM \qquad (11)$$

Remembering that $[(1-F(M^*))H-H\hat{T}f(M^*)dM^*/dT] > 0$, (11) implies

$$p^{-1} = \int_{\underline{M}}^{\bar{M}} \frac{\beta^h}{\bar{\beta}} MRS^h f(M^h)dM \ \{\gtrless\} \int_{M^*}^{\bar{M}} \frac{\hat{\beta}^h}{\hat{\bar{\beta}}} \widehat{MRS}^h f(M^h)dM \quad \text{as} \quad \frac{dM^*(\hat{T})}{dT} \ \{\lesseqgtr\} \ 0 \qquad (12)$$

Thus, at the optimum, the variable population weighted MRS is taken beyond the MRT between private and public goods, if $dM^*(\hat{T})/dT > 0$, and not as far if $dM^*(\hat{T})/dT < 0$. The presence of the term in dM^*/dT shows the way in which recognition of the possibility that some individuals will choose to not purchase G in response to variations in the licence fee drives a wedge between the MRT and the weighted MRS for the population of eventual consumers.

Of course, little information about the relative magnitudes of $\hat{T}$ and $\bar{T}$ can be gleaned from contrasting (9) and (10) directly. Comparisons across behavioural regimes are notoriously difficult. This is particularly true when the outcomes have first-order conditions as complex as (9) and (10) (or (6) and its counterpart). Nonetheless, we thought initially that a result of the form $\hat{T} > \bar{T}$ would be available, arguing as follows.

As modelled, individuals' willingness to pay for the public good increases with their endowments. With the relatively poor excluded from the population, it then seemed reasonable to believe that it would be optimal for the relatively affluent voluntary consumers to pay a higher T than is optimal in the captive population case, especially as this is necessary to ensure they obtain at least as much public good as in the latter case and G is a "luxury". However, this intuition again does not seem universally valid.

In what follows, we refer to the economy with voluntary consumption of G as the voluntary or variable population case. To compare outcomes with the captive and variable populations we use the following procedure. First, we compare the outcomes with the optimal variable population and with a fixed population of identical size and income distribution. (This establishes an inequality between $\hat{T}$ and the $\bar{T}$ for an identical fixed population.) We then consider what happens to $\bar{T}$ as this fixed population is increased to the original captive population by progressively adding back the poorer individuals excluded from the optimal variable population of G consumers.

The optimal population of voluntary consumers is $(1-F(M^*(\hat{T})))H$ satisfying

$$-p^{-1} \int_{M^*}^{\bar{M}} U_1(X^h_{\hat{T}},\hat{G})f(M^h)dM + [(1-F(M^*))H-H\hat{T}f(M^*)dM^*/dT] \int_{M^*}^{\bar{M}} U_2(X^h_{\hat{T}},\hat{G})f(M^h)dM = 0$$

$$\iff -p^{-1} \int_{M^*}^{\bar{M}} U_1(X^h_{\hat{T}},\hat{G})f(M^h)dM + [(1-F(M^*))H] \int_{M^*}^{\bar{M}} U_2(X^h_{\hat{T}},\hat{G})f(M^h)dM$$

$$- H\hat{T}f(M^*)\ dM^*/dT \int_{M^*}^{\bar{M}} U_2(X^h_{\hat{T}},\hat{G})f(M^h)dM = 0 \qquad (13)$$

(and, of course, the condition defining the marginal man).

The analogous FONC for an identical captive population is

$$-p^{-1}\int_{M^*}^{\bar{M}} U_1(X^h_{\bar{T}},\bar{G})f(M^h)dM + [(1-F(M^*))H]\int_{M^*}^{\bar{M}} U_2(X^h_{\hat{T}},\bar{G})f(M^h)dM = 0 \qquad (14)$$

where $\bar{G} \equiv (1-F(M^*(\hat{T}))H\bar{T}$.

