Exchange Rate Bands with Price Inertia

Marcus Miller
Warwick University and CEPR

and

Paul Weller
Cornell University, Warwick University and CEPR

No. 337

February, 1990

We are most grateful to Avinash Dixit for many comments and suggestions, and for the research support provided by Alan Sutherland under ESRC (grant # R000231417). The work in this paper benefited substantially from time spent at the National Bureau of Economic Research on leave in 1988 and at the European Department of the International Monetary Fund as Visiting Scholars in 1989, and we are happy to acknowledge their hospitality. We would like to thank Guillermo Calvo, Bob Flood, Ken Froot, Maurice Obstfeld and especially Paul Krugman for helpful discussions. We also benefited from comments of seminar participants at LSE, MIT, Princeton and Yale. Responsibility for errors remains with us.

Exchange Rate Bands with Price Inertia

Abstract

We formulate a stochastic rational-expectations model of exchange rate determination in which there are random shocks to the process of sluggish price adjustment. We examine the effects of imposing limits upon the range of variation of both nominal and real exchange rates, and describe the intervention policies needed to defend the bands in each case. We consider the possibility that commitment to defend a particular nominal band may be less than fully credible, and analyze the implications of operating certain rules for realignment. We contrast our results with those which arise in the Krugman model of a nominal band.

Introduction

After the demise of the Bretton Woods system of "fixed but adjustable" exchange rates in the early 1970s, there followed a period of free floating characterized by a far greater degree of exchange rate variability than had been generally anticipated (Williamson (1985)). The exchange rate mechanism of the European Monetary System established in 1979 was intended in part to avoid this problem, by committing member states to taking suitable action to prevent cross-parities from straying outside some given, publicly-announced range, and Williamson's proposal for target zones was explicitly aimed at preventing exchange rate misalignment among the principal industrial countries. Subsequently, the Louvre Accord struck between members of the Group of Seven in 1987 involved unpublished commitments designed to constrain fluctuation between the U.S. dollar and the currencies of its major trading partners (Funabashi (1988)).

Given the obvious practical importance of understanding the implications of such policy commitments for the economy in general and the exchange rate in particular, it is perhaps not surprising that the seminal work by Krugman (1988, 1989), in which he presents a simple stochastic model of exchange rate determination within a band, has aroused considerable interest. Under the assumption that the velocity of money follows a Brownian motion process, he shows that it is possible to obtain an explicit solution for the exchange rate as a function of the fundamental. The presence of a commitment to intervene so as to defend the band exerts a stabilizing influence on the exchange rate even before any such intervention takes place.

In their contribution to this emerging literature, Froot and Obstfeld (1989) show that Krugman's minimalist stochastic specification can be derived from the monetary model of exchange rates where prices are perfectly flexible, full employment prevails and the exchange rate preserves purchasing power parity (cf. Mussa (1976)), and that the behavior of the rate inside a currency band can be obtained using results on regulated Brownian motion processes (cf. Harrison (1985). Using the same framework and techniques, Flood and Garber (1989) indicate how the rate will respond to non-infinitesimal intervention; and Svensson (1989) examines the effect of bandwidth on the stochastic behavior of interest rates. Klein (1989) considers the effects of uncertainty about the width of the band.

As these papers demonstrate, the monetary approach has proved attractive both because it yields explicit analytical solutions and allows direct application of results on regulated Brownian motion to the study of currency bands. But it has been severely criticized largely for its reliance on price flexibility and on purchasing power parity as the essential ingredients of exchange rate determination; Dornbusch (1987, 1988). In this paper we extend Krugman's analysis of currency bands to the case where prices adjust sluggishly to shocks in the economy, causing fluctuations in real exchange rates. The specific model used is a stochastic version of that presented by Dornbusch in his famous 1976 paper. Its advantages over that used by Krugman are that it describes the dynamics both of output and the real exchange rate, in addition to those of the nominal exchange rate. This enables us to distinguish between the effects of imposing <u>real</u> and <u>nominal</u> exchange rate bands, and provides a framework within which realignments of a nominal band can sensibly be analyzed. It also enables us to avoid a potential indeterminacy of the equilibrium path in the Krugman model.

The cost of this greater realism is that explicit analytical solutions to the model are no longer available. However, this turns out to be no disadvantage for our purposes, since the solutions we are interested in can be given a complete qualitative characterization.

In the first section of the paper we describe the model and its solution for general boundary conditions. In section two we analyze the effects of imposing a real exchange rate band, or target zone (cf. Williamson (1985)). In section three we examine the (substantially different) implications of imposing a nominal currency band. In section four we modify the assumption that commitment to defend the band is fully credible, and consider the effects of particular expectations in the market that a given nominal band will be realigned. We compare our results with those of Krugman in section five, and make concluding comments in section six.

1. The Model

To analyze the operation of currency bands, we treat the exchange rate itself as a forward-looking asset price which adjusts instantly to "news" regarding economic fundamentals. As for wages and prices, however, we take it as a well-established empirical fact that they do not adjust instantaneously to clear markets (as is assumed in the monetary approach). There are a number of possible explanations for this, ranging from insider-outsider theories, through appeal to moral hazard and efficiency wage considerations to the recent literature on "menu costs".

Though the techniques we develop can be applied to an overlapping-contracts model of wage/price adjustment (cf. Calvo (1983)), for analytical tractability we choose in this paper to represent prices as responding sluggishly to current excess demand. In addition, we include a supply side disturbance to the process of wage/price adjustment. Our model is a stochastic version of that presented in Dornbusch (1976), and consists of the following equations:

(1)	$m - p = \kappa y - \lambda i$	Money Market
(2)	$y = -\gamma(i - \pi) + \eta(s - p)$	Goods Market
(3)	$E(ds) = (i - i^*)dt$	Currency Arbitrage
(4)	$dp = \phi(y - \bar{y})dt + \sigma dz$	Price Adjustment
(5)	$\pi = E(dp)/dt$	Inflation Expectations

The first equation describes the equilibrium condition for the domestic money market, where m is the log of the money supply, p the log of the price level, y the log of GNP, and i the domestic interest rate. The second equation is an IS curve which captures the dependence of output on the log of the real exchange rate, s-p, and on the real interest rate, $i-\pi$, where s represents the log of the price of foreign currency and π is the instantaneous expected rate of inflation. The third equation is an arbitrage condition, in which the expected rate of depreciation of the nominal exchange rate, s, is set equal to the interest differential.[1] The fourth equation captures the idea that prices adjust less than instantaneously to any divergence between actual output y and full employment output $\bar{y}$. The term dz is the increment to a standard Wiener process, whose variance per unit of time is σ^2.

The dynamics of the deterministic system obtained by setting $\sigma = 0$ are well-known. The model possesses a stable saddlepoint equilibrium under parametric restrictions described below, to which the exchange rate will adjust smoothly over time following any unanticipated exogenous shock.

The motion of the stochastic system is more complex. We observe first that the system can be written as

$$\begin{bmatrix} dp \\ E(ds) \end{bmatrix} = A \begin{bmatrix} p\ dt \\ s\ dt \end{bmatrix} + B \begin{bmatrix} m\ dt \\ i^*\ dt \\ \bar{y}\ dt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix} \tag{6}$$

where

$$A = \frac{1}{\Delta} \begin{bmatrix} -\phi(\gamma + \lambda\eta) & \phi\lambda\eta \\ 1-\kappa\eta-\phi\gamma & \kappa\eta \end{bmatrix}$$

$$B = \frac{1}{\Delta} \begin{bmatrix} \phi\gamma & 0 & -\phi\gamma\lambda \\ -1 & -\Delta & -\phi\gamma\kappa \end{bmatrix}$$

[1]This relationship is not strictly consistent with risk neutrality, since if S is the <u>level</u> of the exchange rate, $E(dS)/S \neq E(ds)$. Thus there is an implied risk-premium in the foreign exchange market; but this permits a symmetric representation of the arbitrage condition and so avoids the Siegel paradox (see Miller and Weller (1990)).

where $\Delta = \kappa\gamma + \lambda - \phi\gamma\lambda$, and the saddlepoint property requires det A < 0 or $\kappa\lambda^{-1} > \phi - \gamma^{-1}$.

