FOREIGN DIRECT INVESTMENT AND THE RISK OF EXPROPRIATION*

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ABSTRACT: When an investor, for example a transnational corporation, invests abroad it runs the risk that its investment will be expropriated. The host country although it might have a short-term incentive to expropriate has a long-term incentive to foster good relations to attract more investment in the future. This conflict between short-term and long-term incentives determines the type of contracts agreed by transnational corporations and host countries. In a model of the manufacturing industry with a continuous flow of investment it is shown that investment is initially underprovided, increases over time, tending, for certain parameter values, to the efficient level.

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HEADNOTE: When an investor, for example a transnational corporation, invests abroad it runs the risk that its investment will be expropriated for the simple reason that international contracts are practically impossible to enforce. Any agreements or contracts then undertaken by the transnational company and the host country must be designed to be self-enforcing. It could be possible for the host country and the transnational corporation to find such self-enforcing agreements if there are future gains from trade. Thus although the host country might have a short-term incentive to expropriate it has a long-term incentive to foster good relations with potential investors to attract more investment in the future. This conflict between short-term and long-term incentives determines the type of investment contracts agreed. This paper extends previous work on the general underprovision of investment when contracts are incomplete or only partially enforceable (see e.g. Grout, 1984) to a dynamic context. It is likewise shown that investment is initially underprovided but it increases over time and for certain parameter values it tends to the efficient level. The expected future discounted returns to the transnational company declines over time, extending Vernon's observation of the obsolecing bargain (Vernon, 1971). The model is also extended to allow for capital accumulation and consideration is given to renegotiation-proof contracts.

KEYWORDS: Foreign direct investment, self-enforcing contracts, political risk, less-developed countries, transnational corporations, renegotiation-proofness.

1. INTRODUCTION

Foreign direct investment accounts for a considerable proportion of international capital flows. In 1986 the flow of foreign direct investment from developed market economies to developing countries was $12.5 billion or roughly one-half of all private capital flows from the developed to the developing nations (and roughly one-quarter of the flow of all foreign direct investments). Its significance for developing countries may even grow in the future as debt is swapped for equity (see Pollio and Riemschneider, 1988). The most important sector in volume term is the manufacturing sector, the concern of this paper. In 1978 total stocks of manufacturing foreign direct investment accounted for roughly two-thirds of the total in less developed countries, with just one-eighth devoted to the extractive industries (see Stopford and Dunning, 1983, p.22).

All foreign direct investment, including that going to developing countries[1], is subject to expropriation risk. The legal right of host countries to expropriate foreign-owned property within their territory subject to their own tribunals is well established[2]. Capital exporters nevertheless believe that their property is entitled to some protection under international law. The United States for example was instrumental in setting up the International Centre for the Settlement of Investment Disputes which provides a forum for settling investment disputes. But although its judgements are legally binding it cannot ultimately extract more compensation than the host country is willing to pay[3]. The United States generally expects any compensation to be prompt, adeqate and effective. Its claims could in principle be backed up by using the Export-Import Bank to deny credit to expropriating countries or invoking the Hickenlooper or Gonzalez Amendments (imposing aid sanctions or withdrawing support for loans from multilateral agencies) but in

1. The UK, for example, changed its North Sea basic tax rate several times from a rate of 45% in 1978 to 75% in 1982.

2. This principle is enunciated by the United Nations General Assembly Resolution 3281, 1974, Charter of Economic Rights and Duties of States, which states: "Each State has the right to.... nationalize, expropriate, or transfer ownership of foreign property, in which case appropriate compensation should be paid by the State adopting such measures, taking into account its relevant laws and regulations and all circumstances the State considers pertinent. In any case where the question of compensation gives rise to a controversy, it shall be settled under the domestic law of the nationalizing State and by its tribunals".

3. The newly created Multilateral Investment Guarantee Agency offers somewhat better protection in that all members including members from less developed countries make a direct contribution and a certain peer group pressure may emerge to discourage expropriation but the effect is likely to be marginal.

practice it has been loath to do so for fear of worsening its international relations (see Sigmund, 1980, p.331)[4].

The period up to 1978 saw a large number of expropriations. Between 1960 and 1976 at least 1535 firms were forced to divest (either by direct expropriation, forced sale, forced renegotiation of contract resulting in ownership transfer or through extra-legal acts) in 76 different less developed countries (see Kobrin, 1980, p.73). The book value of these firms represented 4.4% (3.3% if agriculture is excluded) of the total stock, including expropriated assets, of partially or wholly foreign-owned firms in the expropriating countries at the end of 1976. The period 1956-72 was even more striking with nearly 20% of all assets being expropriated without compensation (see Williams, 1975 p.265)[5].

In the post 1978 period in contrast there have been no major expropriations (see Chaudhuri, 1988). There may be many reasons for this. It could be that the political situation is inherently more stable. Certainly many past expropriations were primarily politically motivated following changes in regime, such as in Cuba, 1959-60 or in Chile 1971-2, or following independence, as in Angola 1975 or Tanzania 1967. However we take the view that many expropriations and acts of forced divestment were determined by largely economic considerations and are therefore amenable to economic analysis[6,7]. There may then be

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4. The case of the Hickenlooper Amendment is perhaps instructive. In 1962 the Brazilian government nationalised the telecommunications company, Compania Telefonica Nacional. Its failure to promptly agree compensation led to the passing of the Amendment through the U.S. congress. It required the President to suspend aid to any government that expropriated U.S.-owned investment and failed witin six months to honour its obligations under international law, as seen by the U.S. The Amendment was invoked only once (see Akinsanya, 1980) in 1963 against Sri Lanka which had nationalised the oil/petrol distribution facilities of Texaco, Esso Standard Eastern and Standard Oil in the previous year. The effect of the imposition was that the Sri Lankan government proceeded to nationalise virtually all other petroleum marketing facilities: they had lost their foreign aid anyway so had nothing further to lose. Similar experiences in Peru and Bolivia led to the view that foreign aid sanctions were as likely to endanger U.S. assets abroad as to protect them. The Hickenlooper Amendment was gradually watered down and was to all intents and purposes dropped in 1973.

5. Most of this figure was accounted for by Cuba. In June, 1960, Castro proclaimed, "We'll take and take until not even the nails of their shoes are left."

6. Although this distinction is very difficult to make in practice Kobrin (1980, p.69) concludes that "in the post 1960 period mass-ideologically-motivated expropriations took place in only a small minority of countries that forced divestment of foreign direct investment. In the other cases forced divestment was a means rather than an end. It is one of a number of policy options available to attempt to increase national control over foreign investors".

7. Eaton and Gersovitz (1983, 1984) make the distinction between endogenous expropriation risk, where the expropriation decision is determined by primarily economic factors and exogenous expropriation risk - determined by largely political factors

economic explanations why fewer expropriations took place in the 1980's than in the 1960's and 70's. One possible reason is the general excess supply particularly of primary products which makes expropriation less attractive, together with the perception of a large downside risk which makes host countries more willing to share risk, i.e. potential losses, with transnational corporations. Many so-called new forms of investment emerged in the 1970's, that is those involving less than 50% equity participation by the transnational corporation, although these are again becoming less popular[8]. "Fade-in" agreements that allow for a gradual transfer of ownership to the host country also became popular in the early seventies[9], as did contracts with accelerated depreciation (so the host country gets a smaller income in the early years).

Most studies of the relations between the host country and the transnational corporation use a bilateral monopoly framework. This is, according to Kobrin (1987), "...*the* currently accepted paradigm of HC-MNC relations in international political economy." A clear exposition is given by Kindleberger (1969, lecture 5). The host country has an investment opportunity that it is unable exploit itself, either because it does not have the technical know-how or because its access to capital markets is restricted, but which the transnational corporation can exploit as it has the necessary capital, technology, marketing and managerial skills. The host country has control over access and conditions of operation. It is then argued (see e.g Penrose, 1959) that the actual outcome will depend on the relative bargaining strenghs of the host country and the transnational corporation. The lower bound on the return to the transnational corporation is just sufficient to cover the supply price of capital where the transnational corporation makes no economic profit and the upper bound is that level where the host country would just prefer to leave the opportunity unexploited.

The one period model is, however, inappropriate for studying expropriation risk since if there were just one period, or indeed a final period, the host country would certainly expropriate and knowing this the transnational corporation would not be prepared to invest. We therefore consider an infinitely repeated version of the bilateral monopoly model and suppose the transnational corporation and the host country negotiate a long-run contract specifying how much is to be invested in each period and how much of output

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8. See Oman (1989) on these new forms of investment.

9. The Andean Pact countries required new foreign firms to sell 51% of ownership over a 15-20 year period (see Sigmund, 1980, p.289-90). These clauses have recently been made much less rigid.

is to be transferred to the host country at each date-state. The host country can, because of its sovereign status, renege on the contract and confiscate the whole of output without legal sanction. The transnational corporation can choose to withdraw and not to invest in the future. The only feasible contracts then are self-enforcing in which the long-term benefits from adhering to the contract exceed any short-term gains to be had by reneging. Such self-enforcing or implicit contracts are enforced by the threat to return to autarky following any infringement. Even though such contracts represent a sub-game perfect equilibrium of the repeated game between the host country and the transnational corporation, the threat to return to autarky if carried out may be subject to renegotiation. Renegotiation-proof contracts will be examined in Sectin 6. Self-enforcement, however, is a minimal requirement and the main purpose of this paper is to study these long-run, self-enforcing, investment contracts[10].

If host countries generally have a short-term incentive to expropriate they have a long-term incentive to foster good relations in order to encourage investment in the future. In practice host countries have tried numerous ways to encourage investment (see Reuber, 1973, p.126-30 and Guisinger, 1985, p.19-33) including tarrif and import quotas, duty-free import of inputs, direct subsidies and tax holidays which exempt the transnational corporation from tax obligations for a limited period. But the more the transnational corporation is encouraged to invest the greater is the temptation for the host country to expropriate[11]. Transnational corporations therefore must and do develop strategies to forestall expropriation. There are a number of possibilities. The cultivation of local support is one way. Others include locating different parts of an activity in different countries or using a technology that is difficult to operate without outside help (see Eaton and Gersovitz, 1984, p.27-28) or continuously producing new innovations (Gabriel, 1966) or holding back crucial investments. Moran (1985, p.113) for example cites evidence from the chemical and petrochemical industries that "corporate strategists deviated from both

10. Self-enforcing wage contracts are considered in Thomas and Worrall (1988), but in that model no investment decision is made.

11. Bronfrenbrenner (1954-55, p.215) saw no resolution of this problem and suggested that the best policy for the developed world was to withdraw into an economic "neo-isolationism".

engineering and economic optimality to stage investments in a series that would provide them something new to offer when host authorities pressed in"[12].

Despite this evidence it may be thought that since the underlying structure of the model is stationary, the production function is independent of time and states are i.i.d., the optimal contract will itself be stationary. Indeed as the production function is concave there is clearly a cost to having investment change over time. Suppose then that the contract is stationary, that investment is constant and that, following the bargaining strength model, the host country gets a certain constant percentage share of profits each period. Ideally investment should be at the efficient level where expected marginal revenue equals marginal cost. This would be sustainable if the share going to the host country and the discount factor were high enough so that future benefits were attractive enough. Otherwise the host country will have an incentive to confiscate current output rather than wait for the future returns. Such a contract is not self-enforcing. The contract can be made self-enforcing by reducing the investment level; it never pays to increase the investment level since this increases the temptation to renege and at the same time reduces the level of profit to be shared out. This then is a stationary self-enforcing contract with underinvestment. It is however, possible to do better. If the host country is risk neutral (Section 3) it is only interested in the value of discounted payments and not when they actually occur. Then delaying payments does not affect the current investment level but as the time approaches when the payments are due the temptation to renege is diminshed since seen from that date expected future payments are higher and so future investment can be raised without causing the host country to renege. Since investment in the future is higher so too are the transnational corporation's profits and thus a better contract has been found. Of course investment can not rise without limit and eventually either the efficient investment level will be attained or the host country will end up taking all the expected profits - depending on the discount factor. Essentially this policy of delaying payments and investments makes the threat to return to autarky more effective by increasing the cost of any deviation. If the host country is risk averse then it will care about the timing of the transfers made by the

12. Jenkins (1986, p.163-64) quotes the case of the Canadian Cold Lake Tar Sands project sponsored by Imperial Oil. Imperial cancelled the project in 1981 following falling oil prices and the introduction of the National Energy Program. The project was later resumed but investment was to proceed in six separate stages. As one executive explained; "There are a lot of uncertainties in these investments. We want to make sure the incentives are still there".

transnational corporation. Nevertheless it is shown in Section 4 that the same result goes through provided the host country is not too risk averse.