With a captive population, now $(1-F(M^*(\hat{T})))H$, the choice of T to maximise the relevant social welfare (denoted $W^{captive}$), which yields (14), is unambiguously a concave programme. Inspection of (13) and (14) reveals that at identical T, in particular $T = \hat{T}$,

$$\left.\frac{\partial W^{captive}}{\partial T}\right|_{\hat{T}} \{\gtrless\} \left.\frac{\partial W^{variable}}{\partial T}\right|_{\hat{T}} (= 0) \quad \text{as} \quad \frac{dM^*(\hat{T})}{dT}\{\gtrless\} 0 \qquad (15)$$

Thus, given the concavity of $W^{captive}$, we have the following result:

<u>Proposition 3</u>. $\bar{T}\ \{\gtrless\}\ \hat{T}$ as $dM^*(\hat{T})/dT\ \{\gtrless\}\ 0$.

We have already established conditions for signing $dM^*(\hat{T})/dT$ in P1. The case $dM^*(\hat{T})/dT = 0$ would only occur where $M^*(\hat{T}) = \underline{M}$ - i.e., in the uninteresting case where the optimal T was such that all the initial population H would be voluntary consumers of the public good. Note, also, that as the aggregate welfare of those comprising the voluntary population $(1-F(M^*(\hat{T})))H$ of G consumers and those in the identical captive population coincide at a given T, welfare of the latter group must be increased by varying T from $\hat{T}$ to $\bar{T}$. This is simply a matter of revealed preference. Of course, whether $\bar{T} < \hat{T}$ or $\bar{T} > \hat{T}$, this increase in aggregate welfare is achieved while reducing the welfare of the relatively poor "for the greater good" as compared with the voluntary population case. E.g., when $\bar{T} < \hat{T}$ $(dM^*(\hat{T})/dT < 0)$, the reduction in T actually decreases the number of voluntary consumers, just as when $\bar{T} > \hat{T}$. Both cases illustrate how "choice" can reduce aggregate welfare: it is precisely the extra "choice" constraint that consumers of G must be voluntary which results in $W^{captive} > W^{variable}$ at identical populations.[9/]

We will now increase the captive population from $(1-F(M^*(T)))H$ to H by adding back the relatively poor non-consumers of the public good in the voluntary consumption case. This is equivalent to decreasing $M^*(\hat{T})$ in (14) to $\underline{M}$, treating M^* as parametric. Let $\bar{T}(M^*) \equiv$ the optimal T as a function of M^* in this process. Then, as M^* varies, as (14) must remain satisfied, after differentiating and rearranging, we have (suppressing inessential functional arguments),

$$[p^{-1}U_1((M^*-\bar{T})/p,(1-F(M^*))H\bar{T})-(1-F(M^*))HU_2((M^*-\bar{T})/p,(1-F(M^*))H\bar{T})]f(M^*)$$

$$-\ p^{-2}\int_{M^*}^{\bar{M}}\{U_{11}-pU_{12}f(M^*)H\bar{T}+p^2f(M^*)HU_2\ -p(1-F(M^*))HU_{21}+p^2U_{22}f(M^*)H\bar{T}\}f(M^h)dM$$

$$=[-p^{-2}\int_{M^*}^{\bar{M}}\{U_{11}-2pU_{12}(1-F(M^*))H-p^2U_{22}(1-F(M^*))H\}f(M^h)dM]\frac{d\bar{T}}{dM^*} \qquad (16)$$

The coefficient of $d\bar{T}/dM^*$ is positive, by the second-order conditions for the concave fixed population optimization problems. Thus $\mathrm{sgn}(d\bar{T}/dM^*)$ is that of the LHS (16). The first term of this gives the opposite of the effect of a T increase on the welfare of the poorest man in the population. This term, as already argued, is certainly positive. Sufficient conditions for the remaining term to be positive are that $U_{12} \geqq 0$ and $-f(M^*)H\{U_2[(M^h-\bar{T})/p,(1-F(M^*))H\bar{T}]$ $+\ \bar{T}U_{22}[(M^h-\bar{T})/p,\ (1-F(M^*))H\bar{T}]\} \geqq 0$ for all men. The last condition, which requires $-(1-F(M^*))H\bar{T}\ \frac{U_{22}}{U_2}[(\ \frac{M^h-\bar{T}}{p}),(1-F(M^*))H\bar{T}] \geqq (1-F(M^*)H$ is actually rather stringent, especially if H is a large and if Arrow's hypothesis on relative risk aversion being approximately unity is valid. Thus $d\bar{T}/d\bar{M}^*$ is ambiguous in general.