In order to simplify notation, redefine the variables p and s as deviations from the long run equilibrium of the deterministic system. Then we may write

$$(7) \qquad \begin{bmatrix} dp \\ E(ds) \end{bmatrix} = A \begin{bmatrix} p\ dt \\ s\ dt \end{bmatrix} + \begin{bmatrix} \sigma\ dz \\ 0 \end{bmatrix}$$

To find solutions to (7), we begin by postulating a deterministic functional relationship s = f(p). Applying the rules for stochastic differentiation we obtain

$$ds = f'(p)dp + \frac{\sigma^2}{2} f''(p)dt$$

from which it follows that

$$E(ds) = f'(p)E(dp) + \frac{\sigma^2}{2} f''(p)dt.$$

Substituting for E(ds) and E(dp) from (7) gives

$$(8) \qquad \frac{\sigma^2}{2} f''(p) + (a_{11}p + a_{12}f(p))f'(p) - (a_{21}p + a_{22}f(p)) = 0$$

where a_{ij} denotes the appropriate element of the matrix A. This second-order, nonlinear differential equation has no closed form solutions in general, but it is possible to characterize completely the qualitative features of the solutions we shall be examining.

Note first that the only linear solutions to (8) correspond to the stable and unstable arms of the deterministic saddlepath. This is not surprising since imposing linearity has exactly the same effect in (8) as setting σ equal to zero, in that it knocks out the term $\frac{\sigma^2}{2} f''(p)$. The solution corresponding to the stable arm of the saddlepath is of particular interest, since it is the natural candidate for the "free float" solution to the model. It will satisfy the stochastic stability condition that the expected movement of the exchange rate is always in the direction of long-run equilibrium. And it is the unique solution possessing this property, which can be shown to guarantee the existence of a stationary distribution for the exchange rate.[2] Uniqueness is easily established by observing that the loci of stationarity for the deterministic system become loci of expected stationarity in the stochastic system; and the qualitative analysis of solution paths described below reveals that all other solutions have unbounded segments on which the expected movement of the exchange rate is away from equilibrium.

Now consider all solutions to (8) satisfying the boundary condition $f(0) = 0$. Because a second-order differential equation requires two boundary conditions to tie down a unique solution, there will be an infinite family of trajectories passing through the origin. In Figure 1

---

[2]The existence of a stationary distribution associated with the free float solution follows from applying a result of Has'minskii (1980) (Chapter 4, Theorem 4.1.)

we illustrate the general qualitative features of these trajectories,[3] for the "overshooting" case in which the stable saddlepath SS is negatively sloped. This is equivalent to the condition $\kappa\eta + \phi\gamma < 1$.

Since all trajectories are antisymmetric about the origin, we need only consider the two quadrants in which $p \geq 0$. In the region lying between the unstable saddlepath UU and the vertical axis, a typical path is concave and then convex, becoming asymptotically parallel to UU. Between UU and SS, paths are convex and then concave, again becoming asymptotically parallel to UU. Finally, below SS, paths are everywhere concave.

One can think of this set of solutions as that obtained by imposing the boundary conditions $f(0) = 0$, $f'(0) = a$, $-\infty < a < +\infty$. It is of interest to us as it turns out to be the relevant set to consider when we examine the behaviour of the exchange rate within a real or nominal band positioned symmetrically about long-run equilibrium.

2. A Real Currency Band or Target Zone

In advocating target zones for exchange rates, Williamson (1985) proposed that the monetary authorities in the major industrial countries be prepared to adjust the stance of monetary policy so as to keep their real effective exchange rates within broad bands (of ± 10% around the equilibrium levels consistent with sustainable current account flows). It was intended that adjustments be made only at the edges of these bands,

---

[3] Further details together with proofs of the assertions below are given in Miller and Weller (1988, 1989a).

however, so that monetary control could be aimed at domestic anti-inflationary objectives within these target zones.

Suppose, in the model just specified, the monetary authorities make no adjustments to the money stock except as necessary to keep the real exchange rate, defined as $v = s - p$ within a predetermined range $[\underline{v}, \bar{v}]$.

In order to describe the equilibrium path for the exchange rate, it is useful to be able to work entirely with real variables. Thus we reformulate (7) as the equivalent system

$$(9) \qquad \begin{bmatrix} d\ell \\ E(dv) \end{bmatrix} = C \begin{bmatrix} \ell dt \\ vdt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where $\ell = m - p$, and

$$C = \frac{1}{\Delta} \begin{bmatrix} -\phi\gamma & -\phi\lambda\eta \\ -1 & (\kappa - \phi\lambda)\eta \end{bmatrix}$$

The fundamental is now the level of real balances, and the arbitrage equation imposes the requirement that expected depreciation of the real exchange rate be equal to the real interest rate differential. Assuming that there exists a deterministic functional relationship between real balances and the real exchange rate, which we denote $g(\ell)$, the fundamental differential equation takes the form

$$(10) \qquad \frac{\sigma^2}{2} g''(\ell) + (c_{11}\ell + c_{12}g(\ell)g'(\ell) - (c_{21}\ell + c_{22}g(\ell)) = 0$$

where c_{ij} are the elements of C.

Those solutions to (10) which satisfy the condition $g(0) = 0$ are qualitatively identical to those already discussed in the previous section. But now the stable arm of the deterministic saddlepath is always positively sloped.

In order to identify the solution to (10) which corresponds to the case of a target zone, appropriate boundary conditions must be imposed. These require that the path for the real exchange rate be smoothly tangent to the edges of the band.

Krugman was the first to identify these "smooth pasting" conditions as the correct ones to apply in a model which we discuss in Section 5. In order to understand the formal justification for these conditions in the context of real bands, it is necessary first to describe the actions which the monetary authorities undertake to defend the target zone.

By assumption, the money stock m is held fixed so long as the exchange rate lies strictly within its permitted range of variation. At the instant when the rate hits either edge of the band, any price shock tending to push it further from equilibrium is fully accommodated, so that real balances, $\ell = m - p$, are held constant. Thus if the real exchange rate hits $\underline{v}$, any positive price shock is accommodated, whereas if it hits $\bar{v}$ any negative price shock is accommodated. The actions of the monetary authorities, in constraining the range of variation of v, simultaneously limit the range of variation of ℓ. The model is therefore formally equivalent to one in which ℓ is regulated as illustrated in Figure 2, so long as there exists a unique solution to (10) satisfying the smooth pasting boundary conditions.[4] We discuss the circumstances under which this is true below. For the moment we simply assume it to be the case.

The economic argument for the form of the boundary condition runs as follows. Suppose that v has just reached the lower edge of the band, and that the solution path is not tangent to the edge, but cuts it. In the absence of intervention the real exchange rate will fall below $\underline{v}$ if there is a positive shock to the price level. Intervention to hold the real exchange rate at $\underline{v}$ by accommodating a positive price shock, but doing nothing in the event of a negative price shock (which would take the rate back into the interior of the band) will imply that the current level of domestic real interest rates will be too low to be consistent with arbitrage. In other words, any policy intervention designed to prevent v from falling below $\underline{v}$ will increase E(dv) without affecting current real interest rates. Only if the path of the exchange rate within the band is smoothly tangent to the edges will the intervention rule be consistent with arbitrage.

The amount of time that the real exchange rate is observed at the limits of its permitted range of variation is "short". The reason for this is to be found in the effect of the (unsterilized) intervention upon domestic interest rates. This will be to check any further rise in interest rates (real and nominal) when $v = \underline{v}$ and the currency is strong, and conversely when the currency is weak. In other words, interest rates are <u>momentarily</u> stabilized at current levels. But suppose that the

---

[4]For a heuristic derivation of the smooth pasting boundary conditions see Dixit (1989). They are also discussed in Dumas (1989). A more formal treatment is given in Harrison (1985). All the above work with Brownian motion, but the results can be extended straightforwardly to the more general Ito process in our model (Miller and Weller (1989a)).

exchange rate were expected to remain on the edge of the band for more than an instant. Then for that period of time it must be true that $E(dv) = 0$. For this to be compatible with the arbitrage condition, real interest rates would have to make a discrete jump to world levels. What this discussion reveals is that at the moment when the monetary authorities intervene, $E(dv) > 0$ at $\underline{v}$ and $E(dv) < 0$ at $\bar{v}$. This is so because of the one-sided nature of the intervention, in which at $\underline{v}$, the real exchange rate v is prevented from falling further, but not from rising.[5]

Both the nominal and the real exchange rate display a "local" stability, in the sense that given the current level of the money stock, which remains fixed so long as the exchange rate is in the interior of its band, there is always a tendency for the rate to return to its long-run equilibrium level in the middle of the band. However, because the changes in the money stock needed to defend the band are locally irreversible, both money and the price level will follow globally non-stationary processes. But of course expected inflation (defined as π above) remains strictly bounded, within limits determined by the width of the target zone.