Another feature of the optimal contract is that investment moves pro-cyclically and transfers are positively serially correlated. When output is high there is a greater temptation for the host country to confiscate output. To offset this more must be offered by the contract in the future. A contrasting argument is made by King (1988). He argues that governments best maintain office by managing a steady growth in income. In that case a positively serially correlated pattern of tax revenues is likely to lead to unstable governments. This is of course an empirical question. King finds some support for his hypothesis that tax revenues are counter-cyclical from data on bauxite mining in Jamaica.

We make contributions to two strands of the the literature; first to the literature on the obsolescing bargain (see Vernon, 1971) and tax holidays, and second to the literature on the sub-optimality of investment when contracts are incomplete or not fully enforceable. In particular we obtain a strong characterization of the time structure of investment whereas in previous models investment is chosen just once.

Kindeleberger (1969, lecture 5) suggested that the bargaining strengh of the host country might tend to increase over time. The theme was taken up by Vernon (1971,p.46-59) under the title of "the obsolescing bargain" where the sunk investment of the transnational corporation is held to ransom by the host country (for empirical support see e.g. Moran, 1973). This model was initially applied only to the resource sector. However doubt has been cast on its applicability to the manufacturing sector (see Kobrin, 1987). Bennett and Sharpe (1979) even suggest that bargaining power may shift in favour of the transnational corporation. They cite evidence from the Mexican car industry that local capital becomes increasingly dependent on the transnational corporation and provides a powerful lobby for the transnational corporation's cause. Our results suggest that the obsolescing bargain argument does apply to the manufacturing sector and that where there are countervailing effects that these must be especially strong to overcome the general tendency of the host countries returns to increase over time.

The taxation of foreign direct investment is considered in Gersovitz (1987). He notes that tax holidays, which concentrate the benefits to the transnational corporation in the early periods, have a number of shortcomings, such as the difficulty of distinguishing between old and new projects, the encouragement they give to projects with short gestation periods and with rapidly depreciating equipment and concludes that they are probably not desirable. But their existence suggests that they might have offsetting benefits. An explanation along the lines of the obsolescing bargaining model can be found in Doyle and van Wijnbergen (1984). They argue that once a firm has entered the host country and incurred the set up costs the bargaining power of the host country increases and it exploits the lock-in effect to increase taxes. Bond and Samuelson (1986) offer a slighly different interpretation. They argue that tax holidays act as a signal. Firms are unable to directly identify high from low productivity countries until they are actually located there. High productivity countries offer tax holidays as a signal, which low-productivity countries cannot always mimic becuase the future rise in tax rate required to recoup the initial subsidy would drive the firm out of a low productivity country. Our model in contrast does not rely on any fixed costs or information asymmetry or any change in the underlying economic structure to motivate tax holidays although their length is history dependent rather than pre-determined. Further it allows for uncertainty and an endogenous investment decision.

As stated earlier we show that investment tends to the efficient level or stabilizes below it so there is certainly underinvestment in the initial periods. This underinvestment is caused by the host country's inability to commit not to expropriate. That specific assets tend to be underinvested when their quasi-rents can be appropriated has been a concern of the transactions cost literature (see e.g. Williamson, 1975 and Klein, Crawford and Alchian, 1978)[13]. Grout (1984) provides a formal analysis (see also Tirole, 1986). In Grout's model binding contracts are too costly to write or enforce and the division of the rents between labour and stockholders is determined ex post by a Nash bargain. If labour has any bargaining power at all it will appropriate some of the economic returns to capital, and so ex ante stockholders will provide too little capital.

Klein, Crawford and Alchian (1978) suggest that the risk of appropriation may be tempered by vertical integration of buyer and seller or by explicit or implicit contractual arrangements. Grossman and Hart (1986) consider vertical integration and Hart and Moore (1988) study explicit but incomplete contracts in which trade and non-trade prices are enforceable. They show that investment is also generally under-

13. Klein, Crawford and Alchian (1978) note the "relevance for private investments in underdeveloped, politically unstable, that is 'opportunistic' countries".

provided. (A similar conclusion is obtained by Crawford (1988) where the investment is contractable but buyer and seller are risk averse). Our model has many similar features; investment is not contractible and risk aversion does not play a key role. Here we examine the implicit contractual arrangements between the transnational corporation and host country (vertical integration is impossible and we have argued that explicit contracts are difficult to enforce across national boundaries). This has the added advantage that investment is repeated (capital accumulated in Section 5), so the time structure of investment can be considered, whereas in all previous models investment occurs only once.

Eaton and Gersovitz (1983, 1984) provide one of the few analyses of endogenous expropriation risk[14]. In the static version of their model Eaton and Gersovitz (1984) show that for certain production functions and parameter values the transnational corporation will underinvest capital. The static nature of the model however, leads to the rather forced assumption that the host country decides whether to expropriate or not before the transnational corporation commits itself to supply the managers necessary to operate the already installed capital equipment. In Eaton and Gersovitz (1983) a dynamic model is presented in which the threat of withdrawl of future capital is used to forestall expropriation. This is the approach adopted here. However in their model each foreign investor makes only a negligible contribution to output so the strategic interactions between the transnational corporation and the host country which we examine here are excluded.

The paper proceeds as follows: Section 2 sets up the model; Section 3 examines the case of a risk neutral host country and presents a simple example; Section 4 deals with the risk averse case. Section 5 introduces capital and Section 6 looks at renegotiation-proof contracts. An appendix justifies the dynamic programming approach used and provides proofs.

2. MODEL

At each date $\tau = 1,2,...,\infty$ there is a state of nature $s = 1,2,...,N$. The state of nature is i.i.d. over time and the probability of state s is p_s independent of time. There are two goods: a capital and a consumption good. The price of the capital good is constant over time and set equal to unity. (One possible interpretation of the state of nature is however as a variable consumption good price.) The consumption good can only be

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14. The implications of exogenous expropriation risk - determined by largely political factors - for capital flight is considered in Kahn and Haque (1986) and for natural resourse extraction rates in Long (1975).

produced in the host country with the help of foreign capital. The transnational corporation provides the capital good to the host country when it invests. Together with the investment, I provided by the transnational corporation the state of nature determines output at each date through the production/restricted profit function $r(I;s)$. Investment is chosen before the state of nature is known. It is assumed that $r(I;s)$ is twice continuously differentiable in I, increasing and concave in I, $r(0:s) \leq 0$, and by convention increasing in s. Further it is assumed that $E[r(I;s)]$ is strictly concave, with $E[r(I;s)]-I$ positive for some $I > 0$ and bounded above, so that there is a unique, positive solution I^* satisfying $E[r'(I^*;s)] = 1$ with positive per-period profits. Output is non-durable and must be consumed straightaway. In this section it is assumed that capital completely depreciates in one period. Section 5 allows for less than complete depreciation.

The transnational corporation transfers t_s to the host country in state s, or more acurately the host country retains an amount $r(I;s)-t_s$. More than the entire output of the consumption good cannot be retained by the host country[15] and neither can the transnational corporation take more than current output out of the host country, that is, it is assumed

$$r(I;s)-t_s \geq 0 \qquad s = 1,2,...,N \tag{2.1}$$

$$t_s \geq 0 \qquad s = 1,2,...,N. \tag{2.2}$$

The sequence of events at any date τ is illustrated in Figure 1. At date τ, the first decision is taken by the transnational corporation when it decides if and how much to invest, I being contingent upon what has happened in the past. Then nature chooses state s and the output produced is $r(I;s)$. The host country then decides how much the transnational corporation has to transfer to it, again contingent on history, and may if it wishes confiscate the whole of output. Considering just date τ the host country will want to do just that and keep all of the output for itself. If it does so the transnational corporation is unlikely to invest in the future and the host country will lose the future output.

Consider then an infinite horizon contract which specifies I and t_s at each date. This contract cannot be enforced at law because of the host country's sovereign status. The contract then must be self-enforcing.

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15. This constraint does not bind in the model of Section 3.

It is assumed that once the host country deviates from the agreed contract by confiscating output the transnational corporation will not invest again in the future and if the transnational corporation deviates from the agreed on investment the host country will confiscate. Such a policy is a credible perfect equilibrium strategy in the repeated game between the transnational corporation and the host country: it drives both parties down to the autarky level, which also must be the most severe punishment (the minimax and hence the "optimal" punishment).

Nothing yet has been said about the attitude to risk of the transnational corporation and the host country. It will be assumed that the transnational corporation is a well diversified enterprise with a large number of independent projects and thus is risk neutral and discounts the future by a constant factor $\alpha < 1$. The host country is assumed to have a utility function v(t), defined over transfer payments, normalized so that $v(0) = 0$, and to discount future utility by the same factor α as the transnational corporation. In the next section it is assumed that v(t) is linear and in Section 4 it is assumed that it is strictly concave.

Some comments are in order about the assumptions of this model. We have modeled the problem as a two person game. As remarked in the introduction this is the traditional approach. We also saw there is good evidence to suppose that the transnational corporation cannot in general expect concrete support from either its home government or other transnational corporations. This is somewhat different from the situation with international debt where numerous cross-default clauses interlink investors. Indeed the incentive structure with debt is different from that of foreign direct investment: the host country might for example wish to save out of output so as to be able itself to finance projects in the future, or indeed it may choose to consume part of the current loan; these possibilities do not exist when the transnational corporation is providing not only capital but also technology and expertise not otherwise available to the host country[16]. Further the informational problems seem more severe: the lender may have to monitor to ensure that the host country is actually devoting the money lent to the specified project.

Let V_s denote the maximum future utility the host country can get, assuming it abides by the contract, from the beginning of date $t+1$, discounted to date $t+1$, when the state at date t was s. The host country will not wish to seize output at if

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16. Another reason for not interpreting this model in the debt context is that it would be subject to the criticism of Bulow and Rogoff (1989) that if the expected future value of debt were positive the host country could do better by reneging and investing in cash-in-advance contracts.

(2.3) $$v(t_s) + \alpha V_s \geq v(r(I;s)) \qquad s = 1,2,\ldots,N$$

The RHS of this inequality is simply the one-period utility of output. Here we are making the assumptions that the host country is excluded from future investment if it expropriates and that it cannot operate the project itself even if some investment is in place. If the host country could use a less efficient technology than the transnational corporation it would be necessary to add further terms to the RHS, but this would not qualitatively affect the analysis. We shall add a term to the RHS when we consider renegotiation-proof contracts in Section 6.

Let $U(V_s)$ be the corresponding future utility at date $t+1$ of the transnational corporation. It will be prepared to invest at date $t+1$ provided it gets something out of the contract in the future, i.e. provided

(2.4) $$U(V_s) \geq 0 \qquad s = 1,2,\ldots,N$$

The optimal value function is a fixed point of the dynamic programming problem of choosing $(I,(t_s,V_s))$ to maximize $-I + E[r(I;s) - t_s + \alpha U(V_s)]$ subject to (2.1)-(2.4) and

(2.5) $$E[v(t_s) + \alpha V_s] \geq V$$

The latter constraint is a contract consistency constraint that says that what was promised in the past must be delivered. We shall let $p_s\theta_s$, $p_s\pi_s$, $p_s\mu_s$, $\alpha p_s\varphi_s$, and σ be the multipliers for these constraints. Then starting from some initial value of V, say V^0, the initial values of investment and transfers must solve this dynamic programming problem and the value V^1 will be determined by V_s from the actual state that occurs; the problem is then solved given V^1 to determine the second period values, and so on[17].