In the special case when $U[\cdot] = \log X + G$ and F is uniform, things are slightly more transparent. Then (14) is

$$(\frac{1}{\bar{M}-\underline{M}})\int_{M^*}^{\bar{M}}\{-p^{-1}(M^h-\bar{T})^{-1}\ p+(\frac{\bar{M}-M^*}{\bar{M}-\underline{M}})H\}dM=0$$

or

$$- (\bar{M}-\underline{M})^{-1}\log[(\bar{M}-\bar{T})/(M^*-\bar{T})] + [(\bar{M}-M^*)/(\bar{M}-\underline{M})]H = 0 \qquad (17)$$

Now,

$$d\bar{T}/dM^*[(\bar{M}-\bar{T})^{-1} - (M^*-\bar{T})^{-1}] = 2(\bar{M}-M^*)H(\bar{M}-\underline{M})^{-1} - (M^*-\bar{T})^{-1} \qquad (18)$$

Thus, as $[(\bar{M}-\bar{T})^{-1} - (M^*-\bar{T})^{-1}] < 0$,

$$d\bar{T}/dM^* \{\gtrless\} 0 \text{ as } 2(\bar{M}-M^*) H(\bar{M}-\underline{M})^{-1} - (M^*-\bar{T})^{-1}\{\lessgtr\} 0 \qquad (19)$$

(19) neatly captures the two opposing forces which determine whether increasing the population by adding progressively poorer individuals results in the licence fee increasing or decreasing. These opposing forces are the incentive to exploit scale economies in the consumption of the public good (akin to the usual local public good non-convexity effect) and the concern for egalitarianism. The former is reflected in the size of the population, $(\bar{M} - M^*)H(\bar{M} - \underline{M})^{-1}$. The larger this is, the greater is the marginal productivity of any given fee, hence the greater the incentive to increase T . The latter is reflected in $(M^*-\bar{T})^{-1}$, which captures the significance of the licence fee in the poorest man's expenditure. If the force of egalitarianism is dominant, then $d\bar{T}/dM^* > 0$ - i.e., as we are actually <u>decreasing</u> M^*, adding the poorer individuals back into the population reduces the optimal licence fee. If, conversely, scale economy considerations are dominant, then $d\bar{T}/dM^* < 0$.

In sum, taking our arguments above in conjunction with P3 we can conclude, if $\bar{T}(\underline{M})$ is the licence fee with the captive population of size H and $U[\cdot] = \log X + G$,

<u>Proposition 4</u>. If $dM^*(\hat{T})/dT > 0$, then $2(\bar{M}-M^*)H(\bar{M}-\underline{M})^{-1} - (M^*-\bar{T})^{-1} \geq 0$, all $M^* \varepsilon[\underline{M}, M^*(\hat{T})]$, is sufficient for $\bar{T}(\underline{M}) > \hat{T}$. If $dM^*(\hat{T})/dT < 0$, then $2(\bar{M}-M^*)H(\bar{M}-\underline{M})^{-1} - (M^*-\bar{T})^{-1} \leq 0$, all $M^* \varepsilon[\underline{M}, M^*(\hat{T})]$, is sufficient for $\bar{T}(\underline{M}) < \hat{T}$.

Obviously, if $\bar{T} \geq \hat{T}$, the level of provision in the captive population case will exceed that with voluntary consumption of G. Otherwise, the levels of provision cannot be compared.