We show in Figure 3 that the effects of this non-stationarity will be to generate a family of curves describing the path of the nominal exchange rate where the origin represents long-run equilibrium at time zero. Now (long-run) equilibria will always be on the PPP line, and their position

---

[5]Technically, the process m is almost everywhere differentiable with respect to time, with derivative equal to zero. The sample paths of m increase or decrease on a set of measure zero.

will reflect the cumulative effect upon m of defending the target zone in the manner described above.

Thus far we have examined the case in which there exists a unique solution to (10) which also satisfies the boundary conditions. However, for certain parameter values there will exist a second solution, as shown in Figure 4, which is stochastically unstable in the sense that $E(dv) > 0$ when $v > 0$, and conversely. The parameter restriction $\kappa\lambda^{-1} > \phi$ is sufficient to rule out this possibility. In other words, the feedback effect of exchange rate on the price level should not be "too large". If this condition is not satisfied, and in general there is no reason to suppose that it will be, it may be necessary for the authorities to announce that they will intervene within the target zone to prevent the exchange rate from settling on the unstable path. One way in which this could be done is by committing to a policy in which, if the market moved on to the unstable adjustment path, the net interest sensitivity of the demand for money would be reduced to zero by suitable continuous adjustment of the stock of money. This produces a vertical LM curve, and is equivalent to setting λ equal to zero, in which case the necessary parameter restriction given above is automatically satisfied. A credible threat to adopt this policy would deter the market ever from settling on to the unstable path. Fortunately the two possibilities are easy to discriminate between, since the unstable solution involves a perverse response of v to changes in p, so that an increase in the price level will lead to an increase in competitiveness, and vice versa.

The parameter restriction $\kappa\lambda^{-1} > \phi$ also implies that the solution path for v can be represented as

$$v(t) = E[\int_t^\infty e^{(\phi\lambda-\kappa)\eta\tau}\ell(\tau)d\tau]$$

Thus the real exchange rate is an "expected discounted value" of fundamentals.[6]

3. A Nominal Currency Band

The target zone analyzed in the previous section is not an accurate representation of the most durable of the broad-band currency regimes actually implemented, namely the exchange rate mechanism of the EMS. This arrangement is operated as a nominal currency band, with occasional realignments of central parities. It is thus of some practical policy interest to investigate the performance of a nominal band in our framework.

We consider first the question of how to analyze a fully credible nominal currency band. It is clear that the policy of regulating the level of real balances by means of small accommodating adjustments in the money stock will not be applicable here. The implied global non-stationarity of money could not conceivably be consistent with a fixed nominal band.

Formally, however, the structure of the problem is similar to that analyzed above, except that we now impose limits upon the range of variation of the nominal exchange rate s. Let us denote the band by

[6] This follows from a simple extension of Corollary 4, page 83 in Harrison (1985).

$[\underline{s}, \bar{s}]$. Just as before, we assume that the money stock is held fixed while the exchange rate is in the interior of the band. This means that we are looking for a solution to (8) subject to appropriate boundary conditions. Again they take the form of smooth pasting onto the edges of the band. However, the formal justification is different, as are the economic implications.

Now, when the exchange rate hits the edge of the band, it is necessary to think of there being a <u>switch</u> to a new regime, in which the rate is held on the edge so long as the price level is further from equilibrium than at the switch point.

The support regime requires that $E(ds) = 0$, and therefore that domestic interest rates be set at the world level. But at point A in Figure 5a where the solution path $f(p)$ touches the upper edge of the band, although $f' = 0$, f'' is negative, implying that $E(ds)$ is negative and $i < i^*$. So at A there must be a <u>discontinuous</u> adjustment in the money stock, causing an instantaneous jump of interest rates to the world level. Money is then continuously adjusted in response to price shocks to hold the nominal interest rate constant at i^*, so long as $p > \bar{p}$. Once p has drifted back again to $\bar{p}$, the money stock is returned to its original level.

The money stock is plotted as a function of the price level in Figure 5b. At $\bar{p}$ the currency is weak, and monetary policy has to be tightened to raise interest rates to the world level. This is the undershooting case, in which $\kappa\eta + \phi\gamma > 1$. In contrast to the case of a real band, it is possible to show that the solution is unique, and that no instability problem arises (see Miller and Weller (1989)). On the other hand, if

$\kappa\eta + \phi\gamma < 1$, the exchange rate overshoots, and the unique stable solution is negatively rather than positively sloped. The relative insensitivity of the trade balance to a change in competitiveness means that the monetary tightness resulting from inflation forces up interest rates and so the nominal exchange rate.

Formally, what we are describing is "diffusion across a boundary" (see Whittle (1983) ch. 37). Note that although the boundary conditions are the same - namely asset value matching and smooth pasting -, there is no process being regulated. We have a true switch of regime from one of a free float within the band to a fixed rate at the edge. However, what is crucial for our qualitative analysis of a currency band is that the switch be reversible. It is this feature which distinguishes our discussion from the early pioneering work in this field by Flood and Garber (1983), and the more recent treatment of Froot and Obstfeld (1989).

Thus our model of a fully credible nominal currency band carries sharply contrasting implications for the conduct of monetary policy, and for the resulting behaviour of the exchange rate. Both the money stock and the price level will follow globally stationary processes, and the exchange rate will spend random periods of time at the limits of its range of variation before drifting back into the interior of the band.

4. Realignment of a Nominal Currency Band

If we assume a nominal currency band to be fully credible we have seen from the discussion of the previous section that in our model it follows that the price level will be globally stationary. But in practice no such arrangement will ever be completely credible in the eyes of the market. It will always be possible, for example, that a sufficiently long series of adverse shocks will so depress the economy that the government would prefer to realign the band rather than continue to defend it.

The way we choose to represent this possibility is as follows. When the exchange rate hits the edge of the band, the market views the occurrence of some specified realignment as an event following a Poisson process. So long as the rate remains in the interior of the band, the probability of realignment is perceived to be zero. Now suppose that any realignment, if it occurs, involves a shift in the position of the band by one bandwidth. In other words, in an upwards realignment the top of the original band becomes the bottom of the new one. The effects on exchange rate movement are depicted in Figure 6. The original band is centered about equilibrium at the origin 0. The solution path for the exchange rate within the band will be exactly as for the fully credible case. If the exchange rate hits the upper edge, and a realignment occurs at some point, then the money stock is increased so as to shift the new long run equilibrium from 0 to 0'. Simultaneously, the exchange rate jumps on to the new solution path, which is identical to the old one but for the shift of origin.

This solution[7] has a number of interesting qualitative features. First, the exchange rate may spend time being supported at the edge of the band, and return to the interior of the band without a realignment having occurred. Second, interest rates must move more dramatically to compensate for the expectation of jumps in the exchange rate. At the top

[7]Solutions which involve a Poisson process for determining policy changes are examples of what has been referred to as the "peso problem" (see Salant and Henderson (1978), Dornbusch (1982).)

of the band, when the currency is weak, interest rates have to be raised above world levels in order to satisfy arbitrage.[8] Conversely, interest rates are below world levels on the bottom edge, in anticipation of a possible revaluation. Third, the realignment rule introduces global non-stationarity into the levels of prices and money stock, just as was true in the case of a target zone. (Indeed, it is possible to reinterpret the target zone case as one of a nominal band whose edges are automatically indexed only when the rate hits the edge.) However, here too these variables display local stationarity.

At first sight it may appear odd that the "smooth pasting" characteristic of the boundary condition is preserved, despite the imperfect credibility of the band. The explanation lies in the fact that the two distinct "assets" held in the support regime in the two cases, namely "one unit of currency with zero realignment probability" and "one unit of currency with (given) positive (instantaneous) realignment probability" are in fact perfect substitutes. The latter is earning a higher (lower) interest return which exactly compensates for the possible loss (gain) upon realignment.