This is not a straightforward dynamic programming problem because of the presence of the optimal value function in the constraint set, and the usual contraction mapping arguments cannot be used. Nevertheless it is shown in the Appendix (Lemma 1) that the optimal value function can be found by repeated application of a mapping starting from the first-best frontier.

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17. For discussion of this method of solution the reader is refered to Thomas and Worrall (1988). The optimality equation actually characterizes the Pareto-frontier, U(V), of the equilibrium pay-off set of the repeated game between the host country and the transnational corporation, and is a part of the set-valued mapping which defines all equilibrium pay-offs (see Abreu, Pearce and Stachetti, 1986). Attention can be restricted to the Pareto-frontier as the contiuation pay-offs must also be Pareto efficient and because in this case the optimal punishments do not depend on knowledge of the entire pay-off set.

3. RISK NEUTRAL HOST COUNTRY

In this section we assume the host country to be risk neutral. We state this as

ASSUMPTION A: *The host country is risk neutral:* $v(t) = t$.

This would be an appropriate assumption for a country which does have access to capital markets so that it can insure itself against fluctuations in the terms of trade but does not have the technical skills needed to exploit its investment opportunities, or if the project is small relative to national output.

The shape of the value function U(V) is illustrated in Figure 2. In the range $[0,V_{min}]$, U is horizontal at $U(V_{min})$. Equation (2.5) is a strict inequality in this range; if $V = 0$, for example, and (2.5) held as an equality then each V_s and t_s must equal zero. But as any positive level of investment will yield a positive level of output that could be expropriated the transnational corporation must offer the host country at least some positive level of expected future utility to prevent expropriation. At V_{min}, the constraint (2.5) begins to bind and thereafter U declines as a function of V to the point V_{max}, where $U(V_{max}) = 0$. The function U is concave in this range (Lemma 2); essentially a convex combination of any two self-enforcing contracts can be made self-enforcing by transfering any extra output directly to the host country and it will offer the host country and the transnational corporation at least the average from the original contracts. It is this range of the function that represents the Pareto frontier because to operate below it or in the range $(0,V_{min})$ would be inefficient: both parties could gain by a move to the frontier.

As the transnational corporation is interested in maximizing profits it will initially choose the contractual terms such that U is at a maximum and hence it is efficient to set $V^0 = V_{min}$. This is consistent with a situation in which the transnational corporation has all the bargaining power or where there are more investment opportunities than there are resources capable of exploiting them. Other distributions of the surplus are found by choosing the initial value of V to be higher than V_{min}.

It will be helpful as a benchmark case to consider the first-best situation where the self-enforcing constraints are ignored. Investment each period will maximize $E[r(I;s)]-I$, and hence satisfy $E[r'(I^*;s)] = 1$. By assumption there is a unique solution I^*, where $E[r(I^*;s)]-I^* > 0$ so that per-period profits are positive; we shall call I^* is the *efficient level of investment*. The Pareto-frontier will be a straight line with slope -1. The path of transfer payments associated with any point on this frontier is not uniquely determined (though it must satisfy $0 \leq t_s \leq r(I^*;s)$) as the host country is risk neutral and both parties discount at the same rate.

The first step in unravelling the second-best problem is to notice that the constraints (2.1) can be ignored (see Lemma 3, Appendix). With this in mind the first order conditions for the dynamic programming problem are, together with the complementary slackness conditions,

(3.1) $E[r'(I;s)(1-\mu_s)]=1$

(3.2) $\mu_s+\sigma=1-\pi_s$ $\quad s=1,2,...,N$

(3.3) $-(\mu_s+\sigma)/(1+\varphi_s)\in\partial U(V_s)$ $\quad s=1,2,...,N$

The notation $\partial U(V_s)$ represents the set of superdifferentials of the value function at V_s. If $\partial U(V_s)$ is differentiable at V_s then the set consists of a single number, say $-\sigma_s$, with $\sigma_s(1+\varphi_s)=(\mu_s+\sigma)$ and $\sigma_s=-U'(V_s)$. There is also an "envelope condition"

(3.4) $-\sigma\in\partial U(V)$.

This implies that if there is a unique value of σ for which (3.1)-(3.3) hold, then U(V) is differentiable at V. The relationship between σ and V is a non-decresing u.h.c. correspondence. It may however have horizontal sections where U is linear or vertical parts where U is not differentiable. It will be useful for the moment to assume that U is diffentiable so (3.1)-(3.4) are equalities. Then σ_s equals the value of σ in the following period's maximization problem. It will be shown that U is not diffentiable for all parameter values; nevertheless each choice variable is continuous in V even at points of non-differentiability. Further it will be shown that the value function has an absolute slope less than or equal to 1 and is strictly concave whenever the absolute slope is less than 1 (Lemmas 3 and 4).

It can be seen straightaway that $\sigma_s\geq\sigma$ since $\mu_s\geq 0$ and if $\varphi_s>0$ then σ_s is at its maximum value, σ_{max}, and by definition $\sigma_{max}\geq\sigma$. Thus the absolute slope of the value function is non-decreasing over time and from (3.3) will only increase if $\mu_s>0$, where s is the state occuring today (if $\sigma<\sigma_{max}$ and $\mu_s>0$ then $\sigma_s>\sigma$, and if $\sigma=\sigma_{max}$ and $\mu_s>0$ then $\sigma_s=\sigma=\sigma_{max}$ and $\varphi_s>0$).

Translating this updating rule for σ into one for V^τ, the value of V at date τ, we have that in those states where the host country's self-enforcing constraint is not binding ($\mu_s=0$), V^τ remains constant ($V_s=V$); and for states in which it does bind ($\mu_s>0$) and when $V<V_{max}$, V^τ increases ($V_s>V$): if V^τ is already at its maximum value V_{max} then it stays there. Since V can be regarded as the state variable of the process determining the optimal solution, and as t_s, V_s and I are monotonically related to V (Lemma 7), we have the fundamental observation that the contract evolves according to a "ratchet effect", sometimes increasing, sometimes staying the same, but never falling[18].

Next we want to relate the transfer, t_s and investment, I to V. To do this we need to know something of the value of σ. In Lemma 3 it is shown that $\sigma \leq 1$. Intuitively, in the first-best contract the trade-off would be one-to-one. The first-best may be obtained but if so only for that part of a contract starting from high values of V where the host country gets sufficient utility from the first-best contract so that its self-enforcing constraint never binds. We shall say that the first best level of investment is sustainable for such a V. If the self-enforcing constraint binds in no state, then $\sigma = 1$ and I* is sustainable.. Let V* be the smallest level of V such that I* is sustainable (see Lemma 4). As V is reduced below V* the utility the host country gets falls below the utility obtained by confiscating the output, r(I*;s). Thus a one unit reduction in V will lead to a less than one unit gain in U. So σ falls below 1. If the first-best cannot be sustained even for high values of V (remember U has to be non-negative) then σ is always less than 1. In principle there are three possible cases. Firstly no non-trivial contract may exist. This will be true under certain circumstances if α is small (see Proposition 2 below) If a non-trivial contract does exist there are two cases depending on whether it is possible to sustain the efficient level of investment in a self-enforcing way or not. If I* is sustainable, U has a slope of -1 on the interval $[V^*, V_{max}]$ and U is strictly concave, differentiable and $I < I^*$ on the interval $[V_{min}, V^*)$. If efficiency is not sustainable, U is strictly concave on $[V_{min}, V_{max}]$, though not everywhere differentiable, has a slope whose absolute value is less than 1 and I is always less than I* (see Lemmas 4, 7 and 8).

Consider some value of σ strictly less than 1 (the starting value is by assumption 0). If $\sigma_s < 1$ and $\varphi_s = 0$ then from (3.3) $\mu_s + \sigma < 1$, and from (3.2) $\pi_s > 0$ so that $t_s = 0$. (If $t_s > 0$ then $\pi_s = 0$ and from (3.2) $\mu_s > 0$ and from (3.3) either $\sigma_s = 1$ or $\sigma_s < 1$ and $\varphi_s > 0$.) That is unless $\sigma_s = \sigma_{max}$ (which follows from $\varphi_s > 0$) or $\sigma_s = 1$, the transfer is zero. So the optimal rule is to make no transfers to the host country until either V_{max} or efficiency is attained. Intuitively it does not matter for discounted utilities when the host country receives

18. This is different from the ratchet effect identified by Laffont and Tirole (1988). There an agent who reveals too much good information in the first period faces a stiffer incentive scheme in the second period.

the transfers. But the presence of the host country's self-enforcement constaint means that it pays to delay the transfer to offer a "carrot" to prevent reneging. Once V_{max} is reached, however, any further postponement would make it worthwhile for the transnational corporation to renege at some future date when the transfer is positive. Thus the contract becomes stationary with the transfer positive such that the transnational corporation's expected profits are zero each period. Alternatively if efficiency is attainable, then once it is reached postponement of the transfer has no further benefit and positive transfers can be made (the solution is not unique in this case).

The above discussion can be summarized in terms of a simple updating rule which characterizes the optimal contract. Take any value of V below its steady state level - be it V_{max} or V^* - and the corresponding value of I. Then V remains constant in state s ($V_s = V$) and $t_s = 0$ unless the host country's self-enforcing constraint is violated ($r(I;s) > \alpha V$) in which case utility must be increased to just the confiscation utility ($t_s + \alpha V_s = r(I;s)$). Overall expected utility must equal V, so there is only one value of I consistent with this rule. So the investment level can be easily calculated. It follows that I is a strictly increasing function of V and thus increases with positive probability each period until either I^* or $I_{max} = I(V_{max}) < I^*$ is reached depending on which case is applicable. Of course investment remains the same if V does not change. Note that when I^* is sustainable it will be attained with probability one in the optimal contract but this does not mean the the optimal contract is first-best, since investment is intially underprovided and efficiency is only attained in the long run. Not only does investment increases over time but it is procyclical. To see this notice that a high value of s will produce a large temptation to renege leading to a larger increase in V_s and hence a large increase in I next period.

PROPOSITION 1: *Investment is non-decreasing over time, attaining a maximum value in the steady state with probability one which may be less than the efficient level. The discounted utility of the host country is also non-decreasing and transfers are zero until the period before the maximum value of investment is attained.*

Once V_{max} or V^* is reached, the contract is stationary; V remains constant with the transfer chosen appropriately to satisfy the host country's self-enforcing constraints. Therefore if a non-trivial contract exists at all, there must be a non-trivial stationary contract and if it is possible to attain the efficient level of investment, there must exist an efficient stationary contract. So the questions of existence and efficiency can be answered by looking for a non-trivial and an efficient stationary contract. Proposition 2 shows that if an

Inada condition on the production function holds there is always a non-trivial contract. If on the other hand $E[(r(I;s)/I)]$ is bounded above in I then there will be a critical value of the discount factor below which no non-trivial contract exists and above which one always exists. Likewise the efficient level of investment will be attainable if and only if the discount factor is above some critical value.

PROPOSITION 2: *(i) There exists an α^*, $0<\alpha^*<1$, such that a stationary contract at I^* exists if and only if $1>\alpha\geq\alpha^*$. (ii) If $r(0;s)=0$ and $r'(I;s)\to\infty$ as $I\to 0$ for all s, then there exists a non-trivial stationary contract for all $\alpha\in(0,1)$. (iii) If $E[(r(I;s)/I)]$ is bounded above then there exists an α', $0<\alpha'<1$, such that a non-trivial stationary contract exists if and only if $1>\alpha>\alpha'$.*

Since V does not decreases over time, the optimal value, U(V) depends only on its properties above V. This means it is possible to calculate the optimal value function by working backward from V_{max}.