It remains to compare the optimal levels of aggregate welfare in the variable population and in the captive population cases. Here the basic difficulty is that neither of the two sets of distributions of welfare available to the population under the alternative financing schemes encompasses the other. It is readily apparent that the distribution of welfare will be, in general, much more unequal with involuntary consumption of G than with only voluntary consumption. In the latter case the participation constraints identifying the voluntary consumers of G impose a floor on the individuals' welfares. All those choosing to not pay the licence fee to consume the public good receive their reservation utilities $U[M^h/p, 0]$. In the captive population, some individuals (the "involuntary" consumers of G) can have their utilities reduced below reservation levels. Simultaneously, note that $U[M^h-T)/p, HT] \geq U[M^h-T)/p, (1-F(M^*(T)))HT]$ with equality <u>iff</u> $M^*(T) = \underline{M}$. Thus, at any given T, the utility of voluntary consumers of G would be increased by having the level of provision that would occur in the fixed population at that T, unless everyone were voluntary consumers. Hence, at given T, the welfare of someone actually benefitting from having the public good in the captive population case exceeds his welfare in the voluntary consumption case.

It can be shown also (see Appendix B) that the graphs of social welfare plotted against the licence fee for the fixed and variable population of consumers of G intersect. The variable population's aggregate welfare is initially below the fixed population's welfare but exceeds it for sufficiently high T. Thus we cannot say, a priori, whether the population with voluntary consumption of G has greater welfare at the optimal $\hat{T}$ than that with captive consumption of G at its optimum $\bar{T}(\underline{M})$.

IV. Conclusions

Previous analyses of voluntariness w.r.t. public goods have focused on voluntariness in the financing of public goods (including philanthropy - see Sugden), the mechanisms for mitigating the free rider problem and the "theory of clubs". (See, e.g., Bagnoli and Lipman, Bergstrom, Blume and Varian, Cornes and Sandler, and the references therein.) In this paper we have sought to link voluntariness in financing with voluntariness in consumption in situations where, unlike in clubs, congestion is not of the essence. This has been done by considering the very important cases where individuals can only legitimately consume a non-rivalrous good if they pay a licence fee or poll tax which is used to finance provision of the good. These are cases where the free rider problem is unimportant. The area is of obvious policy relevance, at least in the U.K., Here, there is a trend towards financing many activities with public good features (e.g., the Arts, museums, broadcasting) by specific user or subscription charges rather than by levies from general taxation. We have compared the optimal licence fee and the level of provision which emerges from this scenario with those arising when the good is financed by a tax imposed on all the population. In this latter situation some of the population might well have preferred to have not had the tax and the associated public good. To that extent, they are involuntary consumers. Of course, having involuntarily paid the tax, they can do no better than to consume the associated good.

We have seen that two opposing forces, in particular, are at work in determining the relative levels of the licence fee and of overall provision in the captive and voluntary situations. These can be termed scale economy effects and the force of egalitarianism. The more dominant is the former, the more likely are the captive population's licence fee and public good provision to exceed those in the case where consumption of the public good is voluntary.

Our model was deliberately kept simple to focus on the crucial feature, common to all voluntary public goods, that if you do not pay, you do not consume.[10/] Other real world complications can be introduced into our model. Of special importance, we feel, is the possibility of non-convexities introduced by the fact that the enabling expenditure required to consume the public good might include a fixed cost. This could correspond to, say, the annualised expenditure on a television set, separate from the licence fee.

Then, even if the provision of broadcasting is financed from general taxation, the issue of voluntariness remains because only those households choosing to purchase or share in the purchase of a set will be able to consume. This case will be explored elsewhere. For the time being, should anyone be bothered by the absence in this paper of fixed costs of the form indicated, they can just imagine that the model is being applied in a situation where the government is comparing the merits of the alternative financing schemes with all relevant fixed capital inherited and already installed.

Other extensions in the direction of greater realism would include embedding our analysis in a general equilibrium framework with non-constant returns and the consideration of the private provision of voluntary public goods (e.g., private sector broadcasting). These extensions will also be pursued elsewhere.