It is noteworthy that realignments of the kind described above, when they have occurred within the EMS, typically are not associated with significant movement of the currency. This must imply that the realignment has effectively been anticipated by the market. It is thus of

[8]We solve explicitly for the implied money supply rule in Miller and Weller (1989a).

some practical relevance to pose the question "What happens if a realignment is perfectly anticipated?"[9]

We consider a simple rule of the following form: "When the exchange rate hits the upper (lower) edge of the band, there will, with probability one, be an upwards (downwards) realignment of the band by an amount b". In Figures 7a and 7b we show the solutions for b equal to one half and one bandwidth respectively. In the first case the exchange rate will wander along the solid curve which curls around the PPP line, and is the unique solution to (8) satisfying the endpoint conditions necessary to rule out a discontinuity in the path of the exchange rate. When the rate reaches the upper edge of the band at 0′, a realignment is triggered, and by construction 0′ becomes the new long run equilibrium. The exchange rate now moves along the dotted path. Of particular significance is the fact that here there is no smooth transition from one regime to another.[10] However, smooth pasting reappears when the realignment is of a full bandwidth. This is because we are again back in the framework of (reversible) diffusion across a boundary.[11]

---

[9] Driffill (1989) argues that this is the most plausible model of the EMS on the reasonable grounds that it bounds the possible deviation of output from full employment.

[10] This solution closely parallels the case of discrete regulation of Brownian motion analyzed by Harrison (1985). See also Miller and Weller (1989b, 1989c), Dixit (1989) and Flood and Garber (1989).

[11] Bertola and Caballero (1989) present evidence of exchange rate behavior within the EMS in support of the solution paths in Figures 7a and 7b.

5. Monetary Models

In this section we compare the properties of the stochastic Dornbusch model we have elaborated above with the monetary model used by Krugman (1988, 1989). Krugman uses a single equation model of the form

(11) $$s = m + v + \lambda E(ds)/dt$$

where s and m are, as before, the logs of the exchange rate and money stock. The variable v measures the velocity of money, and is assumed to follow a driftless Brownian motion. Thus

(12) $$dv = \sigma dz$$

where dz is the increment to a Wiener process, and v has a variance of σ^2 per unit of time.

The reduced form in (11) can be derived from a model with the following structure:

(13) $$m - p = \kappa\bar{y} - \lambda i - v$$

(14) $$s = p - p*$$

(15) $$E(ds) = (i - i*)dt$$

All variables are as defined before. In addition, $\bar{y}$ represents the log of "full employment" GNP, and p* the price level in the "rest of the world". Thus we have assumed away any output dynamics. In addition, we need to suppose that purchasing power parity always holds (equation (14)), which implies that the domestic price level will always move exactly in step with the nominal exchange rate.

The advantage of working with this highly stylized model of exchange rate determination is that an explicit solution for the path of the exchange rate as a function of fundamentals can be obtained. Krugman (1988, 1989) shows that it takes the form

$$(16) \qquad s = m + v + A\,[e^{-\rho v} - e^{\rho v}]$$

where $\rho = (2/\lambda\sigma^2)^{1/2}$, and A is a constant to be determined by the "smooth pasting" boundary conditions.

This solution is qualitatively similar in some respects to the one we derive for the case of a <u>real</u> band. The path for the exchange rate is an S-shaped function of the fundamental m + v, which is assumed to be regulated in exactly the same way as real balances, m - p, in our model of a target zone.

But the fundamental v follows an autonomous process, unlike the price level in our model. Thus the solution s = m + v, which is simply <u>assumed</u> by Krugman to correspond to the free float, has the somewhat unattractive feature of following a Brownian motion, which is a non-stationary process with infinite asymptotic variance. If any form of mean reversion is assumed for the process driving v, one must immediately forego Krugman's elegant analytical solution.

The monetary model is unable to treat the case of a target zone, since the real exchange rate is assumed fixed throughout. For this reason also it does not seem to be a framework within which one can very sensibly talk about realignments of a nominal band. Motivation for any discussion of realignments must surely come from a presumed desire on the part of governments to avoid long periods defending over- or under-valued exchange rates.

Finally, there is a problem which has not received any attention, but represents perhaps the most serious limitation in any attempt to generate empirical predictions. This is the problem of indeterminacy of the equilibrium path for the exchange rate. In order to derive (16) one needs to impose a <u>symmetry</u> assumption which has no very plausible economic justification. In other words, the smooth pasting boundary conditions alone determine an infinite family of possible solution paths. Since the fundamental follows a Brownian motion, the choice of origin is arbitrary, and yet for any <u>given</u> nominal band will generate a different solution.

It would seem that there are two ways to resolve this difficulty. One is to reinterpret the model explicitly as one of (unanticipated) transition from free float to nominal band. Then the value of the nominal exchange rate (and fundamental) at the instant that the band is imposed constitutes the natural origin. If the band is symmetric about that value, the model predicts that we will observe the Krugman solution. If it is not, the solution path will not be symmetric about the origin. Indeed, it will not pass through the origin, and the exchange rate will have to make a discrete jump.

The other is to introduce mean reversion into the process generating velocity shocks. But then much of the attraction of the model is sacrificed, since the elegant analytical solution in (16) is no longer valid, and one must resort to numerical computation, or the kind of qualitative analysis which we have shown to be possible.

All discussions of the empirical implications of the Krugman model (see Froot and Obstfeld (1989), Svensson (1989) and Bertola and Caballero (1989)) have chosen to consider the symmetric solution to the Brownian motion case. Only if the scenario described above is satisfied is this warranted.

Concluding Comments

We have shown how Krugman's model of a nominal exchange rate band can be extended to incorporate endogenous dynamics in output and the real exchange rate. By using a stochastic version of the Dornbusch model of exchange rate determination we are able to distinguish between the effects of a real band, or target zone, and a nominal band, and to show that the intervention policy required to support the band in each case is radically different, and implies important differences in the behaviour of the exchange rate. We also avoid the problem of indeterminancy of the equilibrium path which arises in the standard interpretation of the Krugman model.

For real exchange rate bands the analysis can be cast in terms of regulating an economic variable (real balances) within predetermined limits by infinitesimal intervention. As the monetary accomodation involved leads to more than one solution trajectory, we discuss how the unstable trajectory can be ruled out in principle (by adopting an appropriate state contingent rule for monetary policy within the band). For a nominal currency band, however, the smooth pasting to be observed at the edges of the band arises not from regulation but from a reversible switch of regime, from floating to staying, possibly for some time, on the edge. The smooth pasting solution is found to apply even when the prospect of realignment generates a peso problem at the edge of the band but not if locally irreversible realignment rules are triggered when the currency reaches its limits.

Though the type of price inertia assumed in this paper excludes forward-looking behavior from the process of wage and price setting, the techniques used can be extended[12] to incorporate overlapping contracts, where the forward-looking element (the current contract) can be solved simultaneously with the exchange rate as a function of the fundamental (the average of existing contracts). Such contracts will of course anticipate changes of policy in defense of exchange rate bands and transmit the effects into wages and prices before the changes are actually triggered.

In this paper we have not dealt with the role played in practice by foreign currency reserves. This is consistent with a view of monetary intervention as a means of influencing domestic interest rates and so rational portfolio decisions and not as a means of warding off a speculative attack. We recognize this is only part of the story, albeit an important part. A recent paper by Krugman (1989) introduces reserve

[12] With two forward looking elements, however, recourse to numerical computation is required.

accumulation and decumulation explicitly into the monetary model with variable velocity, and it remains an important topic for further research to see how the possibility of speculative attacks may be incorporated into a wider class of stochastic models.

REFERENCES

Bertola, Giuseppe and Ricardo J. Caballero, (1989) "Target Zones and Realignments", mimeo, December.

Calvo, Guillermo (1983) "Staggered contracts and exchange rate policy", in J.A. Frenkel (ed.) Exchange Rates and International Economics, Chicago, IL, University of Chicago Press.

Dixit, Avinash, (1989) "A Simplified Exposition of the Theory of Optimal Control of Brownian Motion", Princeton University, June.

Dornbusch, Rudiger, (1976) "Expectations and Exchange Rate Dynamics", Journal of Political Economy, 84, December.

Dornbusch, Rudiger, (1982) "Equilibrium and Disequilibrium Exchange Rates, Zeitschrift fur Wirtschafts - und Sozial Wissenschaften, (102), 573-599.

Dornbusch, Rudiger (1987) "Exchange Rate Economics: 1986" Economic Journal, 97, March 1-18.

Dornbusch, Rudiger (1988) "Real Exchange Rates and Macroeconomics: A Selective Survey, mimeo, MIT.