Consider then a simple example with two equi-probable states in which $r(I;1)=0$ and $r(I;2)=4\sqrt{I}$, so $I^*=1$. The assumption that output is always zero in state one makes everything much simpler since $t_1=0$ and the host country's constraint does not bind, implying $V_1=V$. Further it can be shown that it is optimal to set $I=\min(1,V^2(2-\alpha)^2/16)$, $V_2=\min(V^*,\beta^{-1}V)$ and $t_2=\max(0,\alpha(\beta^{-1}V-V^*))$, where $\beta=\alpha/(2-\alpha)$ and that the value function is

$$U(V)=V_{max}-V \qquad\qquad V\in[V^*,V_{max}]$$

$$U(V)=\beta^n V_{max}+(n-1)V-\alpha(1/8)(\Sigma^n_{i=1}\beta^{-i})V^2 \qquad\qquad V\in[\beta^n V^*,\beta^{n-1}V^*]$$

for $n=1,2,\ldots,m$, where m is the number such that $V_{min}\in[\beta^m V^*,\beta^{m-1}V^*]$. It can be checked that this function is continuous and concave.

The example does not quite meet the conditions of Proposition 2(i), nevertheless it is easy to show that a non-trivial contract exists for all α so that there are just two cases to consider depending on whether the efficient level of investment is sustainable or not. First consider the value of V_{max} and suppose $I=1$. Since $U(V_{max})=0$, $t_2=2$ so that $V_{max}=1/(1-\alpha)$. For this to be feasible requires $t_2+\alpha V_{max}\geq 4$ or $\alpha\geq 2/3$. Further the constraint will not bind provided $t_2+\alpha V_2\geq 4$. But since $t_1=0$ and $V_1=V$, $t_2+\alpha V_2=V(2-\alpha)$ so that $I=1$ for any $V\geq 4/(2-\alpha)$. Therefore for $\alpha\geq 2/3$, U(V) is linear in the range $E_0=[V^*,V_{max}]$ where

$V^* = 4/(2-\alpha)$. This means for $V \in E_0$ the choice of V_2 and t_2 is not uniquely defined. Further from Lemma 8, U(V) is everywhere differentiable as can be easily checked. An example for the discount factor $\alpha = 7/8$ is drawn in Figure 3a. In this case $m = 3$, $V_{min} = 1.81$ and the regions of V are: $E_0 = [3.55,8]$, $E_1 = [2.77,3.55)$, $E_2 = [2.15,2.77)$, $E_3 = [1.81,2.15)$.

In the other case $\alpha < 2/3$ it is not possible to attain the efficient level of investment and the self-enforcing constraint binds at $V = V_{max}$. Then $V^* = V_{max} = 8\alpha/(2-\alpha)^2$ and $V(2-\alpha) = t_2 + \alpha V_2 = 4\sqrt{I}$ or $I = V^2(2-\alpha)^2/16$. For this value of V^*, U(V) is not differentiable at $\beta^n V^*$ but is still concave. The value function for a discount factor of $\alpha = 5/8$ is drawn in Figure 3b. Here $m = 2$, $V_{min} = 0.91$ and $F_1 = [1.2,2.65]$ and $F_2 = [0.91,1.2)$.

4. RISK AVERSE HOST COUNTRY

In this section it is assumed that the host country is risk averse. This is a natural assumption to make for a developing country engaging in a large project. It is likely to have limited technical resources and limited access to capital markets. Further export earnings may be heavily dependent on one or two unstable markets.

The appendix shows how to extend the results of the previous section to the case of a risk averse host country. A condition on the relative curvatures of the production and utility functions is, however, needed to prove that U is concave.

ASSUMPTION B: *The host country has a C^2, strictly concave, per-period, utility function $v(t)$ defined over the transfer t, $v(0) = 0$; uncertainty is multiplicative, $r(I;s) = g(s)r(I)$, and $\{((r(I')-r(I))/r'(I))-(I'-I)\}$ $-E[((v(g(s)r(I'))-v(g(s)r(I)))/v'(g(s)r(I)))-g(s)(r(I')-r(I))]$ is negative for all $I \neq I'$.*

Roughly this says that the host country cannot be too risk averse relative to the concavity of the production function: in the neighbourhood of unconstraind efficient investment the coefficient of absolute risk aversion of the host country should be smaller than the coefficient of absolute risk averion of the transnational corporation.

The results are not too different from those given for the risk neutral case. First unlike the risk neutral case the transfer is not necessarily zero in the initial periods and although the expected value of the transfer is non-decreasing over time, the actual transfer may fall if a bad state occurs. Second to prove that investment is lower than the efficient level in the risk averse case is more difficult than in the risk neutral case. In fact it is no longer clear what is meant by this comparison since even in the absence of the self-enforcing constraints investment will vary with the level of utility given to the host country because of the non-negativity constraints (2.1)-(2.2). A comparison will therefore be made between the second-best contract which gives the host country a net future utility of V and the first-best contract which gives the host country V in the absence of the self-enforcing constraints (2.3) and (2.4). The latter function will be denoted by $I^*(V)$. In the risk neutral case this was a constant function $I^*(V) = I^*$. Consistent with the notation of the previous section we let V^* be the lowest value such that it is possible to sustain the first-best and let $I^* = I(V^*)$

PROPOSITION 3: *$I(V) < I^*(V)$ for $V < V^*$.*

Investment is still non-decreasing over time, indeed it can be shown that investment increases with positive probability in each period if efficiency is sustainable, so that the efficient level of investment $I(V^*)$ is approached but never quite reached. If efficiency is not sustainable then $I(V_{max})$ is attained with probability one.

PROPOSITION 4: *i) If a static efficient contract is sustainable, that is there exists a $V^* \leq V_{max}$, then $V^\tau \rightarrow V^*$ and $I^\tau \rightarrow I^*$ with probability one, where $I^* = I(V^*)$, and each increasing with positive probability in each period (so $V^\tau < V^*$ and $I^\tau < I^*$). ii) If no static efficient contract is sustainable then $V^\tau = V_{max}$ eventually with probability one.*

5. CAPITAL ACCUMULATION

Capital accumulation can be introduced in a simple manner by assuming that start of period investment adds to current capital stock and that a constant fraction $\delta > 0$ of the inherited capital stock depreciates. Capital stock at time τ is then $K^\tau = (1-\delta)K^{\tau-1} + I^\tau$ and the model of Section 3 is covered by the special case of $\delta = 1$[19]. To choose the optimal capital stock the transnational corporation must take into account its user cost, $c = \alpha(r + \delta)$, where $r = (1-\alpha)/\alpha$ is the interest carrying cost and δ is the depreciation cost, and the sum is multiplied by α to convert it into current period dollars. As before it is assumed that

19. For simplicity we treat only the risk neutral case in this section.

there is a unique positive K^* maximizing $Er(K;s)-cK$ such that the expected value of output covers investment costs, $I^* = \delta K^*$, that is $Er(K^*;s)-\delta K^* > 0$. The maximand in the dynamic programming problem becomes[20] $-cK + E[r(K;s)-t_s + \alpha U(V_s)]$ and the transnational corporation's participation constraint in state s is $U(V_s) + (1-\delta)K \geq 0$ which includes the value of the future capital stock[21].

The host country when it expropriates inherits the capital stock. We shall let $D(K;s)$ be the benefit to the host country when it expropriates if the capital stock is K and the state is s. The self-enforcing constraint for the host country in state s is $t_s + \alpha V_s \geq D(K;s)$. If, for example, the host country is unable to use the capital without the transnational's expertise, $D(K;s) = r(K;s) + (1-\delta)K$, the value of current output plus the scrap value of future capital in a perfect market. For simplicity it is assumed that $D''(K;s) \geq r''(K;s)$ for each K and s. Then Lemma 2 can be used mutatis mutandis to prove that $U(V)$ is concave.

As before there will be some maximal capital stock which can be sustained by a self-enforcing contract. Provided $D(K;s)$ is bounded above the efficient capital stock, K^* will be sustainable for a high enough discount factor, although it should be noticed that in this case K^* is itself increasing in α since an increase in α decreases the interest carrying costs of capital. It will be assumed that the initial capital stock is less than the maximum sustainable level, $K^0 < \min(K^*, K_{max})$ where K_{max} is the maximum attainable capital stock if K^* cannot be sustained. This then justifies not introducing a non-negativity constraint on investment, $I \geq 0$, since such a constraint will not be binding at the optimum. Equations (3.2) and (3.3) apply unaltered so it is easy to see that the capital stock increases ratchet like with positive probability each period up to its maximum value (that is investment always covers depreciation and net investment is positive with positive probability)[22]. In the long-run a steady-state is attained with $I = \delta\min(K^*, K_{max})$. That the efficient capital stock is not attained instantaneously is often attributed to adjustment costs (see e.g. Gould, 1968). The slow adjustment here is caused by the absence of a legally binding contract.

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20. This can be shown by explicitly treating K as a state variable in the value function and integrating the envelope condition for K, taking the first order conditions into account. Then the value function given V^T and K^{T-1} equals $U(V^T) + (1-\delta)K^{T-1}$, where $U(V^T)$ is the value function when the inherited capital stock is zero. Intuitively, for each value of V^T there is a best level of K^T, so an amount $(1-\delta)K^{T-1}$ of current investment is saved, and this is added to discounted profits.

21. It can thus be seen that V_{max} is increasing in K.

22. It does not pay to increase capital above K^*. The only reason for doing so would be to relax the transnational corporation's participation constraint. But since $\sigma \leq 1$, any gain in V would be matched by at least as big a fall in U.

6. RENGOTIATION-PROOFNESS

The solution identified above is not renegotiation-proof despite being confined to the Pareto frontier of the set of all equilibrium payoffs. The reason is simple: the punishment meted out to the host country when it reneges, to be cut off from all future investment, is Pareto dominated by points on the second-best frontier[23], and would therefore be subject to renegotiation. The most severe punishment which can be imposed is V_{min}; anything lower by definition also gives the transnational corporation a lower payoff.

It is nevertheless possible to find a renegotiation-proof set of equilibria by replacing zero on the RHS of (2.3) by V_{min}. Any fixed point of the mapping corresponds to a set of payoffs which is weakly renegotiation-proof (Farrell and Maskin, 1987) since no payoff Pareto dominates any other and each payoff corresponds to an equilibrium in which all continuation payoffs also belong to the set.

As an example we solve the one state case with $r(I) = \surd I$ and a risk neutral host country. Then $I^* = 1/4$, and attention will be restricted to the case where efficiency is sustainable at some point in the set. Working backwards as in the example of Section 3 for a given value of V_{min}, and computing the new value of V_{min}, it is straightforward to show that there is a unique fixed point which, using the same notation as before, falls in the interval E_2. The value function is $U_{RP}(V) = ((1/4(1-\alpha))-V)$ for $V \in E_0 = (0.5 + \alpha V_{min}, 1/4(1-\alpha)]$ and $U_{RP}(V) = \Sigma^n{}_{i=1}(-V^2\alpha^{-i+1} - V_{min}{}^2\alpha^{i+1} - V_{min}\alpha^i) + V(2nV_{min} + n-1) + \alpha^n/4(1-\alpha)$ for $V \in E_n = (\max\{V_{min}, \alpha^n(0.5 + \alpha V_{min})\}, \alpha^{n-1}(0.5 + \alpha V_{min})]$ and for $i = 1,2$, where $V_{min} = \alpha/2(1 + \alpha - 2\alpha^2)$. Notice that this solution is only valid (E_0 is an interval) if $\alpha \geq \surd 0.5$, whereas without imposing renegotiation-proofness the efficient level of investment is sustainable for $\alpha \geq 0.5$. Since the set of payoffs (the graph of $U_{RP}(V)$ for $V \in [V_{min}, 1/4(1-\alpha)]$) includes part of the unconstrained first-best frontier it satisfies the stronger definition of renegotiation-proofness given by van Damme (1987) which additionally requires that no point in any other weakly renegotiation-proof set Pareto dominates the whole set[24].

What is interesting here is that the equilibrium outcome path has exactly the same qualitative characteristics as that analysed in earlier sections, since it is derived from the same dynamic programm except for the addition of a constant into the constraint (2.3).

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23. This is not true of the wage-contracts model in Thomas and Worrall (1988): it is shown in Asheim and Strand (1989) that the solution identified there does satisfy renegotiation-proofness.