Appendix

A. Proof of Proposition 1(ii)

When $F(\cdot)$ is uniform, $f(M) = [\bar{M}-\underline{M}]^{-1}$. If $U[X,G] = \log X + G$, N_1 in the definition of dM^*/dT in the text satisfies

$-N_1 = -(M^*-T)^{-1} + (\bar{M}-M^*)H(\bar{M}-\underline{M})^{-1}$ (< 0 by the proof of Proposition 1(i)).

Now, we know from (13) in the text that $(1-F(M^*))H-H\hat{T}f(M^*)dM^*/dT$

$$= \frac{(1-F(M^*))H[p^{-1}\{U_1(X^*_{\hat{T}},.)-U_1(X^*_N,.)\}]-H\hat{T}f(M^*)U_1(X^*_{\hat{T}},.)p^{-1}}{[U_1(X^*_{\hat{T}},.)-U_1(X^*_N,.)]p^{-1}-U_2(X^*_{\hat{T}},.)\ H\hat{T}f(M^*)} \equiv \frac{N_2}{D_2} > 0$$

Thus N_2 and D_2 must have the same sign. In this case,

$$N_2/D_2 = N_2/D_1 = \left(\frac{\bar{M}-M^*}{\bar{M}-\underline{M}}\right)H - \left(\frac{H\hat{T}}{\bar{M}-\underline{M}}\right)\left[\frac{M^*(\bar{M}-\underline{M}) - M^*(\bar{M}-M^*)(M^*-\hat{T})H}{\hat{T}(\bar{M}-\underline{M}) - HM^*(M^*-\hat{T})}\right]$$

$$= \left(\frac{H}{\bar{M}-\underline{M}}\right)\left[\frac{(\bar{M}-M^*)\{(\bar{M}-\underline{M})-HM^*(M^*-\hat{T})\}-\{M^*(\bar{M}-\underline{M})-M^*(\bar{M}-M^*)(M^*-\hat{T})H}{(\bar{M}-\underline{M})-HM^*(M^*-T)}\right]$$

Now, $(\bar{M}-M^*)[(\bar{M}-\underline{M})-HM^*(M^*-\hat{T})]-[M^*(\bar{M}-\underline{M})-M^*(\bar{M}-M^*)(M^*-\hat{T})H]$

$= (\bar{M}-M^*)(\bar{M}-\underline{M})-M^*(\bar{M}-\underline{M}) = (\bar{M}-\underline{M})(\bar{M}-2M^*)$. Thus

$N_2/D_2 = H(\bar{M}-\underline{M})^{-1}(\bar{M}-\underline{M})(\bar{M}-2M^*)[\hat{T}(\bar{M}-\underline{M})-HM^*(M^*-\hat{T})]^{-1}$

$= H(\bar{M}-2M^*)[(\bar{M}-\underline{M}) - HM^*(M^*-\hat{T})]^{-1} > 0$. Hence, $D_1 = D_2 \{\gtrless\} 0 \iff N_2\{\gtrless\} 0$, i.e. $(\bar{M}-\underline{M}) - HM^*(M^*-\hat{T}) \{\gtrless\} 0 \iff (\bar{M} - 2M^*) \{\gtrless\} 0$. Combining this with $N_1 > 0$ yields the results.

B. The Graphs of Welfare in the Captive and Voluntarily Consuming Populations

As the captive population's welfare is concave in T, it must coincide with the aggregate status quo utility $\int_{\underline{M}}^{\bar{M}} HU[M^h/p,0]f(M^h)dM$ at T = 0 and increase from that level if the optimal licence fee in this case, $\bar{T}(\underline{M}) > 0$. Suppose the poorest man is marginal at some $T^1 > 0$. At T^1, as everyone is a voluntary consumer of G, the level of provision in the voluntary and involuntary consuming cases will coincide at HT^1, as will the aggregate welfares at $\int_{\underline{M}}^{\bar{M}} HU[(M^h-T^1)/p,HT^1]f(M^h)dM$. For an infinitessimal increase in T from T^1, the poorest man ceases to be a voluntary consumer of G assuming $dM^*/dT \geq 0$. In the voluntary case he becomes a non-contributor to G's financing, instead taking status quo utility. In the fixed population case, he has to continue to contribute, although his utility falls below status quo level. Does the social gain from having a floor to this man's utility outweigh the loss to the rest of the population from not having his contribution to financing G ?