Driffill, John (1989) "Discussion of "Exchange Rate Bands and Realignments in a Stationary Stochastic Setting" by Miller and Weller," in M. Miller, B. Eichengreen and R. Portes (eds.) Blueprints for Exchange Rate Management, New York: Academic Press, 175-177.

Dumas, Bernard, (1989) "Super Contact and Related Optimality Conditions: A Supplement to Avinash Dixit's "A Simplified Exposition of Some Results Concerning Regulated Brownian Motion", NBER Technical Working Paper No. 77, April.

Flood, Robert P. and Peter M. Garber, (1983) "A Model of Stochastic Process Switching", Econometrica, 51, May, 537-552.

Flood, Robert P. and Peter M. Garber, (1989) "The Linkage Between Speculative Attack and Target Zone Models of Exchange Rates", NBER Working Paper No. 2918, April.

Froot, Kenneth A. and Maurice Obstfeld, (1989) "Exchange-Rate Dynamics under Stochastic Regime Shifts: A Unified Approach", NBER, June.

Funabashi, Yoichi (1988) Managing the Dollar: From the Plaza to the Louvre, Washington, D.C.: Institute for International Economics.

Harrison, J. Michael (1985), Brownian Motion and Stochastic Flow Systems, John Wiley and Sons: New York.

Has'minskii, R.Z., (1980) Stochastic Stability of Differential Equations, Sijthoff and Noordhoff, Alphen aan den Rijn, The Netherlands.

Klein, Michael W., (1989) "Big effects of small interventions: the Informational Role of Intervention in Exchange Rate Policy", Clark University, October.

Krugman, Paul, (1988) "Target Zones and Exchange Rate Dynamics", NBER Working Paper No. 2481, January.

Krugman, Paul, (1989) "Target Zones with Limited Reserves", MIT, August.

Miller, Marcus and Paul Weller, (1988) "Solving Stochastic Saddlepoint Systems: A Qualitative Treatment with Economic Applications", NBER, October, and CEPR Discussion Paper No. 308.

Miller, Marcus and Paul Weller, (1989a) "Qualitative Solutions for Stochastic Saddlepaths and the Analysis of Exchange Rate Regimes", Warwick University, January.

Miller, Marcus and Paul Weller, (1989b) "Exchange Rate Bands and Realignments in a Stationary Stochastic Setting", in M. Miller, B. Eichengreen and R. Portes (eds.) Blueprints for Exchange Rate Management, New York: Academic Press, 161-173.

Miller, Marcus and Paul Weller, (1989c) "Currency Bands, Target Zones and Cash Limits: Thresholds for Monetary and Fiscal Policy", IMF, September.

Miller, Marcus and Paul Weller (1990) "Currency Bubbles Which Affect Fundamentals: A Qualitative Treatment" Economic Journal (forthcoming).

Mussa, Michael (1976) "The Exchange Rate, the Balance of Payments, and Monetary and Fiscal Policy under a Regime of Controlled Floating" Scandinavian Journal of Economics, 78, May, 229-248.

Salant, S. and Dale Henderson (1978) "Market Anticipations of Government Policies and the Price of Gold " Journal of Political Economy, 86.

Svensson, Lars (1989) "Target Zones and Interest Rate Variability" mimeo Institute for International Economic Studies, Stockholm, December.

Whittle, Peter (1982) Optimization over Time: Dynamic Programming and Stochastic Control, vol. 2 (New York: John Wiley and Sons).

Williamson, John (1985), The Exchange Rate System (Revised edition). Policy Analyses in International Economics No. 5., Washington, D.C.: Institute for International Economics.