24. We have not been able to establish strong renegotiation-proofness, although we suspect it to be true.

The impact of renegotiation-proofness on utilities is illustrated for $\alpha=0.9$ in Figure 4[25]. It is also interesting to consider what happens for high discount factors. First multiply payoffs by $(1-\alpha)$ to normalise. Then $V_{min}=\alpha/2(1+2\alpha)$ which tends to 1/6 as α tends to one, while $V_{max}=1/4$ for all $\alpha\geq\surd 0.5$, so U_{RP} converges to that part of the first-best frontier on [1/6,1/4] whereas U converges to the entire frontier by the folk-theorem.

APPENDIX

Define $V^{\#}$ as the largest discounted utility the host country can receive in the first-best (unconstrained) problem, subject to giving the investor zero utility. In the space of bounded functions on $[0,V^{\#}]$ consider some decreasing concave not necessarily differentiable function P and define the mapping L as follows.

(A.1) $$L(P)(V) = \max_{I,(t_s,V_s)} \{-I+E[r(I;s)-t_s+\alpha P(V_s)]\}$$

subject to:

(A.2)	$E[v(t_s)+\alpha V_s]\geq V$		$:\sigma$
(A.3)	$v(t_s)-v(r(I;s))+\alpha V_s\geq 0,$	$s=1,2,...,N$	$:p_s\mu_s$
(A.4)	$P(V_s)\geq 0,$	$s=1,2,...,N$	$:\alpha p_s\varphi_s$
(A.5)	$r(I;s)-t_s\geq 0,$	$s=1,2,...,N$	$:p_s\theta_s$
(A.6)	$t_s\geq 0,$	$s=1,2,...,N$	$:p_s\pi_s$

The first order conditions are:

(A.7) $$E[r'(I;s)(1-\mu_s v'(r(I;s))+\theta_s)]=1$$

(A.8) $$\sigma+\mu_s+(\pi_s-\theta_s)/v'(t_s)=1/v'(t_s) \qquad s=1,2,...,N$$

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25. To find U(V), set $V_{min}=0$ in the definition of $U_{RP}(V)$.

(A.9) $\quad -((\sigma+\mu_s)/(1+\varphi_s))\in\partial P(V_s) \quad\quad s=1,2,\ldots,N$

together with an envelope condition:

(A.10) $\quad -\sigma\in\partial L(P)(V)$

where $\partial P(V)$ denotes the set of the superdifferentials of P at V. This set is a single point for almost every value of V if P is concave. At such points let $\sigma_s\equiv -P'(V_s)$.

This is not a standard concave programming problem even when P is concave because the self-enforcing constraint (A.4) means that the constraint set is not convex, Neither, unfortunately is L a contraction mapping in the supremum metric, despite the presence of strict discounting, because L has more than one fixed point when a non-trivial contract exists, the zero function being one, and U itself another. Technically, the reason for this is the presence of the value function itself in the constraints (in (A.4)). Nevertheless the following can be proved:

LEMMA 1: *Define P^* as the unconstrained first-best Pareto frontier for the problem without constraints (A.3) and (A.4). Then $L^n(P)$ converges pointwise to U as $n\to\infty$.*

PROOF: (i) Notice that when P^* is the first-best frontier $L(P^*)\le P^*$.

(ii) Make the induction assumption that $L^n(P^*)\le L^{n-1}(P^*)$. Compare $L(L^n(P^*))$ and $L(L^{n-1}(P^*))$. The constraint set in the latter case is at least as large as in the former, so $L(L^{n-1}(P^*))\ge L(L^n(P^*))$; i.e. $L^n(P^*)\ge L^{n+1}(P^*)$, thus completing the induction assumption.

(iii) Hence $L^n(P^*)$ is a decreasing sequence, and must therefore converge pointwise to some limit function , say U°.

(iv) U° is a fixed point of L. To see this, consider for any fixed V, the sequence of variables chosen at each application of L: $(I^n,(t_s^{\,n},V_s^{\,n}))$, which are a solution to $L(L^{n-1}(P^*))(V)$. Because $L^{n+1}(P^*)\le L^n(P^*)$ the constraint (A.3) does not relax as n increases. Hence the sequence belongs to a compact set and has a convergent subsequence, converging to, say, $(I^*,(t_s^*,V_s^*))$. We have $L^{n-1}(P^*)(V_s^{\,n})\ge 0$, for each n in the subsequence, so in the limit $U°(V_s^*)\ge 0$, for each s, and the limit contract clearly satisfies all other constraints in the problem L(U°)(V), and gives the transnational corporation a utility of U°(V).

Consequently $LU°(V) \geq U°(V)$. However since $L^{n-1}(P^*) \geq L^n(P^*) \geq ... \geq U°$, we have $L^n(P^*) \geq L(U°)$, and taking the limit as $n \to \infty$, $U° \geq L(U°)$. So $U° = L(U°)$.

(v) Every fixed point U^1 of L corresponds to a family of self-enforcing contracts in the sense that there is a self-enforcing contract which gives the host country a discounted utility of V, and the transnational corporation, $U^1(V)$, for any V satisfying $V \geq 0$, $U^1(V) \geq 0$. Consider the contract formed by the repeated application of L, starting from utility V, so that the variables in the first period of the contract are the I and $t_s(1)$'s that solve Problem A from V; the second period contract, contingent upon s(1) occurring in the first period, is then the solution to Problem A from $V_s(1)$, and so on. As in any discounted programming problem, this contract must deliver V and $U^1(V)$ respectively to the two parties, and this same argument guarantees that because constraints (A.3) and (A.4) are satisfied at each point in the future, the self-enforcing constraints proper are satisfied. All other constraints are clearly also met, so this contract is as required.

(vi) Since $P^* \geq U$, $L^n(P^*) \geq L^n(U) = U$, and in the limit $U° \geq U$. From (v) and by definition of U, therefore, $U° = U$. *Q.E.D.*

LEMMA 2: *Under Assumption A, U(V) is stricly decreasing and concave on (Vmin, Vmax].*

PROOF: Assume that $L^{n-1}P(V)$ is concave. For a given V, V' and corresponding contracts $(I,(t_s,V_s))$, $(I',(t'_s,V'_s))$, consider the contract $(I^\delta,(t^\delta_s,V^\delta_s))$ where $I^\delta = \delta I + (1\text{-}\delta)I'$, $V^\delta_s = \delta V_s + (1\text{-}\delta)V'_s$ and $t^\delta_s = \delta t_s + (1\text{-}\delta)t'_s + r(I^\delta)\text{-}(\delta r(I;s) + (1\text{-}\delta)r(I';s))$. This new contract is feasible, it satisfies (A.3), and offers neither the host country or transnational corporation less overall utility. Then $L^nP(V)$ is concave and since P^* is concave and U is the pointwise limit of L^nP^* from Lemma 1, U(V) is itself concave. *Q.E.D.*

LEMMA 3: *Under Assumption A, (i) $\sigma \leq 1$ for $V \in [V_{min}, V_{max}]$ and (ii) $\theta_s = 0$ for all s.*

PROOF: (i) Suppose that $\sigma > 1$. From (A.8) $\theta_s = (\sigma\text{-}1) + \mu_s + \pi_s > 0$ for all s, so that $t_s = r(I;s)$. By concavity if $\sigma > 1$ anywhere then at V_{max}. If $\varphi_s > 0$ then $V_s = V_{max}$ so $U(V_s) = 0$. If $\varphi_s = 0$, then from (A.9) $-(\sigma + \mu_s) \in \partial U(V_s)$ which implies $-\inf \partial U(V_s) > 1$ so that in the next period $\theta_s > 0$ and again $t_s = r(I;s)$ for all s and so on. Thus the transnational corporation cannot make positve profits at any stage and would not invest. Thus $\sigma \leq 1$.

(ii) If $\theta_s > 0$ then as $\sigma \leq 1$, either μ_s or π_s is strictly positive. By complementary slackness $\pi_s = 0$.If $\mu_s > 0$ then (A.3) binds, and as (A.5) binds too, $V_s = 0$, which is inefficient, implying $\mu_s = 0$. *Q.E.D.*

LEMMA 4: *Under Assumption A, if for some V, there exists (t_s, V_s) such that I* is sustainable then U is linear with slope -1 on $(V, V_{max}]$ and if V* is the smallest such V then U is strictly concave on (V_{min}, V^*) with $I < I^*$. If no such V exists then U is strictly concave on (V_{min}, V_{max}) with $I < I^*$.*

PROOF: Suppose U is linear on some interval (V,V') and consider the contracts $(I,(t_s,V_s))$ and $(I',(t'_s,V'_s))$. First suppose $I < I^*$. Then from (A.7), $\mu_s > 0$ for some s and $\sigma < 1$. Then from linearity $\sigma' < 1$ implying $I' < I^*$. Moreover since U is strictly decreasing the contract $(I^\delta,(t^\delta_s,V^\delta_s))$ defined in Lemma 2 satisfies $U(V^\delta) \geq U(V^\delta + E[r(I^\delta)-(\delta r(I;s) + (1-\delta)r(I';s))]) \geq \delta U(V) + (1-\delta)U(V')$ which can only hold with equality if $I = I'$. From (A.7) and using the implicit function theorem μ_s is a continuous function of I and hence $\mu_s = \mu'_s$. If $\mu_s > 0$, $t_s + \alpha V_s = r(I;s) = r(I';s) = t'_s + \alpha V'_s$. If $\mu_s = 0$, then $\pi_s = \pi'_s > 0$ since $\sigma = \sigma' < 1$ from (A.8). Then $t_s = t'_s$ and $V_s \in (V,V')$ from (A.9) and (A.10). But then
$E[\,|(t_s + \alpha V_s)-(t'_s + \alpha V'_s)|\,] \leq |V-V'|$ as $\alpha < 1$, which contradict (A.2). Now consider the interval (V*,V) where at V* investment is at the efficient level. By definition $\sigma^* = 1$. But $\sigma \leq 1$ from Lemma 3 and since U is concave from Lemma 2 it follws that at any $V > V^*$ $I = I^*$ and $\sigma = 1$. *Q.E.D.*

LEMMA 5: *Under Assumption A, if $\mu_q > 0$ then $\mu_s \geq \mu_q$ for all $s > q$. If $\pi_q > 0$ then $\pi_s \geq \pi_q$ for $s < q$.*

PROOF: Suppose $\mu_s < \mu_q$. From (A.8) $\pi_s - \pi_q = \mu_q - \mu_s > 0$. So $\pi_s > \pi_q \geq 0$ and $t_s = 0$. Then $\alpha V_s \geq r(I;s) > r(I;q) = t_q + \alpha V_q \geq \alpha V_q$ and $V_s > V_q$. As $\sigma < 1$ and $\sigma < 1$ and U is strictly concave we have $-\sigma_s < -\sigma_q$ for all $-\sigma_s \in \partial U(V_s)$ and $-\sigma_q \in \partial U(V_q)$. Then $-\sigma_s(1 + \varphi_s) + \mu_s < -\sigma_q + \mu_q$. Therefore equation (A.9) can only be satisfied if $\varphi_q > 0$. But this implies $V_q = V_{max}$ which contradicts $V_s > V_q$.