For an infinitessimal increase in T from T^1, we need to compare $\int_{\underline{M}}^{\bar{M}} \{-p^{-1}U_1[(M^h-T^1)/p,HT^1] + HU_2[(M^h-T^1)/p,HT^1]\}f(M^h)dM$ and $\int_{\underline{M}}^{\bar{M}} \{-p^{-1}U_1[(M^h-T^1)/p,HT^1] + H[1-f(\underline{M})dM^*/dT]U_2[(M^h-T^1)/p,HT^1]\}f(M^h)dM$. These are, respectively, the captive and voluntary population's welfare derivatives w.r.t. T at T^1. Here $dM^*/dT > 0$ definitionally. Thus the former derivative must exceed the latter. Hence, the configurations of the graphs at T^1 must be as shown in Figure A.1.

FIGURE A.1 HERE

Next, consider the T such that the richest man is just marginal in the fixed population case. If such a T exists, call it T^2. Then $U[(\bar{M}-T^2)/p, HT^2] = U[\bar{M}/p, 0]$. In the voluntary consumption case, as provision of G at T^2 must satisfy $G \leqq HT^2$, all individuals would have utility less than or equal to their status quo levels $U[M^h/p, 0]$ were they to consume G at T^2. Thus they would all be non-consumers or just indifferent. Thus the voluntary population would have aggregate welfare at the status quo level. The captive population's aggregate utility would be unambiguously lower. Hence its graph must lie below the voluntary population's one at this T. As aggregate welfare is continuous in T in both cases, there must exist a T at which they coincide. Thus their graphs must intersect. However, any of the configurations shown, among others, might be feasible.

Foonotes

1/ How the exclusion is achieved need not concern us greatly. However, in the case of television we can appeal to the presence of self-financing schemes such as TV detector vans and fines for licence evaders. The important thing is that we should be able to take the "free-rider problem" as insignificant. Thus we are not considering streetlights or defence, say.

2/ The "theory of clubs" (see, e.g., Cornes and Sandler) addresses some of these issues in the context of shared facilities which experience congestion depending on usage - e.g. telephone systems (Artle and Averous) and swimming pools. It does not seem unfair to say that the theory developed so far is largely vacuous when applied to situations without congestion and, as we argue elsewhere (see Fraser and Hollander), is as yet without an entirely satisfactory treatment of club membership. (See also Cornes and Sandler, pp.182-3, on this last point.)

3/ The possibility of the marginal man (or men) being atomic in probability mass is assumed away.

4/ In technical terms, the set of purchasers of the licence need not form a connected interval.

5/ This is of particular interest to those concerned, e.g., at the possibility that the trend towards user charges for museums and other public facilities will both reduce use of them and, more serious perhaps, exclude the poor - maybe those most in need of the benefits from them.

6/ Of course, $\underline{M}$ plays a role because we see (Appendix A) that it helps determine the sign of $D_1 = D_2$ there.

7/ That "one man", of mass $f(\bar{M})H$, might be quite large.

8/ The captive population case obviously corresponds to that where, say, broadcasting is financed from general taxation, the latter here taken to be lump sum.

9/ Note that, alas, this argument does not mean that the resulting welfares for the full populations of H satisfy this inequality. This will be discussed further below.

10/ At least, not without having to look constantly over your shoulder!