Figure 1

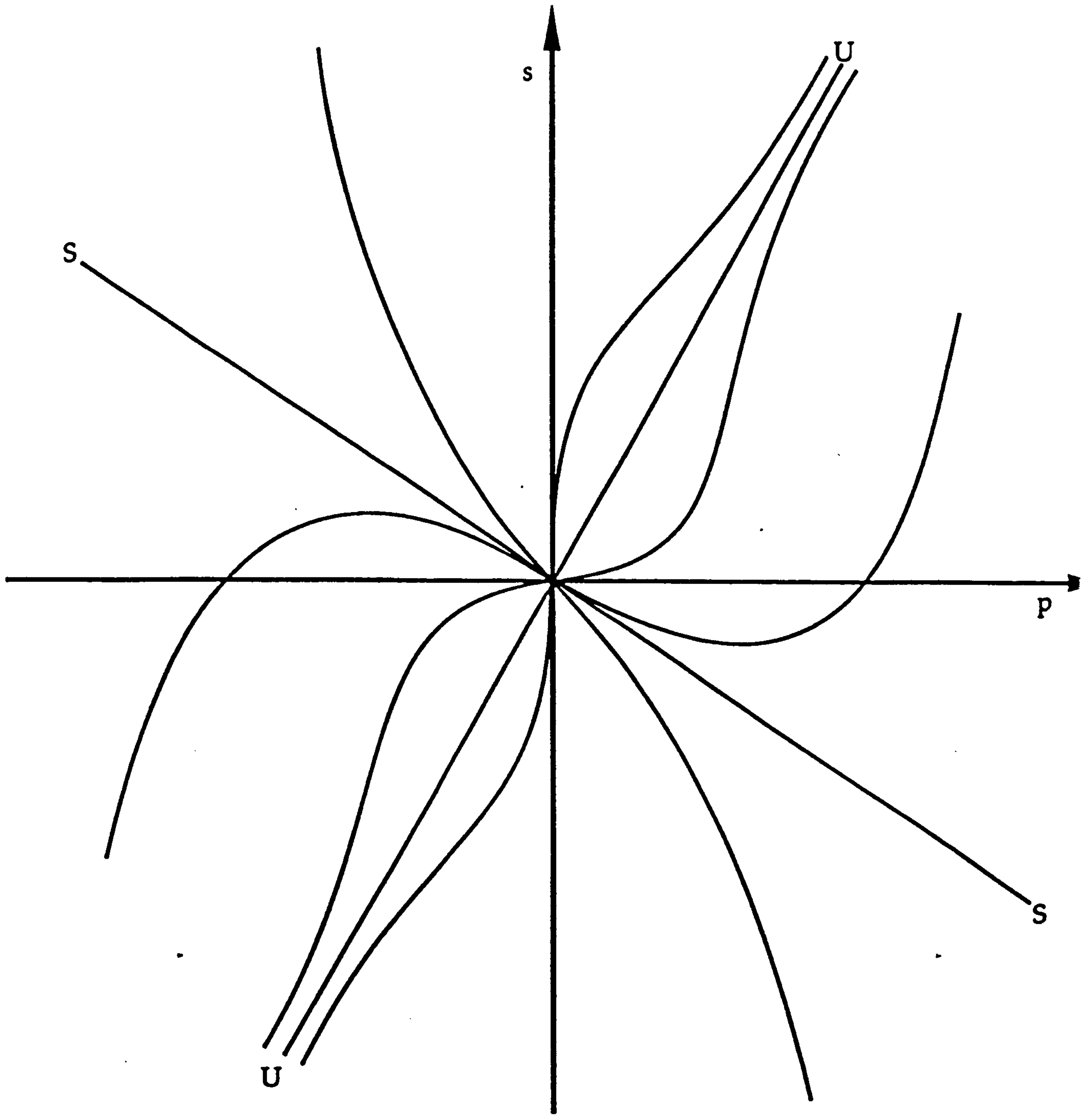


Figure 2

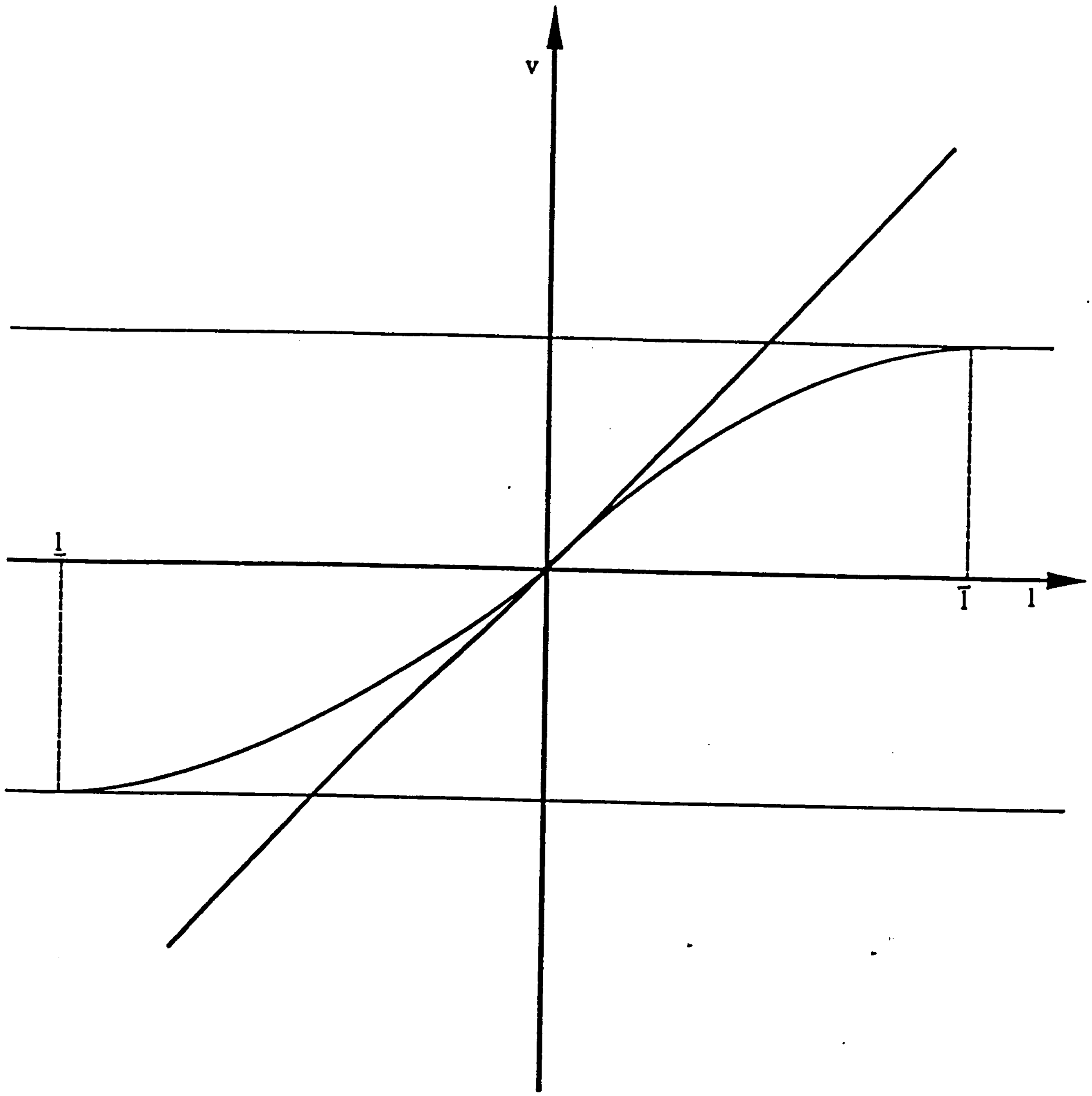


Figure 3

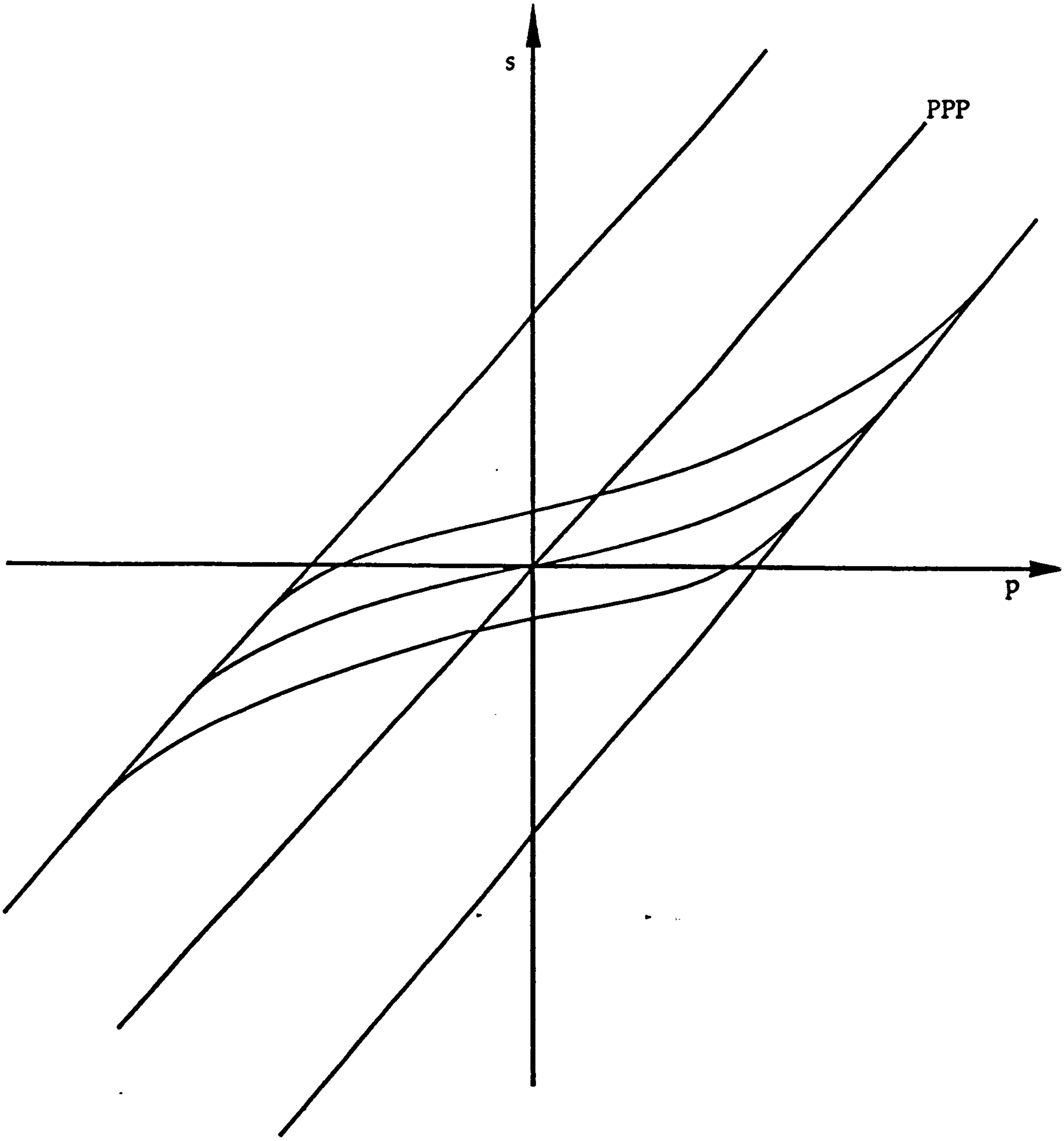