Suppose $\pi_s < \pi_q$. From (A.7), $\mu_s - \mu_q = \pi_q - \pi_s > 0$. So $\mu_s > \mu_q \geq 0$. As $t_q = 0$, $\alpha V_q \geq r(I;q) > r(I;q) = t_s + \alpha V_s \geq \alpha V_s$ and $V_q > V_s$. As before $-\sigma_s > -\sigma_q$ for all $-\sigma_s \in \partial U(V_s)$ and $-\sigma_q \in \partial U(V_q)$. Thus $-\sigma_s + \mu_s > -\sigma_q(1 + \varphi_q) + \mu_q$ so that (A.9) can only be satisfied if $\varphi_s > 0$, implying $V_s = V_{max}$ which contradicts $V_q > V_s$. *Q.E.D.*

LEMMA 6: *Under Assumption A,* $t_s > 0$ *iff* $-1 \in \partial U(V_s)$

PROOF: $\partial U(V_{max}) = [-1, -\sigma_{max}]$. If $t_s > 0$ then $\mu_s = 1-\sigma$. If $\varphi_s > 0$ then $V = V_{max}$ and $-1 \in \partial U(V_{max})$. If $\varphi_s = 0$ then from (A.9) $-1 \in \partial U(V_{max})$. If $-1 \in \partial U(V_{max})$ then $V_s < V_{max}$, $\varphi_s = 0$ and $\mu_s + \sigma < 1$, so that $\pi_s > 0$ and $t_s = 0$ from (A.8). *Q.E.D.*

LEMMA 7: *Under Assumption A, I is strictly increasing in V for* $V < \min(V^*, V_{max})$ *and* $I = I^*$ *otherwise;* V_s *and* t_s *are non-decreasing in V.*

PROOF: Consider $V_{max} \geq V' > V$. First suppose $V'_s < V_s \leq V_{max}$. Then $\varphi'_s = 0$ and since by assumption $\sigma' > \sigma$ we have from (A.9) $\mu_s > \mu'_s \geq 0$. Then from (A.7), $I' > I$. Since $V'_s < V_{max}$, $t'_s = 0$ from Lemma . Therfore $t_s + \alpha V_s = r(I;s) < r(I';s) \leq \alpha V'_s$. This implies $t_s < \alpha(V'_s - V_s) < 0$, a contradiction. Therefore $V'_s \geq V_s$.

Suppose $\mu'_s > \mu_s \geq 0$, then from (A.7), $I' < I$. From the first part of the proof $V_{max} \geq V'_s \geq V_s$, so that $\sigma_{max} \geq \sigma'_s \geq \sigma_s$. First consider $\sigma_s < \sigma_{max}$. Then $t_s = 0$, so $t'_s + \alpha V'_s = r(I';s) < r(I;s) \leq \alpha V_s$. But since $V'_s \geq V_s$ this implies $t'_s < 0$ which is impossible. If $\sigma_s = \sigma_{max}$, then $t'_s - t_s < 0$. But from (A.8) $\pi_s > \pi'_s \geq 0$ since $\sigma' > \sigma$, again implying $t'_s < 0$. Thus $\mu_s \geq \mu'_s \geq 0$ and $I' \geq I$, and from Lemma if $\sigma < 1$, $I < I' \geq I^*$.

Now suppose $t_s > t'_s \geq 0$. From Lemma 6, $-1 \in \partial U(V_s)$, but $V'_s \geq V_s$ from above. If $V'_s > V_s$ then the intersection of $\partial U(V'_s)$ and $\partial U(V_s)$ is empty since U is strictly concave. But $\sigma \leq 1$, so $-1 \in \partial U(V'_s)$, a contradiction. If $V'_s = V_s$ on the other hand, from (A.8) $\mu_s > \mu'_s \geq 0$ as $\sigma' > \sigma$. Then $t_s + \alpha V_s = r(I;s) < r(I';s) \leq t'_s + \alpha V'_s$ or $t_s < t'_s$, a contradiction. *Q.E.D.*

Proof of Proposition 1: If the efficient level of investment is attanable then $\mu_s \to 0$ along all paths since V^τ is increasing over time. If $V^\tau < V^*$ for all τ then $t_s = 0$ for all τ which is impossible. So $V^\tau \in [V^*, V_{max}]$ with probability one. Equally if the efficient level of investment is not sustainable $V = V_{max}$ eventually with probability one. The rest of the Proposition follows directly. *Q.E.D.*

LEMMA 8: *Under Assumption A, if* $\sigma_{max} = 1$, *i.e.* $V^* \leq V_{max}$, *then U is everywhere differentiable.*

PROOF: Since V_s is non-decreasing over time in the interval $V \in [V_{min}, V^*)$, and attains some $V \in [V^*, V_{max}]$ where $\partial U(V)$ is unique and equal to -1 and $\varphi_s = 0$, then σ is also unique. *Q.E.D.*

Proof of Proposition 2: For a stationary contract $(I,(t_s))$ to be feasible it must satisfy

$$(t_s/I)-(r(I;s)/I)+((\alpha/(1-\alpha))E[(t_s/I)]\geq 0. \qquad s=1,2,...,N \tag{A.11}$$

$$-1+E[((r(I;s)/I)-(t_s/I)]\geq 0 \tag{A.12}$$

$$(r(I;s)/I)\geq(t_s/I)\geq 0. \qquad s=1,2,...,N \tag{A.13}$$

(i) The first thing to notice about the constraints is that as at least one $t_s>0$ they are strictly relaxed as α increases. At I^*, $-1+Er(I^*;s)/I^*>0$. So consider (t_s) such that (A.12) and (A.13) are satisfied where at least one $t_s>0$ since $V>0$. Therefore for α near enough one (A.11) will be satisfied. On the other hand for α near enough zero (A.11)-(A.13) cannot hold simultaneously: from (A.11) $(r(I^*;s)/I^*)-(t_s/I^*)\leq((\alpha/(1-\alpha))E[t_s/I^*]$ and from (A.12) $((\alpha/(1-\alpha))E[t_s/I^*]\leq((\alpha/(1-\alpha))E[(r(I^*;s)/I^*)-1]$ which can be made less than one for α small enough, so $(r(I^*;s)/I^*)-(t_s/I^*)<1$ which contradicts (A.12)

(ii) Set $t_s=r(I;s)$, $s<N$ and $t_N=r(I;N)-I/p_N$. Then (A.12) holds with equality, (A.11) holds for $s<N$ and if $s=N$, then $-(1/p_N)+((\alpha/(1-\alpha))E[(r(I;s)/I)-1]\geq 0$, which is satisfied if I is small enough since $E[r(I;s)/I]\rightarrow\infty$ as $I\rightarrow 0$. Likewise (A.13) holds for $s<N$ and for $s=N$ it becomes $r(I;N)/I\geq(r(I;N)/)I-(1/p_N)\geq 0$, which again holds for I small enough. Thus for any $\alpha\in(0,1)$ there is a feasible stationary contract with $I>0$.

(iii) By assumption there is some $(I,(t_s))$ satisfying (A.12) and (A.13) with at least one $t_s>0$. The proof proceeds along the lines of part (i) given that $E[(r(I;s)/I)-1]$ is bounded above. *Q.E.D.*

LEMMA 9: *Under Assumption B, $L^nP(V)$ is concave.*

PROOF: Consider any two values V and V' and the associated contracts $(I,(t_s,V_s))$ and $(I',(t'_s,V'_s))$.

$$L^nP(V')-L^nP(V)=E[(r(I';s)-r(I;s))-(I'-I)]-E[t'_s-t_s]+\alpha E[L^{n-1}P(V'_s)-L^{n-1}P(V_s)] \tag{A.14}$$

Consider each of the three terms on the RHS in turn.

i) Let $\Phi_s(I'-I)=(r(I';s)-r(I;s))-r'(I;s)(I'-I)$. $\Phi_s(I'-I)\leq 0$ with equality iff $I'=I$, since $r(I;s)$ is strictly concave and differentiable in I. Similarly let $\Omega_s(I'-I)=(v(r(I';s)-v(r(I;s))\ v'(r(I;s))(r(I';s)-r(I;s))$. Again $\Omega_s(I'-I)\leq 0$ with equality iff $I'=I$, since v is differentiable. Multiplying both sides of (A.7) by $(I'-I)$ gives

$$(I'-I) = (I'-I)E[(1+\theta_s-\mu_s v'(r(I;s))r'(I;s)] = E[(1+\theta_s-\mu_s v'(r(I;s)))((r(I';s)-r(I;s))-\Phi_s(I'-I))].$$

Then rearranging terms and using the definition for $\Omega_s(I'-I)$ gives

(A.15)
$$E[(r(I';s)-r(I;s))-(I'-I)] = E[\mu_s(v(r(I';s))-v(r(I;s)))]-E[\theta_s(r(I';s)-r(I;s))]$$
$$+E[(1+\theta_s-\mu_s v'(r(I;s)))\Phi_s(I'-I)]-E[\mu_s\Omega_s(I'-I)].$$

ii) From (A.5) and (A.6), $\theta_s(r(I;s)-t_s)=0\leq\theta_s(r(I';s)-t'_s)$ and $\pi_s t_s=0\leq\pi_s t'_s$ by complementary slackness. Therefore $-(t'_s-t_s)\leq-(1-\pi_s+\theta_s)(t'_s-t_s)+\theta_s(r(I';s)-r(I;s))$. But from (A.8), $-(1-\pi_s+\theta_s)=-(\sigma+\mu_s)v'(t_s)$. So $-(1-\pi_s+\theta_s)(t'_s-t_s)=-(\sigma+\mu_s)v'(t_s)(t'_s-t_s)\ \leq-(\sigma+\mu_s)(v(t'_s)-v(t_s))$ since v is concave. Therefore combining terms and taking expectations

(A.16)
$$-E[t'_s-t_s]\leq-E[(\sigma+\mu_s)(v(t'_s)-v(t_s))]+E[\theta_s(r(I';s)-r(I;s))].$$

iii) From (A.4) $\varphi_s L^{n-1}P(V_s)=0\leq\varphi_s L^{n-1}P(V'_s)=0$. So $L^{n-1}P(V'_s)-L^{n-1}P(V_s)$ $\leq(1+\varphi_s)(L^{n-1}P(V'_s)-L^{n-1}P(V_s))$. But from (A.10) $(1+\varphi_s)\partial L^{n-1}P(V_s)=-(\sigma+\mu_s)$ and since $L^{n-1}P(V_s)$ is concave $L^{n-1}P(V'_s)-L^{n-1}P(V_s)\leq\partial L^{n-1}P(V_s)(V'_s-V_s)$. So

(A.17)
$$\alpha E[L^{n-1}P(V'_s)-L^{n-1}P(V_s)]\leq-\alpha E[(\sigma+\mu_s)(V'_s-V_s)].$$

Then substituting (A.15), (A.16), (A.17) into (A.14) gives

$$L^nP(V')-L^nP(V)\leq E[\mu_s(v(t_s)+\alpha V_s-v(r(I;s)))]-E[\mu_s(v(t'_s)+\alpha V'_s-v(r(I';s)))]$$
$$+E[(1+\theta_s-\mu_s v'(r(I;s)))(\Phi_s(I'-I)]-E[\mu_s\Omega_s(I'-I)]$$

But from (A.4), $E[v(t_s)+\alpha V_s]=V$ and $E[v(t'_s)+\alpha V'_s]=V'$ and from (A.4) $\mu_s(v(t_s)+\alpha V_s-v(r(I;s)))=0\leq\mu_s(v(t'_s)+\alpha V'_s-v(r(I';s)))$ and from (A.10) $-\sigma=\partial L^nP(V)$. So

(A.18)
$$L^nP(V')-L^nP(V)\leq\partial L^nP(V)(V'-V)+E[(1+\theta_s-\mu_s v'(r(I;s)))(\Phi s(I'-I)]-E[\mu_s\Omega_s(I'-I)]$$

With the assumption $r(I;s)=g(s)r(I)$, $\Phi_s(I'-I)=g(s)\Phi(I'-I)$, where $\Phi(I'-I)=[(r(I')-r(I))-r'(I)(I'-I)]$. Then

$$E[(1+\theta_s-\mu_s v'(r(I;s)))\Phi_s(I'-I)]=E[(1+\theta_s-\mu_s v'(g(s)r(I)))g(s)]\Phi(I'-I)=\Phi(I'-I)/r'(I)$$

using (A.7). From (A.8) $\mu_s v'(t_s) = 1+\theta_s-\pi_s-\sigma v'(t_s) \leq 1+\theta_s$. Since $\theta_s\mu_s = 0$ and $t_s \leq r(I;s)$ from (A.4), $\mu_s v'(r(I;s)) \leq \mu_s v'(t_s) \leq 1$. Then as $\Omega_s(I'-I) \leq 0$, $-\mu_s\Omega_s(I'-I) \leq -\Omega_s(I'-I)/v'(r(I;s))$. So

$$E[(1+\theta_s-\mu_s v'(r(I;s)))\Phi s(I'-I)]-E[\mu_s\Omega_s(I'-I)] \leq (\Phi(I'-I)/r'(I))-E[\Omega_s(I'-I)/v'(r(I;s))]$$