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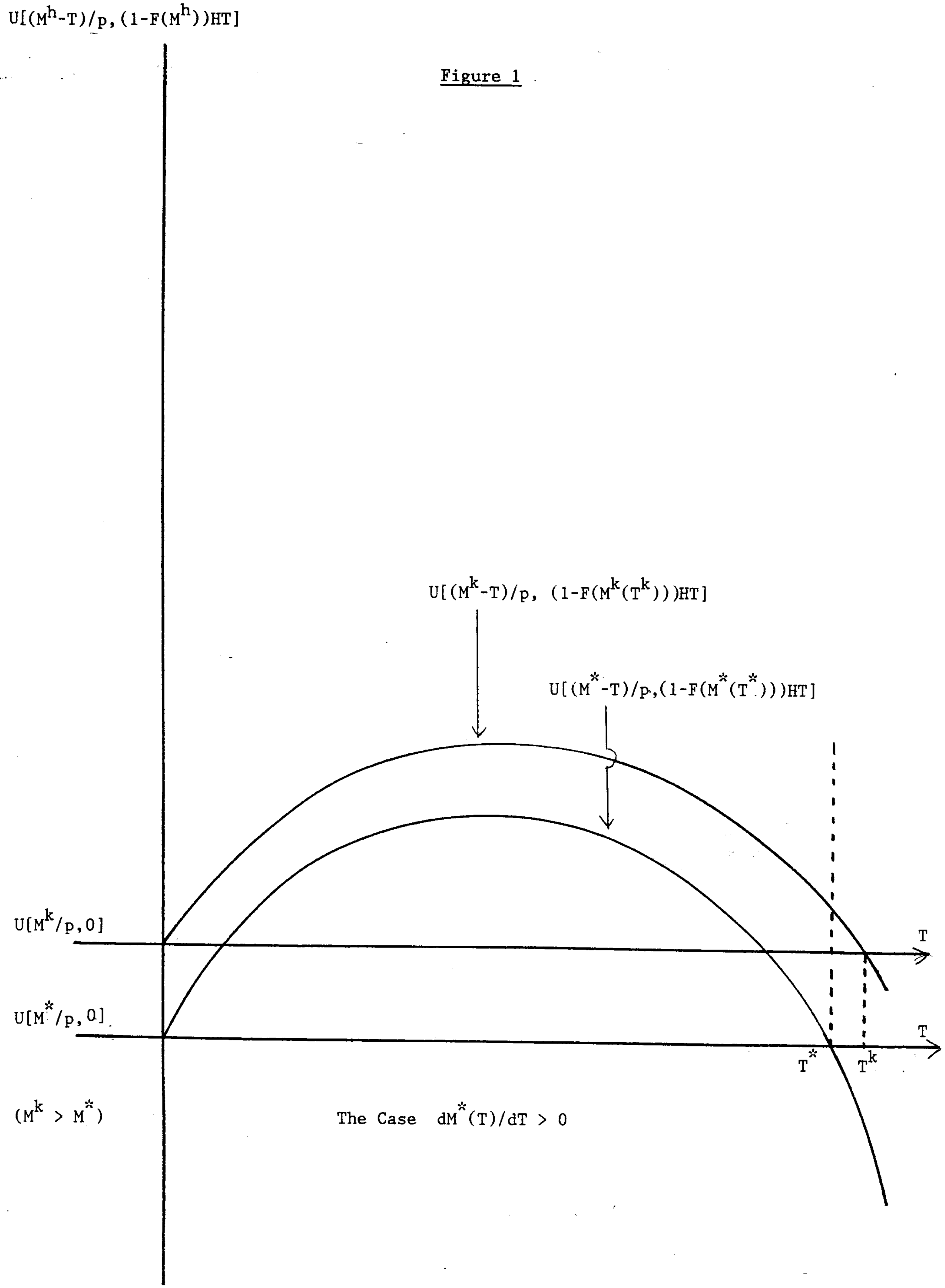
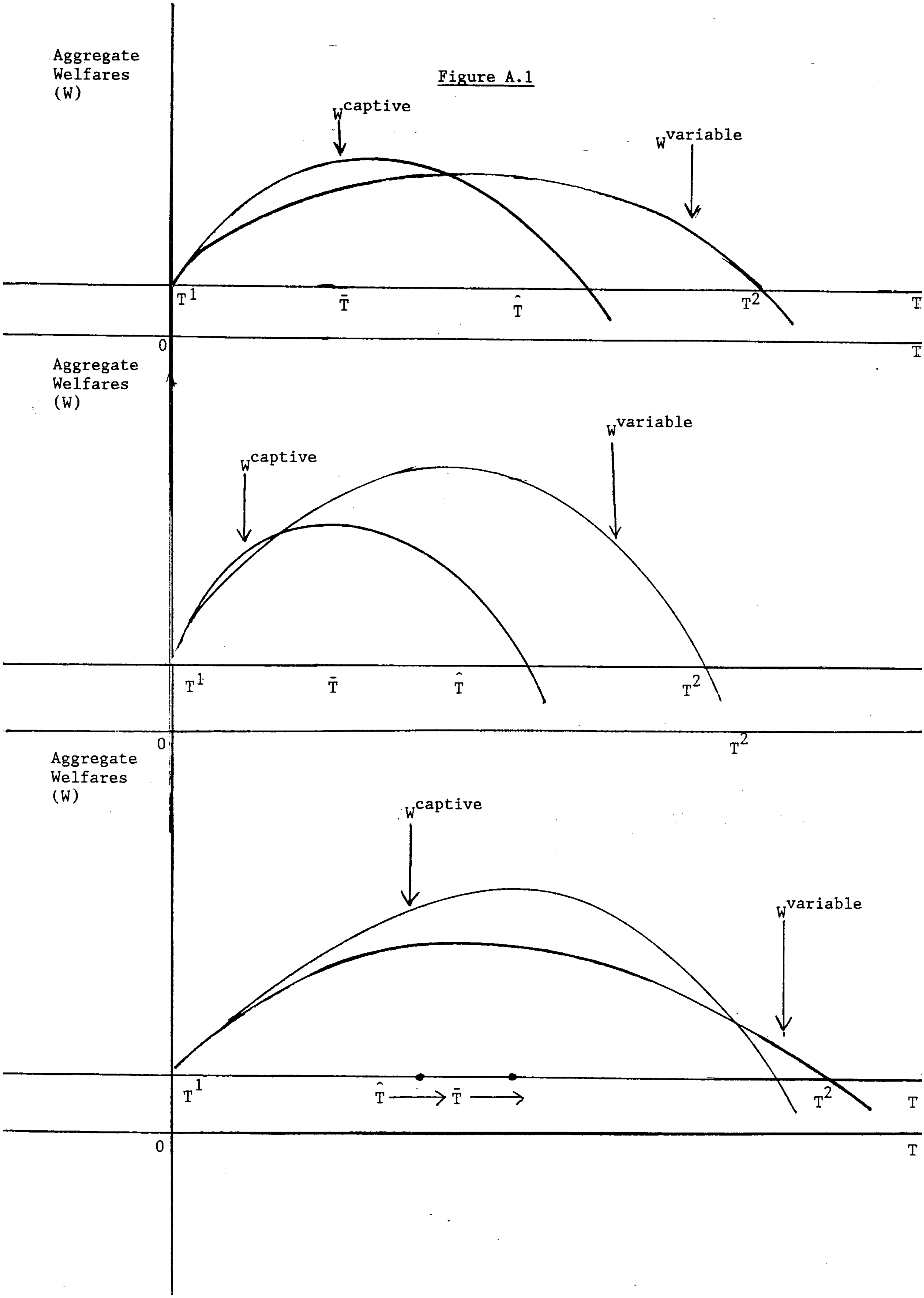
Figure 1



The Case $dM^*(T)/dT > 0$

Figure A.1


Aggregate Welfares (W)
W^captive
W^variable
T^1
T̄
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Aggregate Welfares (W)
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