Figure 4

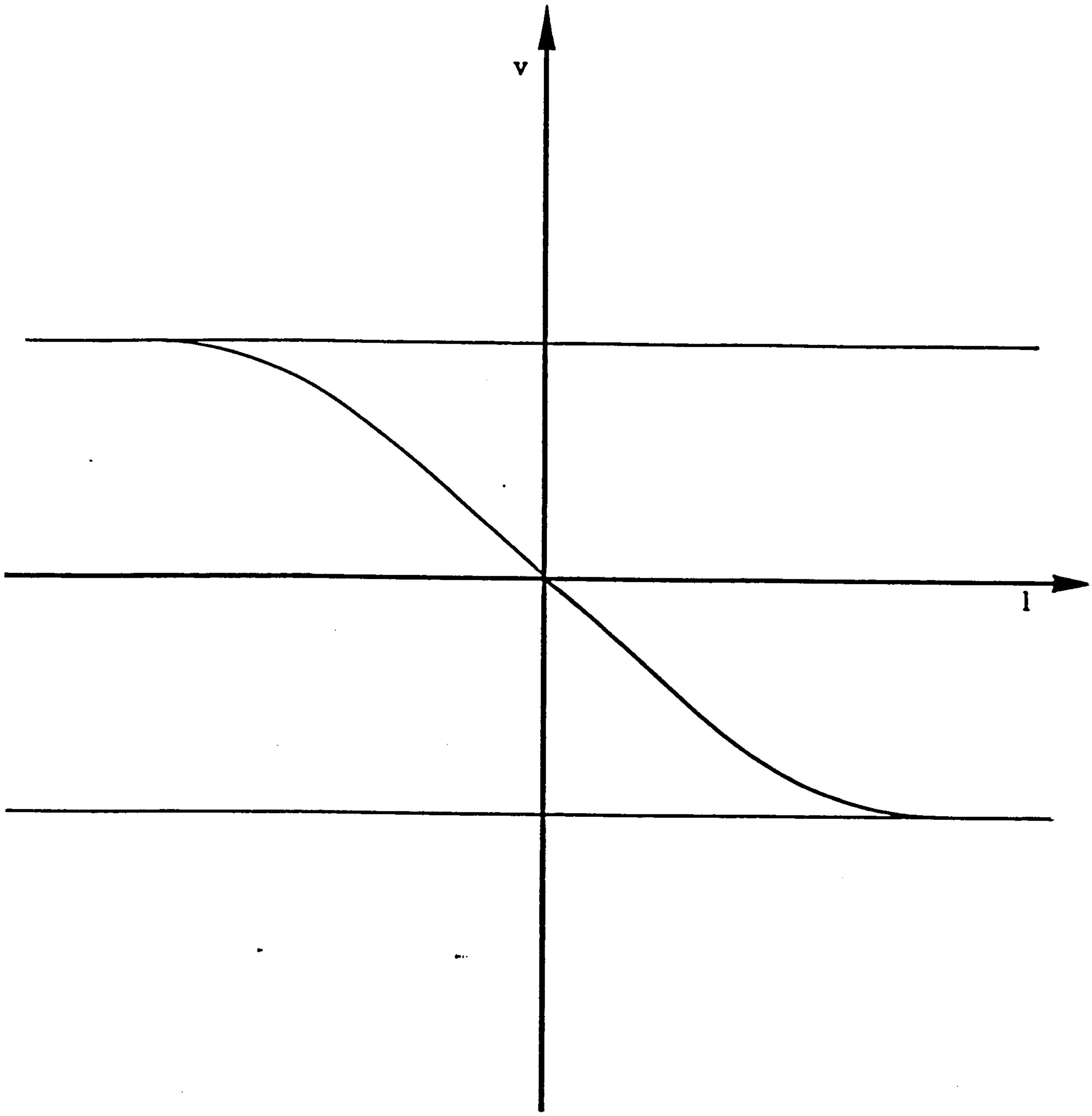


Figure 5a

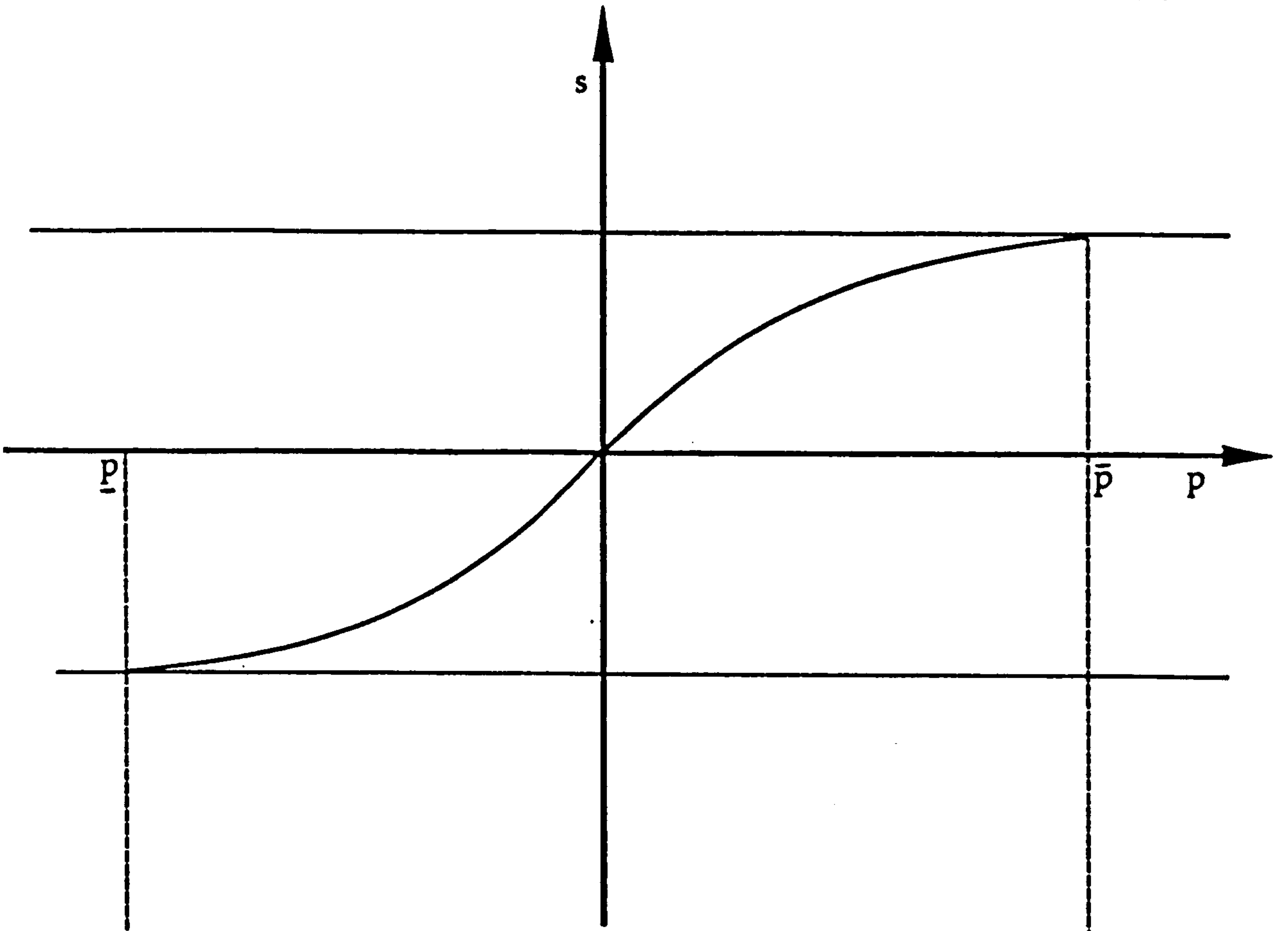
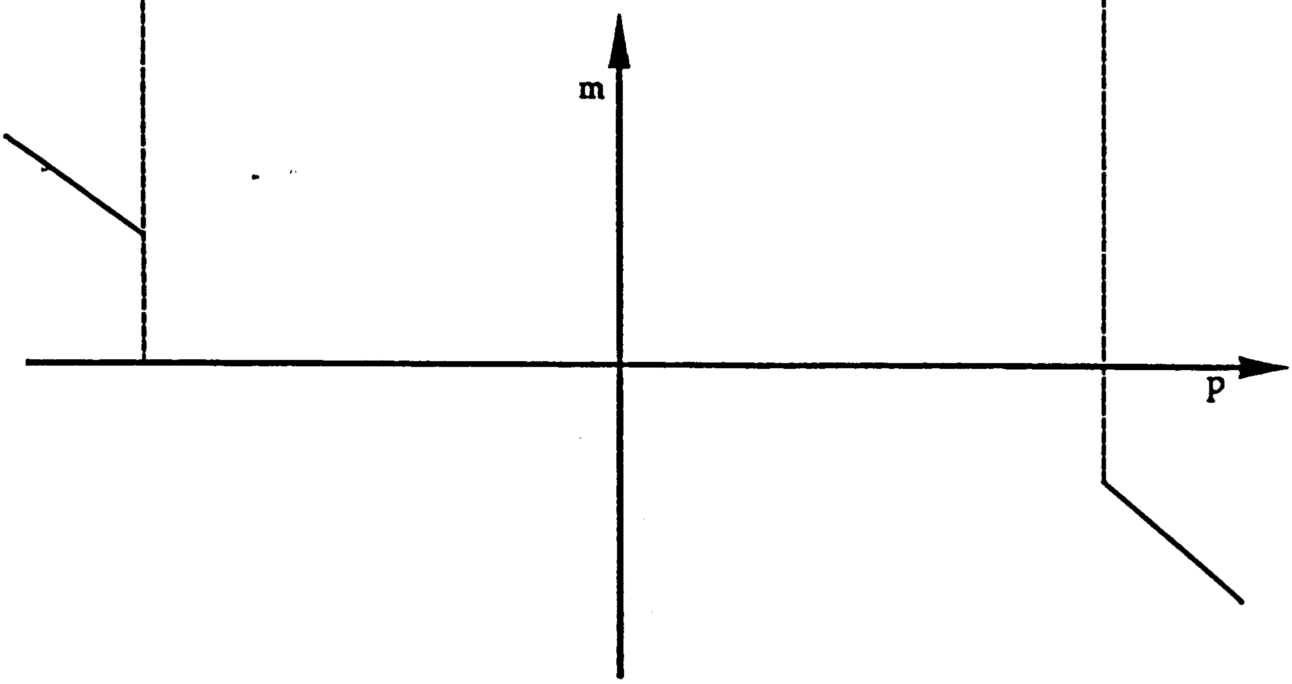