This latter term is non-positive by assumption B with equality iff I' = I. This proves that U itself is concave since P* is concave and from Lemma 1, U is the pointwise limit of a sequence of concave functions. *Q.E.D.*

LEMMA 10: *Under Assumption B, there is a unique value of I which solves the dynamic program and hence I is a continuous function of V.*

PROOF: Setting V = V' in (A.18) implies that there is a unique solution for I. Thus from the maximum theorem I is continuous in V. *Q.E.D.*

LEMMA 11: *Under Assumption B, each μ_s and θ_s are continuous functions of V.*

PROOF: From (A.7) and using the implicit function theorem, given $r''(I;s) < 0$, implies that μ_s and θ_s are continuous functions of I. Then use Lemma 10 that I is a continuous function of V. *Q.E.D.*

Now consider the following sub-problem of choosing (t,V) to maximize $y-t+\alpha U(V)$ subject to: $U(V) \geq 0$, $v(t)+\alpha V-v(y) \geq 0$, $t \leq y$ and $t \geq 0$ with multipliers $\alpha\underline{\varphi}$, $\underline{\mu}$, $\underline{\theta}$ and $\underline{\pi}$ where a bar beneath the multiplier denotes that it refers to this sub-problem and let Q(y) be the maximum function and let $\underline{\sigma}(y) \equiv -U'(V)$. The solution $(\underline{t}, \underline{V})$ corresponds to the optimal way of giving the host country the minimum gain when output is known to be y. Assuming that the undominated part of U is concave, then it is a standard concave programming problem with Q(y) increasing and concave. With $U'(V) > 0$ and $V > 0$, $\underline{\theta} = 0$ and $\underline{\mu} > 0$. If v is strictly concave then $\underline{t}$ and $\underline{V}$ are unique and thus continuous functions of y with $Q'(y) = 1-\underline{\mu}v'(y)$. Some properties of the solution will be useful later

LEMMA 12: *(i) $\underline{t}$ and $\underline{V}$ are non-decreasing functions of y; (ii) $\underline{\mu}$ is a non-decreasing function of y; (iii) If $\underline{\pi} > 0$, $\underline{\varphi} = 0$ then $\underline{V} = v(y)/\alpha$, if $\underline{\pi} = \underline{\varphi} = 0$, then $\underline{V}$ solves $v^{-1\prime}(v(y)-\alpha\underline{V}) = -U'(\underline{V})$, if $\underline{\varphi} > 0$ then $\underline{V} = V_{max}$; (iv) $\underline{t} = v^{-1}(v(y)-\alpha\underline{V})$.*

PROOF: *Straightforward.*

Since from Lemma 10 for each level of V there is a unique optimal value for I, and hence letting $y=r(I;s)$ the above sub-problem can be solved for each s. Use an s subscript to denote the solution: thus $\underline{V}_s$, $\underline{\mu}_s$ etc. The dependence of the solutions upon V may be stessed by writing $\underline{\sigma}_s(V)$ for example.

LEMMA 13: *For a given value of V and hence a given value of I (i) if $V<\underline{V}_s$ then $V_s=\underline{V}_s$ and $\mu_s>0$; (ii) if $V\geq\underline{V}_s$ then $V_s=V$ if U is strictly concave in the neighbourhood of V or $V_s\in\{V_s\in[\underline{V}_s, V_{max}] \mid \partial U(V)=-\sigma\}$*

PROOF: (i) Assume $V<\underline{V}_s$ and $\mu_s=0$. From (A8) $1/v'(t_s)=\sigma+(\pi_s-\theta_s)/v'(t_s)$, so $t_s=\max\{0,\min\{t(\sigma),r(I;s)\}\}$. From the solution to the sub-problem, $1/v'(\underline{t}_s)=(1+\underline{\varphi}_s)\underline{\sigma}_s+(\underline{\pi}_s-\underline{\theta}_s)/v'(\underline{t}_s)$, so $\underline{t}_s\geq t(\underline{\sigma}_s)$ or $\underline{t}_s=r(I;s)$ if $r(I;s)<t(\underline{\sigma}_s)$ or $\underline{t}_s\geq 0$ if $t(\underline{\sigma}_s)<0$. Thus since $\sigma<\underline{\sigma}_s$, $t_s\leq t_s$. In either case $V_s\geq\underline{V}_s$, a contradiction. Consequently $\mu_s>0$ and by the principle of optimality $\partial U(V_s)=\partial U(\underline{V}_s)$ or $V_s=\underline{V}_s$.

(ii) Suppose $\sigma<\sigma_{max}$. Since by assumption $\sigma\geq\underline{\sigma}_s$, we have $\underline{\sigma}_s<\sigma_{max}$, so that μ_s and φ_s cannot both be positive (as it would imply $\sigma_s=\underline{\sigma}_s$ and $\sigma_s=\sigma_{max}$). If $\mu_s>0$, $\sigma_s=\underline{\sigma}_s\leq\sigma$. But from (A11) $\mu_s>0$ implies $\sigma_s>\sigma$, a contradiction, so $\mu_s=0$. Equally, if $\varphi_s>0$, $\sigma_s=\sigma_{max}>\sigma$, while from (A11) $\sigma_s<\sigma$, a contradiction, so $\varphi_s=0$. Thus from (A11), $\sigma_s=\sigma$. If $\sigma=\sigma_{max}$, then if additionally $\sigma_s<\sigma_{max}$, from (A11) we have $\varphi_s>0$, a contradiction, so $\sigma_s=\sigma_{max}$. *Q.E.D.*

LEMMA 14: *Under Assumption B, the limit function U is strictly concave.*

PROOF:Suppose U is linear over some interval [V,V']. Then $U(V')-U(V)=U'(V)(V'-V)$. From Lemma 4 this can only happen if $t_s=t'_s$ and $I=I'$. From Lemma 7, $\underline{V}_s$ is a continuous function of I and hence does not change. Thus if $\mu_s>0$, $V_s=V'_s=\underline{V}_s$, or $|V_s-V'_s|=0$. On the other hand if $\mu_s=0$, then $|V_s-V'_s|\leq|V-V'|$. Thus $\alpha E[V_s-V'_s]<V-V'$ which contradicts equation (A.2). *Q.E.D.*

LEMMA 15: *Under Assumption B, I is increasing in V.*

Proof:Suppose $I'<I$ and $V<V'$. Then $\theta'_s\geq\theta_s$ and $1-\mu'_s v'(r(I';s))\geq 1-\mu_s v'(r(I;s))$. First consider θ'_s and θ_s. Suppose $\theta'_s<\theta_s$. If $\theta'_s>0$ then from equation (A.8) $\sigma=(1+\theta_s)/v'(t_s)$ and $\sigma'=(1+\theta'_s)/v'(t'_s)$. Since $t_s=r(I;s)>r(I';s)=t'_s$, $1/v'(t_s)>1/v'(t'_s)$. But then $(1+\theta_s)>(1+\theta'_s)$ implies $\sigma>\sigma'$ which contradicts $V<V'$ since U(V) is concave. If $\theta'_s=0$ then $\sigma'\leq\sigma'+\mu'_s=(1-\pi_s)/v'(t'_s)\leq 1/v'(t'_s)$. But $t_s>t'_s$ so $1/v'(t_s)>1/v'(t'_s)$ where $1/v'(t_s)\leq(1+\theta_s)/v'(t_s)=\sigma$. Thus again $\sigma'<\sigma$ a contradiction. Now $Q'(y)=1-\underline{\mu}v'(y)$, and since Q is concave $1-\underline{\mu}'v'(y')>1-\underline{\mu}v'(y)$. Since μ_s is equal either to $\underline{\mu}_s$ or zero it can be inferred that

$1\text{-}\mu'_s v'(r(I';s)) \geq 1\text{-}\mu_s v'(r(I;s))$ provided $\mu'_s = \underline{\mu}'_s$ and $\mu_s = 0$ does not occur. But this can be ruled out since $\mu'_s \leq \mu_s$. Then as $r'(I';s) > r'(I;s)$ we have $E[r'(I';s)(1\text{-}\mu'_s v'(r(I';s) + \theta'_s)] > E[r'(I;s)(1\text{-}\mu_s v'(r(I;s) + \theta_s)]$ which contradicts equation (A.7) that both should equal unity. *Q.E.D.*

Proof of Proposition 3: Suppose $I(V) \geq I^*(V)$. The unconstrained case must have a stationary solution. If $\sigma \geq \sigma^*$, from (A.7) $t_s \geq t^*_s$ for all s, and since $\mu_s > 0$, $V_s > V$; so that the borrower's utility is higher in the constrained case - contrary to assumption. Hence $\sigma < \sigma^*$. Then $\theta_s \leq \theta^*_s$; since if $\mu_s > 0$, $\theta_s = 0$ and when $\mu_s = 0$, $\sigma + \mu_s = \sigma < \sigma^*$, which implies $\theta_s \leq \theta^*_s$ from (A.5) and (A.8) (since if (A.5) binds in the constrained case, $t_s \geq t^*_s$). But $\theta_s \leq \theta^*_s$ for all s and $\mu_s > 0$ for some s implies from (A.7) that $I(V) < I^*(V)$, a contradiction. *Q.E.D.*

Proof of Proposition 4: (i)(a) At V^*, by the updating rule of Lemma 13(i), $\underline{\sigma}_s(V^*) \leq \sigma(V^*)$ for all s, since otherwise V^* could not be a steady-state. But $\underline{\sigma}_s$ is strictly increasing in I up to the point where $\underline{V}_s = V_{max}$ and hence, since $I(V) < I^*(V)$ for $V < V^*$, we have $\underline{\sigma}_s(V) < \underline{\sigma}_s(V^*)$ and so $\underline{\sigma}_s(V) < \sigma(V^*)$ for all s. So as $\sigma(V) < \sigma(V^*)$, by the updating rule $\sigma_s(V) < \sigma(V^*)$, and so also $V_s < V^*$.

(b) Next consider V as a stochastic process $\{V^\tau\}_{\tau > 0}$. Note that $\varphi_s = 0$ as $V^\tau < V^*$. So from (A.11) $\mu_s(V^\tau) = \sigma(V^{\tau+1})\text{-}\sigma(V^\tau)$ and as $\sigma \leq \sigma_{max}$ we have $\mu_s(V^\tau) \to 0$ along all paths. Moreover state N occurs infinitely often with probability one, so on almost all paths, by choosing those dates when state N occurs, there is a convergent subsequence of V^τ's such that $\mu_N(V^\tau) \to 0$. For any such path $\mu_N(\lim V^\tau) = 0$ by the continuity of μ_N in V, Lemma 11. This implies $\mu_s(\lim V^\tau) = 0$ for all s since $\mu_N \geq \mu_s$ for all s. Hence efficiency is attained at $\lim V^\tau$ which in view of part (a) must be V^*. So $V^\tau \to V^*$ and $I^\tau \to I^*(V^*)$ with probability one.