Figure 5b



<u>Figure 6</u>

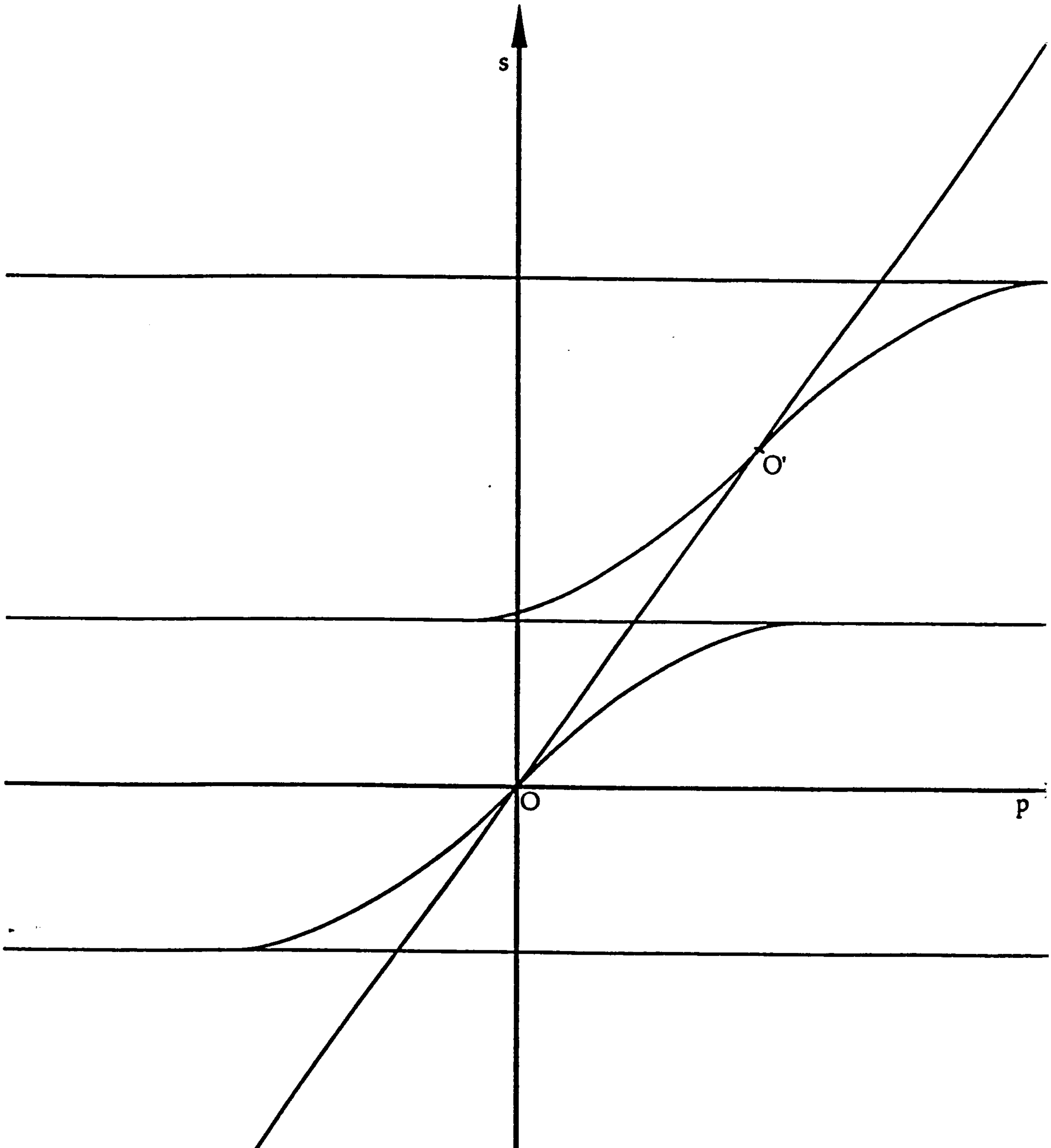


Figure 7a

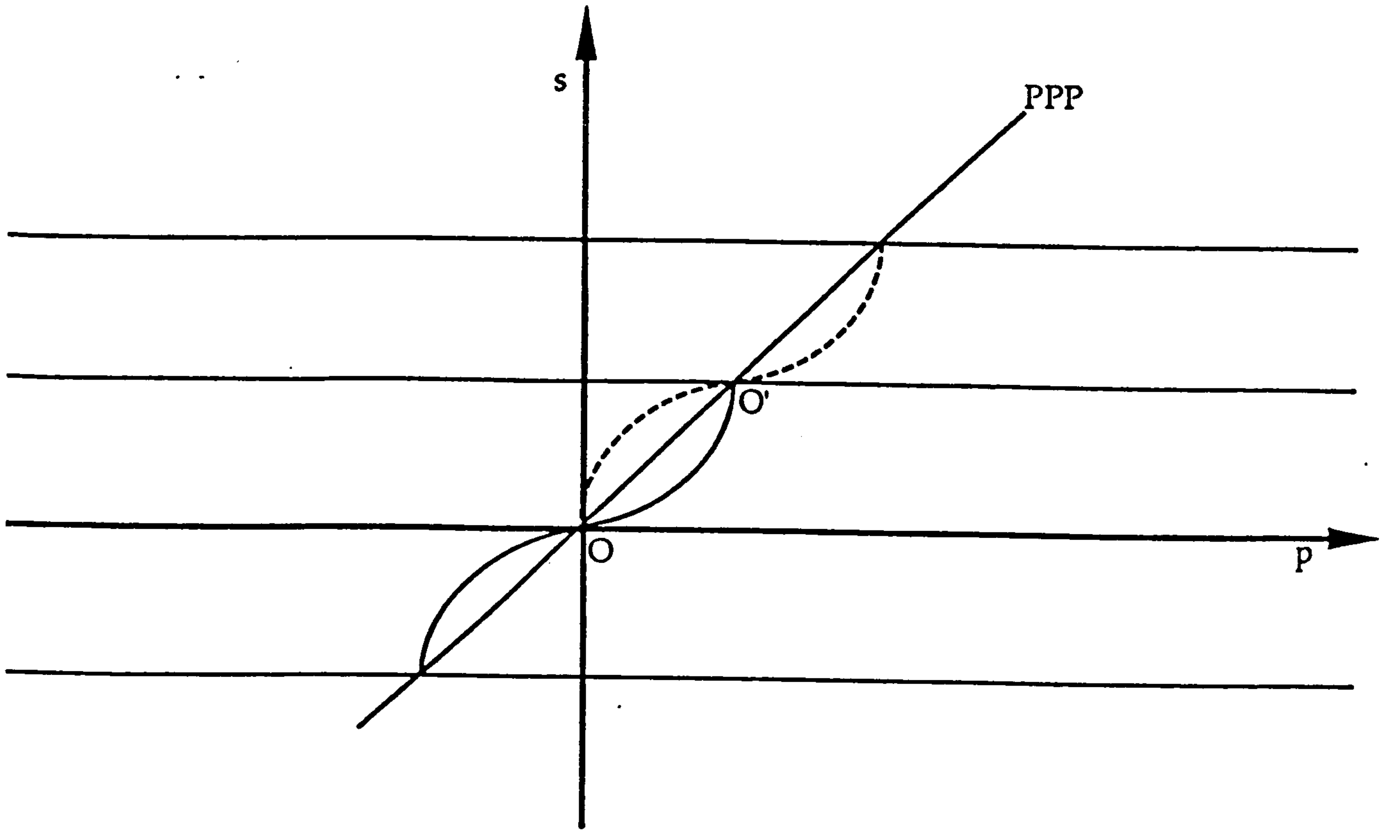


Figure 7b