(ii) Given that V^τ is non-decreasing, $V^\tau = V_{max}$ eventually with probability one unless $\text{Prob}\{V^\tau < V_{max} \text{ all } \tau\} > 0$. The latter implies $\text{Prob}\{\lim \mu_s(V^\tau) = 0\} > 0$, but by the argument of part (b) this means V^τ converges to a value with $\mu_N = 0$ with positive probability. This is impossible since such a point would be efficient, contrary to asssumption. *Q.E.D.*

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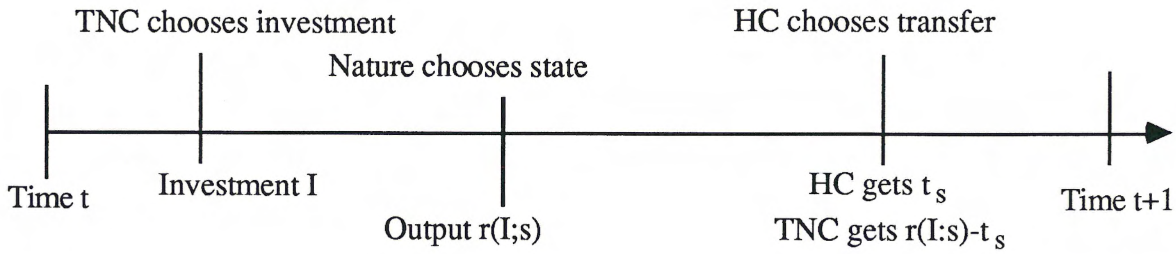


FIGURE 1 - TIME SEQUENCE OF DECISIONS

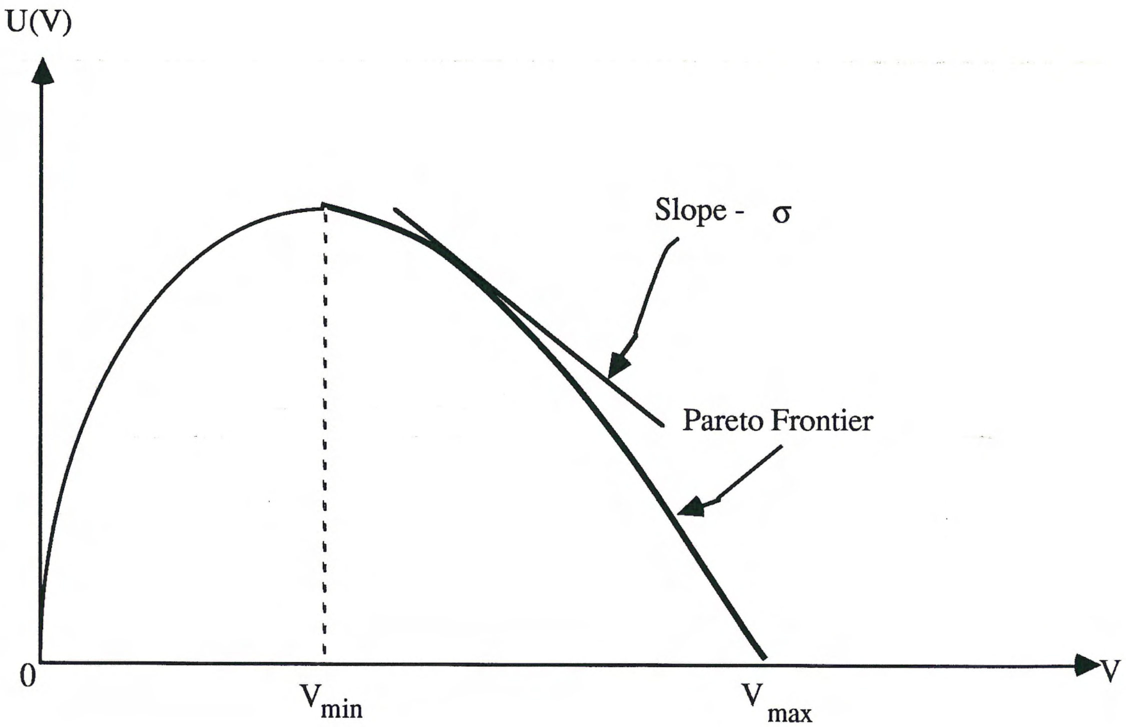
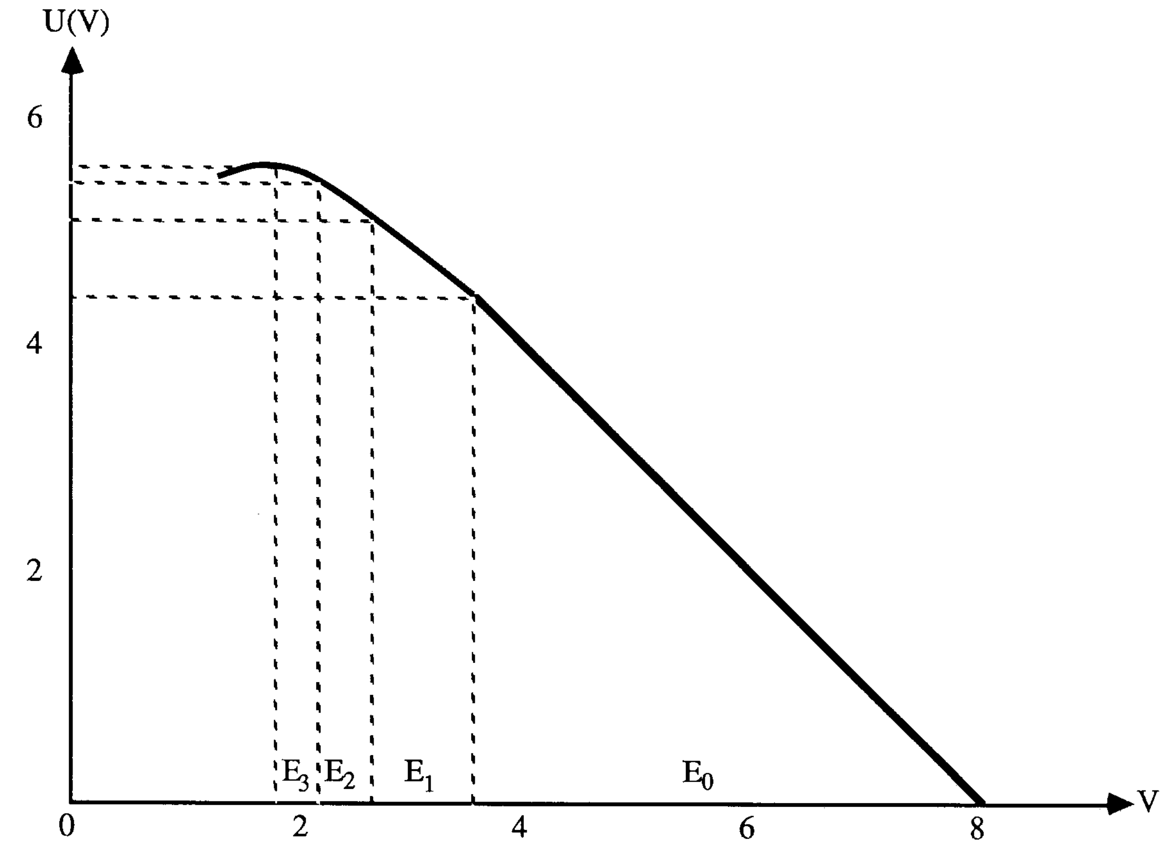
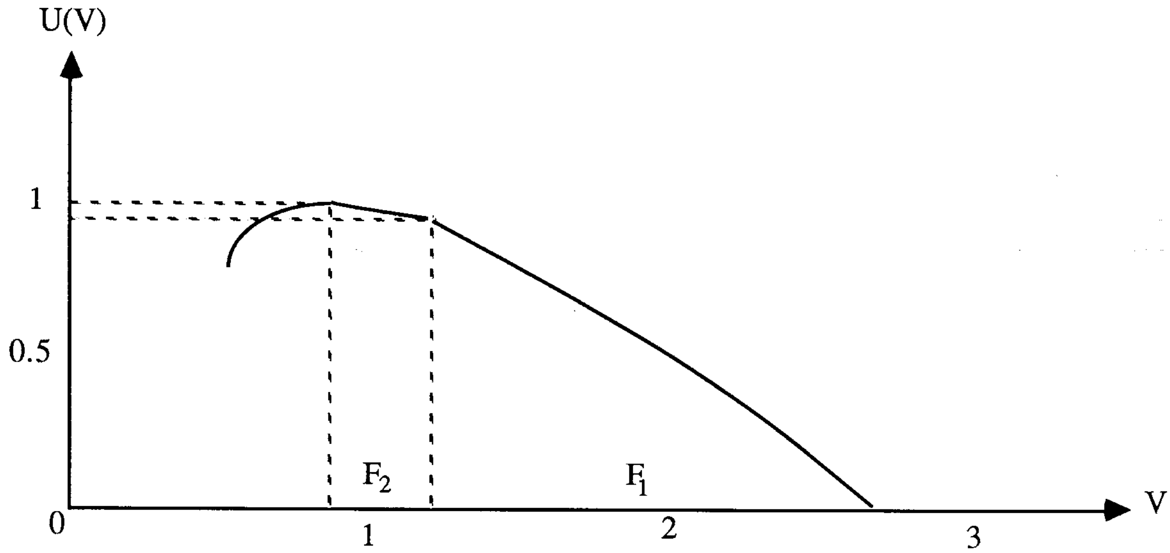


FIGURE 2 - THE VALUE FUNCTION



a) Discount factor 0.875



b) Discount factor 0.625

FIGURE 3 - EXAMPLE OF VALUE FUNCTIONS

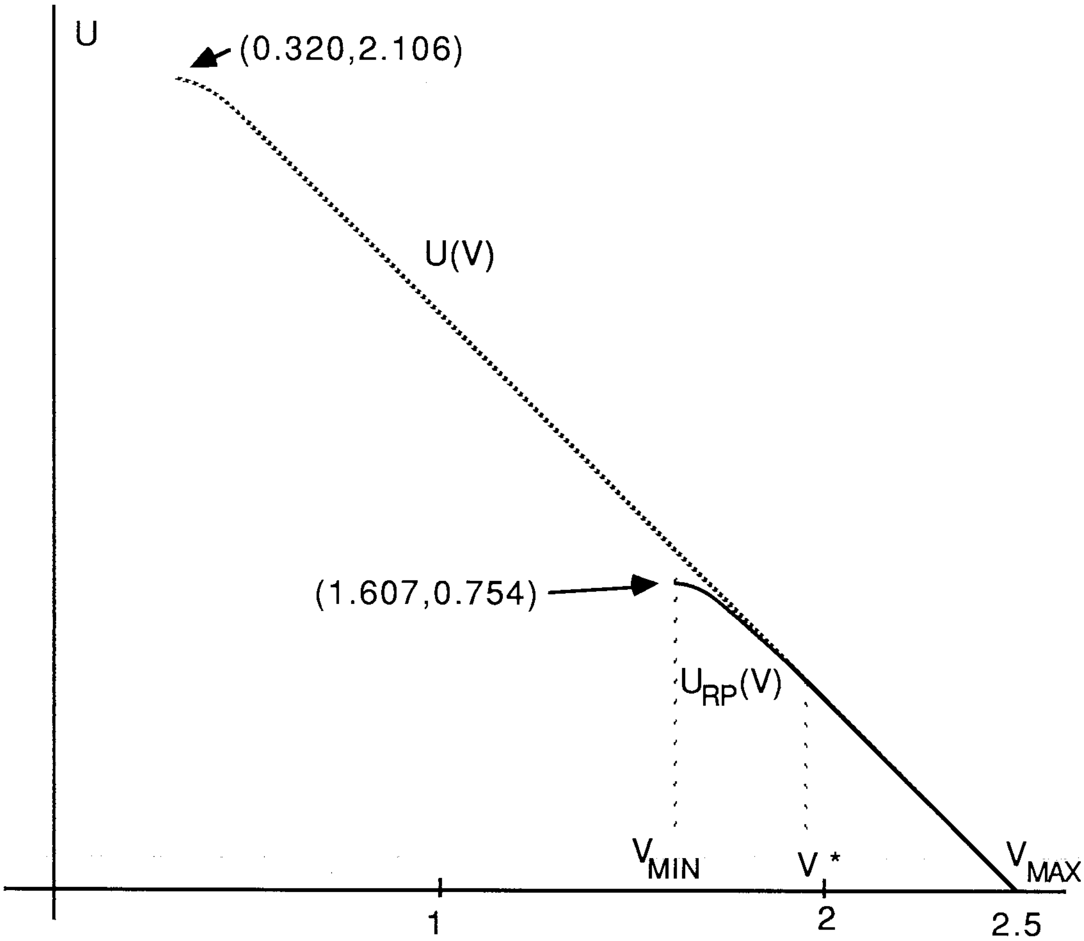


FIGURE 4 - A RENEGOTIATION PROOF SET